NPTEL
NPTEL ONLINE COURSE
Discrete Mathematics Relations
Equivalence Relation – Example 4
With
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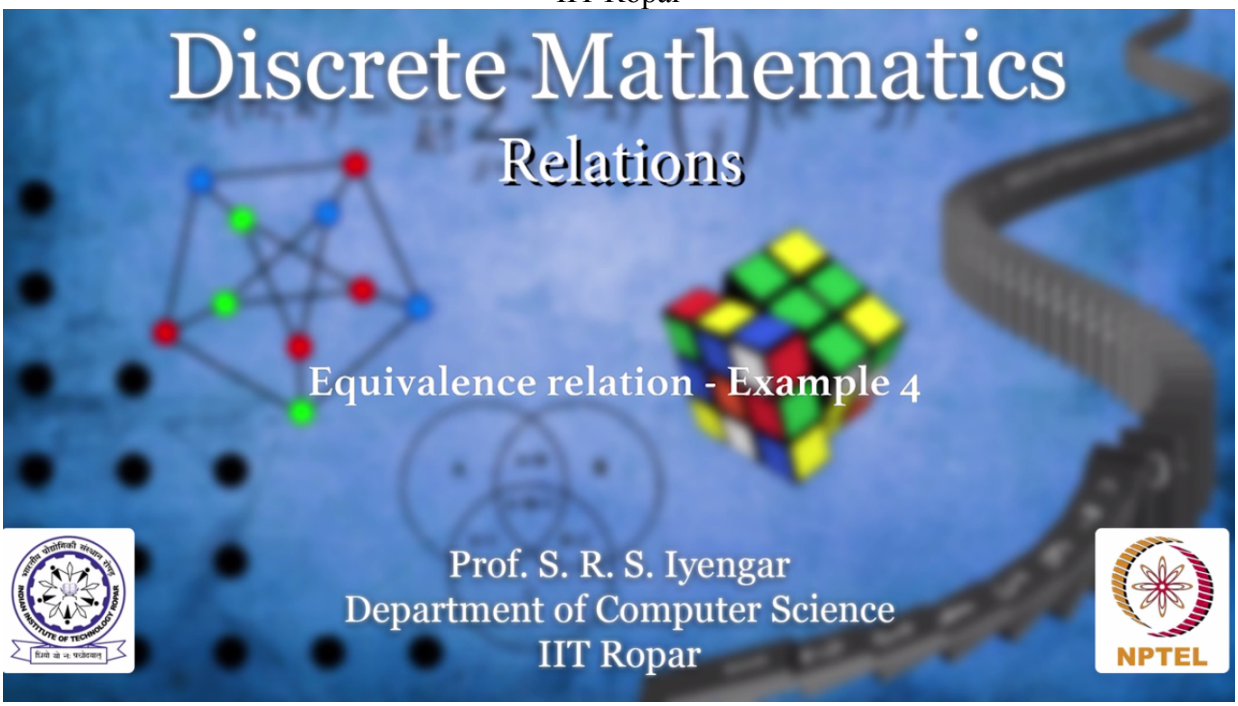
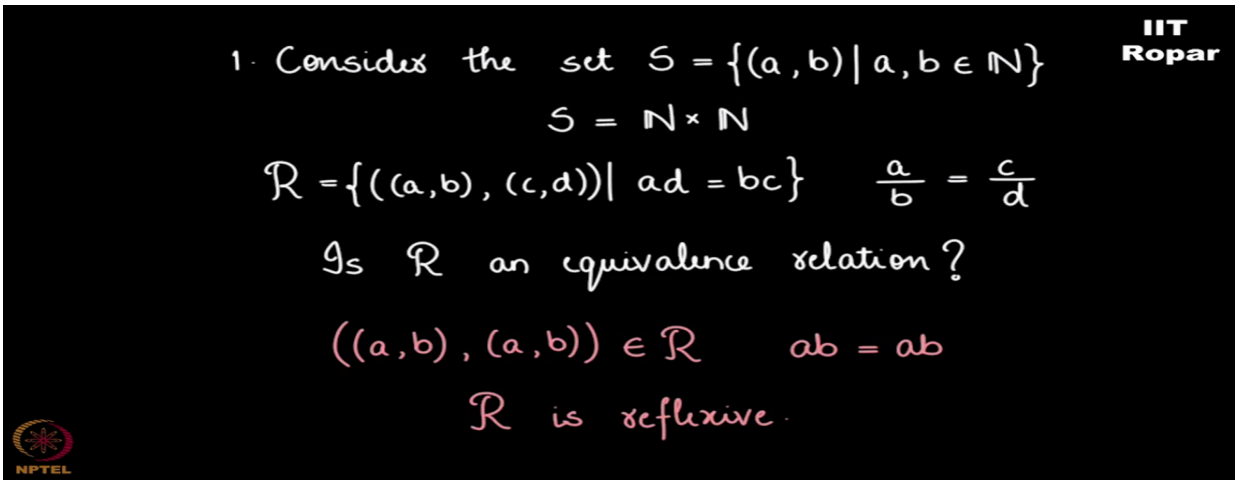
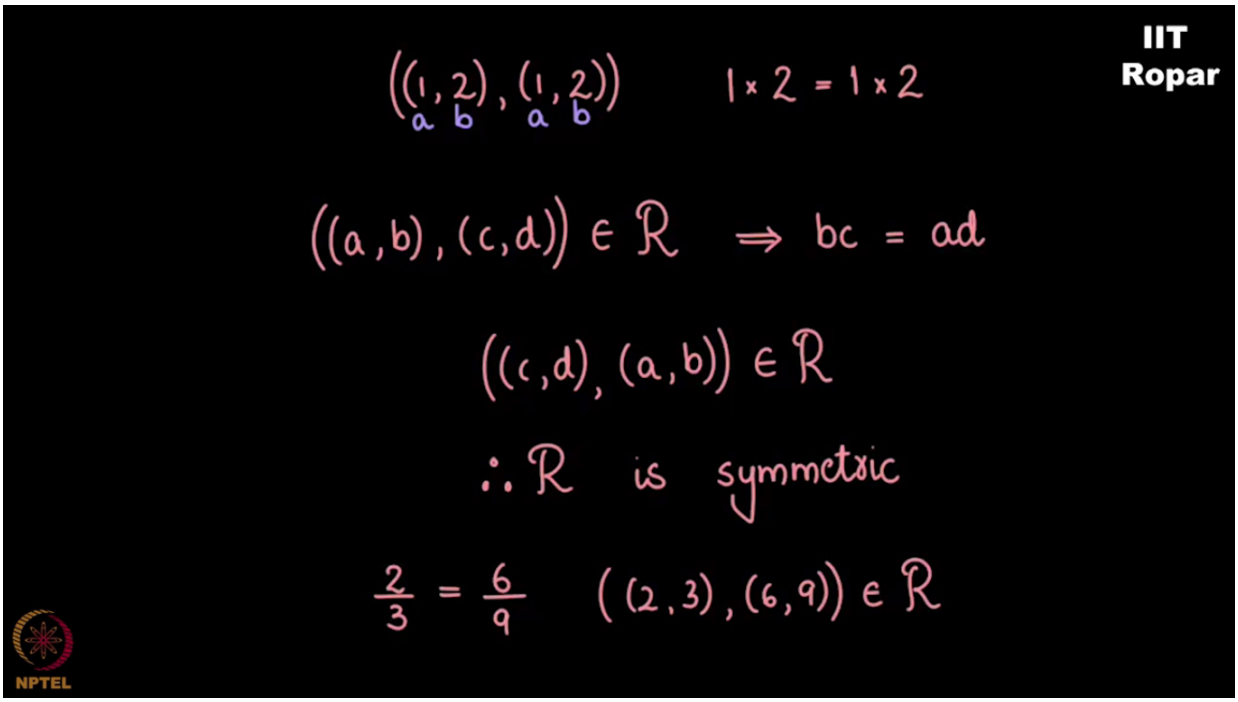


We are now going to see a nice example on equivalence relation. Consider the set S = (a, b) where a and b are natural numbers. So what is S actually? S is simply the cross product of N i.e. S = N x N.

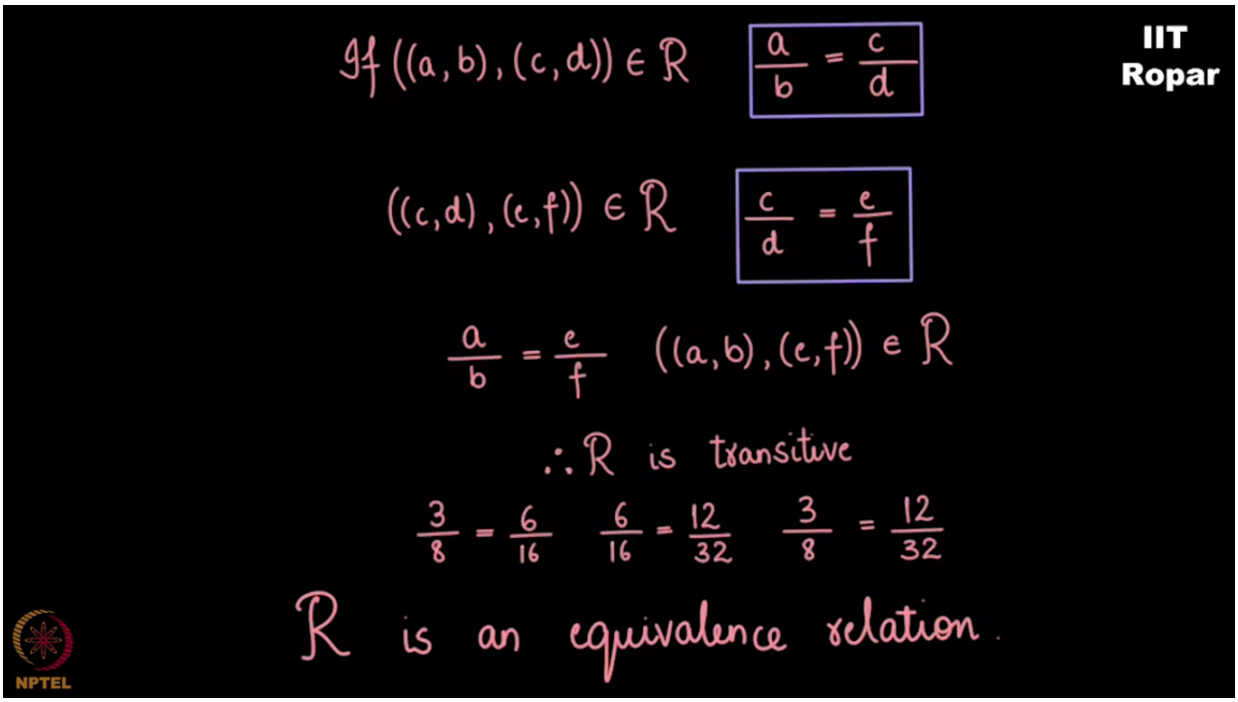
Now I'll define a relation on S like this. R = {((a, b), (c, d) | ad = bc}. Observe this relation very carefully. The elements here are (a, b) is considered to be one element, (c, d) is another element. ((a, b), (c, d)). And this should satisfy ad = bc or a/b = c/d. The ratio of a and b should be equal to the ratio of c and d. Pause here and think for a while.



Let us check if this relation is an equivalence relation. So we have this set R. So for the relation R to be reflexive, (a, a) must belong to R. Observe. ((a, b), (a, b)). I take this element. This must belong to R. We are considering (a, b) to be one element and hence, we have ab = ab according to the condition and hence, R is a reflexive relation.
Let us take an example. ((1, 2), (1, 2)), this is a, this is b, this is a and this is b again. So (a, b) = (a, b) and hence this belongs to R. This is true for every (a, b) and hence R is a reflexive relation.



Now is R symmetric? Let us assume ((a, b), (c, d)) belongs to R. ((a, b), (c, d)) belongs to R, this implies ad = bc. But don't you think it is the same as bc = ad. Think for a moment, but what does this imply.
It implies ((c, d), (a, b)) belongs to R according to our condition and hence R is symmetric.
Let me take an example. 2/3 = 6/9 and hence, ((2, 3), (6, 9)) belongs to R which also implies ((6, 9), (2, 3)) belongs to R.
Well, this must be clear now. The last property to check is transitivity. If ((a, b), (c, d)) belongs to R and ((c, d), (e, f)) belongs to R, does it imply ((a, b), (a, f)) belongs to R? Let us check. So if ((a, b), (c, d)) belongs to R, it implies a/b = c/d according to the condition. Now I am also telling that ((c, d), (e, f)) belongs to R. what does this imply? This implies c/d = e/f. I hope this step is clear.



Now look at these two comparisons. This implies a/b = e/f and hence, ((a, b), (e, f)) belongs to the relation. Hence, R is transitive. Let us take a quick example. We know that 3/8 = 6/16 and 6/16 = 12/32 and hence 3/8 = 12/32.

So we saw that this relation is reflexive, symmetric and transitive and hence and an equivalence relation.

Pause the video at each step and make sure that you've understood every step clearly.

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