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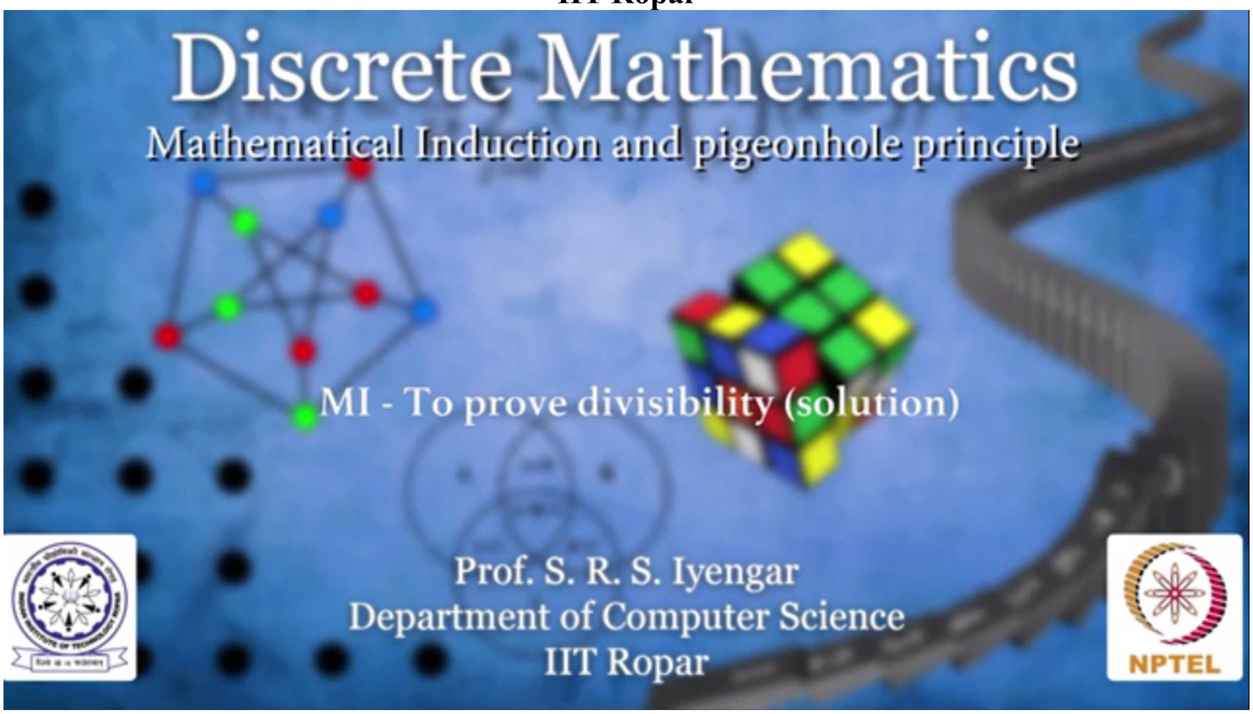
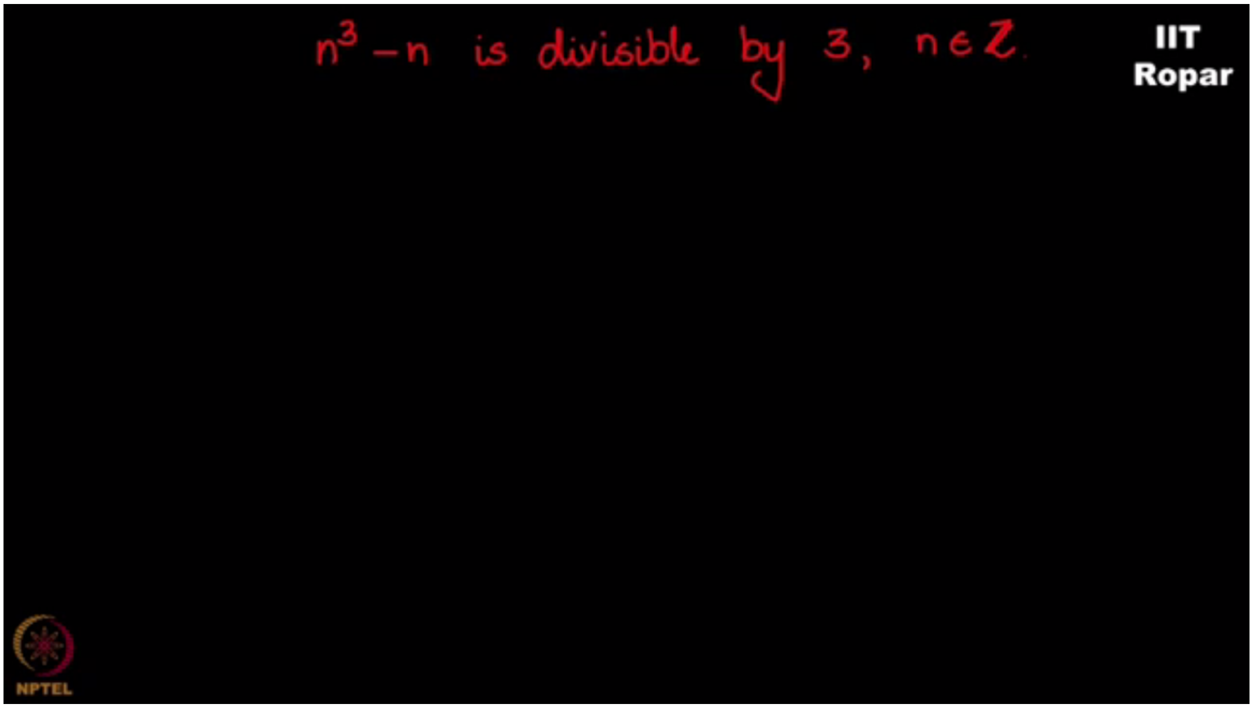
Discrete Mathematics
Mathematical Induction and pigeonhole principle

MI - To prove divisibility (solution)

Prof. S. R. S. Iyengar
Department of Computer Science
IIT Ropar

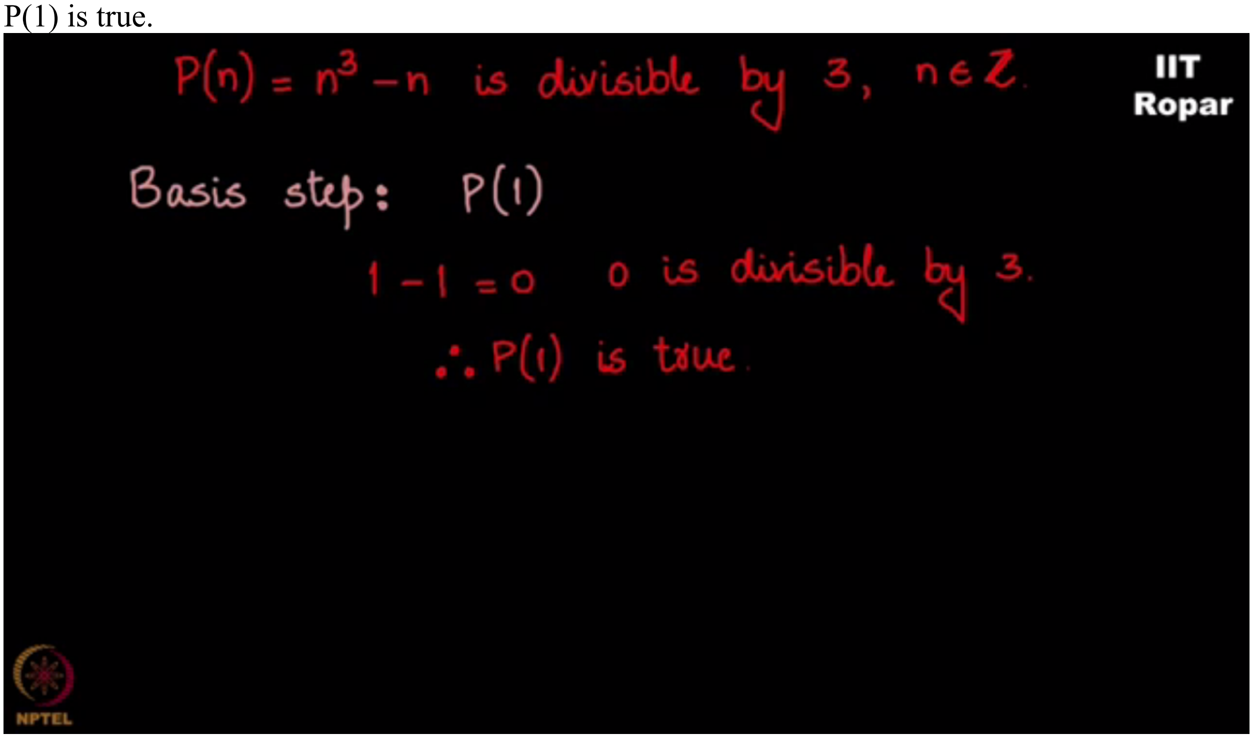


You observe the pattern. It was n^3-n and what was it divisible by? You must jump and tell me that n^3-n is divisible by 3 for any n belonging to the set of integers, right? So this is my statement and I am going to prove this using induction.

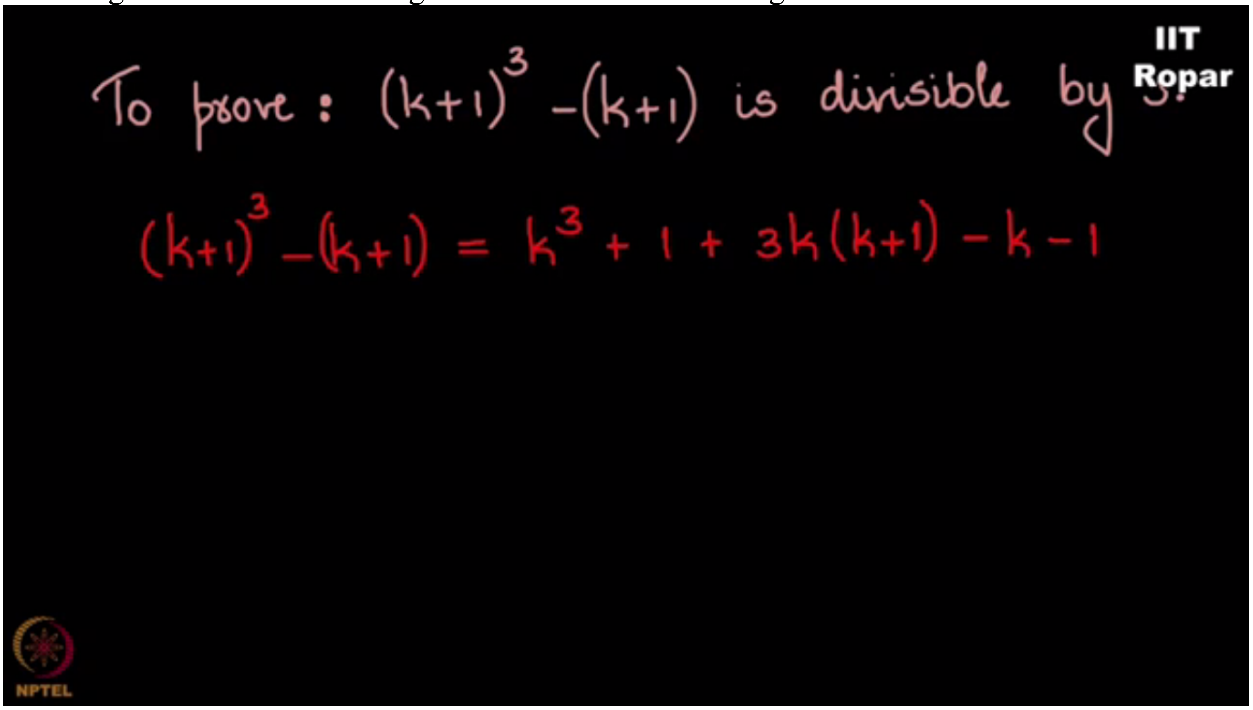


So P(n) goes like this: n^3-n is divisible by 3.

The basis step is P(1), 1-1, 1^3 is 1 you see. So 1-1 is 0. It is divisible by 3. For that matter, 0 is divisible by any number and we are particularly concerned about 3 here. So the basis step is true. P(1) is true.

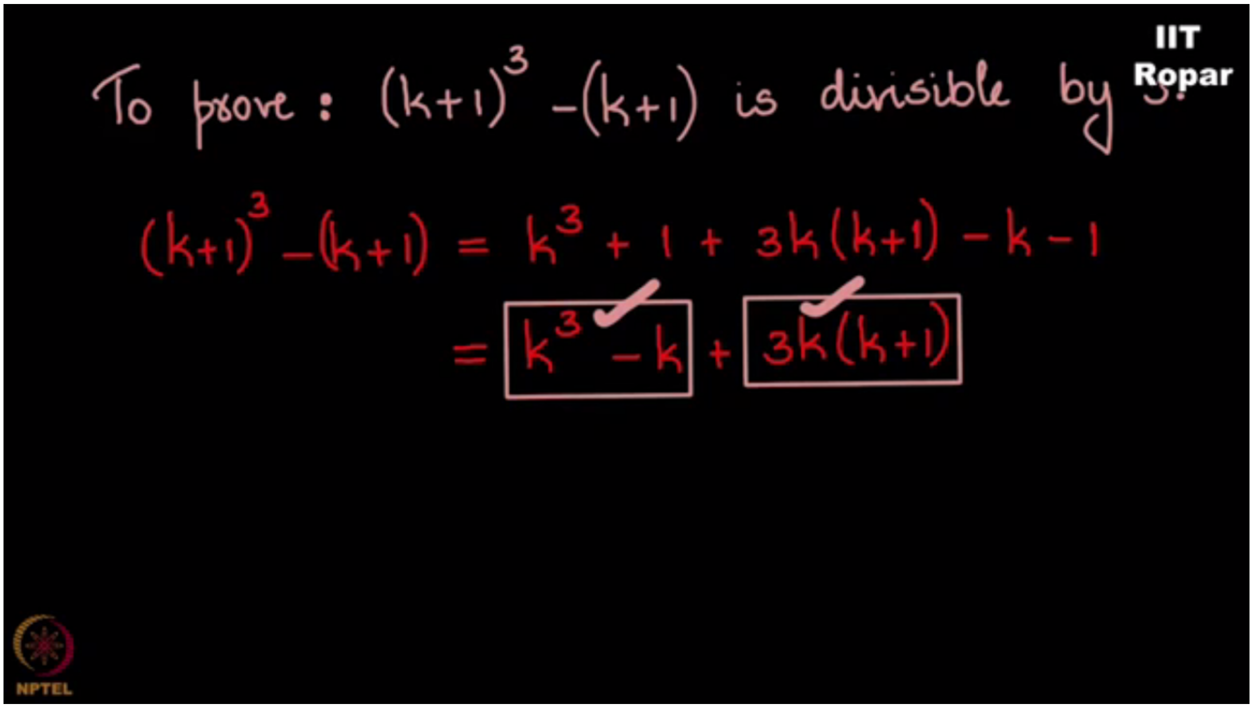


What is the Induction Hypothesis? I assume that k^3-k is divisible by 3 for some integer k. Now if I assume this and then if -- and then I have to prove that $(k+1)^3-(k+1)$ is divisible by 3 by assuming that it is true for k. Right? That is how we do using induction.



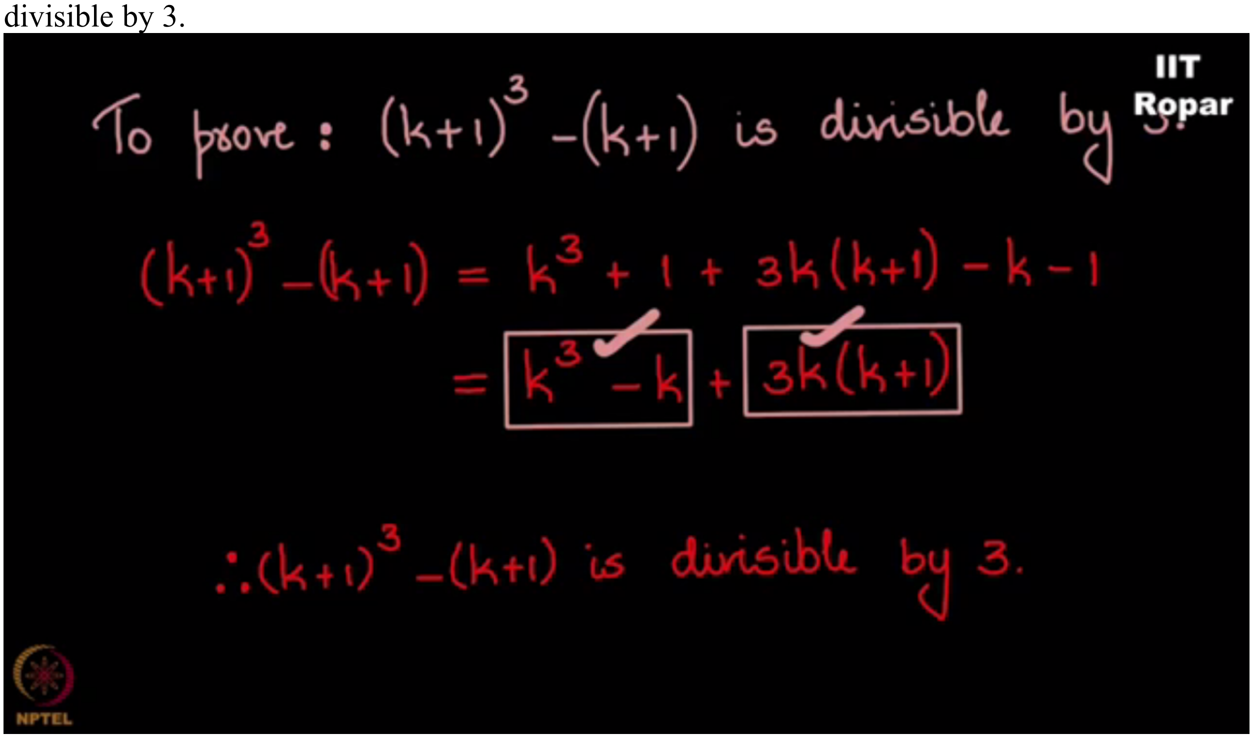
Now let me check if $(k+1)^3-(k+1)$ is divisible by 3. You see (k+1) is in the bracket. What did I do? I replaced k with (k+1) and hence it became $(k+1)^3-(k+1)$. $(k+1)^3$, you do it using $(a+b)^3$ identity and what it becomes? $k^3+1+3k(k+1)$. This is the expansion of $(k+1)^3$ minus I am going to expand it – I am going to expand the bracket and hence what happens? –k-1.

Now let me simplify this. +1, -1 get cancelled and hence what remains is $k^3-k+3k(k+1)$.



Observe this expression: k^3-k+3k(k+1). What do you see here?

What did we say in the hypothesis? We stated that k^3-k is divisible by 3. Hence, this portion in this expression is divisible by 3. This portion must be divisible by 3. Why? Because it is a multiple of 3. You see, it is 3 into k(k+1). If I write k(k+1) as some x, it is nothing but 3x and hence this portion is divisible by 3, and this portion is divisible by 3, and hence the sum must be divisible by 3.



So $(k^3-k)+3k(k+1)$ is divisible by 3 and hence $(k+1)^3-(k+1)$ is divisible by 3, but only when (k^3-k) is divisible by 3. Only if this is true can we say that this is true and hence the proof.

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