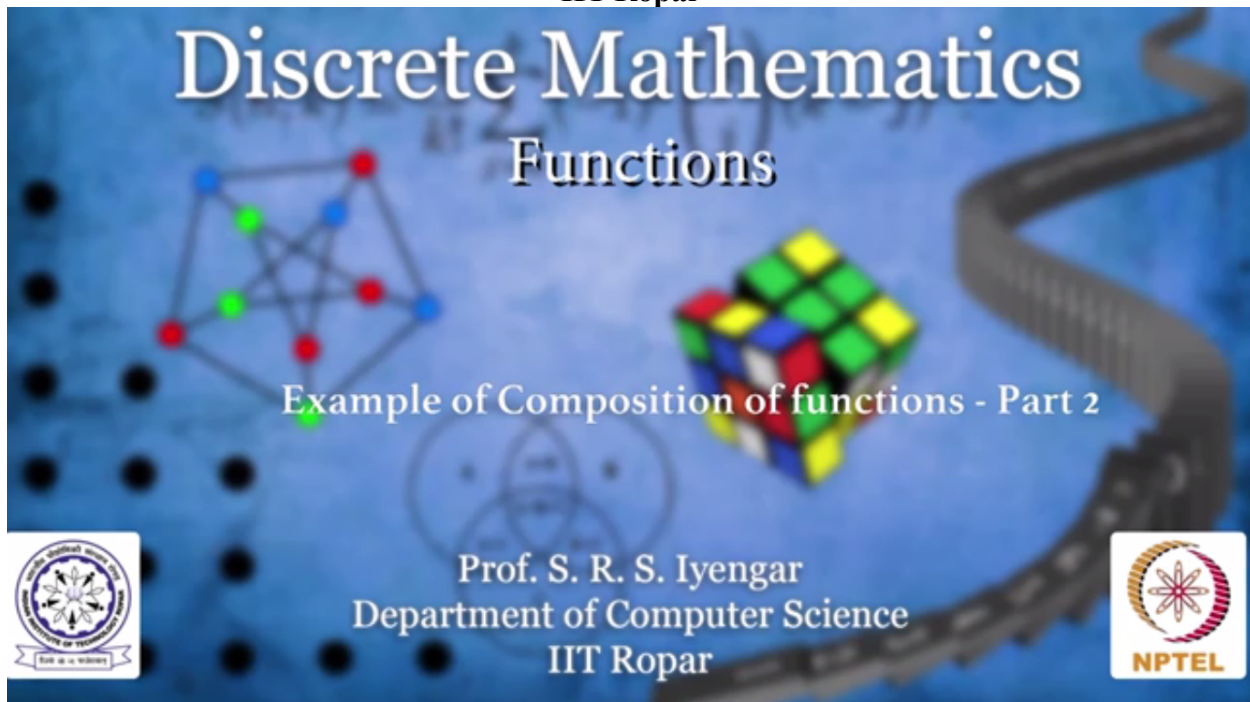


**NPTEL
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**Discrete Mathematics
Functions**

Example of Composition of functions - Part 2

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Let us quickly train our mind with more problems on composition of functions. Consider this function f from the set of all real numbers to real numbers defined as $f(x) = x^2$ and g from \mathbb{R} to \mathbb{Z} , real numbers to integers defined as $g(x) = \text{modulus of } x$.

Now what is f composition g ? f composition $g(x)$ is $f(g(x))$. This is how composition is defined as.

$$1. f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = x^2$$

$$g: \mathbb{R} \rightarrow \mathbb{Z} \quad g(x) = |x|$$

$$f \circ g ?$$

$$f \circ g(x) = f(g(x))$$



Now what is $g(x)$? It is modulus of x and $f(x)$ is x^2 and hence $f(g(x))$ happens to be f of modulus of x . And this is going to be my term now and hence f of modulus of x will be modulus of x the whole square. Correct? Yes.

$$1. f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = x^2$$

$$g: \mathbb{R} \rightarrow \mathbb{Z} \quad g(x) = |x|$$

$$f \circ g ?$$

$$f \circ g(x) = f(g(x)) = f(|x|) = (|x|)^2$$

$$Q: \text{Is } f \circ g \text{ same as } g \circ f ?$$



Now I will end this problem by asking a nice question. Is f composition g same as g composition f ? I leave it to you to try out. You may want to compute g composition f and try proving that both of them are not the same.

The last question. Consider this function f from integers to integers defined as $f(x) = x^2 + 1$ and the function g defined from again integers to integers as $g(x) = 3x$. Let us compute g composition f and f composition g , both of them.

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$$2. \quad f: \mathbb{Z} \rightarrow \mathbb{Z} \quad f(x) = x^2 + 1$$

$$g: \mathbb{Z} \rightarrow \mathbb{Z} \quad g(x) = 3x$$

$g \circ f$ and $f \circ g$?

$$g \circ f(x) = g(f(x)) = g(x^2 + 1) = 3(x^2 + 1)$$

$$f \circ g(x) = f(g(x)) = f(3x) = (3x)^2 + 1$$



g composition f is given to be g composition $f(x)$, which is $g(f(x))$, which is equal to g of what is $f(x)$? It is x^2+1 . Now if this is going to be my term in the domain of g , then what is g of this term? Let's say this is y . So what is $g(y)$ going to be? It is going to be $3y$. So y is x^2+1 and hence it is $3(x^2+1)$.

What is f composition g ? f composition $g(x)$ is equal to $f(g(x))$. $g(x)$ is $3x$. Now this is my y here. What is $f(y)$? $f(y)$ is x^2+1 . Okay. Let me say $f(z)$ because I have already used y . So $f(z)$ is z^2+1 and what is my z here. It is $3x$ actually. I can substitute $3x$ as z and hence f composition $g(x)$ is $(3x)^2 + 1$.

This is my answer and hence what did we observe? g composition f is $3(x^2+1)$ and f composition g is $(3x)^2 + 1$.

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