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Discrete Mathematics
Mathematical Induction and pigeonhole principle

Mathematical Induction - A false proof

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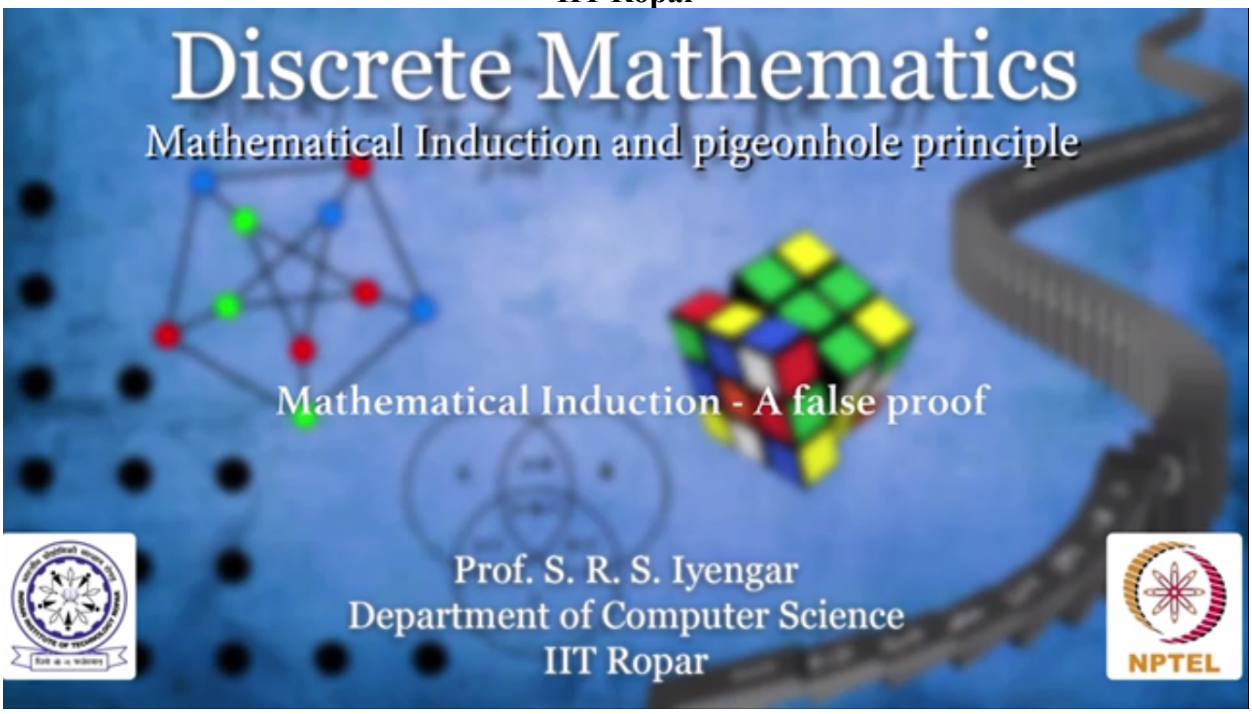
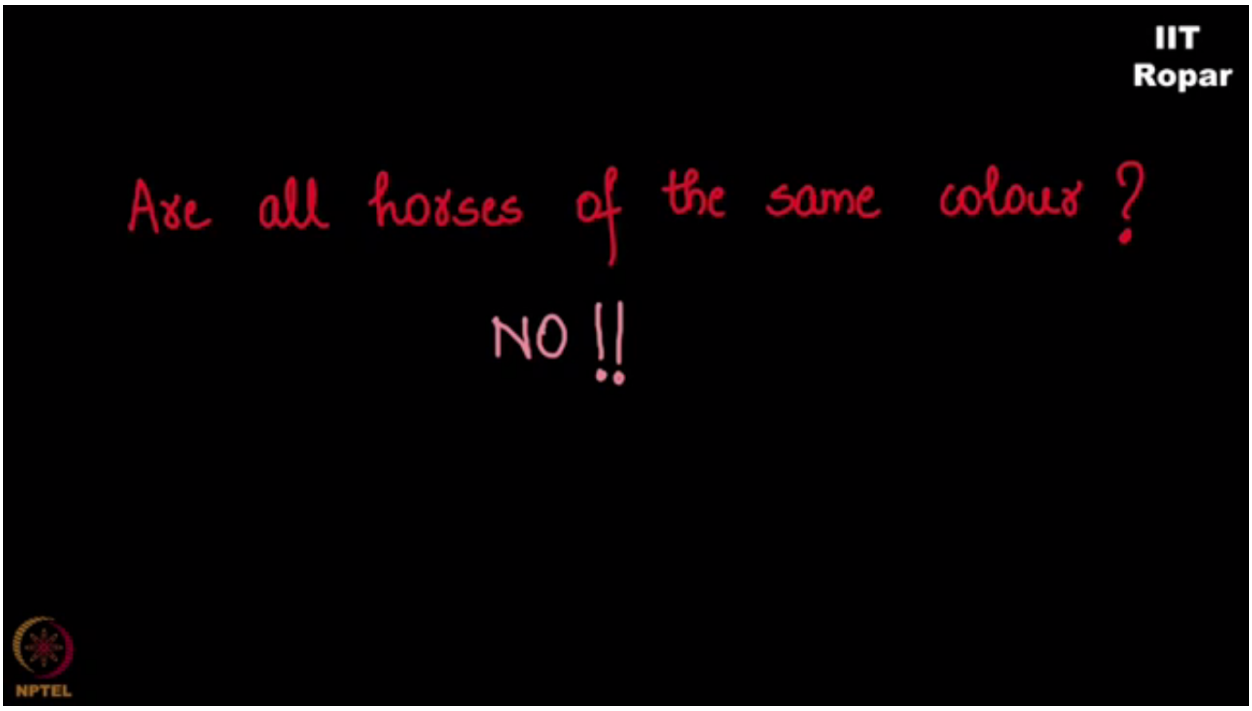
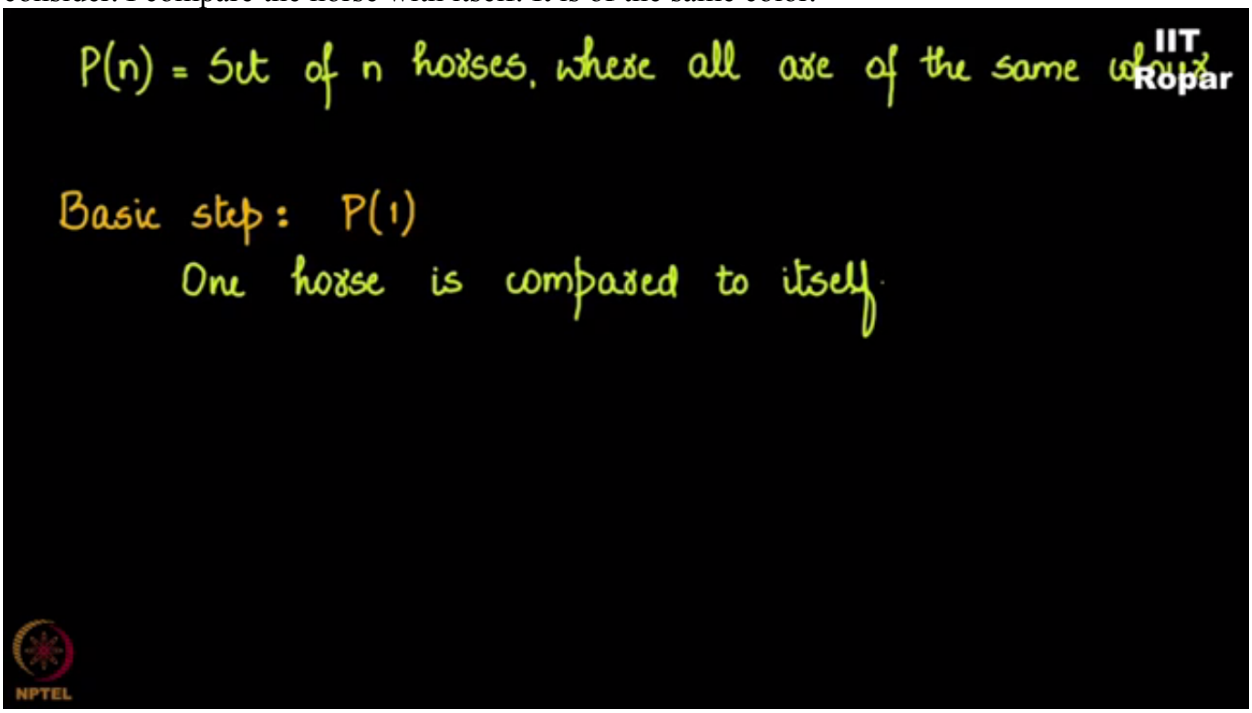


A few days back, a friend of mine asked me, are all horses of the same color? Well, my answer was obviously not.

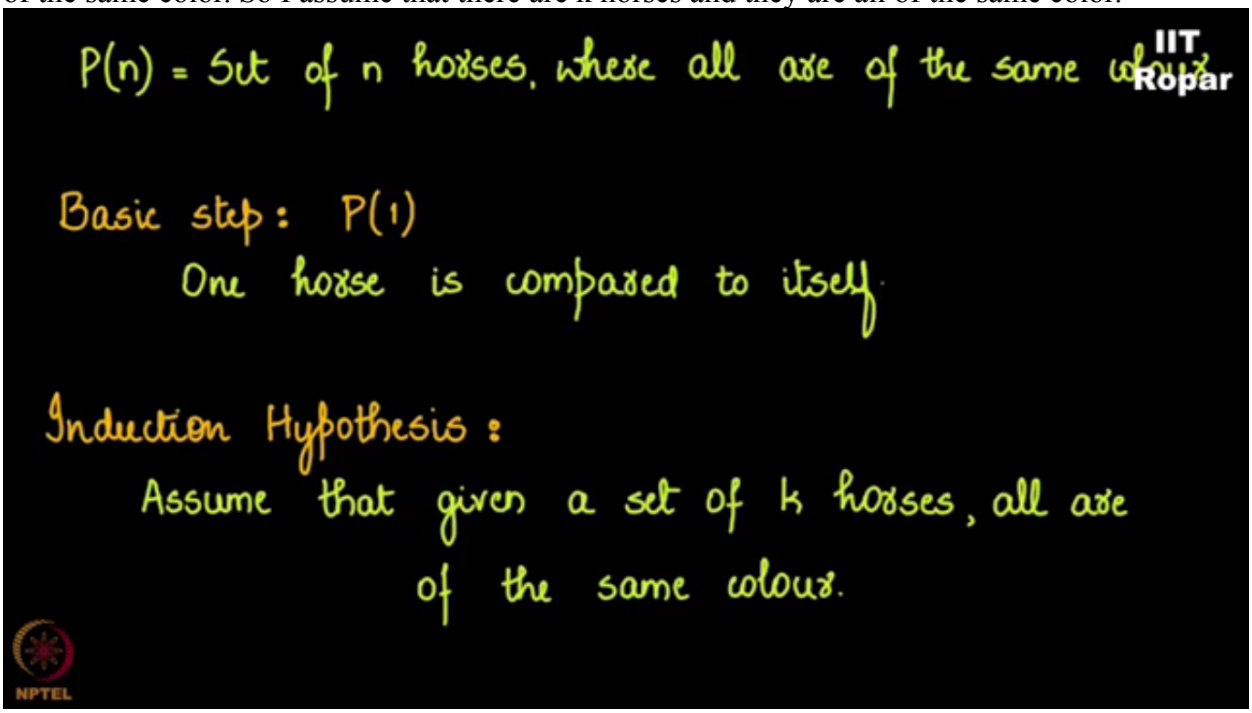


Even you might be guessing the same thing by now, but it kept me thinking once I started doing induction. How? I can prove you that all horses are indeed of the same color. I'm sure most of you must be laughing. Let us see how.

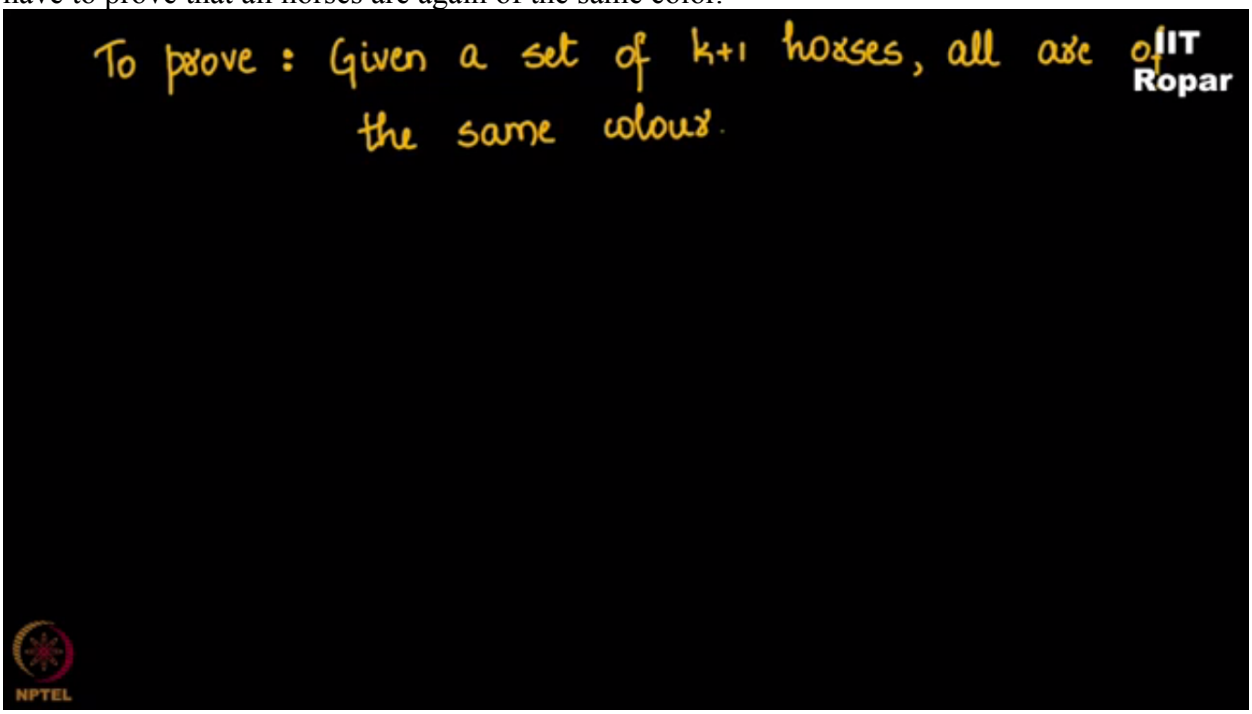
I will consider my P(n) to be a set of n horses are all of the same color. What is my basic step? I will consider basic step as P(1), which says a set of one horse is of the same color. Obviously. I consider. I compare the horse with itself. It is of the same color.



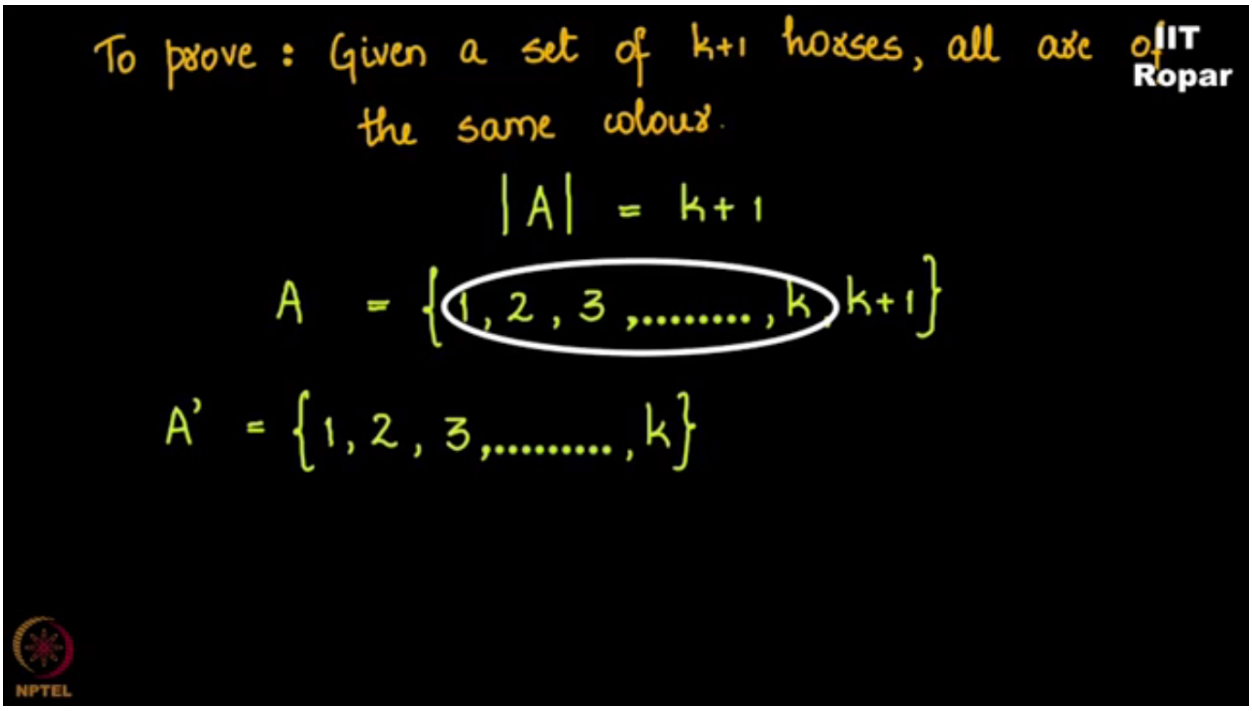
Now I'll move on to the Induction Hypothesis where I assume that given a set of k horses, all are of the same color. So I assume that there are k horses and they are all of the same color.



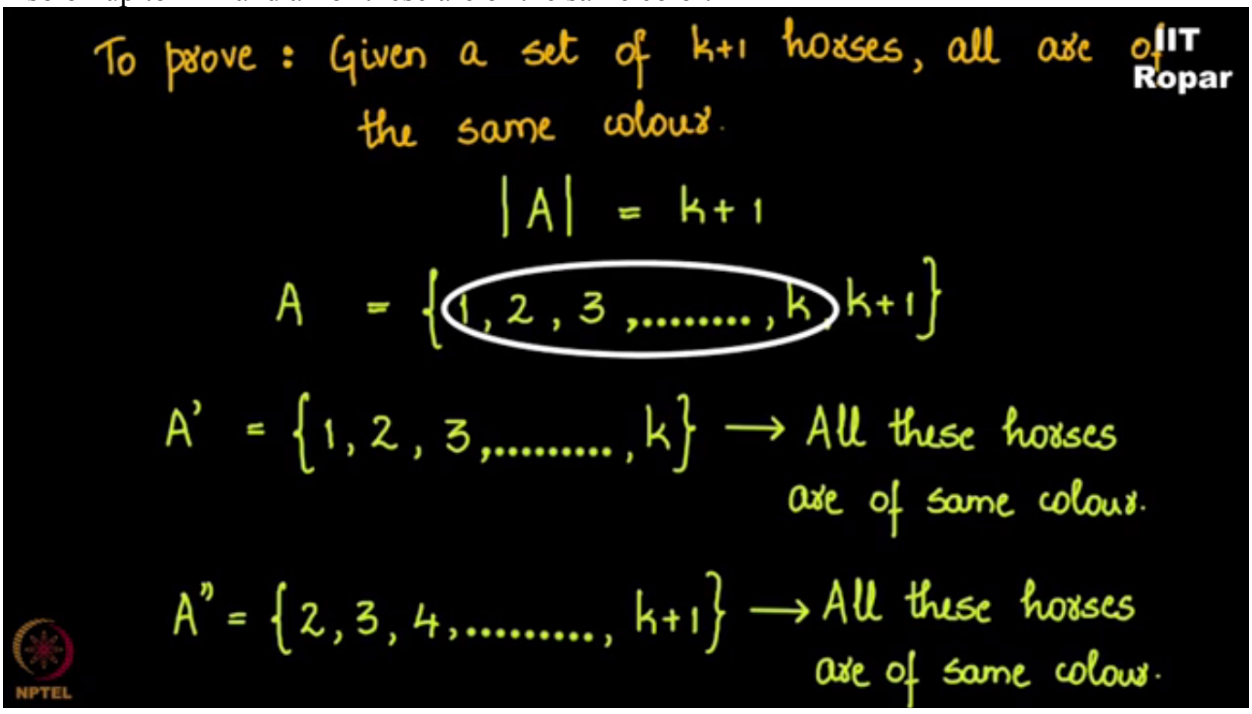
So the next step is the inductive step where we will prove that given a set of k+1 horses, you have to prove that all horses are again of the same color.



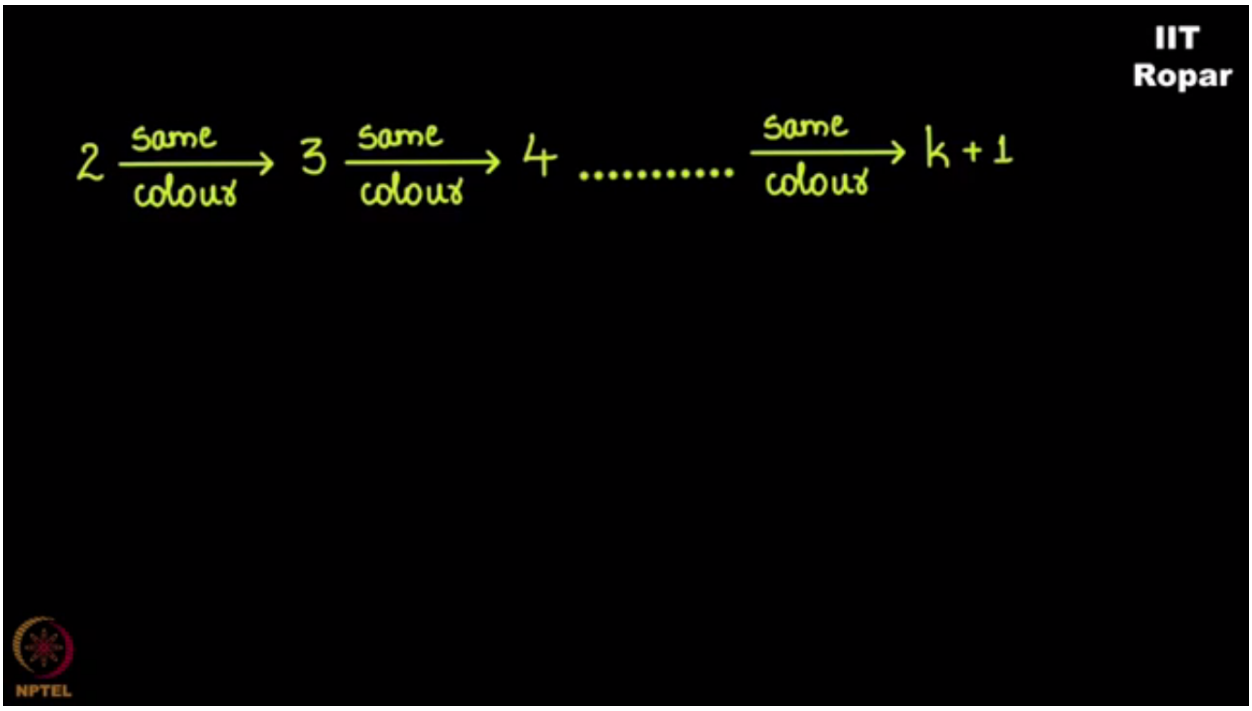
So what I am going to do? I will consider this set A, which has k +1 horses. I will label them 1, 2, 3 and so on up to k+1. So first, I will pick the first k horses. Let me say that as A, A'. A' has k horses 1, 2, 3 up to k. By Induction Hypothesis, what can we conclude? We can conclude that all of these are of the same color. Right?



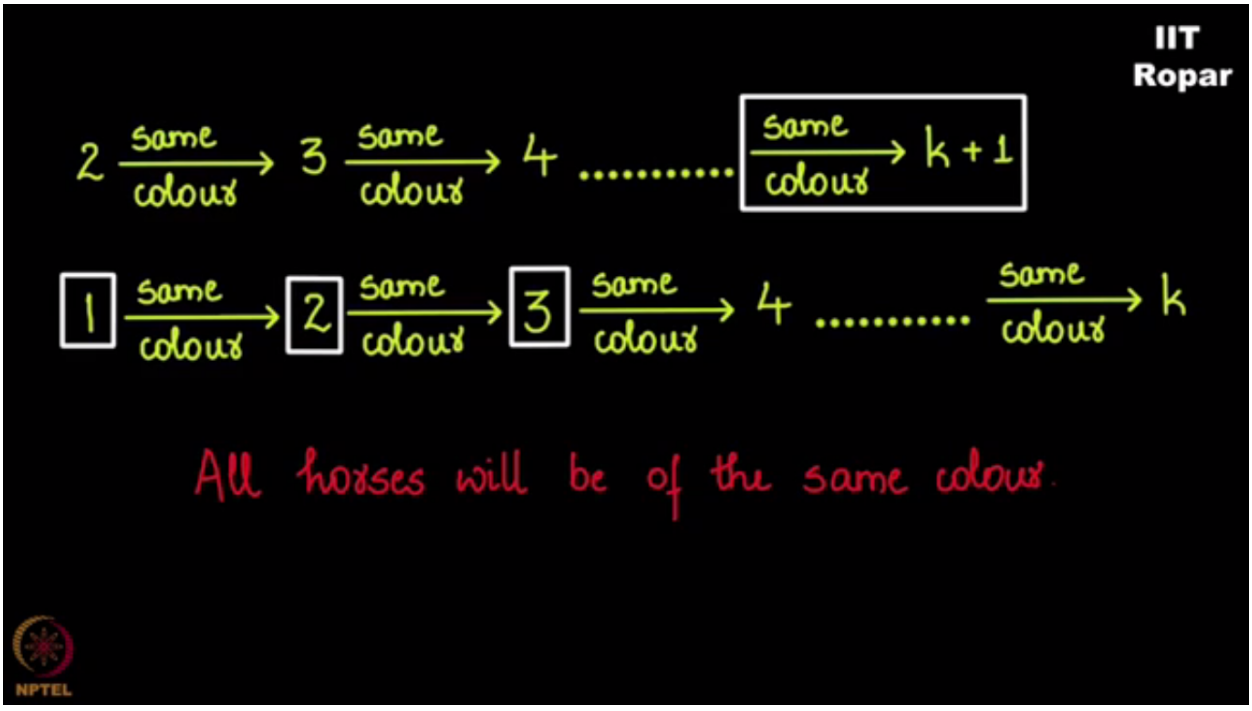
Now I'll consider another subset of A, let me call that A" from 2 to k+1. So A" has elements 2, 3, 4 so on up to k+1 and all of these are of the same color.



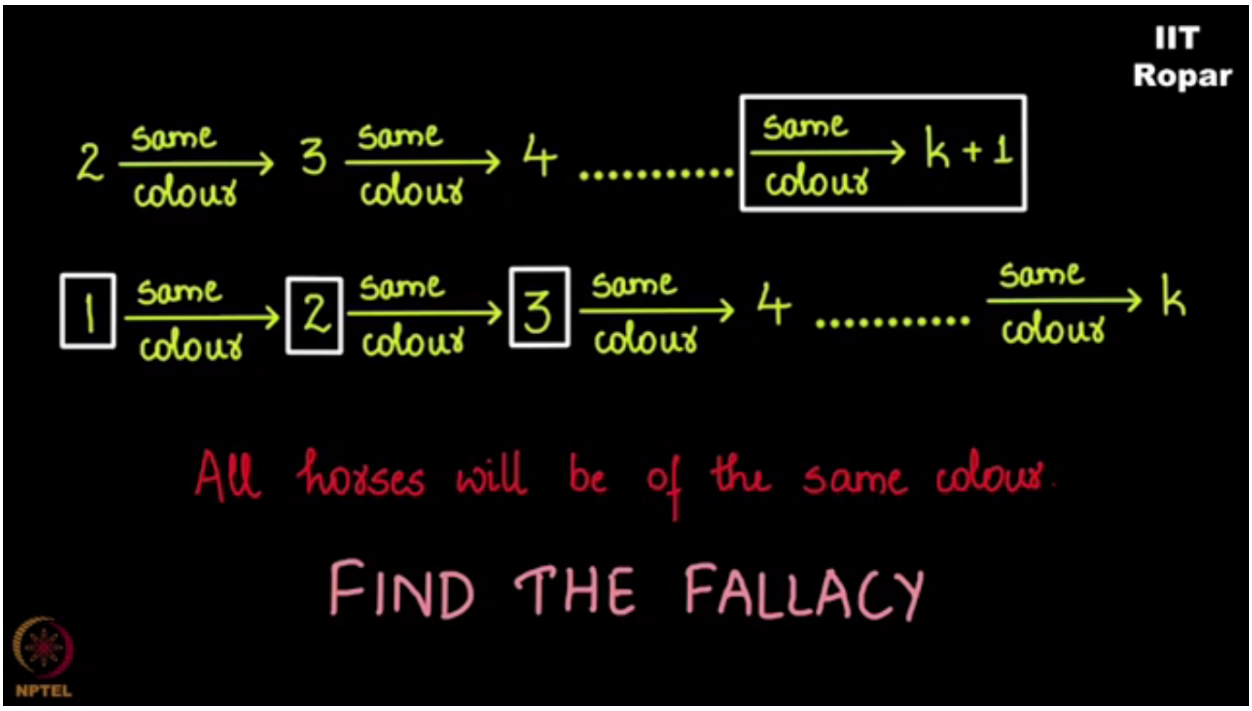
Wait. Did you observe something? 2 has the same color as 3 has the same color as 4 and so on and all of these have the same color as k+1.



Compare this with the horses in A'. 2 has the same color as 1 as 3 as 4 and so on as k. So what can we conclude? We can conclude that 1 has the same color as k+1. So with the case with 2. So as with 3. So all these k+1 horses have the same color.



Well, I must tell my friend that yes, all horses will be of the same color. So if you are all convinced with my argument that all horses are of the same color, let me tell you, I was wrong and I leave it to you to find out the fallacy in the proof.



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