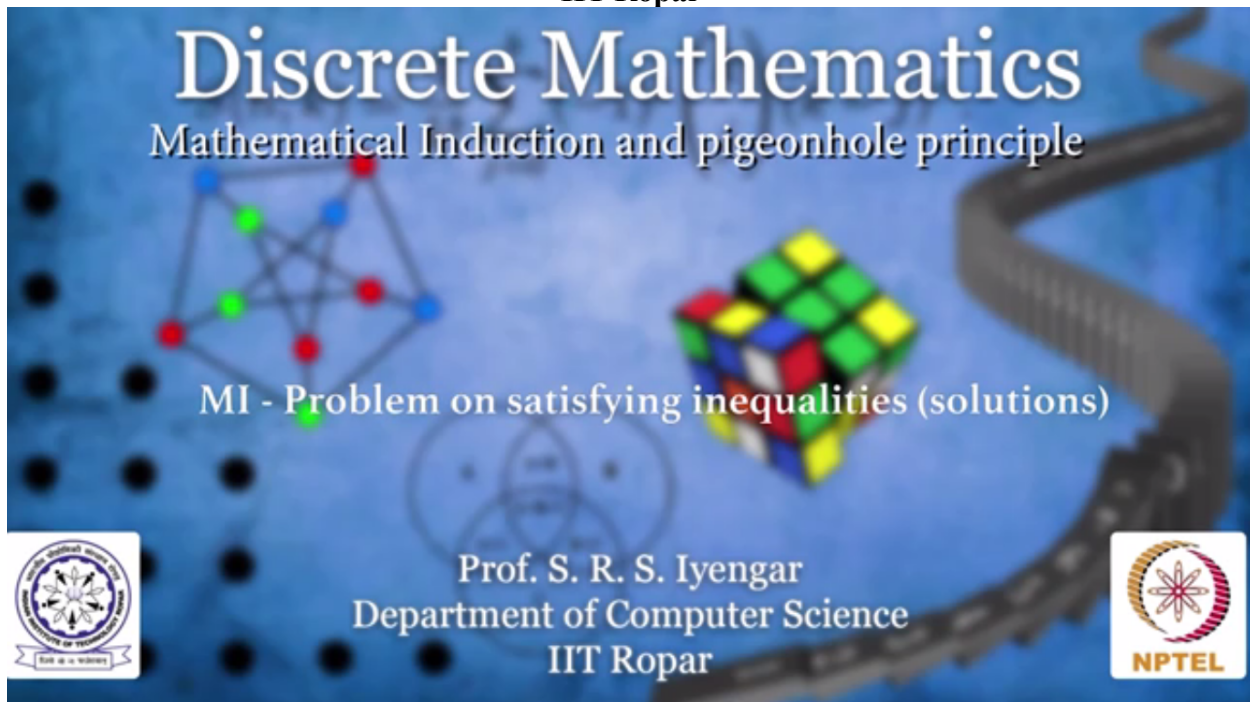**Discrete Mathematics**
Mathematical Induction and pigeonhole principle

MI - Problem on satisfying inequalities (solutions)

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So what did you conclude? I am sure you all know one thing that this chapter is about
Mathematical Induction and hence there should be an application of Mathematical Induction.
And indeed you are right. We can solve this using mathematical induction. How?

Let us induct on the number of inequalities. For n = 1, which is only one symbol, you'll have only two numbers: 1 and 2. Correct? So if you have 1 and 2, you put a less than symbol in between. If you have a 2 and 1, you put a greater than symbol in between and that's it. You can try it for 3 also, 1, 2, 3. Any possible 1, 2, 3 permutations, there are six in number. For any possible thing, you will have a less than, greater than combination, two symbols and you will be able to do it. Okay.

What did you conclude?

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Induct on the number of inequalities.

Basic step: n = 1

$$1 < 2 \qquad 2 > 1$$

Induction Hypothesis:
Assume it is true for m symbols.

So let's assume that this is true for m symbols. Okay. What I do is pretty straightforward. I will take these m symbols and then I have an extra symbol right now. What I will do is I will put 1 to 9 here like this and finally put a 10 at the end on the right provided the extra symbol is a less than symbol.
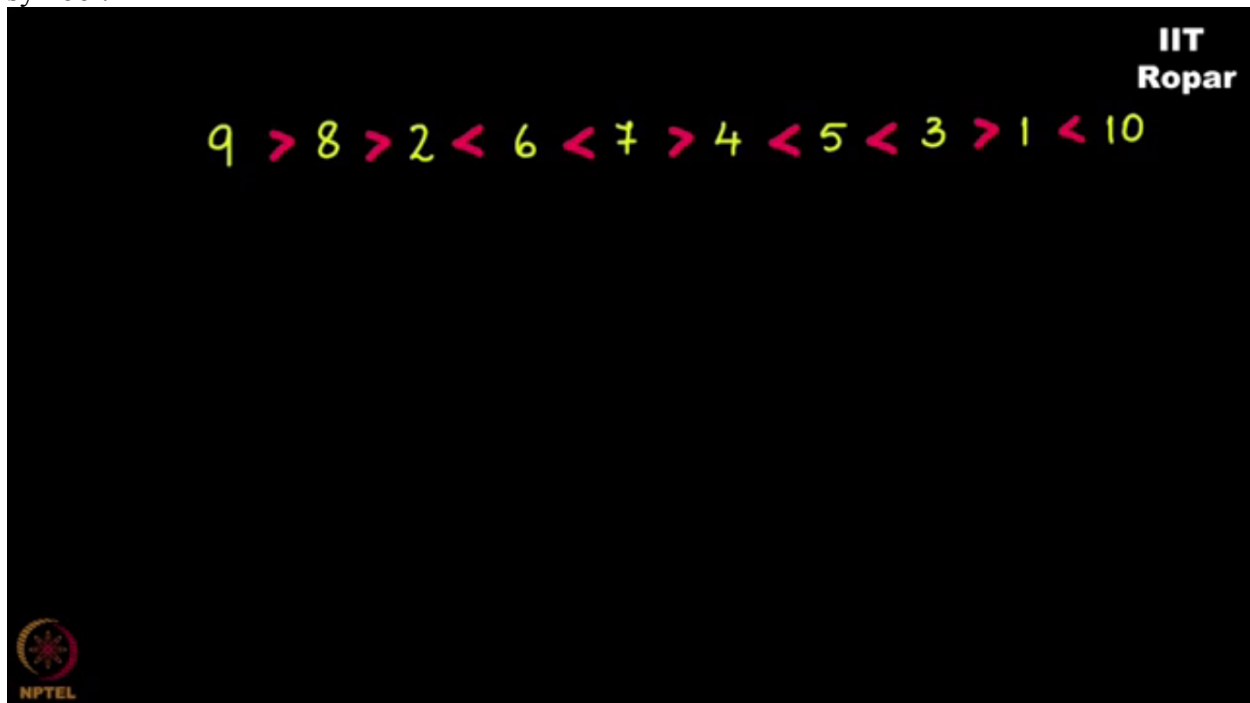
So if it is a greater than symbol, what I do is I put 2, 3, 4, 5, 6 up to 10 and then I put a 1 here. In either case, I can manage no matter what is the symbol. Correct? And thus by induction, I say that give me any sequence, any number of greater than, less than symbols. I can put consecutive 1 to n numbers there. That is if you give me n number of less than, greater than symbols, I can put 1, 2, 3 up to n+1 numbers here in some order such that it respects the given less than, greater than symbols.

$$9 > 8 > 2 < 6 < 7 > 4 < 5 < 3 > 1 < 10$$

$$9 > 7 > 4 < 5 < 6 > 3 < 8 < 10 > 2 > 1$$

For n number of symbols, we can put
1, 2, 3, ..., n+1 numbers in some order,
such that it respects the < > symbols.

I was slightly fast here, but it is because you people are now familiar with induction. I quickly applied induction here. I hope you were able be to see how induction plays its role here. If not, try watching the video once again. You will understand it very well.

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