Discrete Mathematics

Functions

Cardinality condition in Bijection - Part 2

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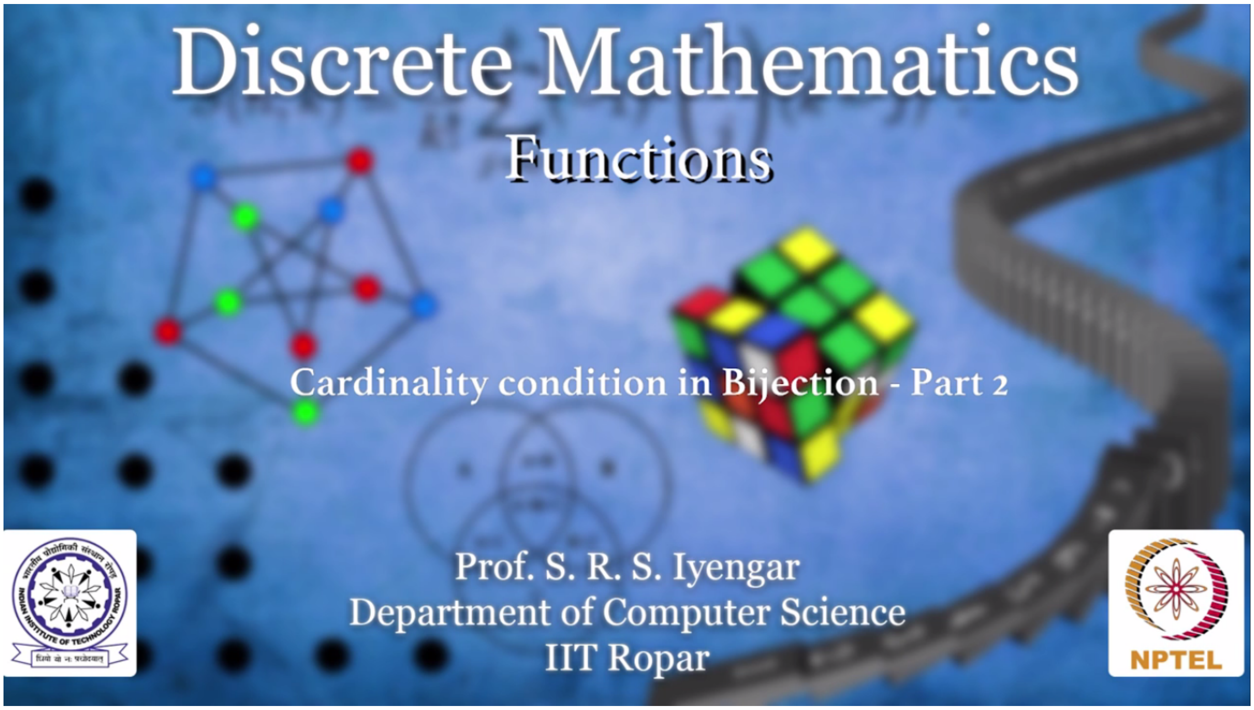
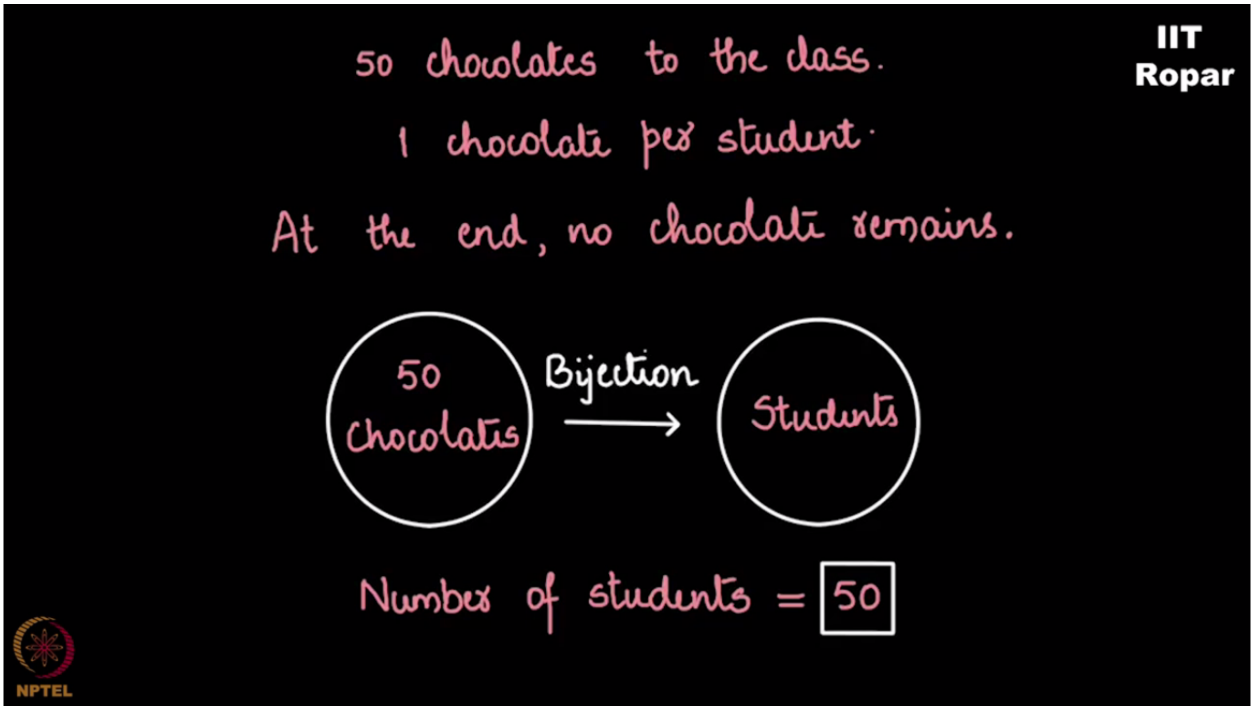


What can we say about the cardinality of a domain and a co-domain where a function f is given to be both one-one and onto? You see when a function is one-one from a domain D domain core domain C you know for sure that the number of elements here on the right wing is greater than or equal to the number of elements in the left wing which means number of elements in C is greater than or equal to number of elements in D given that f is one-one but I'm saying something else as well. I'm saying f is also onto. Now one-one means C is greater than or equal to D I mean cardinality of it right. And onto means no element in the co-domain is left out, okay, which means number of elements in co-domain is less than or equal to number of elements in domain. That's what onto stands for. So look at these two statements. They both put together gives us that the number of elements in C is in fact equal to the number of elements in D. Where did I start from where did I conclude? What did I conclude? Where did I start from? I started from the fact that wherever a function f is one-one and onto from a domain D to a co-domain C both with finitely many elements we call that finite cardinality. Then we conclude that they both should be of the same size.

Alright. So let me give you an intuition of what I am trying to say here. I'm trying to say that whenever you have two sets and you want to show the number of elements here is equal to number of elements here you can simply give a bijection. Bijection means both one-one and onto, a function give some function. Then you are proving that they're of the same cardinality. Conversely speaking if they're of the same cardinality you can actually give a function f which is both one-one and onto from this set to that set.

Let me list it a nice example here. I take fifty chocolates to my class. Distribute one chocolate each to every student in my class. By that I mean I give one chocolate to a student and every student gets a chocolate and at the end of this exercise no chocolate is empty. I repeat every student gets a chocolate, no chocolates are empty. I entered the class with 50 chocolates which means basically I'm creating bijection from 50 chocolates through the students here right. One goes to one and nothing is left empty. So I'm giving a bijection here which means the number of students in my class should be precisely 50.



So exhibiting a bijection comes in as a very handy tool to show the cardinality of a set sometimes you may not know the cardinality of a set. Sometimes you may know the cardinality of set S but you may know the cardinality of set T. In that case you exhibit a bijection from S to T and then you can conclude that given that you know the cardinality of T the cardinality of S is supposed to be the same.