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NPTEL ONLINE COURSE
Discrete Mathematics Relations
Examples of Transitive and Antisymmetric Relation
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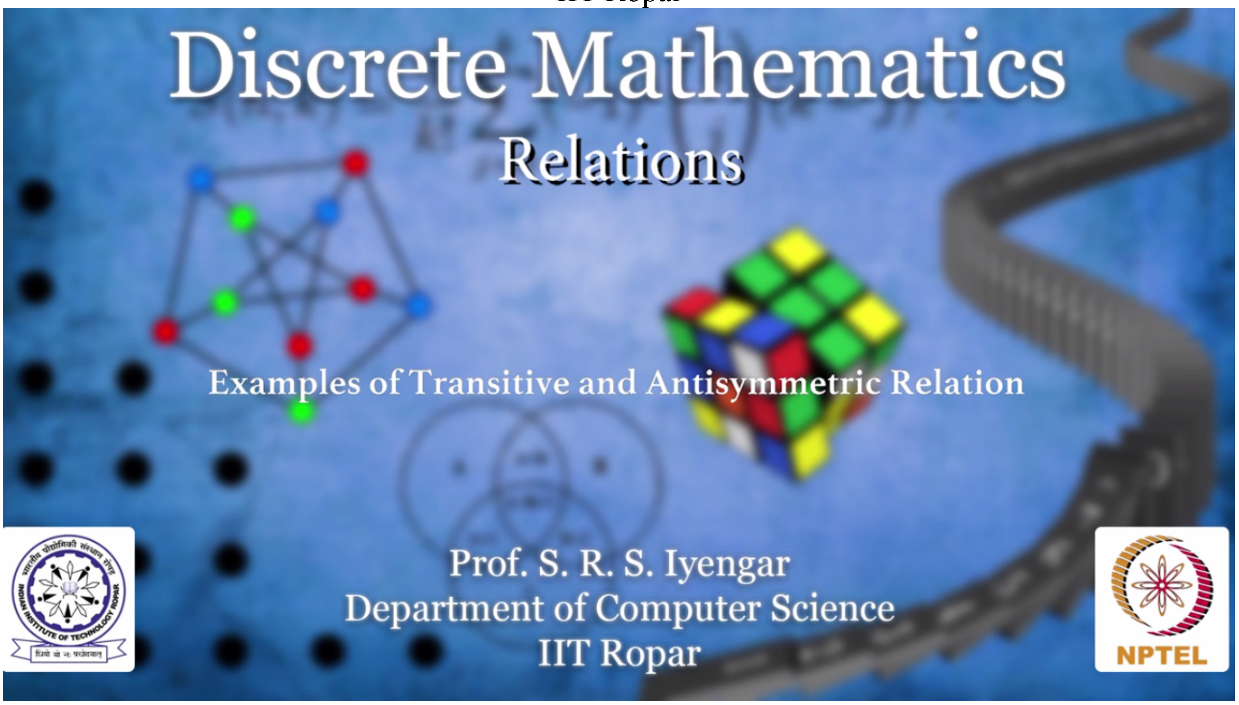
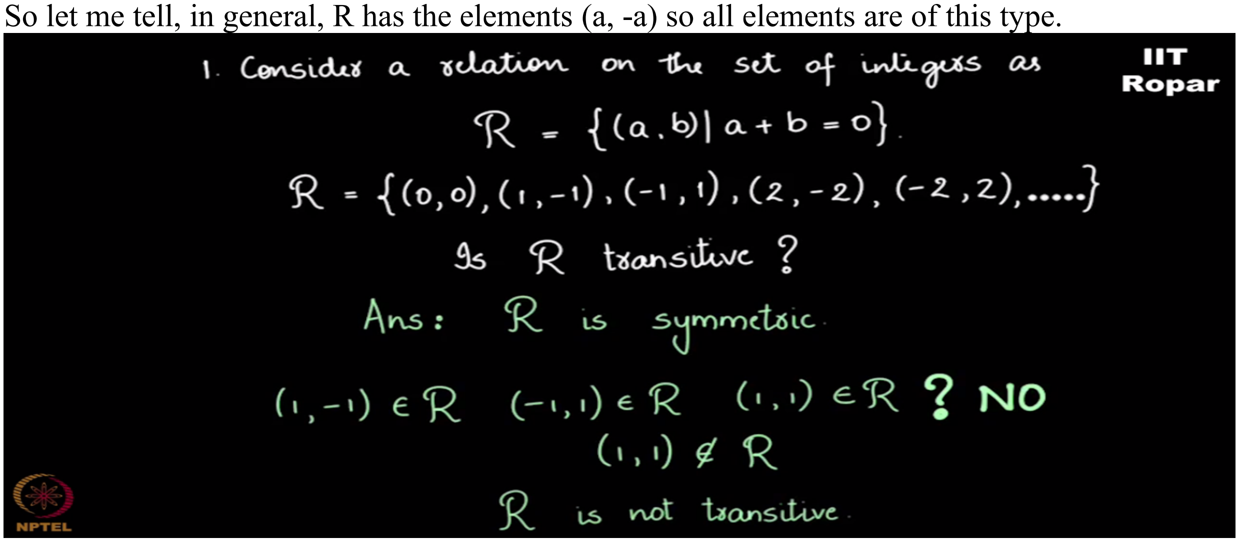
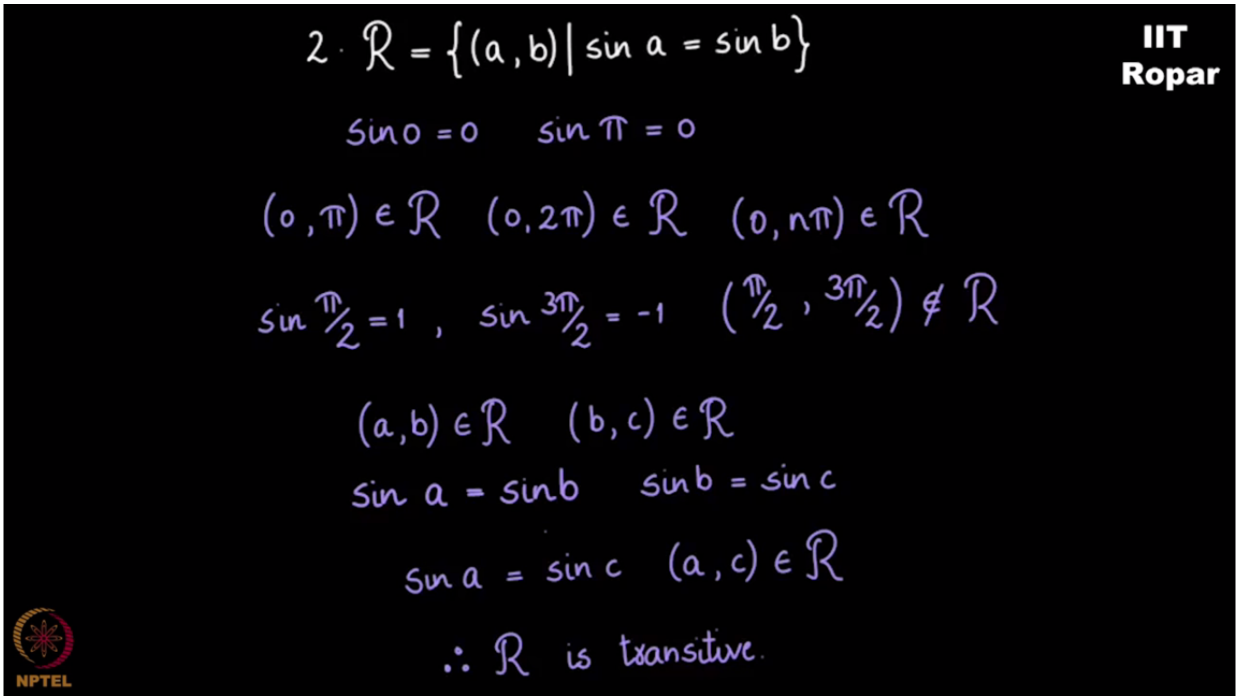


Let us now see some problems on the transitive relation. Consider a relation on the set of integers as (a, b) such that a + b = 0 so what are the elements in R? These will be the elements. So let me tell, in general, R has the elements (a, -a) so all elements are of this type.



Now is this a transitive relation? This relation is definitely symmetric, I'll leave it to you to just observe that. Let me see if it is a transitive relation. If (a, b) belongs to R, and (b, c) belongs to R, we should check if (a, c) belongs to R.

Now (1, -1) belongs to R, (-1, 1) belongs to R but does (1, 1) belong to R? Definitely no because 1 + 1 = 2 which is not 0 and hence, (1, 1) does not belong to R and hence this relation is not transitive.



I'll now be taking up a slightly non-trivial example. Consider this relation (a, b) such that sin a = sin b, the trigonometric function, sine, please concentrate and don't worry, you can pause the video and watch it several times. So what would be the elements in this relation?

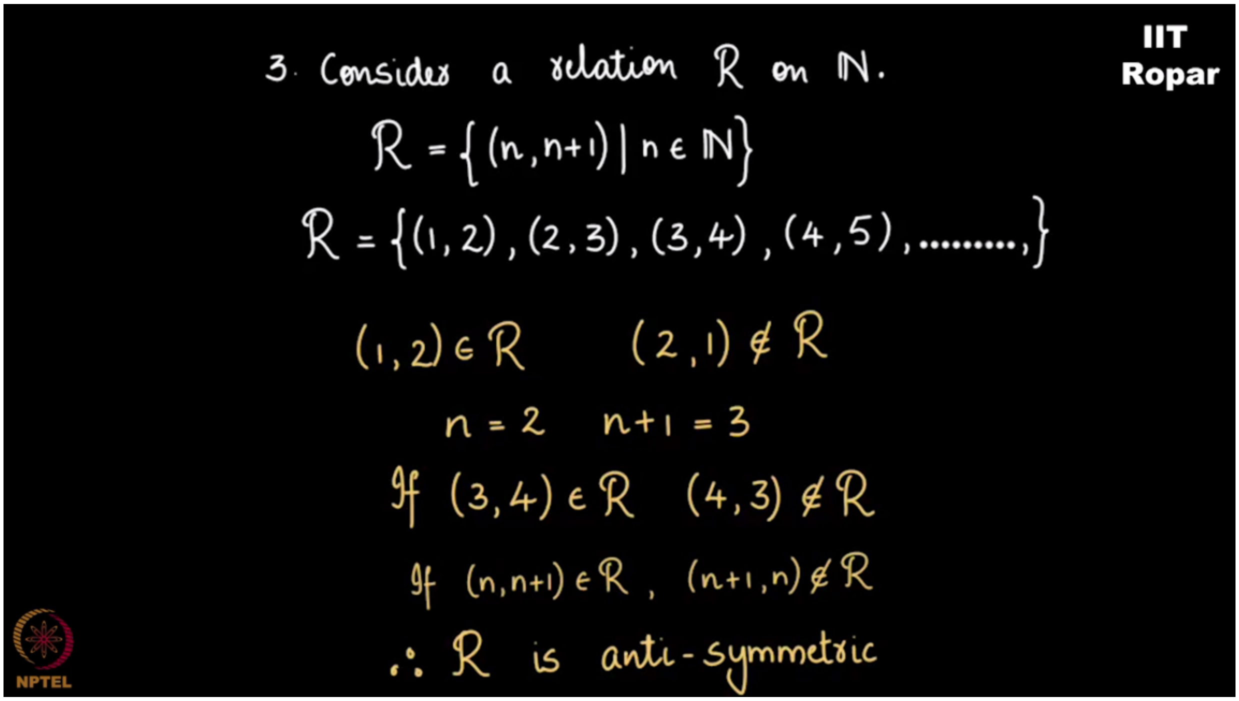
We have to pick those real numbers a and b such that sin a = sin b. let me take 0 for simplicity. Sin 0 is 0 and we know that sin π is also 0 and hence $(0, \pi)$ belongs to R. $(0, 2\pi)$ also belongs to R. In general, any $(0, n\pi)$ will belong to R because sin $n\pi$ is 0.

But let us see sin π /2 is 1, sine 3π /2 is -1 and hence, (π /2. 3π /2) does not belong to R. So let us assume that (a, b) belongs to R, (b, c) belongs to R. So this implies sin a = sin b and sin b = sin c. If sin a = sin b, it means (a, b) belongs to R and similarly, (b, c) belongs to R.

Now by this relation, sin a = sin c and hence (a, c) belongs to R. therefore, the relation is transitive.

I'll leave it as an exercise to you people to check if this relation is reflexive and symmetric. If you remember, we have solved some problems so you must be able to do this quickly.

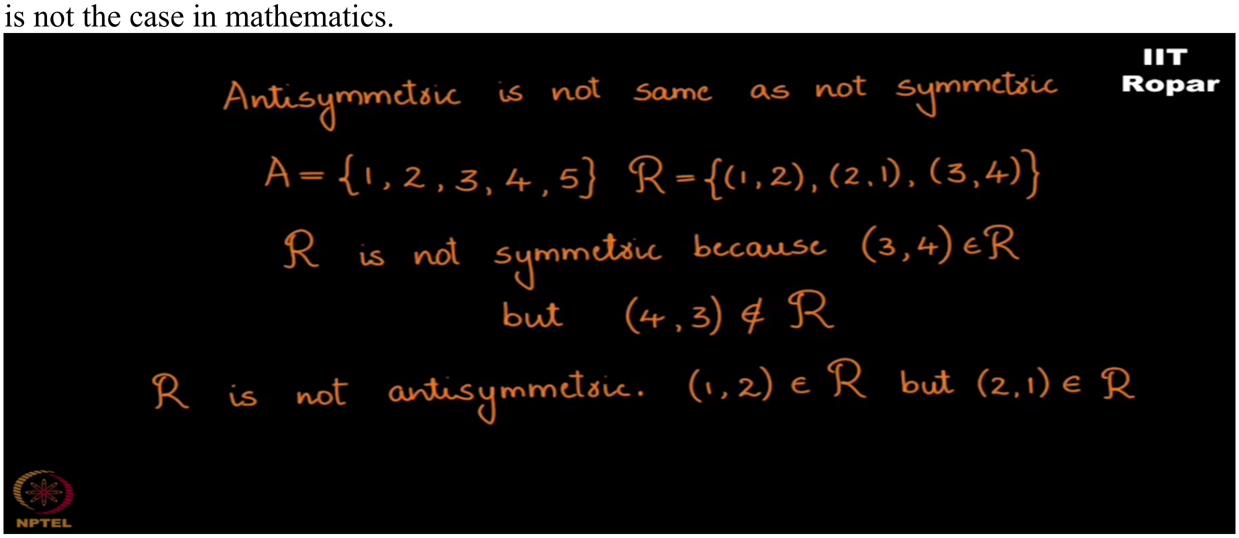
Let us now see a few problems where we can check if the relations are antisymmetric.



Consider a relation R on a set of natural numbers; R is given to be (n, n+1) such that n belongs to natural numbers. Now, let me write down the elements in R. R is going to have elements such has (1, 2), (2, 3) (3, 4) (4, 5) and so on. Observe carefully. (1, 2) belongs to R but definitely (2, 1) does not belong to R because if you consider n as 2 then you cannot have n+1 as 1; it should be 3 here. And hence, (2, 1) does not belong to R. If (3, 4) belongs to R, (4, 3) does not belong to R.

You see, if (a, b) belongs to R, (b, a) does not belong to R, then this relation is said to be antisymmetric. Here, (n, n+1) belongs to R and (n+1, n) does not belong to R and hence, this relation is antisymmetric.

Let me mention something very important here. Antisymmetric is not the same as not symmetric. Often these words are confused; antisymmetric is thought of as not symmetric but it is not the case in mathematics.



Let us see how. I'll give you a simple example. Let the set A = {1, 2, 3, 4, 5} and let R be this relation. This relation is not symmetric. Definitely you must be guessing by now because (3, 4)

is there but (4, 3) isn't there here. Then this relation is not antisymmetric as well. Why? (1, 2) and (2, 1) both are there in R and hence, it's not antisymmetric.
So antisymmetric is different from not symmetric.

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