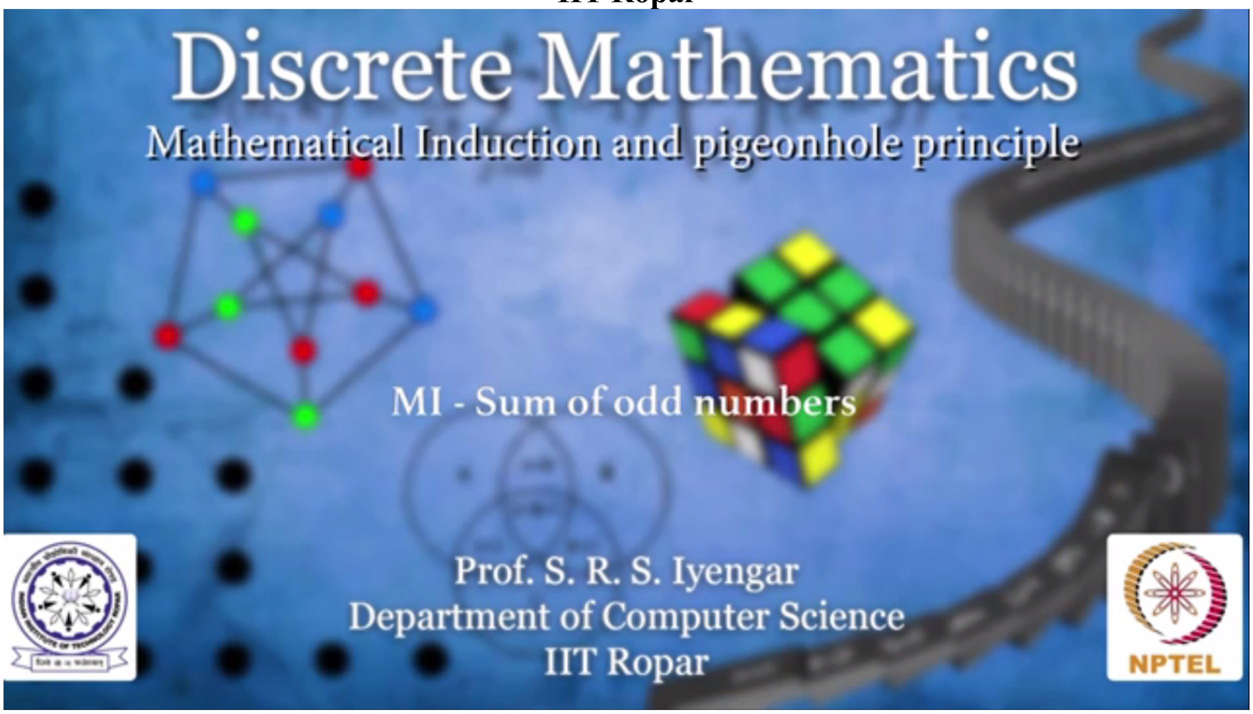
NPTEL
NPTEL ONLINE COURSE

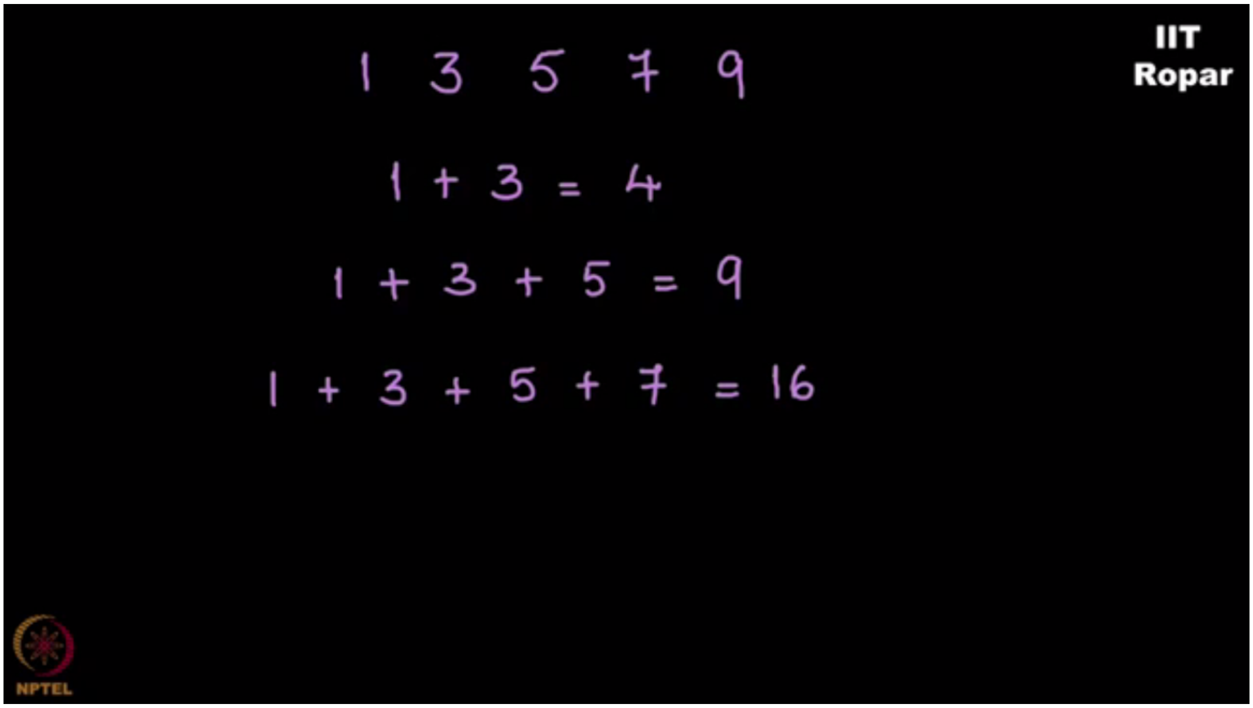
Discrete Mathematics
Mathematical Induction and pigeonhole principle

MI - Sum of odd numbers

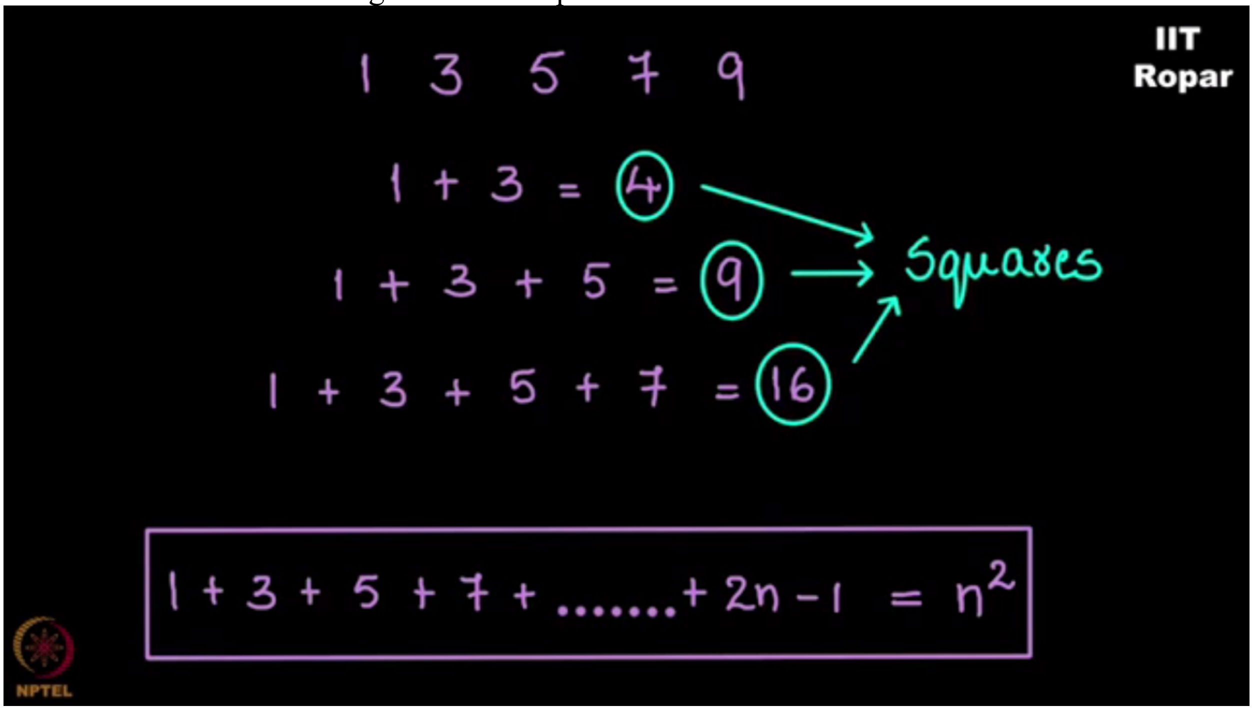
Prof. S. R. S. Iyengar
Department of Computer Science
IIT Ropar



Look at this question. 1, 3, 5, 7, 9 and so on. These are the first few odd numbers and look at this formula. 1 + 3 is giving me 4. 1 + 3 + 5 is giving me 9. 1 + 3 + 5 + 7 is giving me 16 and so on.

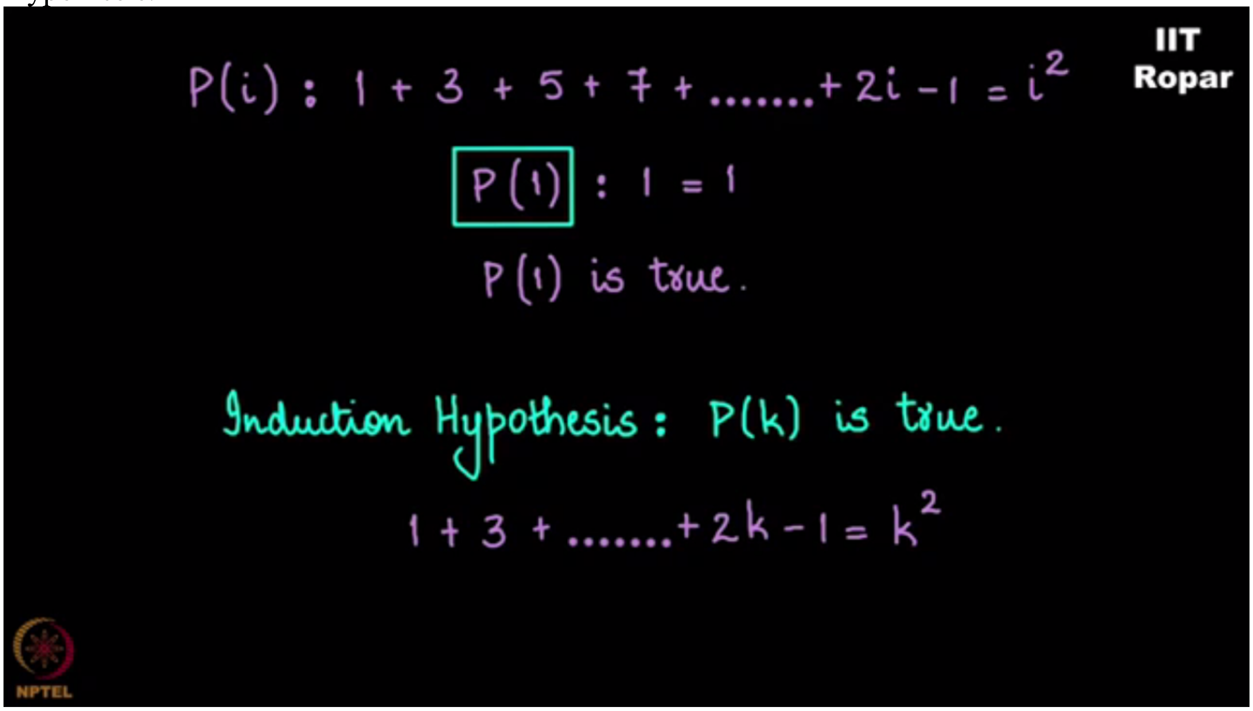


What do you observe here? Do you see that all of them are squares? Now how is this true? How is it true that when you add 1 + 3 + 5 + 7 up to let's say 2n-1 because it's an odd number you see, it is being equal to n^2 always. Always? Is it true that it's true always or was it coincidental for the first few numbers it is turning out to be a square?



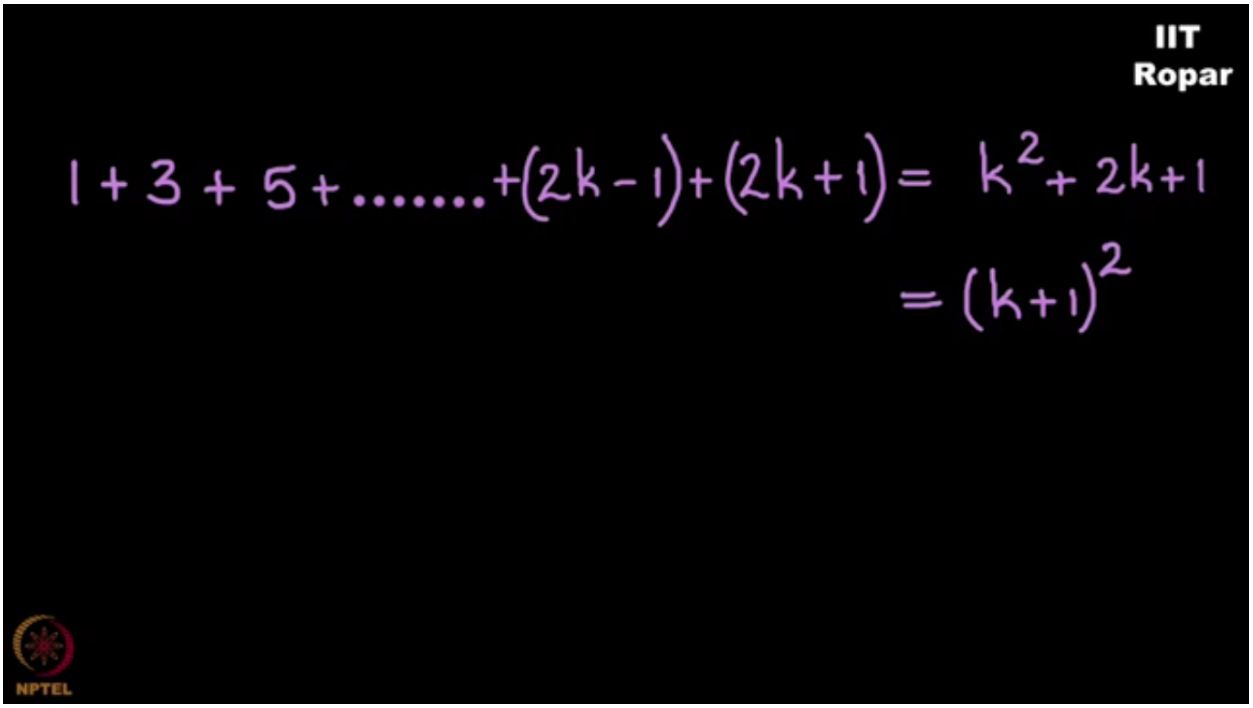
Let us investigate this patiently. We know this sharp instrument called induction. Do you think induction will come in handy for us to solve this? Let me try. So let me call P(i) to be the fact that 1 + 3 + 5 + 7 up to 2i-1 is equal to i^2. Right? So what is P(1)? P(1) is turning out to be 1 = 1. Obviously true, 1 is its square. Perfect.

Now assuming P(k) is true, this is called the Induction Hypothesis, don't worry much, know what – what are – what are the three main things in induction. This is the second main thing, first main thing being the basis step, second main thing being the inductive step, Induction Hypothesis.



P(k) is true means what? 1 + 3 + up to 2k-1 is equal to k^2. We assume this to be true. What do we mean by assume this to be true? It means if we assume it and show that it's true for k+1, we are done, right? That's what induction is all about. If you are confused, maybe you may want to watch the previous few videos.

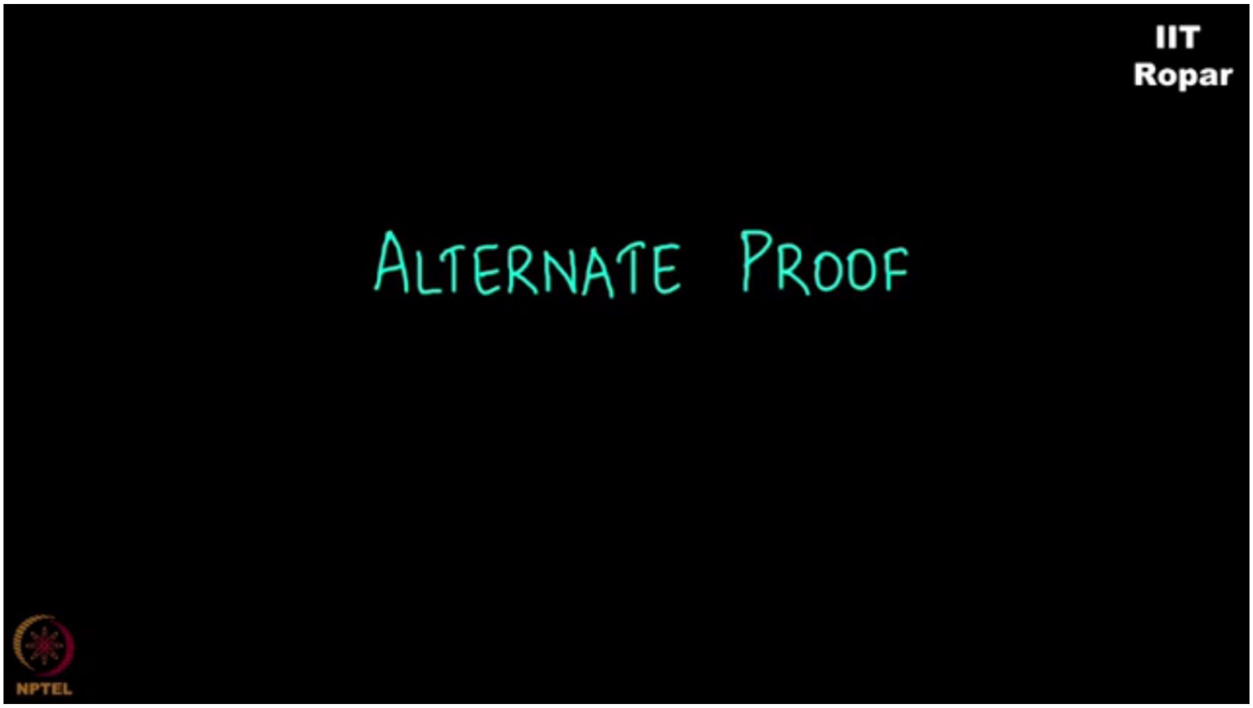
So 1 + 3 up to 2k-1 is k^2. We assume this. Is it true that it's true for k+1? That is when we go up to 2k+1, (2k-1)+(2k+1), will it be equal to $(k+1)^2$? We don't know. Let's check.



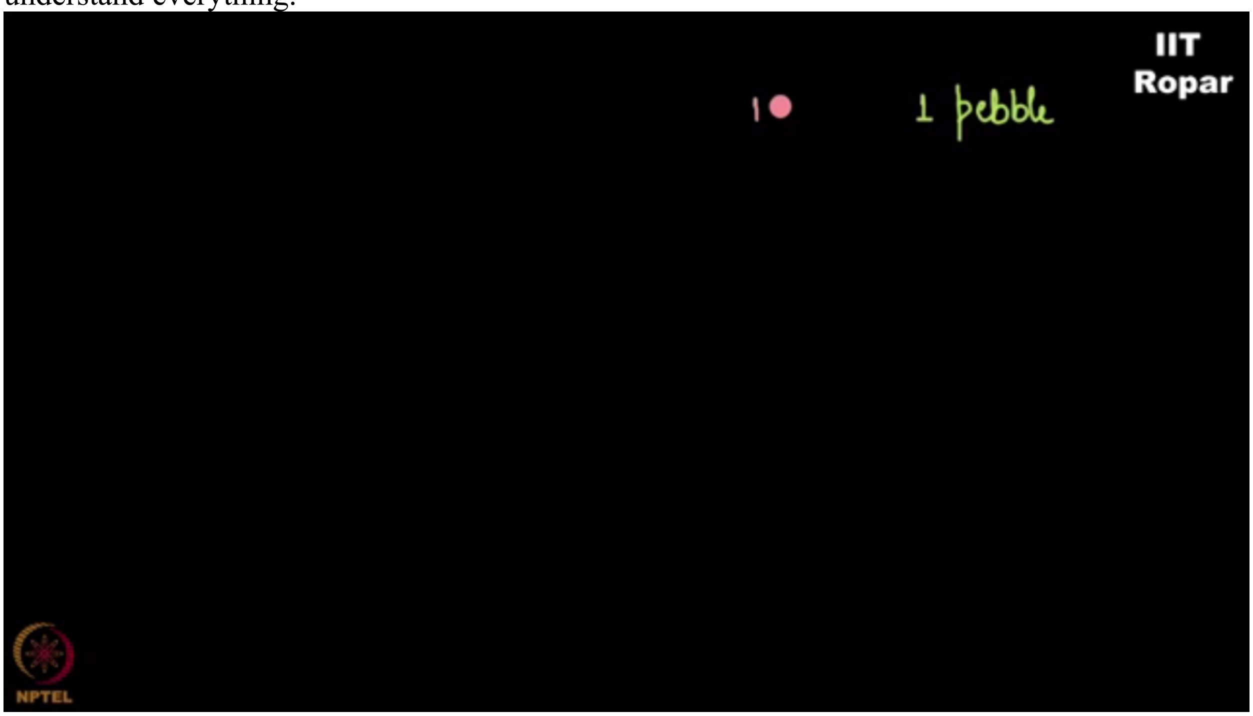
Now look at this. 1 + 3 + 5 up to (2k-1)+(2k+1) is k^2, induction hypothesis, +2k+1 and as you can see k^2+2k+1 is nothing else but $(k+1)^2$. Why? $(a+b)^2$. Expand this. You will get this. Hence for any n, this statement is true.

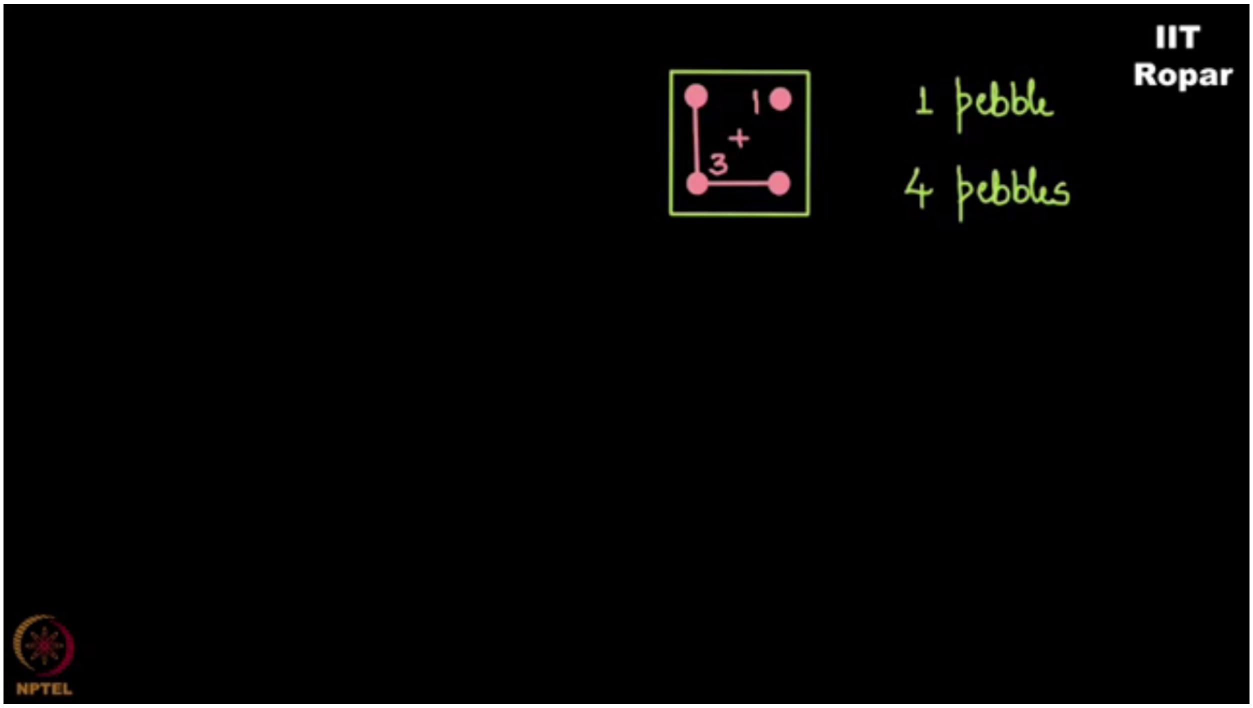
You don’t see the power of induction. You needn’t know what's happening, but you can show that something is true. Anyways, this is a very cute question. We are so tempted to give you another solution for this which doesn't involve induction.

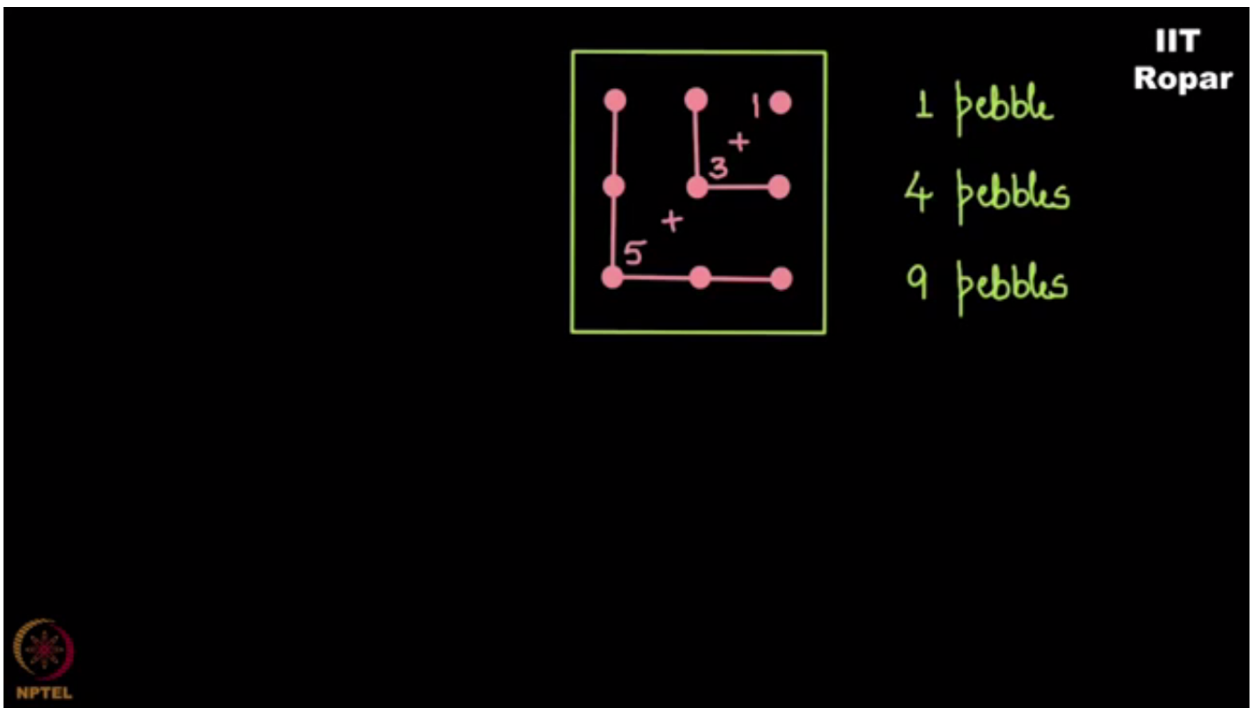
Yes, I agree that this is a chapter on induction and we shouldn’t be talking anything beyond induction, but it's too good that I am tempted to tell you an alternate proof for this, which is so elegant, just observe.

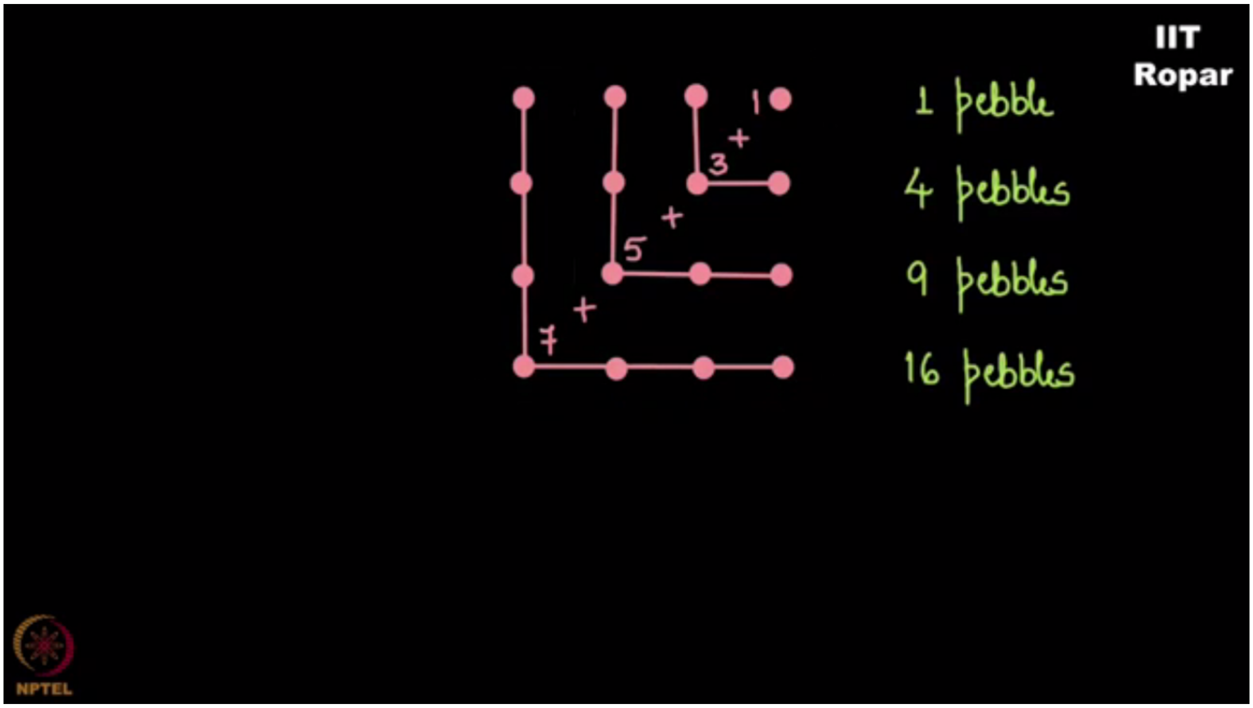


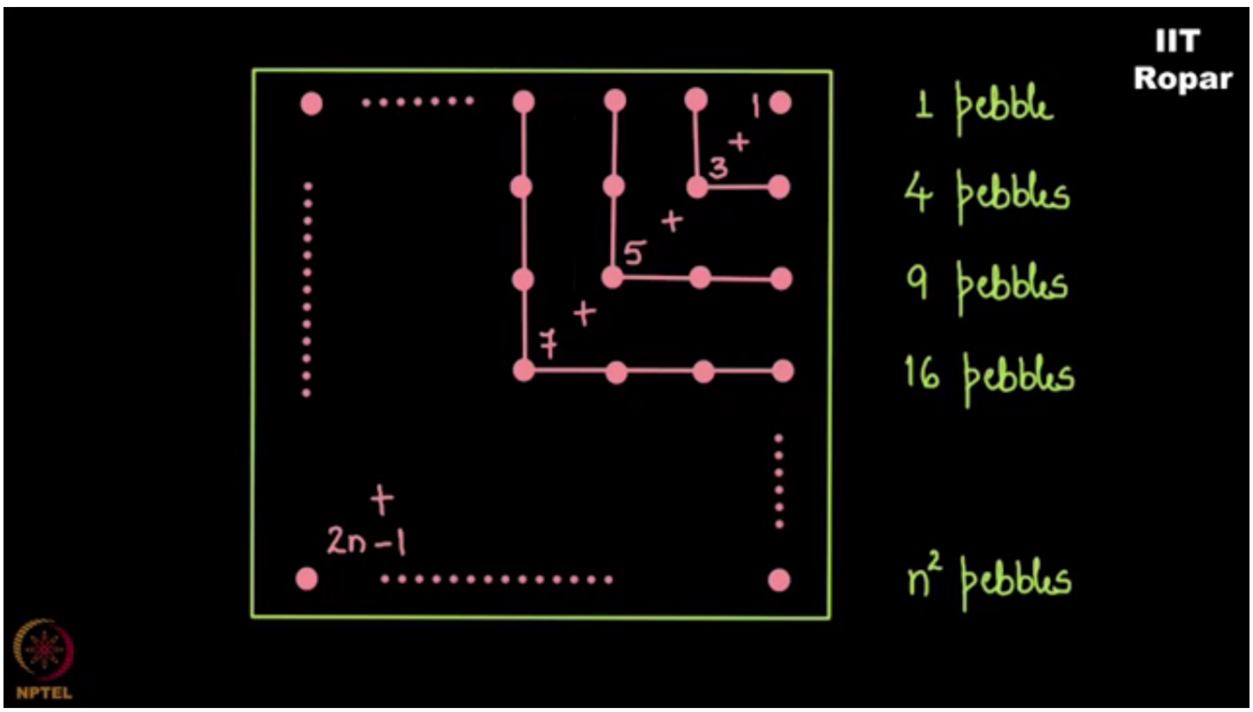
We are just going to animate and show you. I will not talk anything. Still you will be able to understand everything.




IIT Ropar
1
3
+
1 pebble
4 pebbles
NPTEL


IIT Ropar
1
3
+
5
+
1 pebble
4 pebbles
9 pebbles
NPTEL





IIT Madras Production

Founded by
Department of Higher Education
Ministry of Human Resource Development
Government of India