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Discrete Mathematics
Mathematical Induction and pigeonhole principle

Mathematical Induction - Example 10 solution

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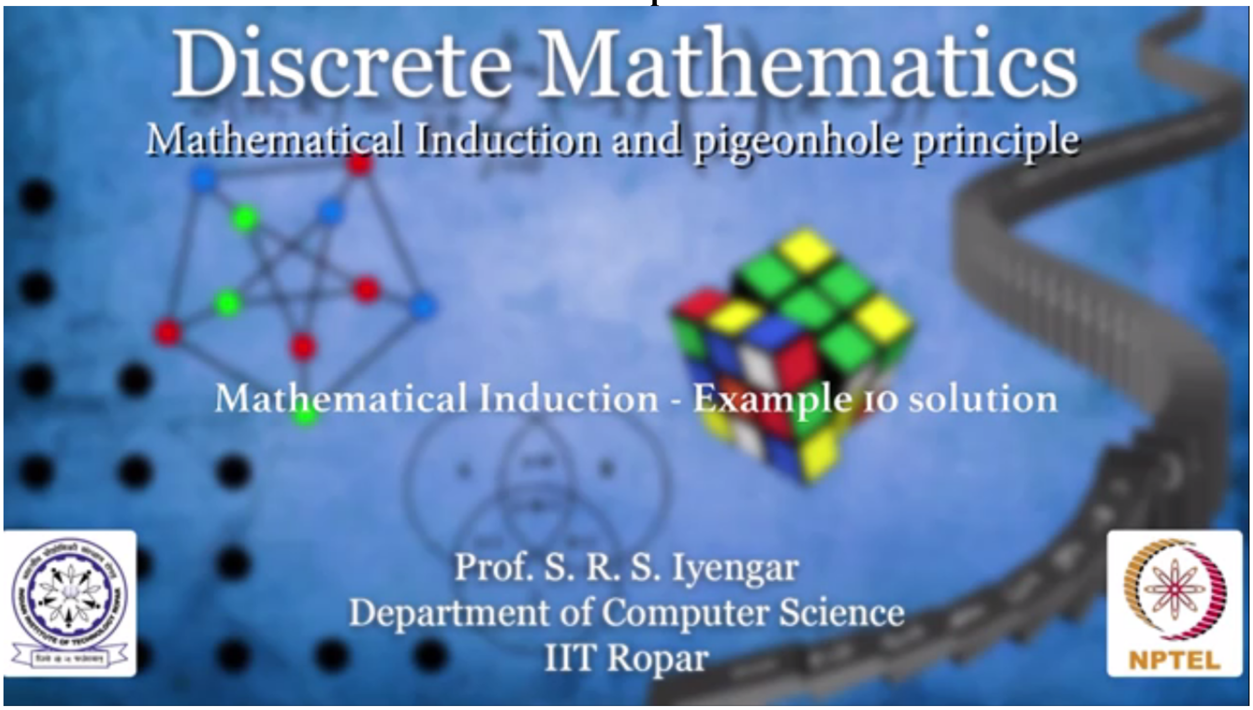
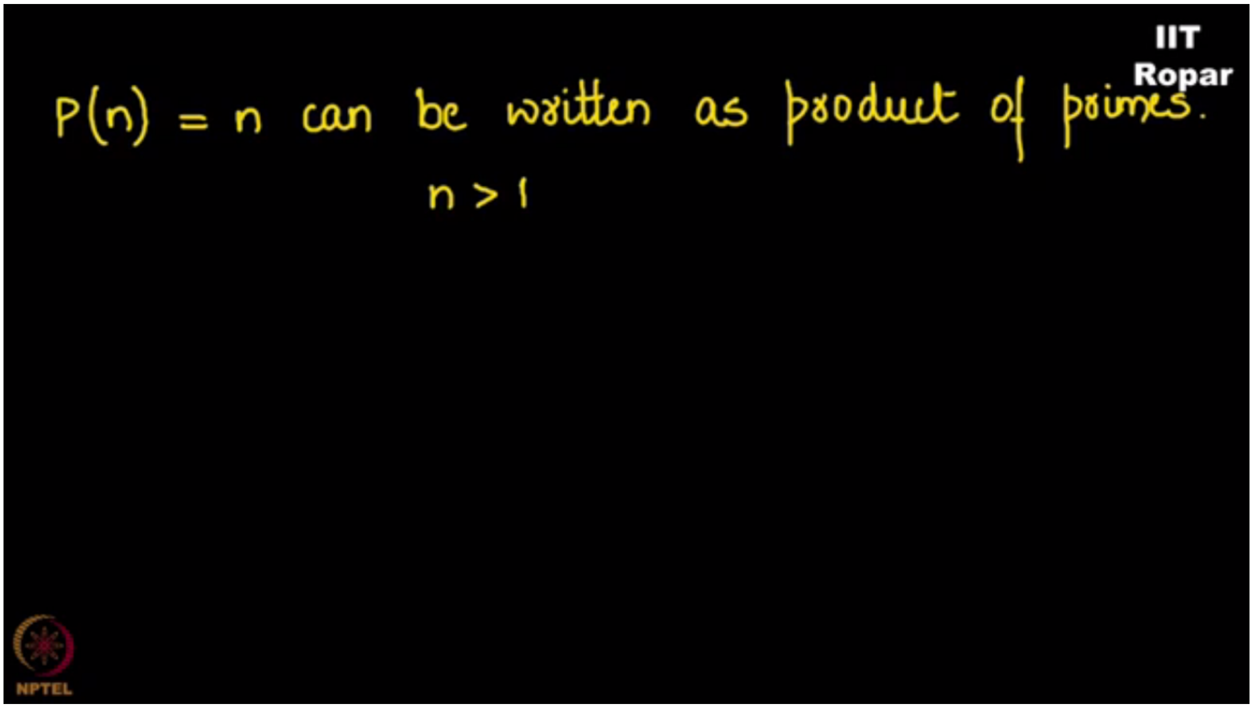
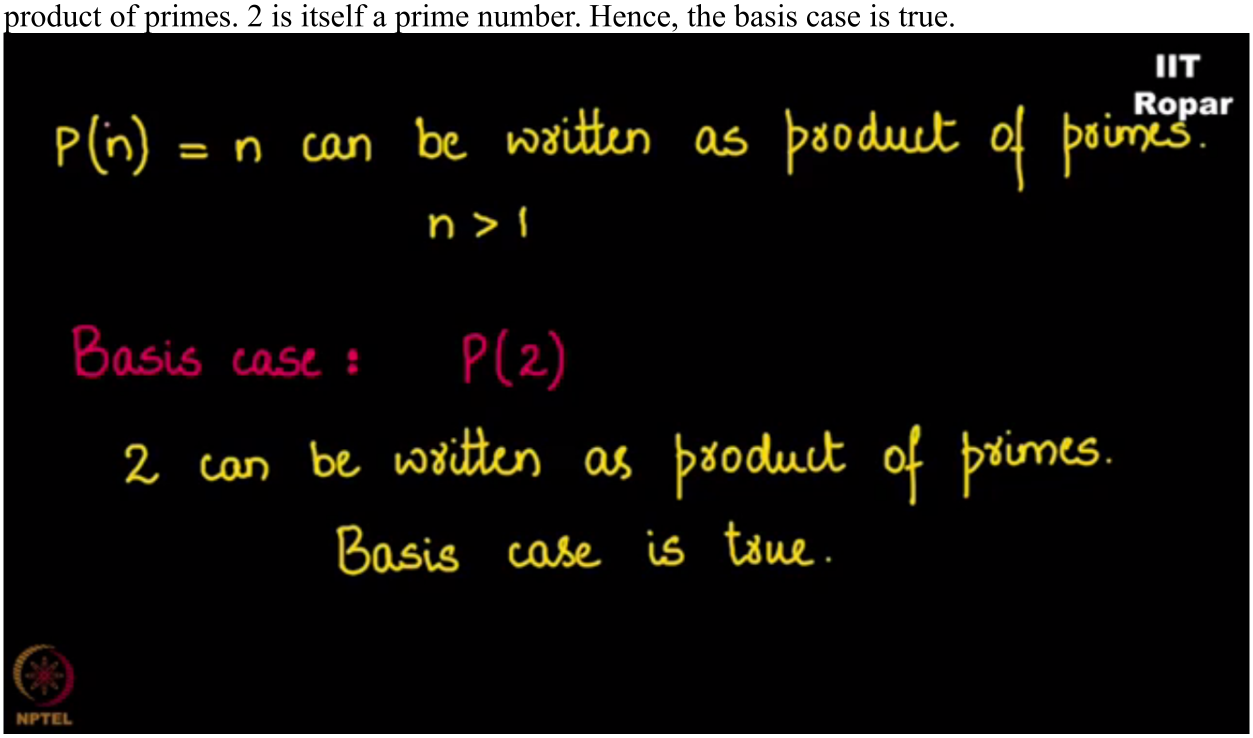


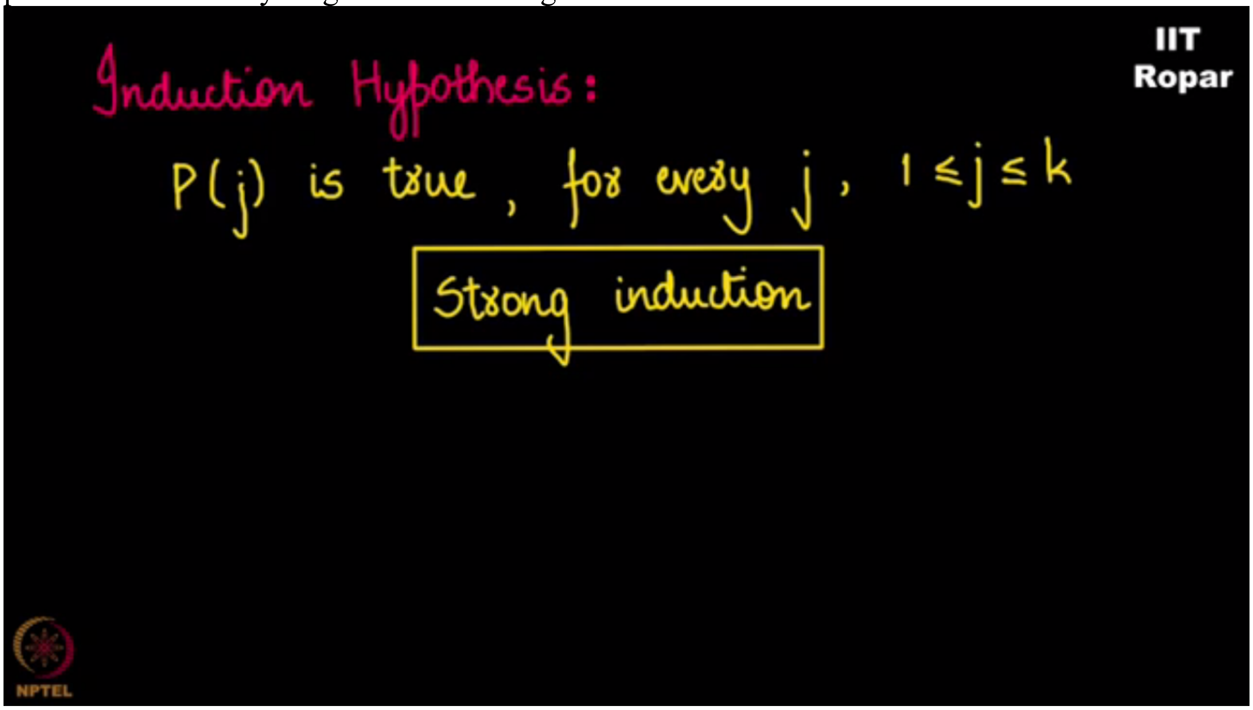
Consider this proposition P(n) as n can be written as a product of primes where n is greater than 1. So this is the proposition. n can be written as product of primes.



Let us prove this by induction. What is the basis case? The basis case as you know since n is greater than, strictly greater than 1, we start with 2. Right? Now P(2) will be 2 can be written as product of primes. 2 is itself a prime number. Hence, the basis case is true.

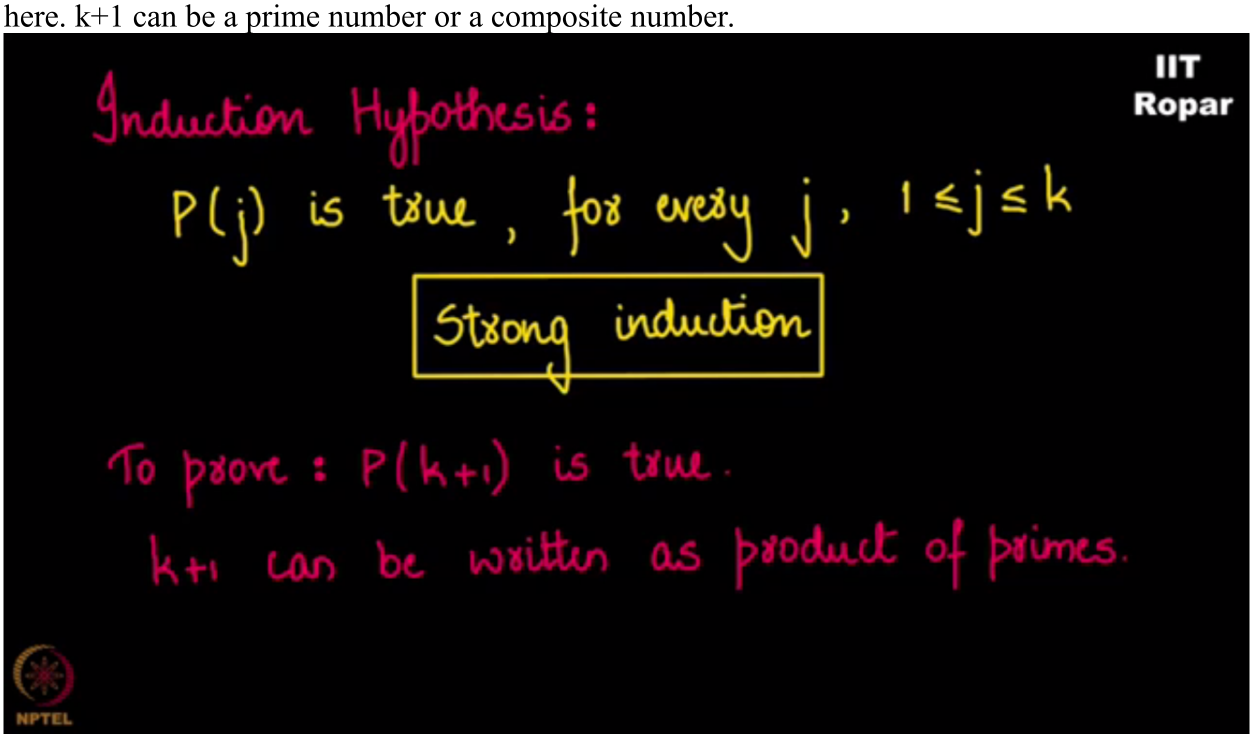


What does the hypothesis states? The hypothesis states that P(j) is true for every j between 1 to k, 1 less than or equal to j less than or equal to k. Every number between 1 to k, every number between 1 to k must be true. The P of that number must be true. That is every number between 1 to k can be written as a product of primes. You see I am using Strong Induction here. The professor has already taught what is Strong Induction.

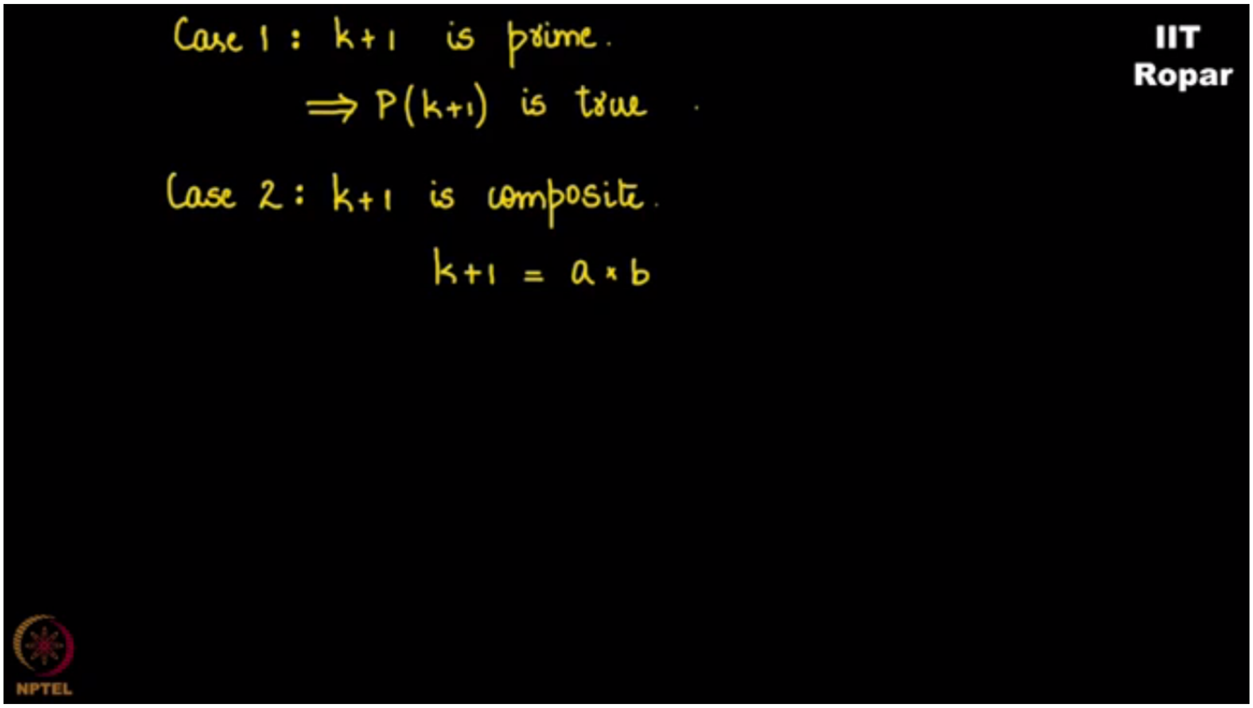


Now assuming that P(n) is true for every number less than k, less than or equal to k, I'll move ahead to prove P(k+1).

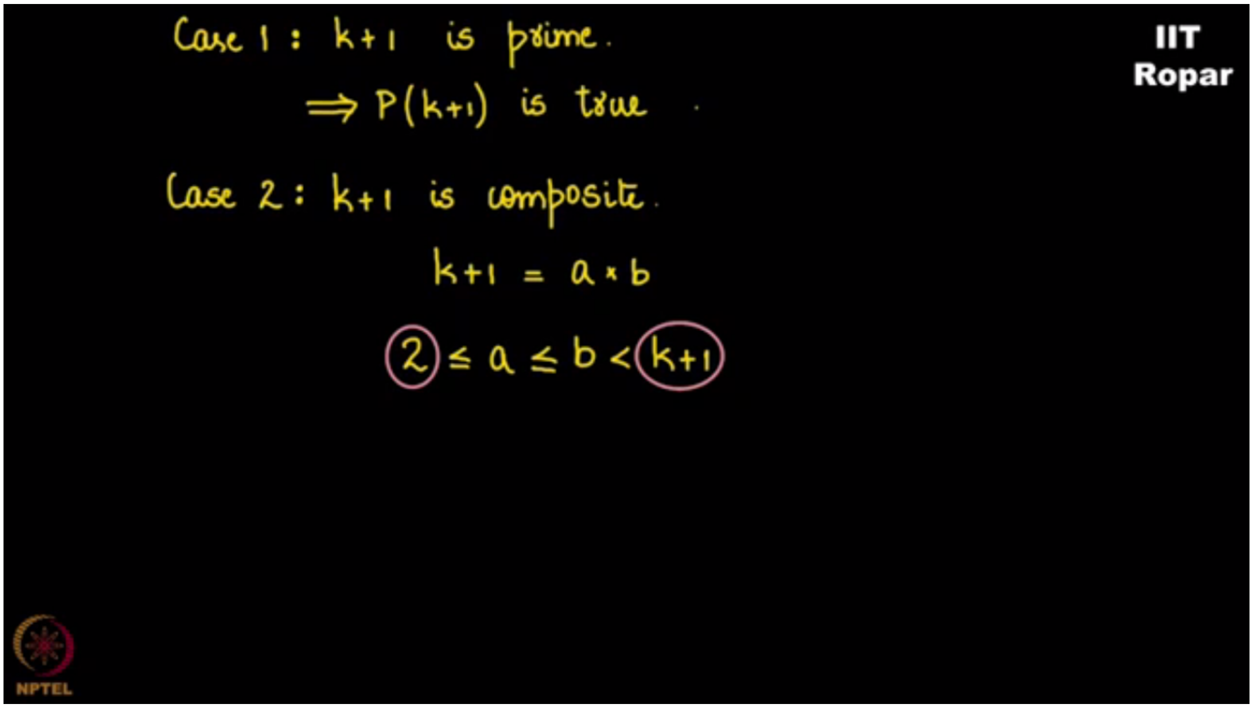
What is P(k+1)? It states that k+1 can be written as a product of primes. There are two cases here. k+1 can be a prime number or a composite number.



Let me take the case where k+1is prime. If it is prime, then we are done. If it is not prime, that is it is a composite number, then let us see how we can proceed. k+1 can be written as since it is composite, it can be written as a x b. a and b are some two numbers whose product is giving me k+1.



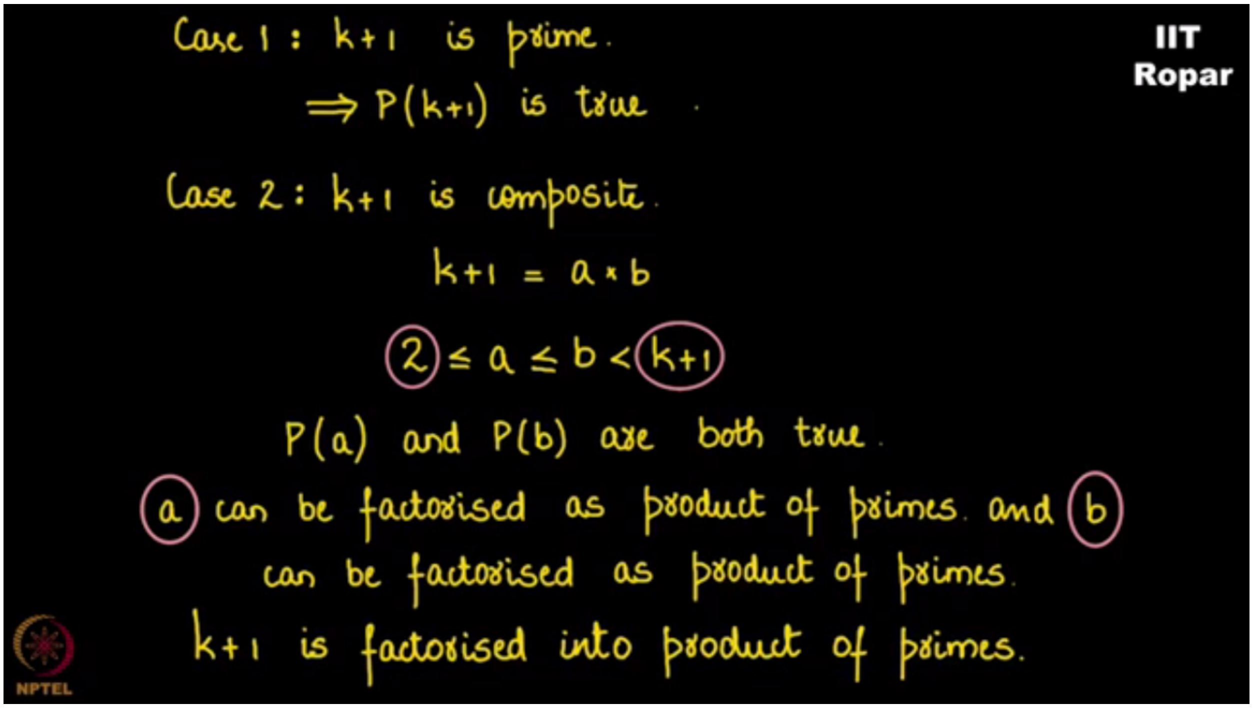
Now what can these a and b be? The condition goes like this. 2 less than or equal to a less than or equal to b strictly less than k +1. Do you see something? a and b lies between or lies in the range 2 to k+1. It cant take the value 2, but cannot take the value k +1 because k+1 we cannot write it as 1(k+1). It should be strictly less than k+1. So a and b lie in this range.



Now what am I going to do here? I will use the Induction Hypothesis, which states -- I will apply Induction Hypothesis to both a and b. What do I do? Which says that P(a) and P(b) are both true.

Did you see why we use Strong Induction here? We want every number below k, that is less than k, to be written as product of primes which we assumed in our hypothesis. That is coming to our help here. So did you see the significance of Strong Induction?

When apply Induction Hypothesis to a and b, I see that a can be factorized as product of primes and b can be factorized as product of primes, and hence if it is true that a and b can both be factorized, which means product of primes into product of primes is nothing but another set of product of primes, right? You get another huge expression, which is a product of primes. Now that is k +1.



So we ended up having a prime factorization of k +1 and hence we see that k +1 can be written as product of primes. We did it in terms of a x b. Right? We used the hypothesis that a can be written as product of primes, b can be written as product of primes and hence we ended up having k +1 factored into product of primes.

Take a minute to watch the video here and there again and try to understand the proof.

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