NPTEL
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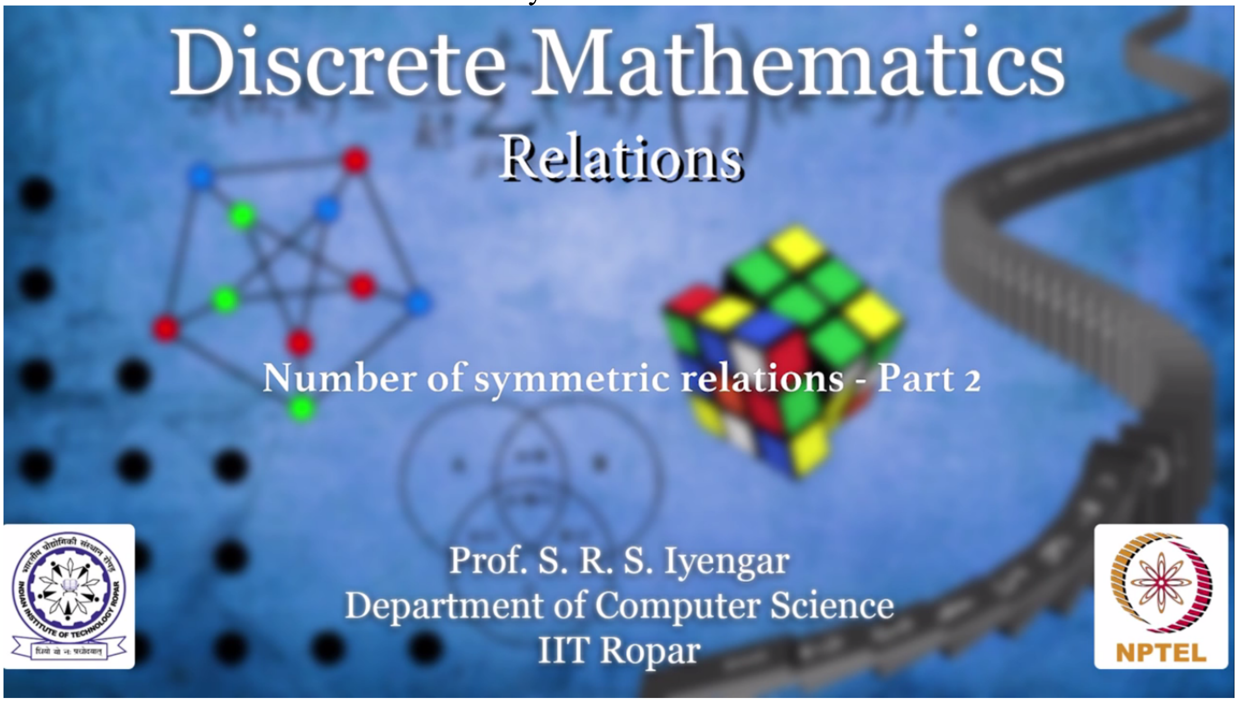
Discrete Mathematics
Logic
Number of symmetric relations - Part 2



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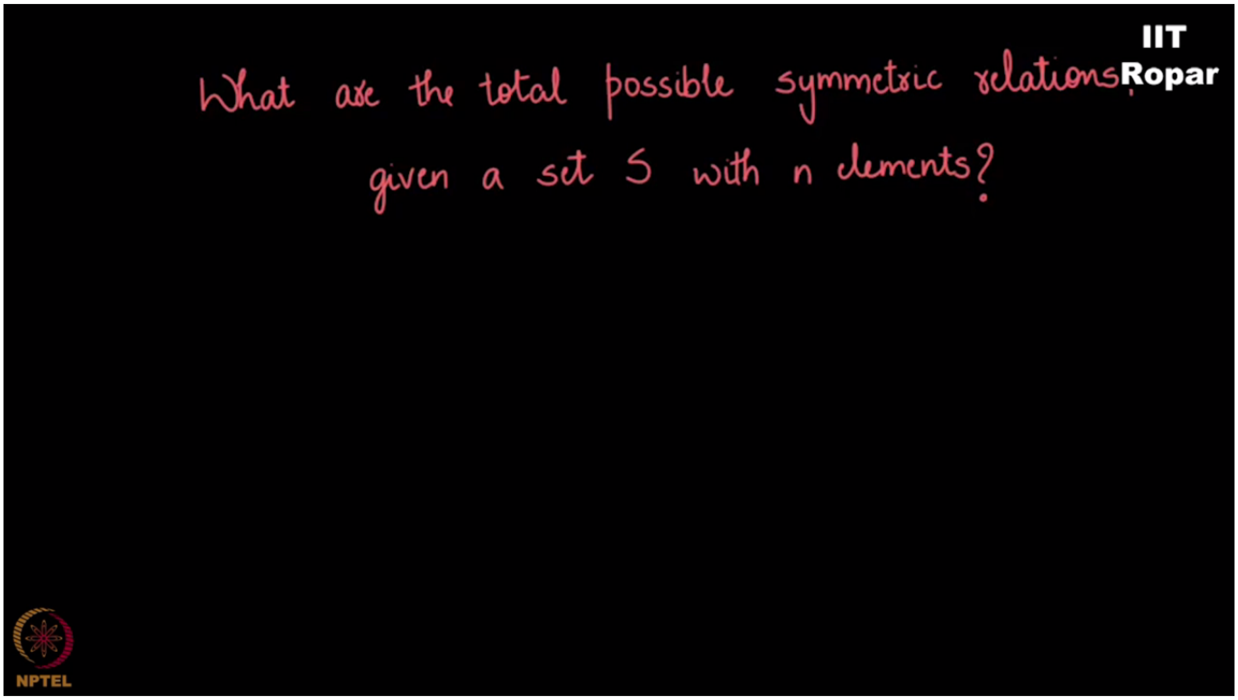
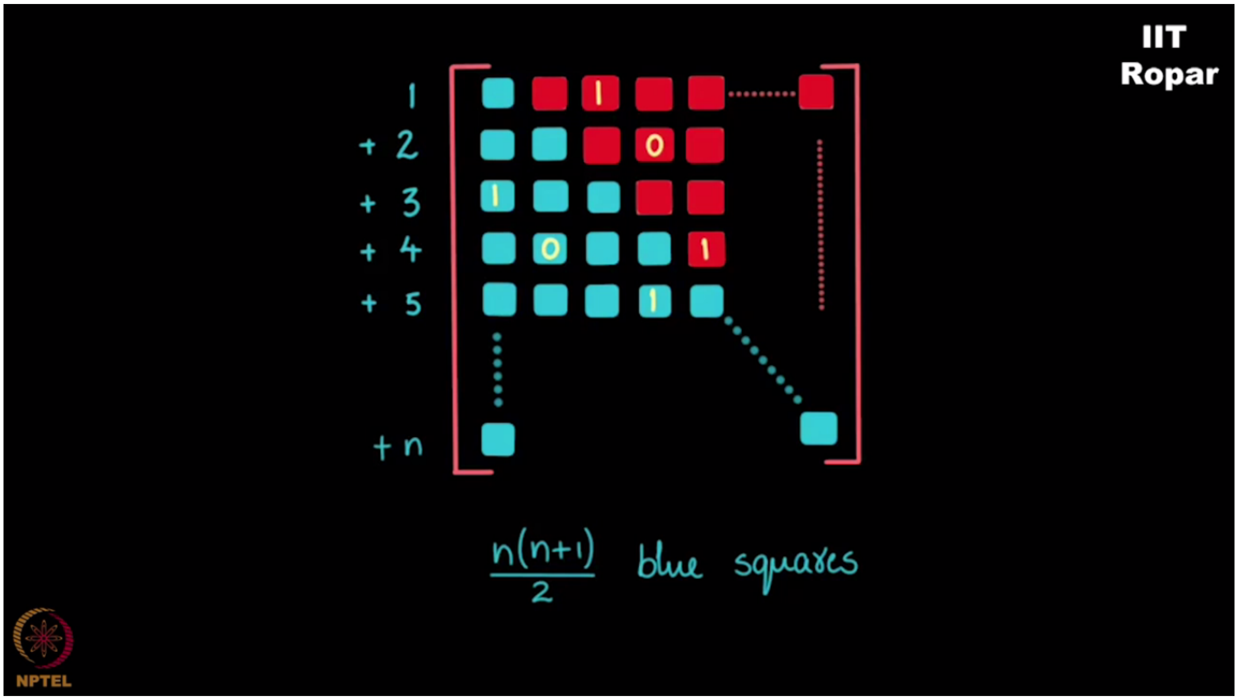
What are the total possible symmetric relations given a set S with n elements. This is a very little trickier question but the answer is very nice once you understand it. It's a little involved. You will take some time to understand this.



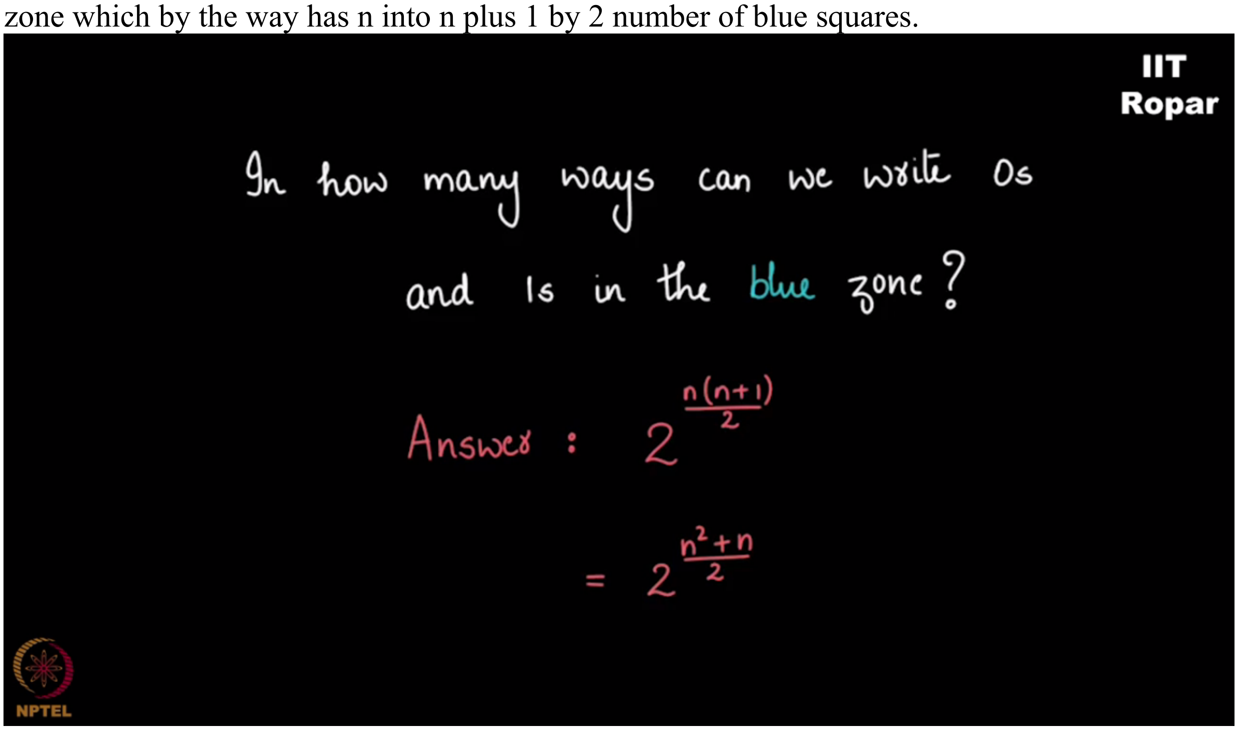
So what are the total possible symmetric relations? Firstly, let us write down the given symmetric relation and its matrix representation. So you have a matrix here and as you know this is called the lower triangle of the matrix and this is called the upper triangle of the matrix and you have the diagonal. Now let me paint the items in the diagonal and the lower triangle. Let me paint this items blue. You see all of them became blue and all the elements above the diagonal, the upper triangle I am going to paint it red. You will soon see why I am doing this.

Now observe closely. Any element in the lower diagonal, any space in the lower diagonal of this matrix if you put one, the corresponding symmetric side on the red zone should also be one. So you don't have a choice for the red zone once you decide on what is in the blue zone. That is lower triangle. Correct. So you have your freedom to put whatever you want in the lower triangle and of course you can put whatever you want in the diagonal. Why? What is the diagonal doesn't matter for a symmetric relation. All that you need to observe, all that you need to ensure is that whatever is below the diagonal symmetrically should be there above the diagonal. If it's zero here, it should be zero there. If there is a one here, it should be a one there. Correct.

Now I have my freedom to put whatever I want in this blue space holders, these blue rectangles basically, small-small rectangles. So in this blue squares, in these blue squares I have a right to put anything I want but of course zeros are ones. So how many blue squares are there to begin with? One plus two plus three plus four plus five so on up to n. So you have in total n into n plus 1 by 2 number of blue squares and you can put whatever you want here. Whatever you put here you will get a corresponding values in your red blocks as you can see and it'll be a symmetric relation.



Our given symmetric relation you can always see that they are symmetric along the diagonal. So all that I need to count right now is in how many ways can I write zeros and ones in the blue zone which by the way has n into n plus 1 by 2 number of blue squares.



So the answer simply is 2 to the power of n into n plus 1 by 2 which is 2 to the n square plus n by 2. N square plus n by 2 in the exponent. Now why is it 2 to the power of because you can take only a part of this blue zone and call it all ones, rest are zeros. It again boils down to you considering a subset of these blue squares. You can consider the subset of these blue squares in

2 to the n square plus n by 2 ways. It's a little involved maybe you should try watching the video once again or maybe try thinking about it with 3 cross 3 matrix example and it will be very clear to you people.