Discrete Mathematics

Functions

Pegionhole Principle - A result

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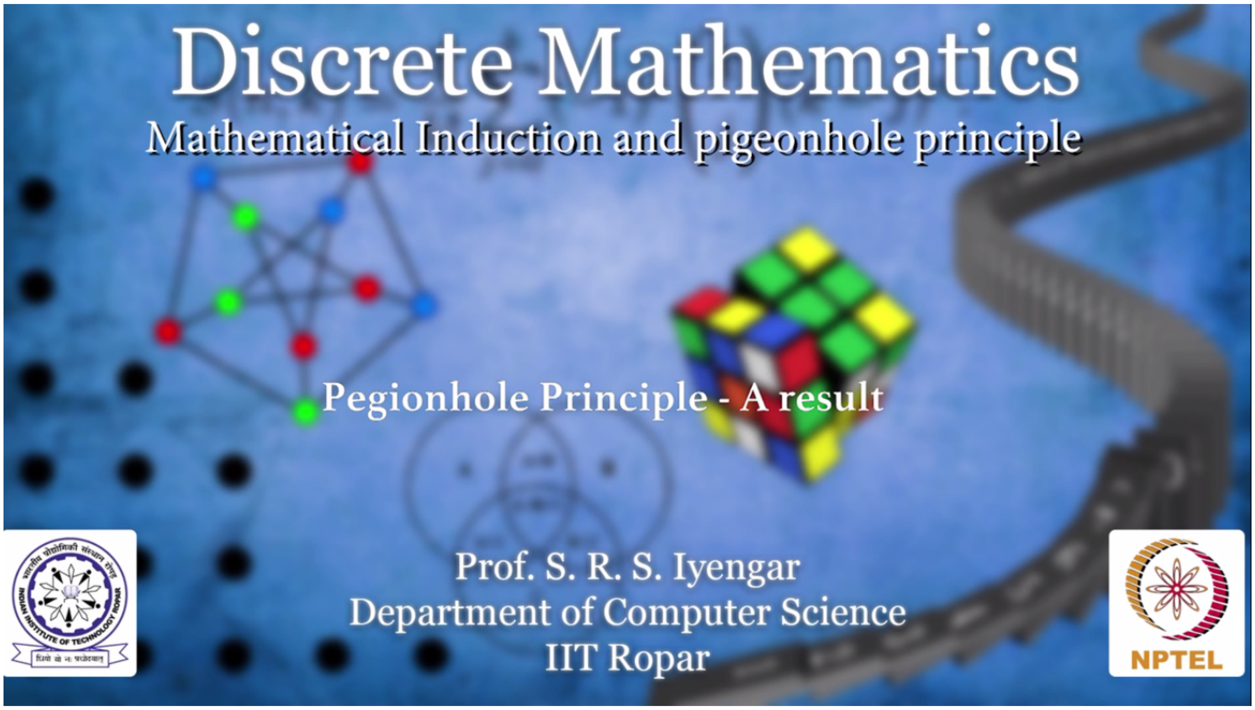
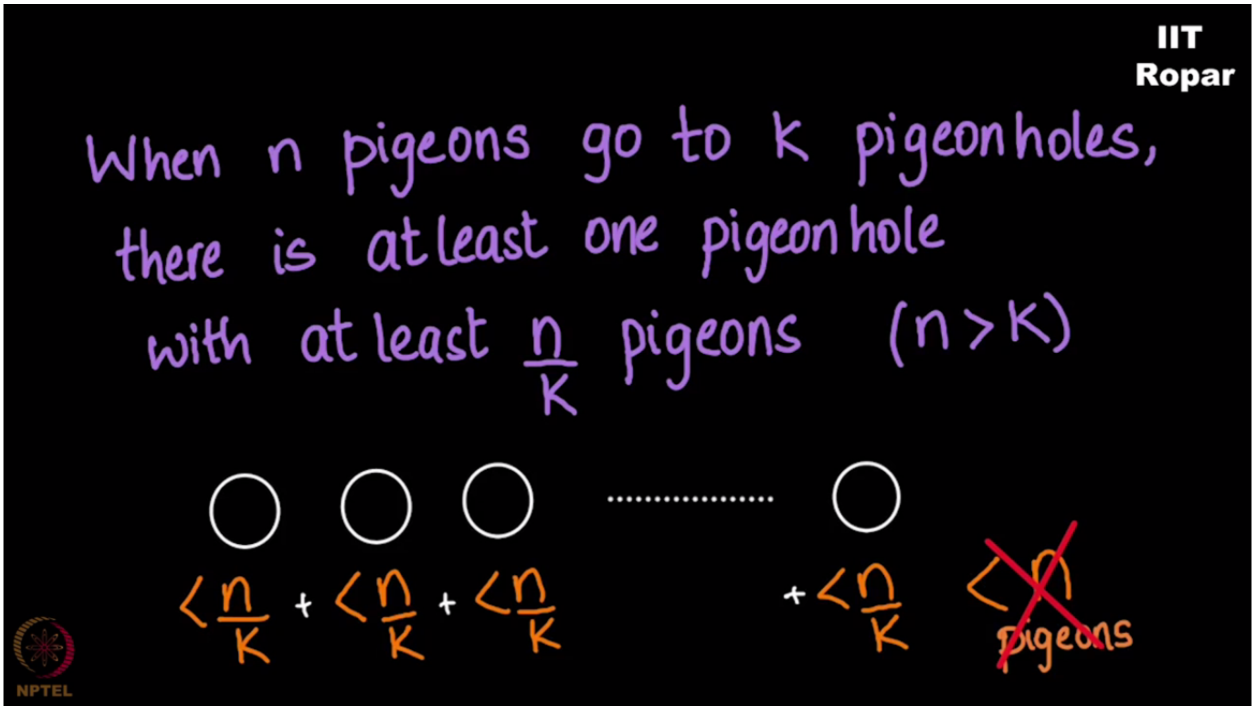


So look at this problem. There are 50 gold coins that a dad decides to distribute to ten of his sons. Son 1, or let's say daughter. Daughter 1, daughter 2, and daughter 10. He has ten daughters and he decides to distribute 50 gold coins to these ten daughters.

Now you see can all of them get 1, 1, 1 gold coin? Not really. Then it doesn't add up to 50. The point here is that 50 gold coins are all distributed. So the dad can give all 50 gold coins to his first daughter D1, and 0,0,0 to the rest. Now I am going to make a statement here. I am going to say that at least it's going to be play onward, please concentrate at least one daughter gets at least five gold coins. What do I mean by that? Amongst these ten people it doesn't so happen that all of them get four or less. At least one of them should get five. Why? If none of them get five or more not even a single person gets five or more then all of them are getting four or less. When each one of them is getting four or less put together the gold coins distributed will be 40 or less. Are you trying to see the simple math here. When every one gets four or less gold coins these ten people put together will have 40 or less gold coins. So it is impossible that at least one daughter gets at least five gold coins doesn't happen. Think about it. This is actually stated as the following theorem.



When n pigeons go to k pigeonholes there is at least one pigeonhole with at least n/k pigeons. The proof is already explained in the previous example. The same thing holds good here too. Just in case every pigeonhole has less than n/k pigeons then the total number of pigeons will not be more than n a contradiction or assumption that all the pigeonholes have than n/k pigeons. That's not possible. And hence I can guarantee that at least on pigeonhole with at least n/k pigeons. Sounds like stating the obvious but all the math notation here sounds slightly geeky but don't worry it is as easy as the gold coins and the daughter example.