Discrete Mathematics

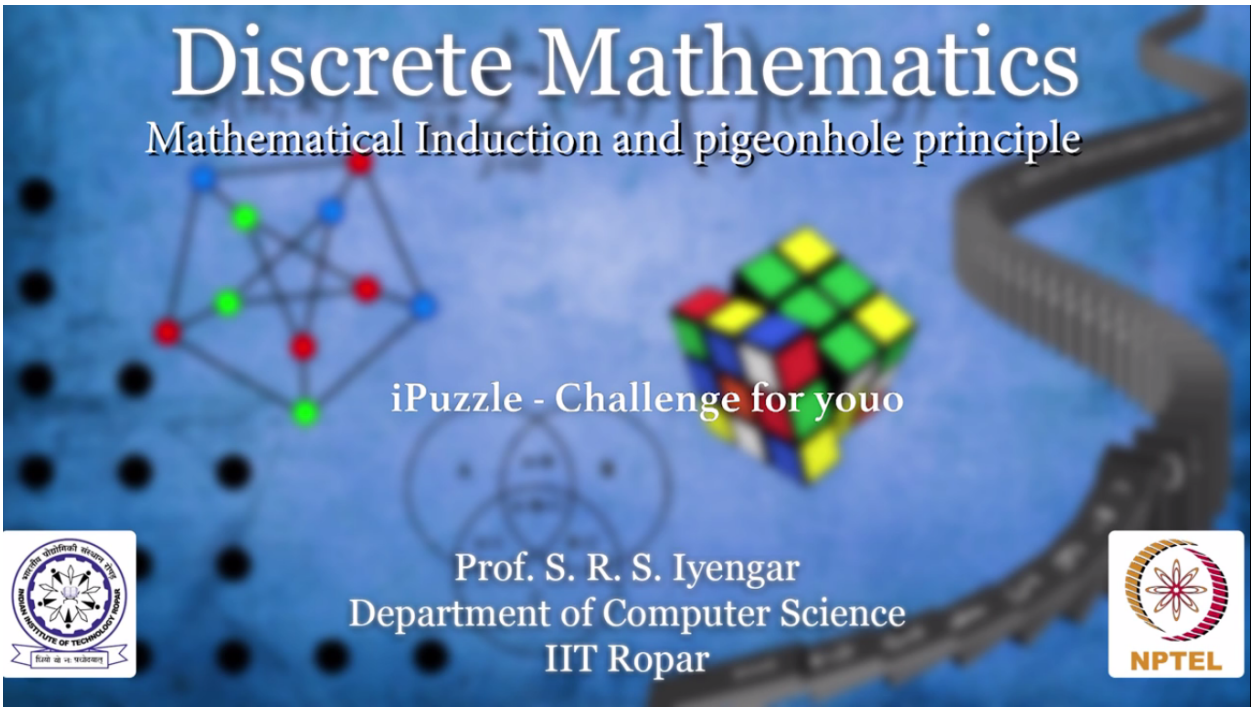
Functions

Puzzle - Challenge for you

Prof S.R.S. Iyengar

Department of Computer Science

IIT Ropar



Here is a very non-trivial application of pigeonhole principle. Let me start with the question. Imagine you have let's say ten different people of different heights. You see they are all of different heights correct. The first person is taller than the second person and third person is taller than the second person but shorter than the next and so on right. So what we will do is I will try writing more such people I said 10 such people okay all of different heights okay.

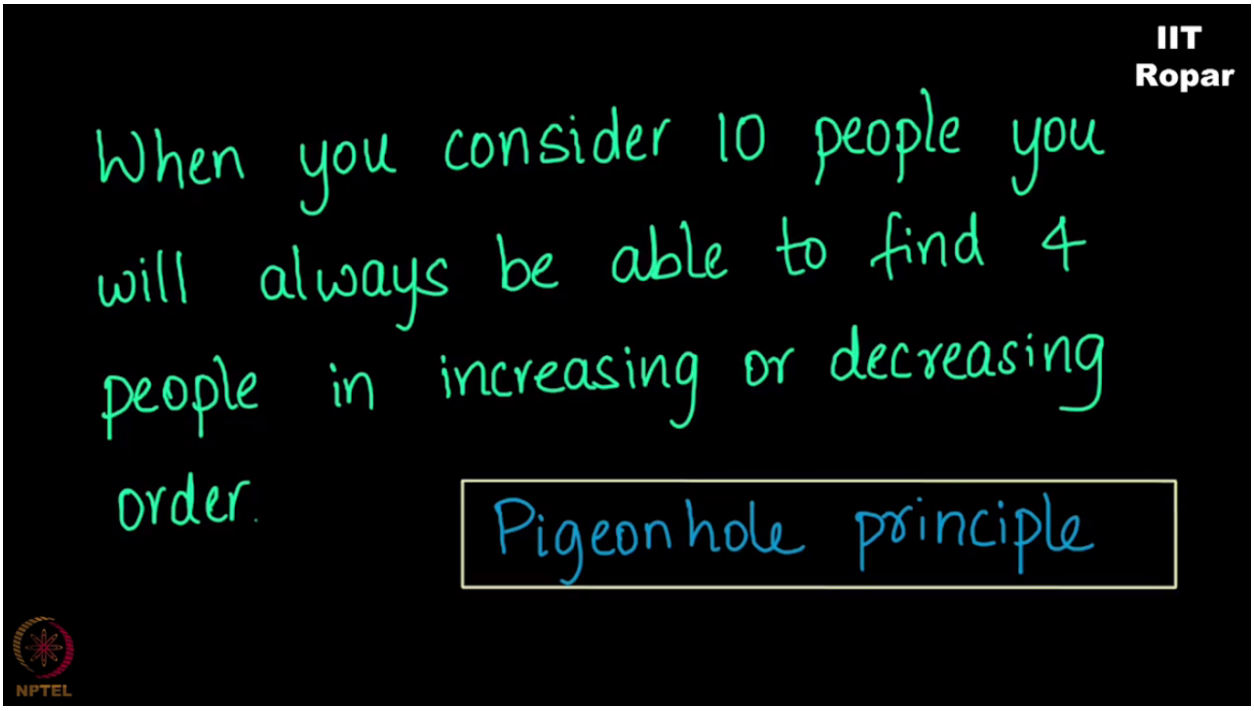
Now it's easy if I were to visualize this in terms of numbers okay. The first person is of height let's say ten, the second person is a height let's say two third person is of height let's say seven and so on let me write some ten numbers like this okay. Yeah ten, two, seven, six, one, four, five, three, and then eight and let's say nine, okay.

You have ten numbers here; one, two, three, four, five, six, seven, eight, nine, ten numbers here right. So what I want you people to observe is that when you write these numbers in any order it so happens that you can always find four numbers which are either in increasing order or decreasing order. By that I mean these people who are here these 10 people, some four of them can be made to step forward in such a way that they are all either in ascending order or descending order.

Let me try to figure that out for this. I think I have found that 10, 7, 6, 5 if they step ahead you will see that 10, 7, 6 and 5 are in descending order. The point is I cannot say 10, 7, 6, and 10, 7, 6, & 2 I cannot take. I should maintain the sequence here. Okay. So I should maintain the sequence and show that there's actually an increasing or decreasing sequence. It can be either of them okay.

Let us try another example. Let me write down. Let's say 1, 7, 6, 2, 10, 8. I'm just randomly writing some numbers, 4 and then let's say I'll write 3 here and is 1, 2, 3, 4, 5 is remaining, 5, 6, 7, 8, and the 9 is remaining so let me put 9 here. Okay.

Now do you think there will be some four people here four people here who will step forward and they will be an increasing or decreasing order. You see I am actually seeing one. Look at this 1, 2, 4, 5 let them step ahead you will now see 1, 2, 4, and 5 for people in increasing order. Now this is always true. Let us write this down neatly and observe what we are trying to say.



All we are trying to say is then you consider, it's sort of important so I'm trying to write this down neatly, when you consider ten people you will always end up you will always be able to find four people in increasing or decreasing order okay. That's what we have observed. Is this always true? Indeed it is always true and trust me you can show this using pigeonhole principle.