Discrete Mathematics

Functions

10 points on an equilateral triangle

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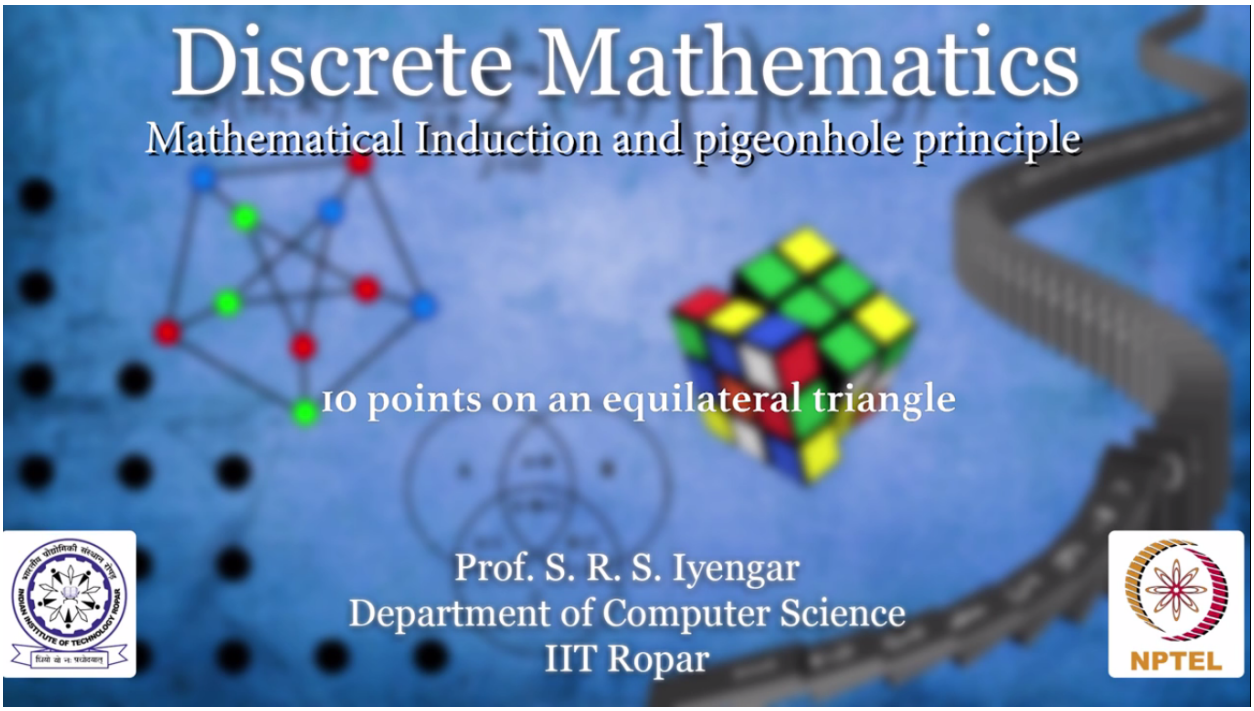
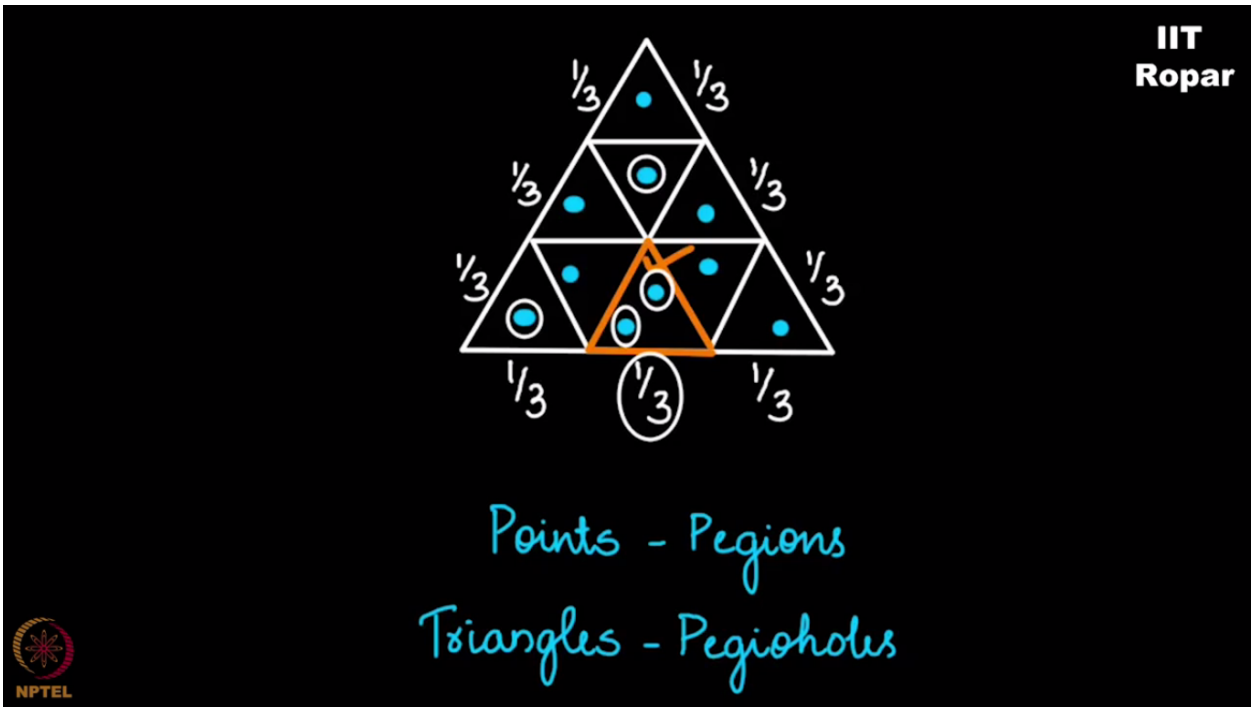


Consider this equilateral triangle. It's a unit equilateral triangle where ten points are placed within it. Now we have to show that there exists two points with distance at most 1/3. So if I give you a equilateral triangle like this you see there are ten points here within this equilateral triangle and it's a unit equilateral triangle. What do I mean by that?

The sides of the triangle are one unit each. Now there are ten points inside this. What I must do is I must show that there are two points. There exist two points which are 1/3 apart. Now the proof is very cute. Let me show you how. I am going to divide this equilateral triangle further to create more equilateral triangles. How am I going to do that? Like this. Do you see. We get 9 equilateral triangles more.

Now you have to select -- you have to just sprinkle 10 points rather 10 points are sprinkled in this equilateral triangle which has further being divided into 9 equilateral triangles. Now do you observe that each equilateral triangle inside is of length 1/3, 1/3, 1/3 each side has length 1/3 here within. There are 9 triangles and we have ten points to sprinkle.



Now you must be very sure here that definitely there exists a triangle which has two points. Once you get a triangle with two points what can you observe about the distance between the two points? Supposing I choose this point and this point, you see we cannot conclude anything about this here between these two points but consider this triangle where there are two points inside this triangle. What can I conclude? I can definitely conclude that the distance between these two points will at the maximum be 1/3. Where did I use pigeonhole principle here? I considered my points to be pigeons and these triangles to be pigeonholes. I sprinkled the points in different triangles and I got one triangle with two points and hence the distance between them is less than or equal to 1/3, hence the proof.