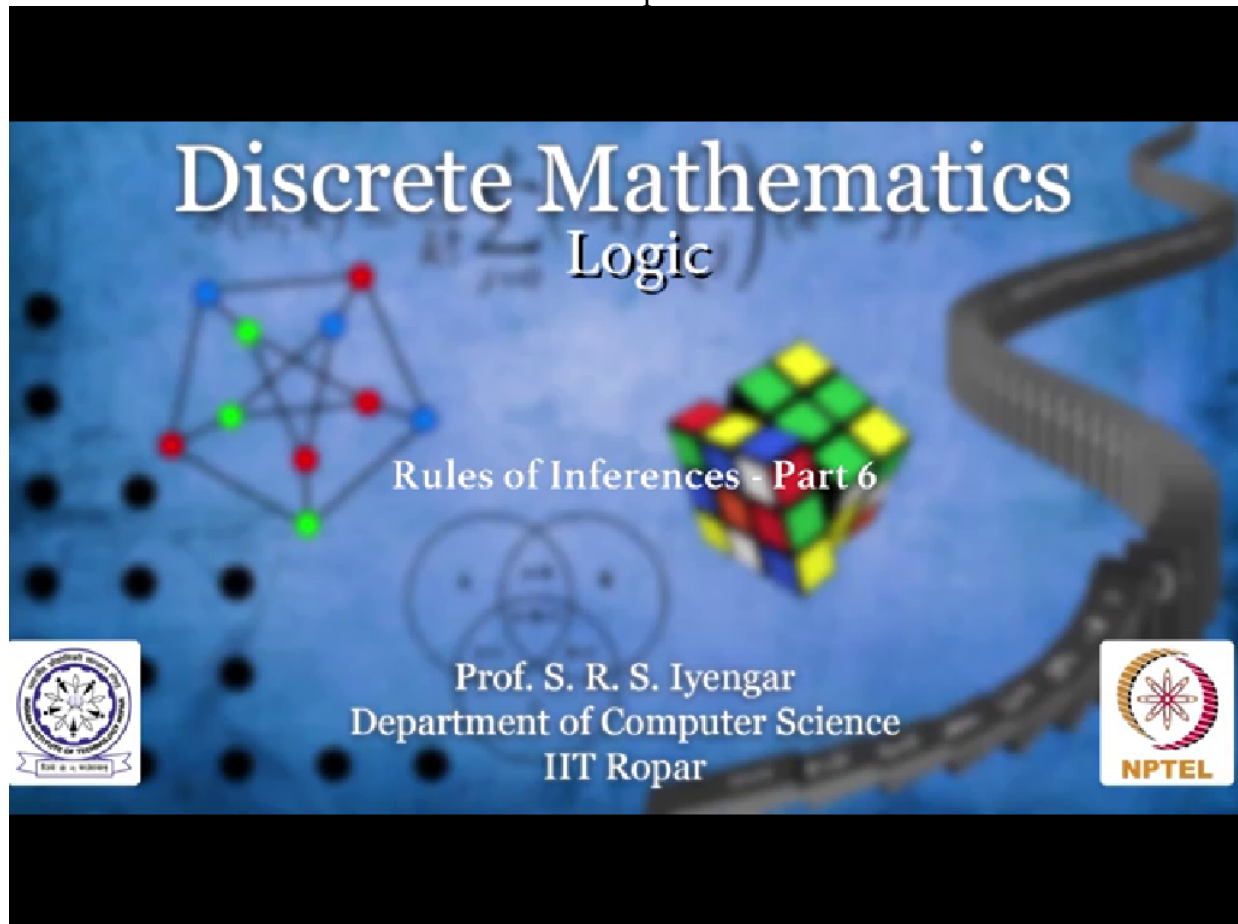


NPTEL
NPTEL ONLINE COURSE
Discrete Mathematics
Logic
Rules of Inferences - Part 6
Prof. S. R. S. Iyengar
Department of Computer Science
IIT Ropar



Let statement p denote Raj does yoga regularly and statement q denote Raj's blood pressure is normal. Right? Now don't you think p implies q is true? We are revisiting p implies q .

p : Raj does yoga regularly.
 q : Raj's BP is normal.

$$(p \rightarrow q)'$$
$$p' \rightarrow q'$$



If you remember, p implies q meant it will not so happen that whenever p is 1, q happens to be 0. This is never possible. Right? When p is 1, q is also 1. Then you say the implication is always true. Correct? 0 0 is 1. 0 1 is 1. 1 0 is 0. 1 1 is 1. The only situation where it is false is when p is 1 and q is 0. Then p implies q becomes false. The statement becomes false you see.

$$p \rightarrow q$$

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1



Now is this statement true, p implies q here? Very true because you observe logically speaking if you are physically active, your blood pressure remains normal, okay, most of the cases. Let us assume this to be true always, right? P implies q is true. Now look at this, but does Raj do yoga regularly? If p is not true, if p is false, you cannot say anything about q , but if p is true, you know for sure q is true because p implies q is true.

p : Raj does yoga regularly.
 q : Raj's BP is normal.

$$\begin{array}{c} (p \rightarrow q)' \\ p' \\ \hline \therefore q' \end{array}$$



Now look at this. If p is true, given that p is true and if p implies q is true, you can conclude that therefore q is true. Observe this boxed stuff deeply. This is not in particular about the given example. In general whenever you say p implies q is true and p is given to be true, you forcefully should conclude that q is true.

Now it may take some time for this idea to sink into your mind as and always you should solve a lot of problems. Don't worry if you don't understand what I am saying. What I'm saying might even sound ambiguous, but let us solve some 5 to 6 problems, and at the end of these problems it will be very clear to you what we are doing. Okay.

We have solved questions with implication being our final inference. Now we will try to show this question with four statements and one final inference, which is therefore r OR s , we have to show that r OR s is true.

Show $\therefore r \vee s$

IIT
Ropar

$$\neg p \vee s$$

$$\neg t \vee (s \wedge r)$$

$$\neg q \vee r$$

$$p \vee q \vee t$$



Now the same old trick. We will try to show this by contradiction. What do we do? We'll assume $r \vee s$ is not true, which means r is false, s is false. The moment you say r is false, I am going to spot all r 's and make them 0. You have one in the second line, one in the third line and then s is also 0, so I put s as 0, one in the second line and one in the first line. Okay. Perfect.

$$\begin{aligned}
 &\text{Show } \therefore r \vee s \\
 &(r \vee s)^0 \\
 &\neg p \vee s^0 \\
 &\neg t \vee (s^0 \wedge r^0) \\
 &\neg q \vee r^0 \\
 &p \vee q \vee t
 \end{aligned}$$



Now you see that look at the first line. S is false. First line says NOT p OR s is true, which means NOT p should be true, which means p is false. Okay. So p should be false everywhere. You see p is false. Fine. I make p as false in the fourth line as you saw. Right? Okay. So p is false and now r is given to be false. NOT q OR r is true. So q has to be false. So I put q to be false and I put r to be false in the fourth line as well. Okay.

Now look at the second line. S is false. R is false. S and r becomes false and NOT t OR this second line, NOT t OR (s AND r) is true, right? But s AND r is turning out to be false. So NOT t should be true and hence t is false. So t is false. Look at the last line. We have a contradiction. P is false, q is false, t is false, but you are saying p OR q OR t is true. That's not possible. So contradiction to our assumption that r OR s is false, which means r OR s should be true and therefore r OR s is our answer.

Show $\therefore \gamma \vee s$

$$(\gamma \vee s)^{\circ}$$

$$(\neg p)^{\uparrow} \vee s^{\circ}$$

$$(\neg t)^{\uparrow} \vee (s^{\circ} \wedge \gamma^{\circ})^{\circ}$$

$$\neg q \vee \gamma^{\circ}$$

$$(p^{\circ} \vee q^{\circ} \vee t^{\circ})^{\uparrow}$$

$$\therefore \gamma \vee s$$

Contradiction
to $(\gamma \vee s)^{\circ}$ 

I am sure you people are getting a hang of it right now.

Look at this question. I got to show that t is true. It is slightly tricky. How do I go about it?

Show : t'

$$\neg p \wedge q$$

$$r \rightarrow p$$

$$\neg r \rightarrow s$$

$$s \rightarrow t$$



Now look at the first line. It says NOT p AND q is true, which means q is 1 and p is 0. Only then will NOT p be true and NOT p AND q be true. Okay. So p is 0. Look at the second line. P is zero. Something which is true cannot imply 0. R implies p is true. So r has to be 0 only. If r becomes 1, 1 cannot imply 0, which is p. P is known to be 0 you see. So r implies p, I very well infer that r is 0 here. So far so good. If r is 0 and NOT r becomes 1, the next line, correct, if NOT r is 1, and this 1 is implying something that something should also be 1, so s is 1. Perfect. S is 1, S implies t, so t has to be 1. So I can conclude that therefore t is true.

Show : t'

$$(\neg p)' \wedge q'$$

$$\delta^0 \rightarrow p^0$$

$$(\neg \delta)' \rightarrow s'$$

$$s' \rightarrow t'$$

$$\therefore t'$$

