Discrete Mathematics

Functions

Examples and Non- examples of One-One function

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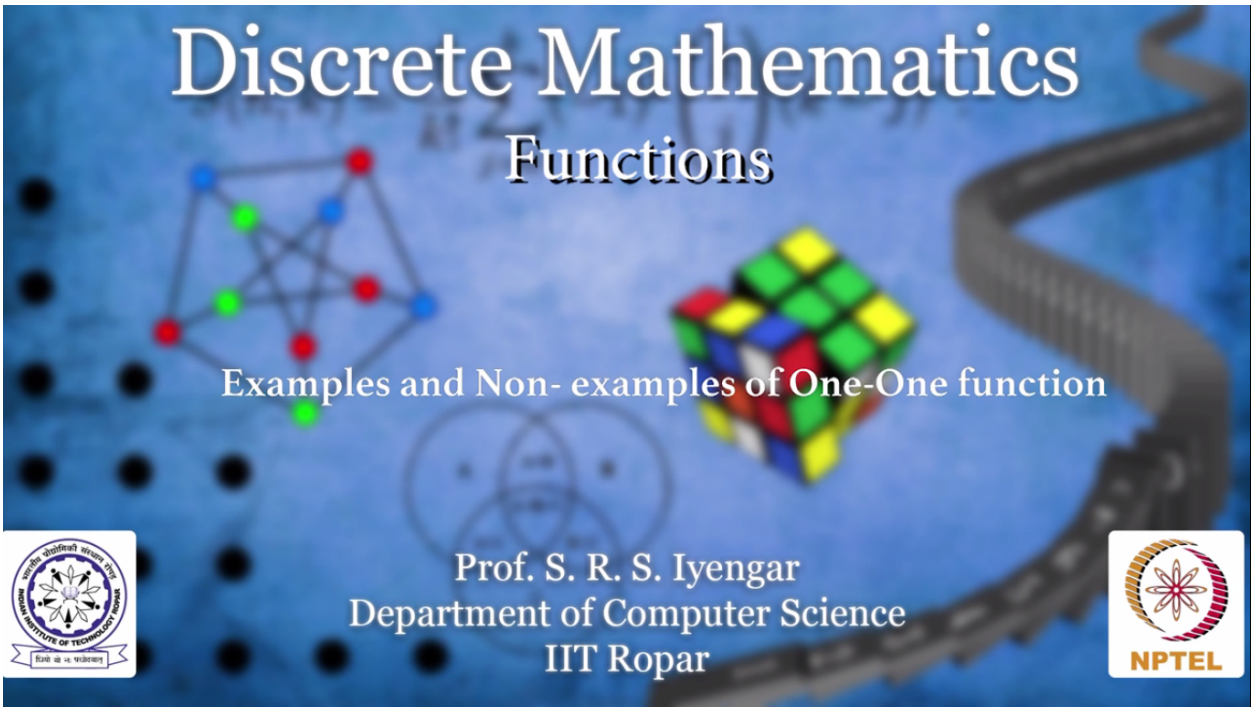
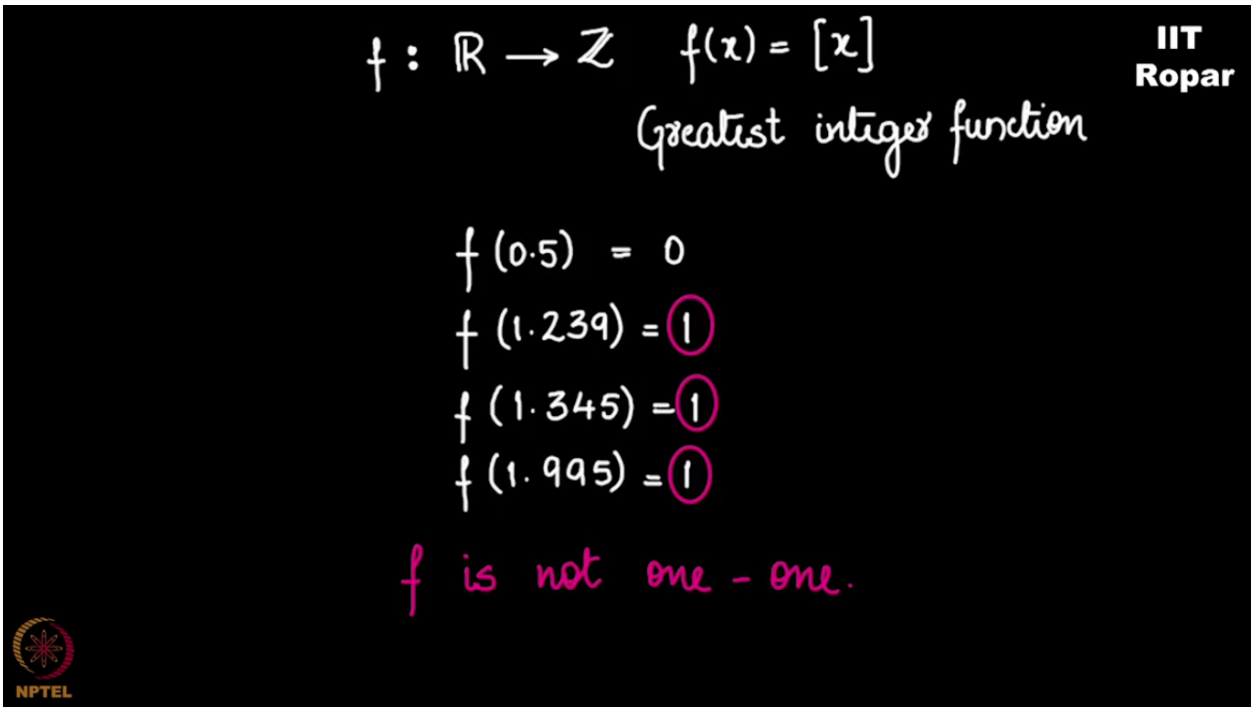


Here are some examples of one-one functions. Consider this function f from the set of all natural numbers to the set of all natural numbers defined as f of x is equal to 2x. 1 goes to 2. 2 is mapped to 4. 3 goes to 6. 4 goes to 8 and so on. There is no other number x belonging to the natural number which is mapped to 2, only 1 is mapped to 2.

Similarly only 2 is mapped to 4 but how do we prove that it's a one-one function. So let us check. Consider the two natural numbers x1 and x2. So f of x1 is equal to 2x1 and f of x2 is equal to 2x2. We have seen the proof technique right. I assume 2x1 is equal to 2x2. What do I get after canceling the twos? x1 is equal to x2 which means if in case 2x1 becomes equal to 2x2 then x1 is equal to x2 and hence f is always a one-one function.

The next example consider f from real numbers to real numbers defined as f of x is equal to 1-x. f of 0 is 1. f of 0.5 is 0.5. f of 2 is -1. f of 1 is 0 and so on. How do we prove this is a one-on function? Let x1 and x2 be two real numbers f of x1 is 1-x1 and f of x2 is 1-x2. I assume f of x1 is equal to f of x2 and hence 1 -x1 is equal to 1-x2 canceling 1 on both the sides give me x1 is equal to x2 which contradicts our assumption and hence f is one-one. So what do I assume every time? I assume that x1 and x2 are different but f of x1 and f of x2 are the same and hence I arrived at the contradiction.



The last example. Consider this function from real numbers to integers defined as f of x is equal to square bracket x which means it is the greatest integer function. What is the greatest integer function? What do I get after applying the function it is the greatest integer less than or equal to this x. So f of 0.5 is the greatest integer which is less than 0.5 is 0 and hence I take it as 0. f of 1.239 is 1. f of 1.345 is also 1. f of 1.995 is also 1 and so on. You observe here I am getting 1 thrice for every value between 1 & 2 I'll get it as 1. Hence f is not one-one. We need not prove since we have arrived at a counter example which says that f is not one-one.