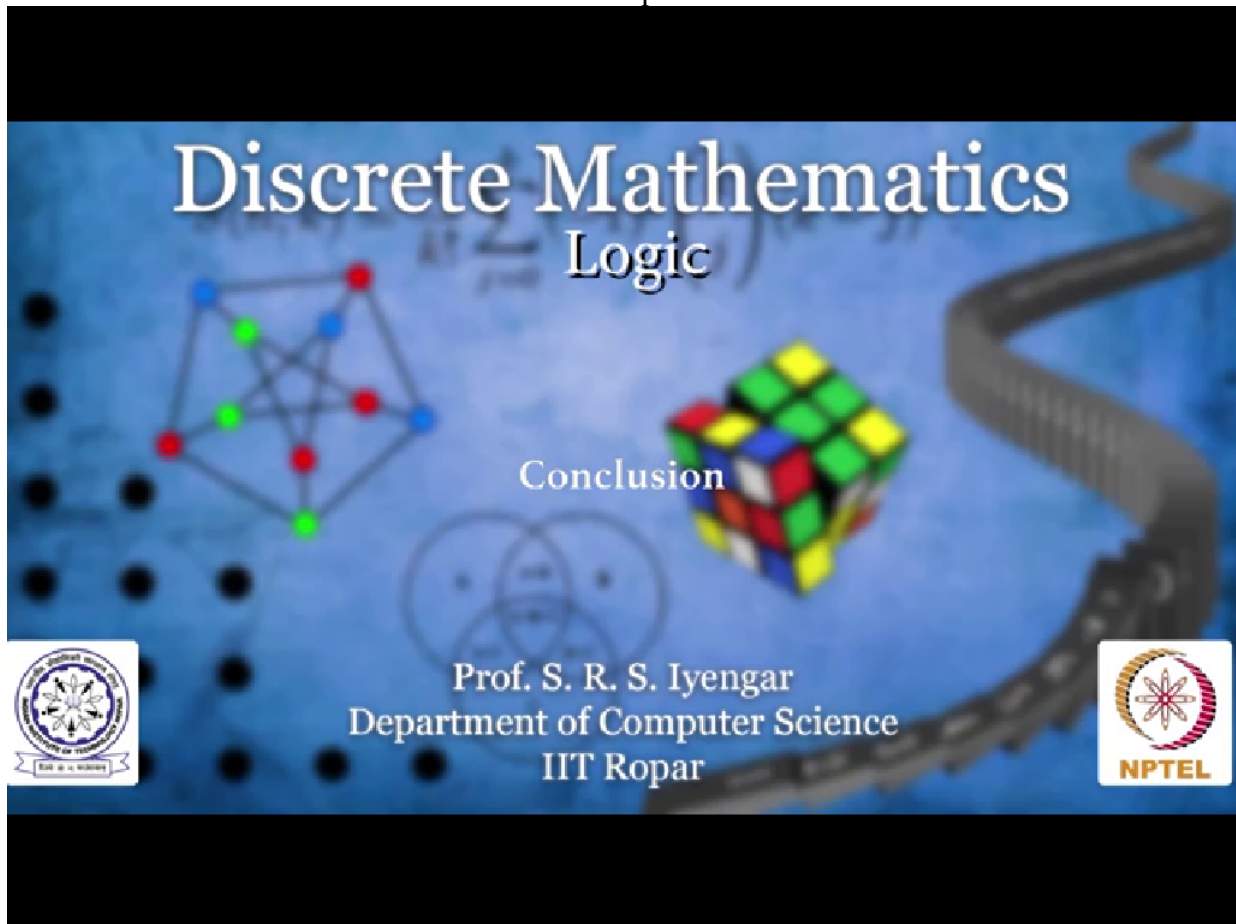NPTEL
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Discrete Mathematics
Logic
Conclusion
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Well, we have now come to the fag end of this chapter, which involves basics of mathematical logic. Let us go back and recollect how we started off with this chapter.

To prove : $\sqrt{2}$ is irrational.

Assumed $\sqrt{2}$ is rational.

We proved that square root 2 was irrational. How did we prove it? We assumed it to be rational and arrived at a contradiction.

To prove : $\sqrt{2}$ is irrational.

p : $\sqrt{2}$ is rational.
$\Downarrow$ → Not true

$\cdots\cdots\cdots$
$\Downarrow$
$\cdots\cdots\cdots$
$\Downarrow$
→ Not a fact

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Now you are probably wondering in what context this was start back then and where did we see its relationship to the chapter? Now is the time for us to tell you the relationship.

Look at this. Let the statement S be square root of 2 is rational. While we had to prove that square root of 2 is irrational, we assume that it is rational and call this statement S and S implies some S_1, right. We did that. Square root of 2 we assumed is equal to some p by q and 2 will be equal to p^2 by q^2 and so on and so forth.

$$\sqrt{2} \longrightarrow \text{Irrational}$$

S: Assume $\sqrt{2}$ is rational.

S_1: $\sqrt{2} = \dfrac{p}{q}$ $2 = \dfrac{p^2}{q^2}$

$\dfrac{12}{16}$

Simplest form

We also assume that p and q are in its simplified form and that it cannot be simplified any further. So S implies S_1, S_1 implies S_2 and so on we proceed until we conclude something. What do we conclude? We conclude that p and q is not in its simplest form with the conclusion statement B, C, right?

S_2: $\quad 2 = \dfrac{p^2}{q^2} \quad \boxed{p^2} = \boxed{2q^2} \longrightarrow$ Even

$\downarrow$

Even

p^2 is even $\implies$ p is even

$\Downarrow$

p^2 is a multiple of 4

$2q^2$ is a multiple of 4 $\qquad$??

C: $\quad q^2$ is even $\implies$ q is even

We start with S that square root of 2 is rational and root 2 is equal to p by q in its simplest form, and we arrive at a conclusion C, which says otherwise that p and q is not in its simplest form, which is absurdity as you can know. S implies C and C is absurd.

S_2:

$$2 = \frac{p^2}{q^2} \qquad \boxed{p^2} = \boxed{2q^2} \longrightarrow \text{Even}$$

$$\downarrow$$

$$\text{Even}$$

p^2 is even $\Rightarrow$ (p is even)

$$\Downarrow$$

p^2 is a multiple of 4

$2q^2$ is a multiple of 4 $\qquad$??

C: $\qquad$ q^2 is even $\Rightarrow$ (q is even) $\qquad$ absurdity

Absurd means false. It is not true. Something is not true. When S implies false, what can you say about S? You now have the wisdom to answer this question. If something implies falsity, what is that something? That something is also false. I am sure you understand implication really well now.

What was S? S was square root of 2 is rational and you are saying that this is not true, which means S complement is true. If S is square root of 2 is rational, what is S compliment? S compliment stands for square root of 2 is irrational and that is how the proof went.

What we just now did is we dissected the math behind a mathematical proof. Isn't that awesome? Do you see a math hidden in doing math? Logic actually helps you do this. Its relevance in computer science is in the sense that you can use logic to deduce some conclusions. It is a very highly applied subject in modern computing.

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