NPTEL
NPTEL ONLINE COURSE

Discrete Mathematics
Mathematical Induction and pigeonhole principle

A false proof - Solution

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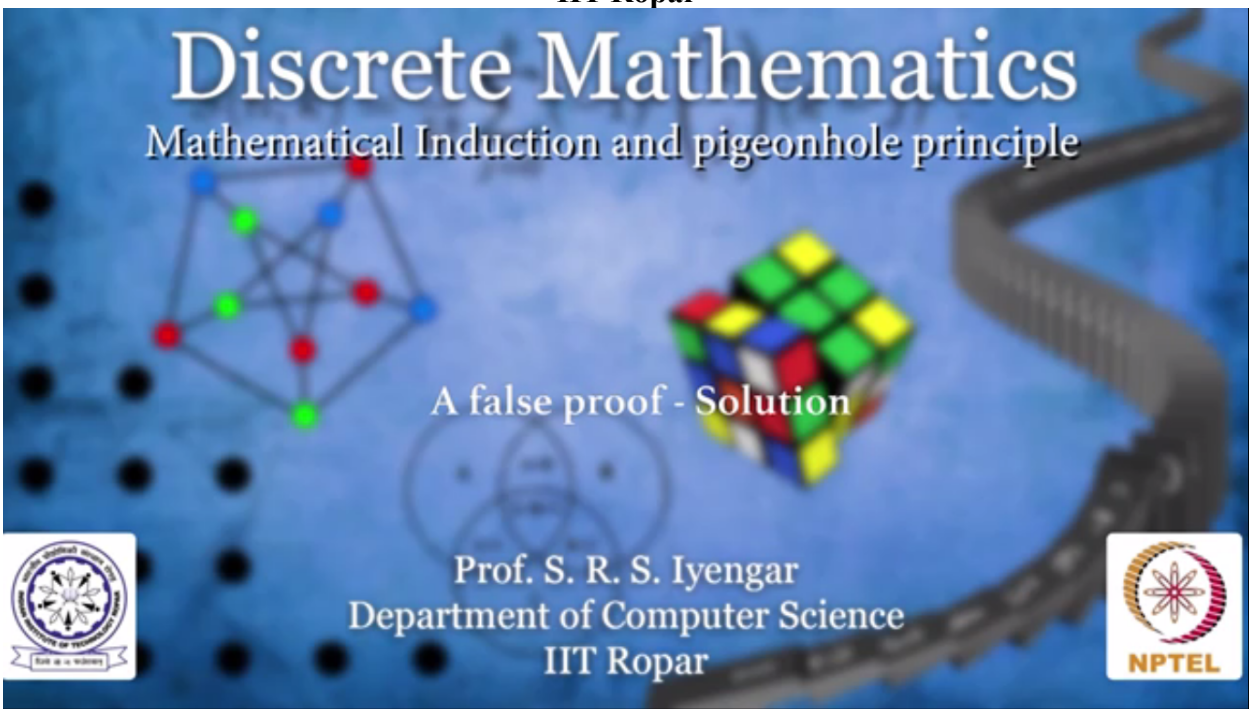
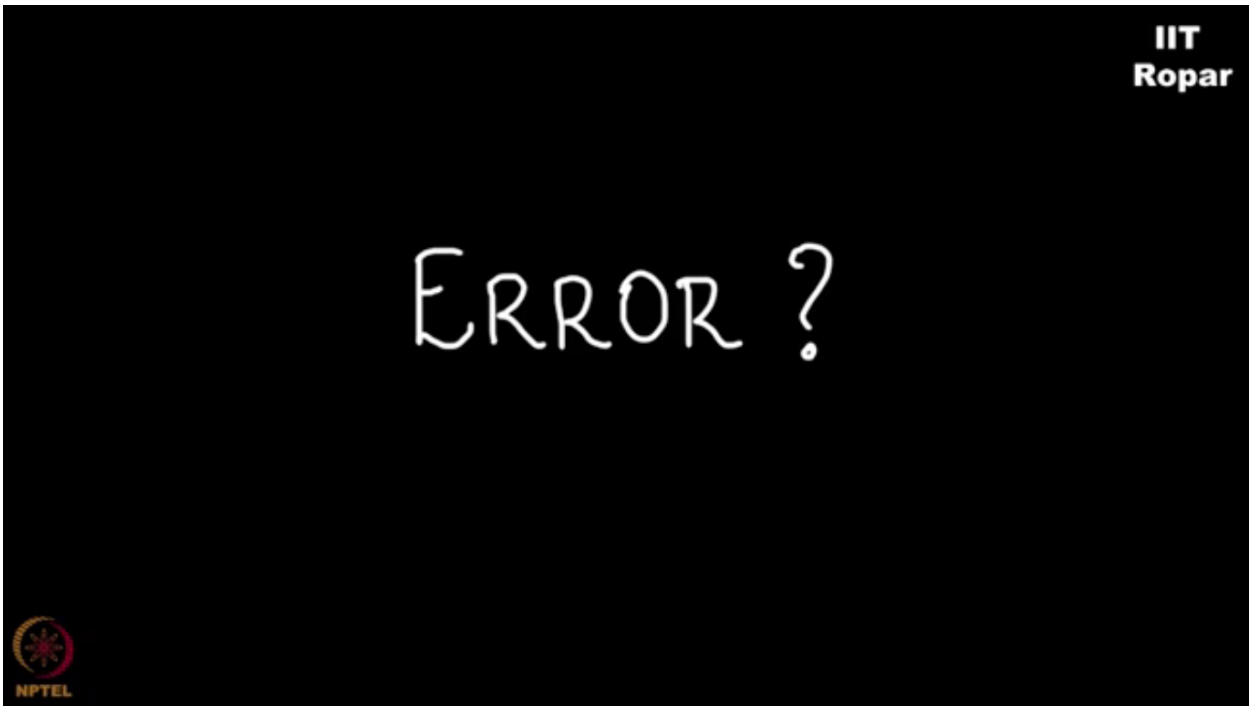
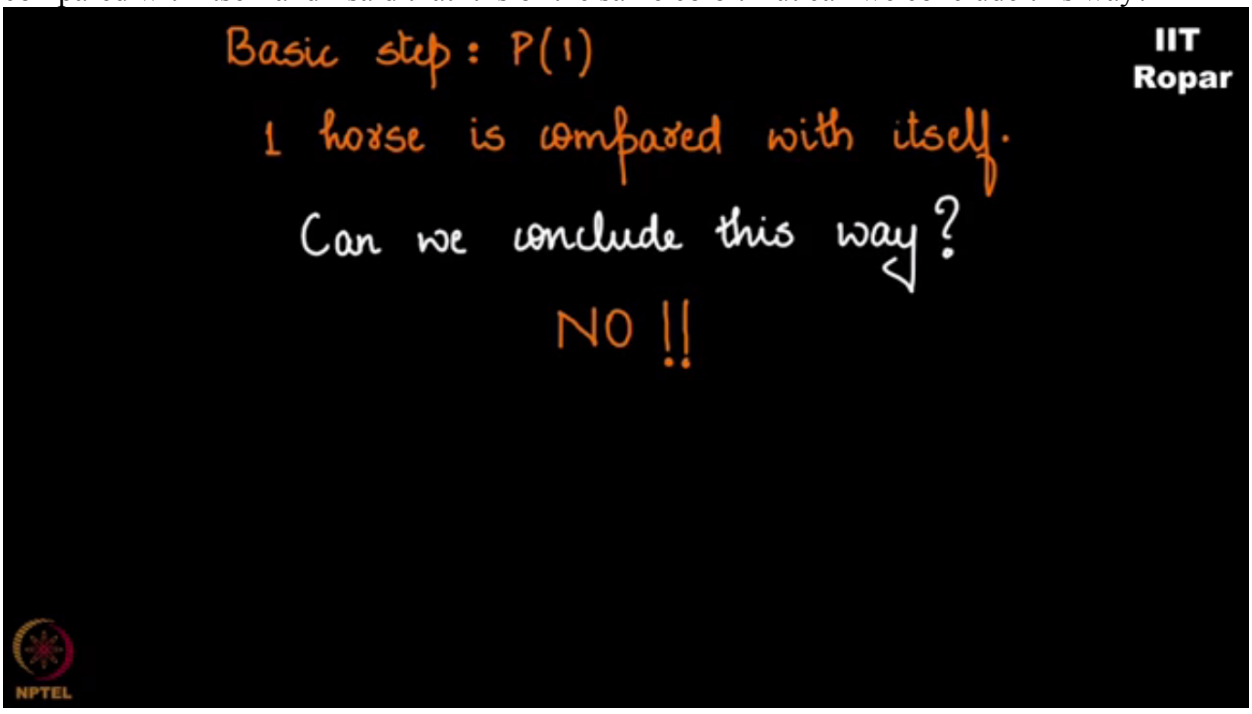


Have you all figured out the error? If you haven't, once I tell you the error, you will tell, "Ah! Yes, this was the mistake." It is very subtle.

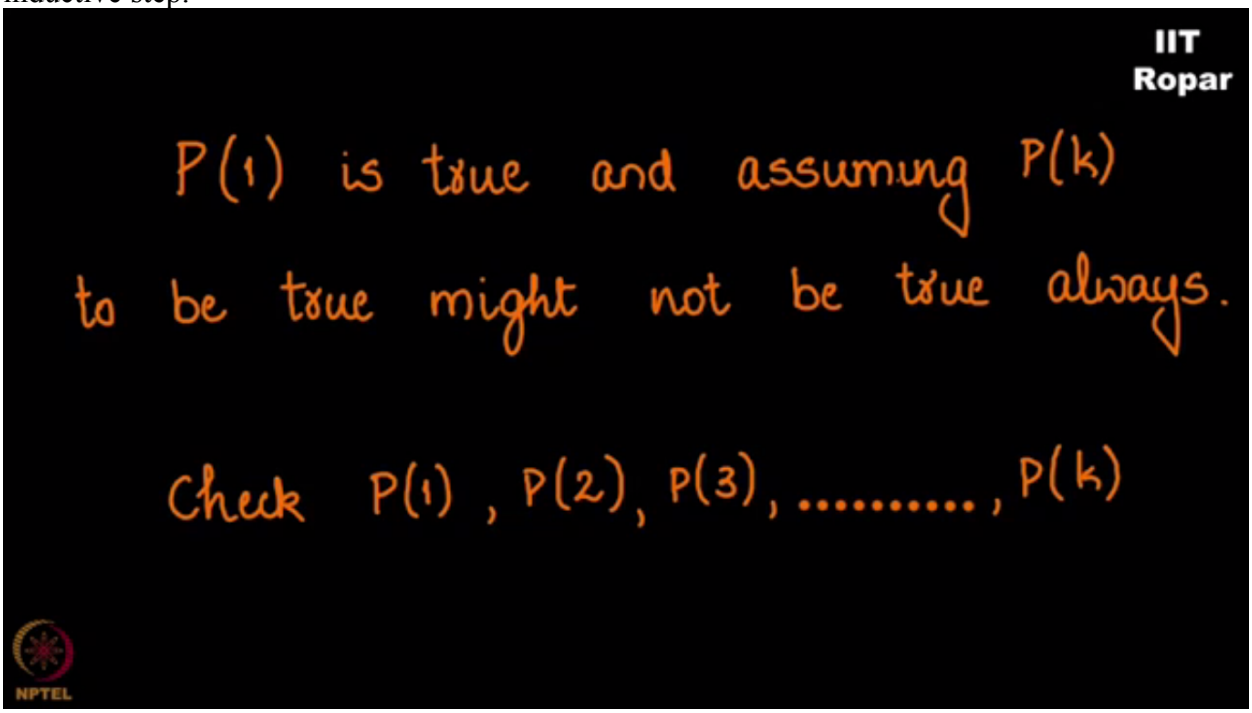


So listen here. The basic step which I considered was P(1) and what was that? One horse is compared with itself and I said that it is of the same color. But can we conclude this way?



No, because considering the basic step to be P(1) which says comparing one horse with itself and saying that it is of the same color and starting from here and assuming P(k) to be true is where lies the mistake because if I take k to be 2, which is P(2), two horses are of the same color. Is it true always? Need not be and hence telling that P(1) is true and assuming P(k) to be true, this transition might not be true always. We need to carefully check P(1), P(2), P(3) and then move

ahead to P(k), which means you have to be very careful when you are moving ahead with the inductive step.



You will find several problems, which is very easy to prove by Mathematical Induction, but you might end up being wrong and it is very difficult to find out or it is very difficult to reason out where the error lies.

Apart from all this, though induction is deceptive, it is a very, very powerful tool in mathematics to prove several theorems. In the coming weeks, you will see its significance and you will see where it is used.

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