NPTEL
NPTEL ONLINE COURSE
Discrete Mathematics
Relations

Partition - Part 6

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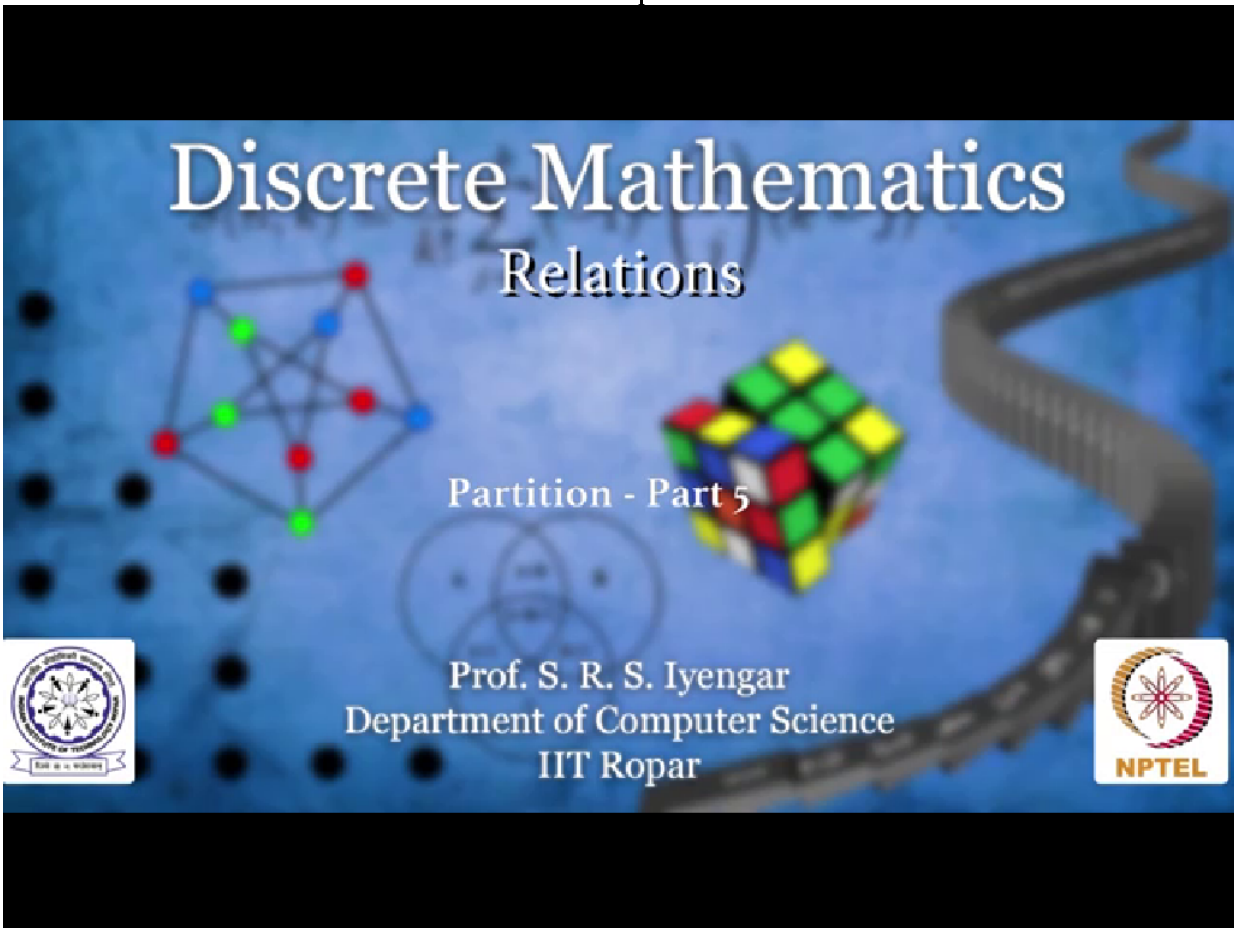
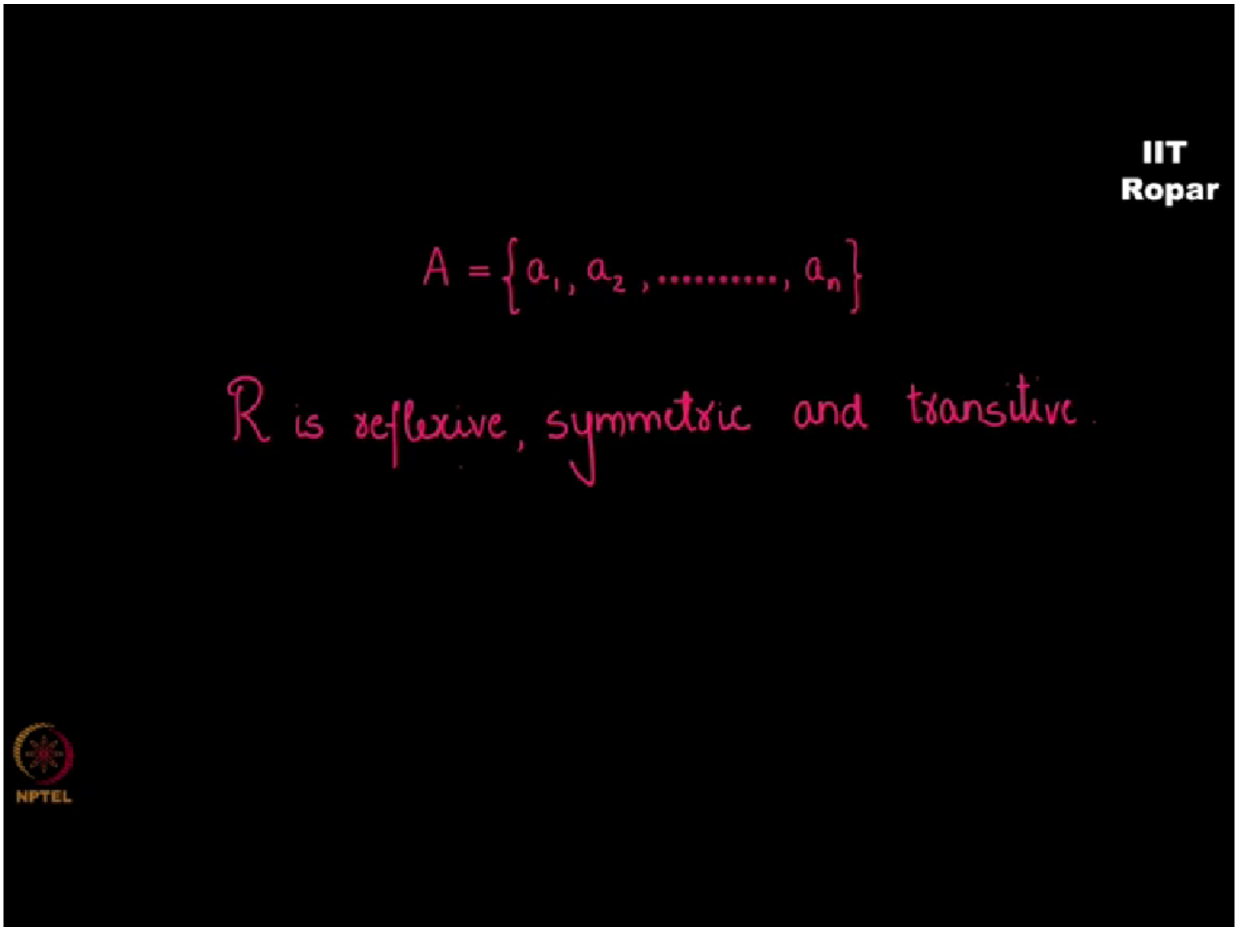


The example that we just illustrated simply has the following to say. Mathematically speaking, given a set A comprising of some n elements, let's say a_1, a_2 up to a_n, we are considering the set is finite, and then there is a relation R defined on it, and this relation R is known to be reflexive, symmetric and transitive. Okay.



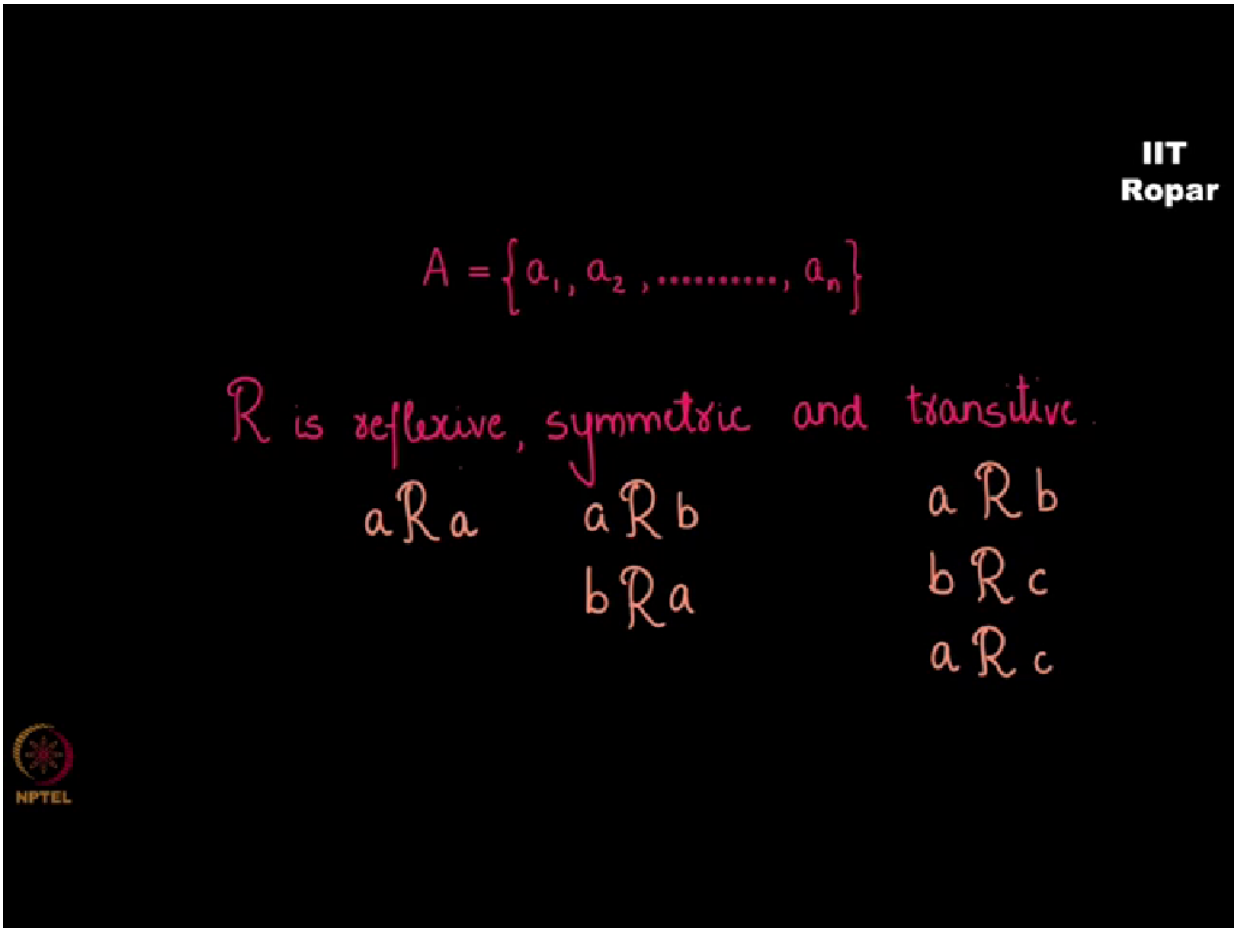
Then what does this relation do to the set? What do I even mean by this sentence, relation do to the set? I mean, it sounds like some complicated English usage, you know, what can a relation do to a set? You see it's a, I mean, there is nothing that a relation does to a set. You define a relation on a set, but by that I mean this relation classifies the set into type 1, type 2, type 3. Okay. So what do I mean by that?

You saw the recent example of friendships, right? Okay. There is a bunch of 100 friends gets classified into several types or rather what is called partitions. You partition these 100 friends into some disjoint sets. You see a friend either belongs to the set, a smaller cluster and knows everyone there or he is completely outside that cluster. Okay. So let's see mathematically what this means.

Given an equivalence relation R, which is known to be reflexive, symmetric, and transitive, let us see what happens to this set. You will even realize what I mean by what happens very soon. So it's reflexive, which means for every element, that element R a R b belongs to the relation. Correct? 'a' is related to 'b' is what I mean, right? Okay. I am sorry, a is related to a because it is reflexive. For any element a, a is related to a. Correct? We have been seeing that quite obvious.

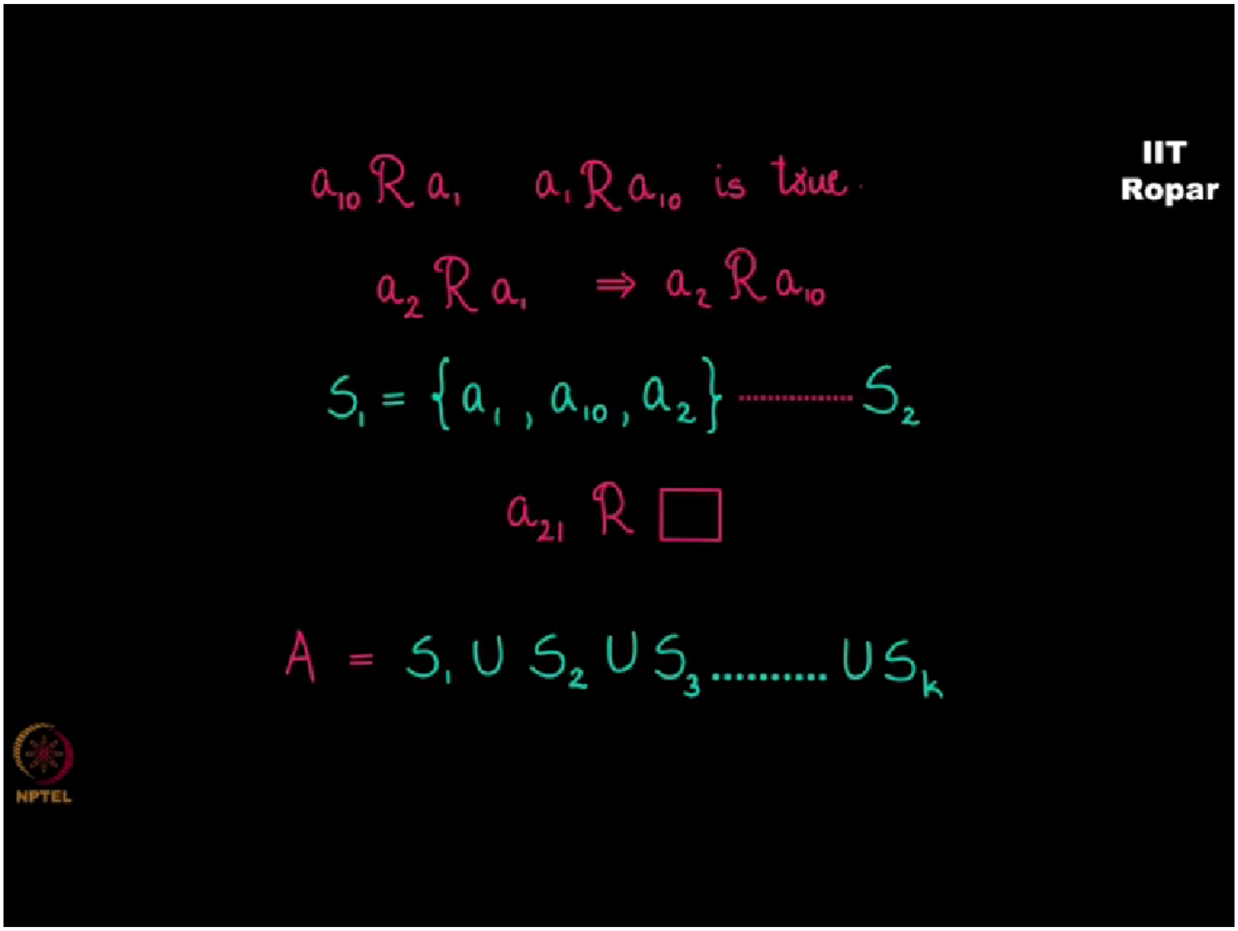
Second, symmetric. Whenever a is related to b, b is also related to a. Right? There are many such relationships. A is friends with b. Then b is also friends with a.

The last one is transitive, which means whenever a is related to b and b is related to c, a is related to c.



Now let me consider this a_1 here coming from a. Okay, a_1 belongs to A. Now just in case a_{10} and a_1 were related, let me write this as a_{10} R a_1, then I realize that a_1 R a_{10} is also true because the relation is given to be symmetric.

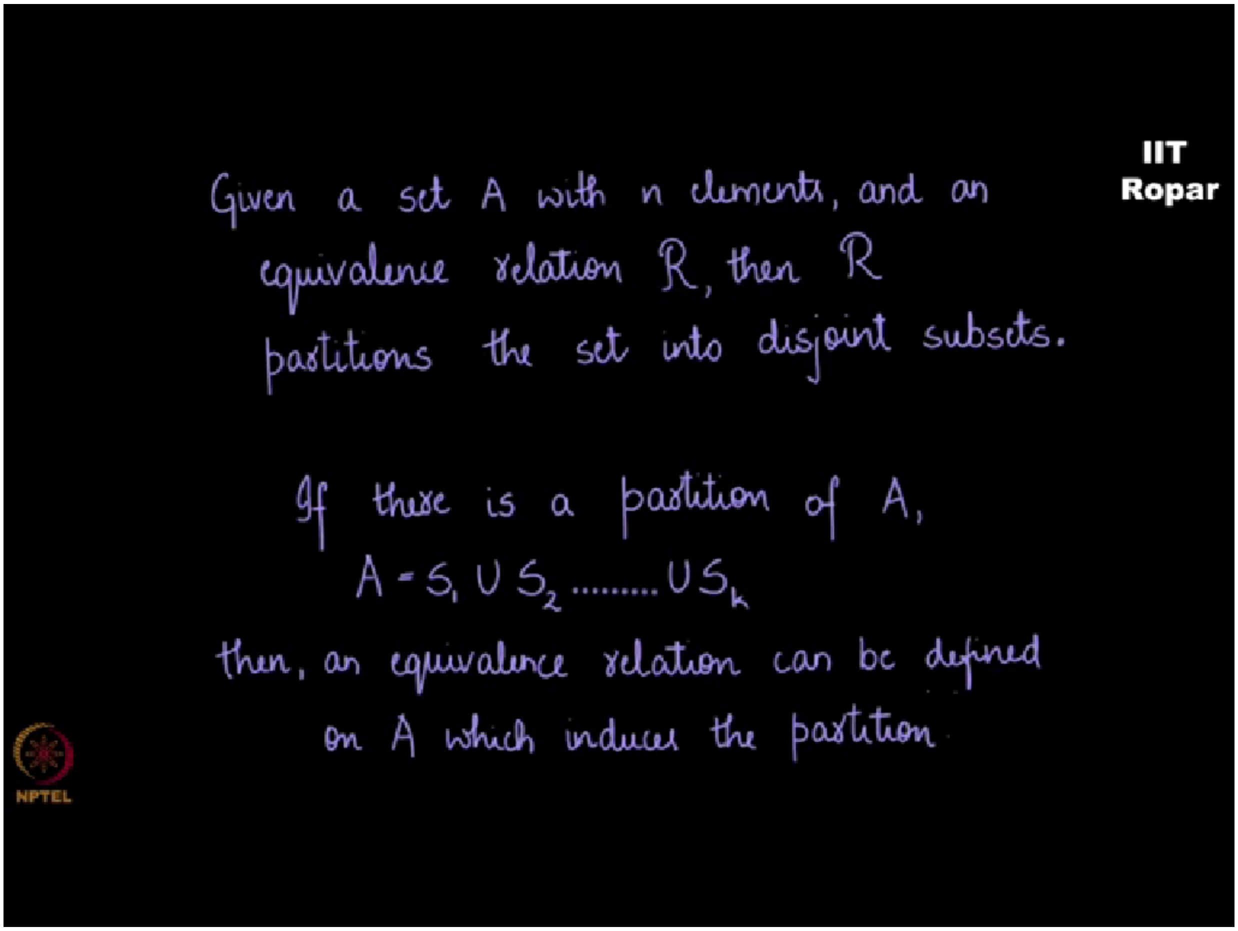
Now assuming a_2 is related to a_1, it implies that a_2 is related to a_{10} as well, right? So recollect the previous example. A friend's friend is a friend is all we mean by transitive relation. A relation R satisfying this, what it does is whenever a_1 is related to a_{10}, we will put a_1 and a_{10} into a set. Let us call this set S_1. Okay. When a_1 is related to let's say a_2, we will put that also inside and we say, every element here is related to each other because it's a transitive relation. Do we remember from the previous example that when a friend knows someone, when E knew D, E knew the whole of A, B, C in the previous example, right? So on and so forth you will get the set S_1 to be equal to some elements where everyone know each other.



And then take an element that is not inside S_1. Okay. Let's call it let's say some a_{21}, and see what a_{21} is related to, and put that element here, and keep going, and you will get a brand-new set. As you can see, none of the elements of S_1 will be related to none of the elements of S_2. Why? If in case any element of S_1 is related to an element of S_2, then the whole of S_2 will be related to that element in S_1 and S_1, S_2 will not be two different sets.

Please observe that these S_1 and S_2 are obviously subsets of A. Now as I keep doing this S_1, S_2, S_3 so on, I should stop somewhere because the set is finite. Set A is finite. So I will basically be writing A as S_1 U S_2 U S_3 up to let's say S_k where, where a small bouncer I had, although it's not that big a bouncer, you can understand it, but you will find it difficult to make sense of what I'm saying. I see that this S_1, S_2 intersection S_3 intersection S_4 and so on intersection S_k is empty. By that I mean not just is the case that all these sets don't have a common element. No two sets have anything in common. In fact, S_i intersection S_j is an empty set, right?

So what is it that we are saying? This here goes the theorem. Given a set A with let's say n elements and a relation R, which is known to be reflexive, symmetric, and transitive, we conclude that such a relation partitions the set into disjoint subsets.



So, as and always, whenever there is a property, a theorem, the proof looks a little complicated, but the idea will generally be simple. So I told you the simple version of what the theorem is. You can refer to any book, any standard textbook. The proof will go on the lines that I told you. Okay. So if you are confused probably you are confused about the notations. The idea is fairly simple. Whenever you have a set and you have a relation R that is an equivalence relation, this R partitions the set A into disjoint sets. Okay.

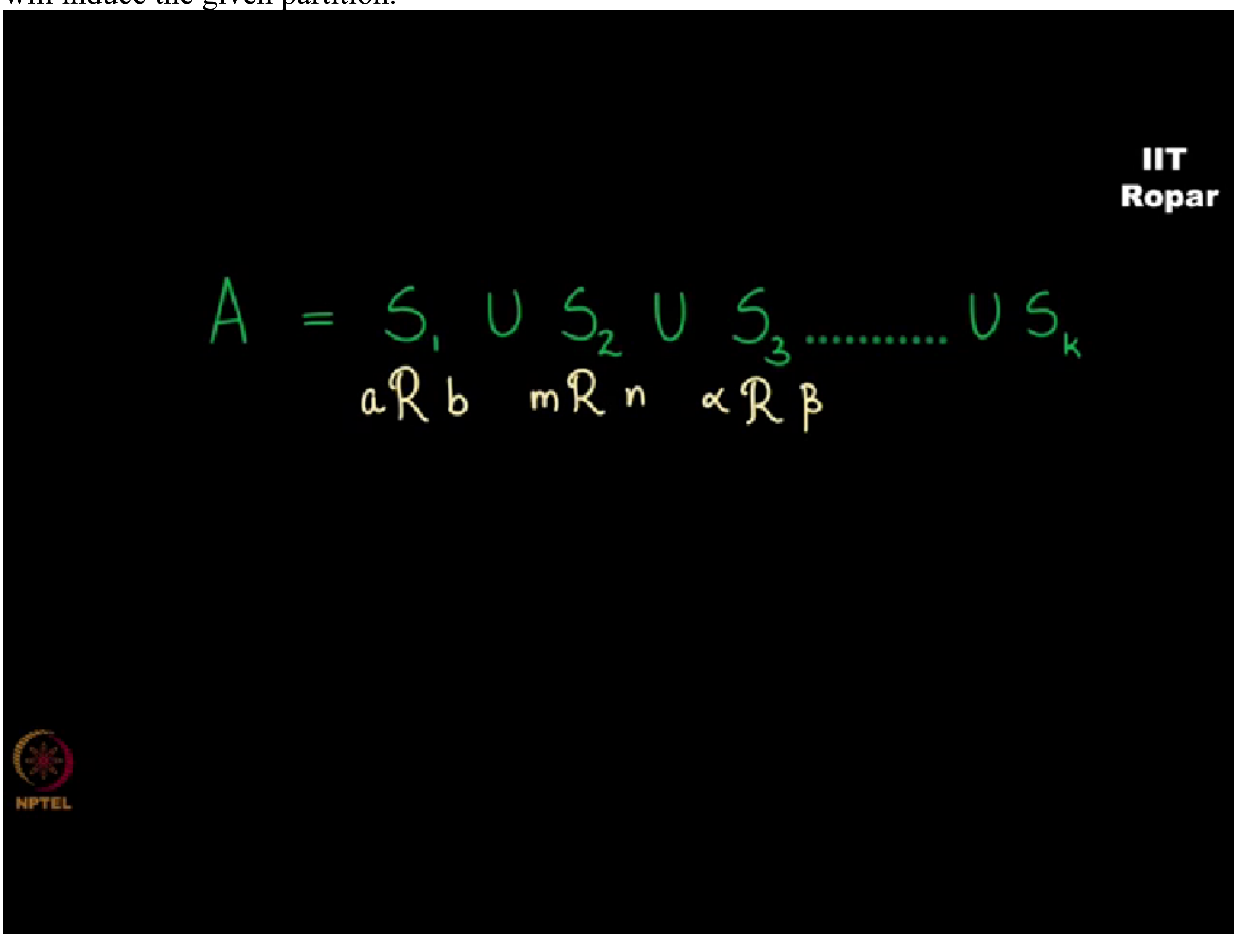
Now they say that converse is also true. What do we mean by that? By that we mean if you are given a partition of A as S_1 U S_2 up to let's say S_K, some partition of A, forget about the previous [indiscernible 00:06:48] that we wrote, that was in a different context, you simply take A. You classify A as S_1, S_2 up to S_k.

Let’s say, for example, you take all citizens of India and classify them based on the states that they belong to. Okay. A person belonging to Maharashtra, people belonging to Karnataka, people belonging to Tamil Nadu are all classified as these sets S_1, S_2 up to S_k. K is the total number of states in India. Okay.

Now the converse says what does the theorem state? If there is an equi valence relation, then A gets partitioned into subsets. One can write it as a partitioning. The relationship itself induces a partition. The converse would be if A can be written as disjoint union of some subsets, then you can define an equivalence relation on A such that that induces the given partition. What do I mean by that? Once you see what I am saying, what I am going to write next you will understand this statement.

The converse simply says, A is equal to S1, S2 up to Sk. You can actually define a relation R, which is reflexive, symmetric, and transitive and hence an equivalence relation, which induces this partition.

How do you do that? Very simple. First take S1 and define any two elements in S_1 as related. Go to S2 and define any two elements in S_2 as related. Okay. Go to S_3. Declare -- ensure that any two elements of S_3 are related. So if two elements alpha and beta are present in S_3, you say alpha is related to beta and beta is related to alpha. This is your relation and what you do is to make it reflexive, you for every element a in A, you say a is related to a. Okay. Such a relation will induce the given partition.



I will not spoon-feed any further. This is actually pretty straightforward. As and always, if it is -- if you're finding it difficult, the problem is with notations. Okay. Go through it a couple of times. You will find it very easy.

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