NPTEL
NPTEL ONLINE COURSE
Discrete Mathematics
Logic
Rules of Inferences - Part 5
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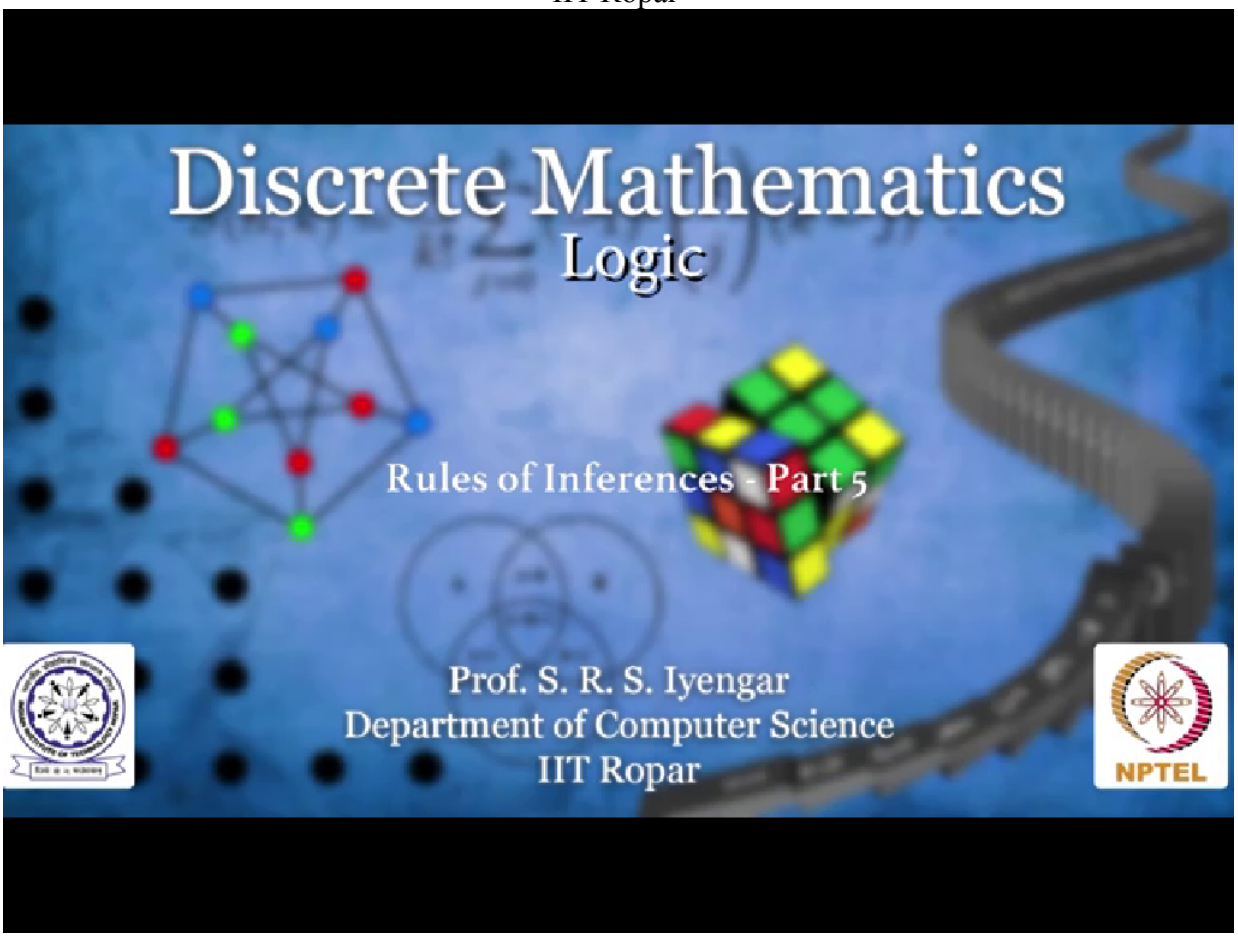
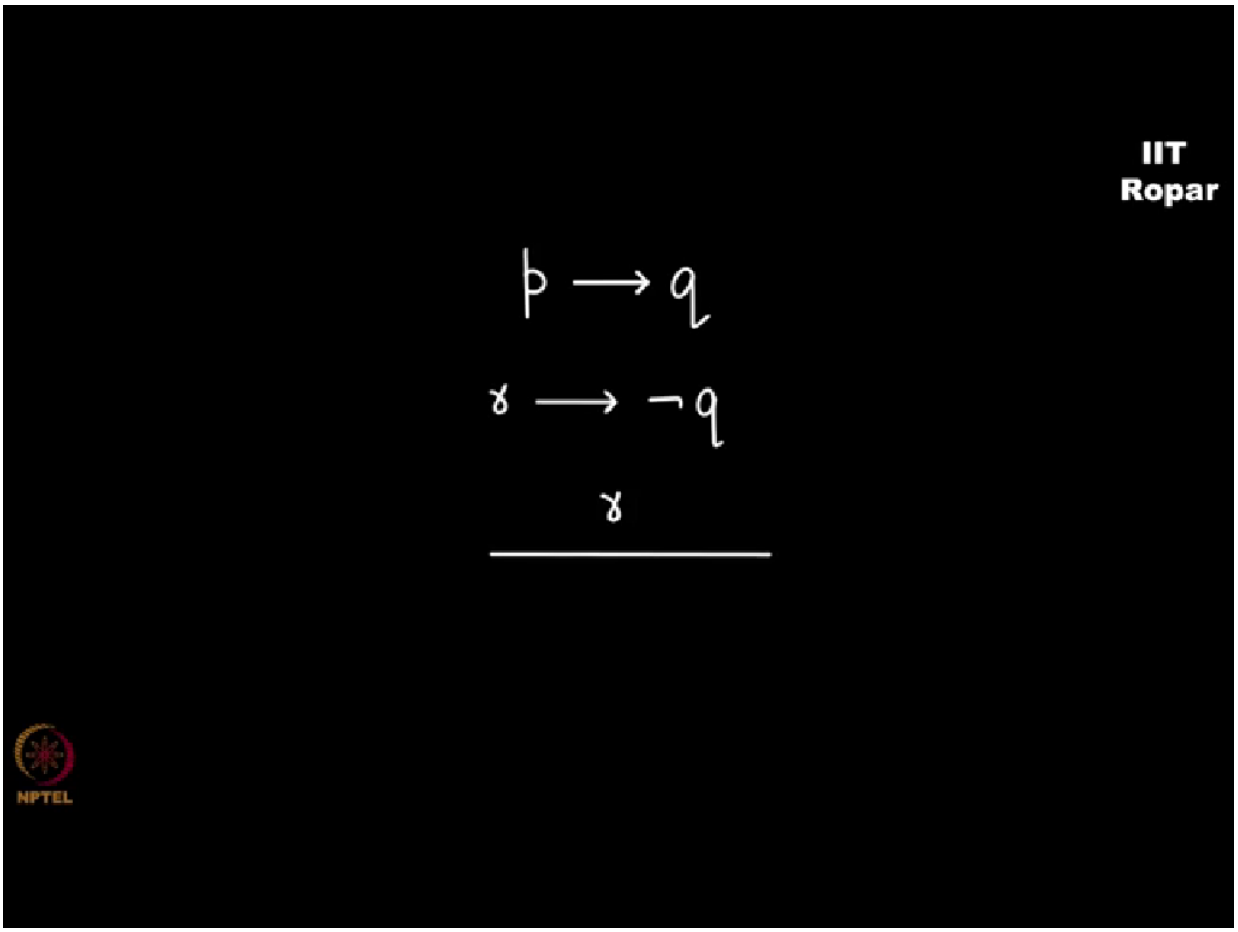
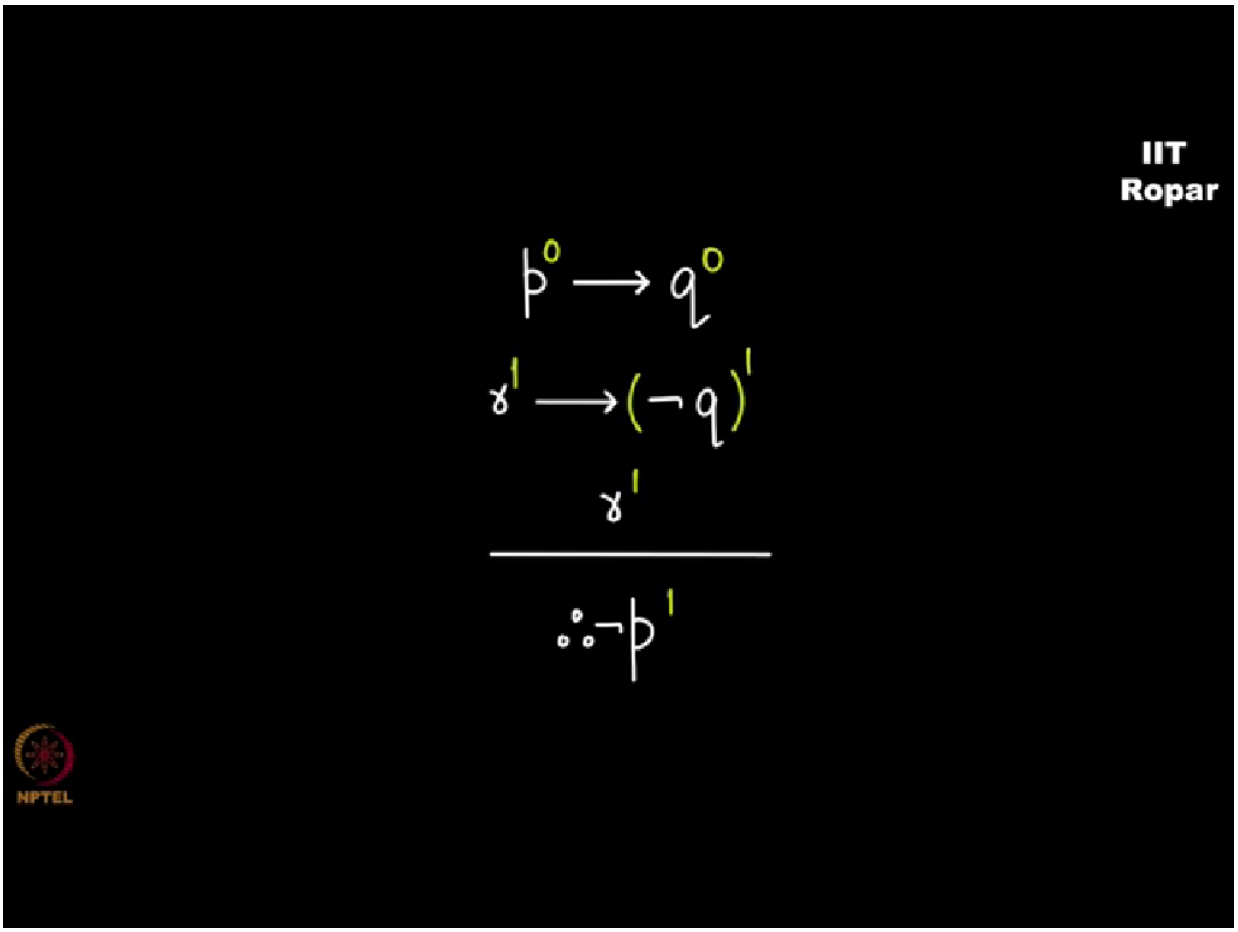



Look at this question, p implies q, r implies NOT q, and then r is given to be true. What can you conclude from these three statements?

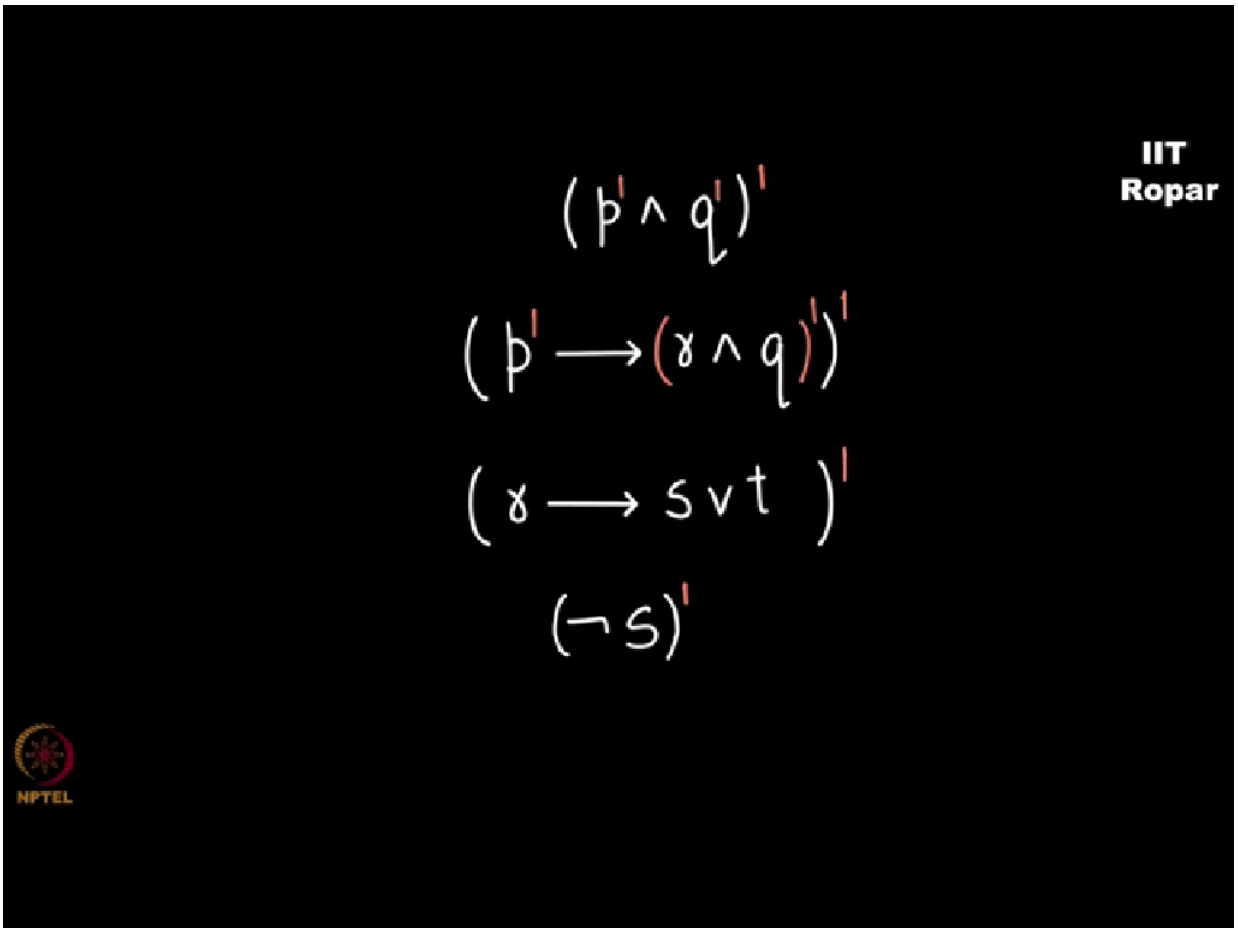



Well, observe TT. R is given to be true, all right, which means r is true. Let me write 1 on top of r to denote that r is true. Once r is true, r is implying something. That something has to be true if the implication is true you see. So NOT q is true, which means NOT q is true means what? Q is false. So when q, NOT q becomes 1, q becomes 0. If q is 0 and p is implying q, p cannot be 1 because 1 cannot imply 0. So p has to be 0, so I can say therefore p is false or in other words, NOT p is true.



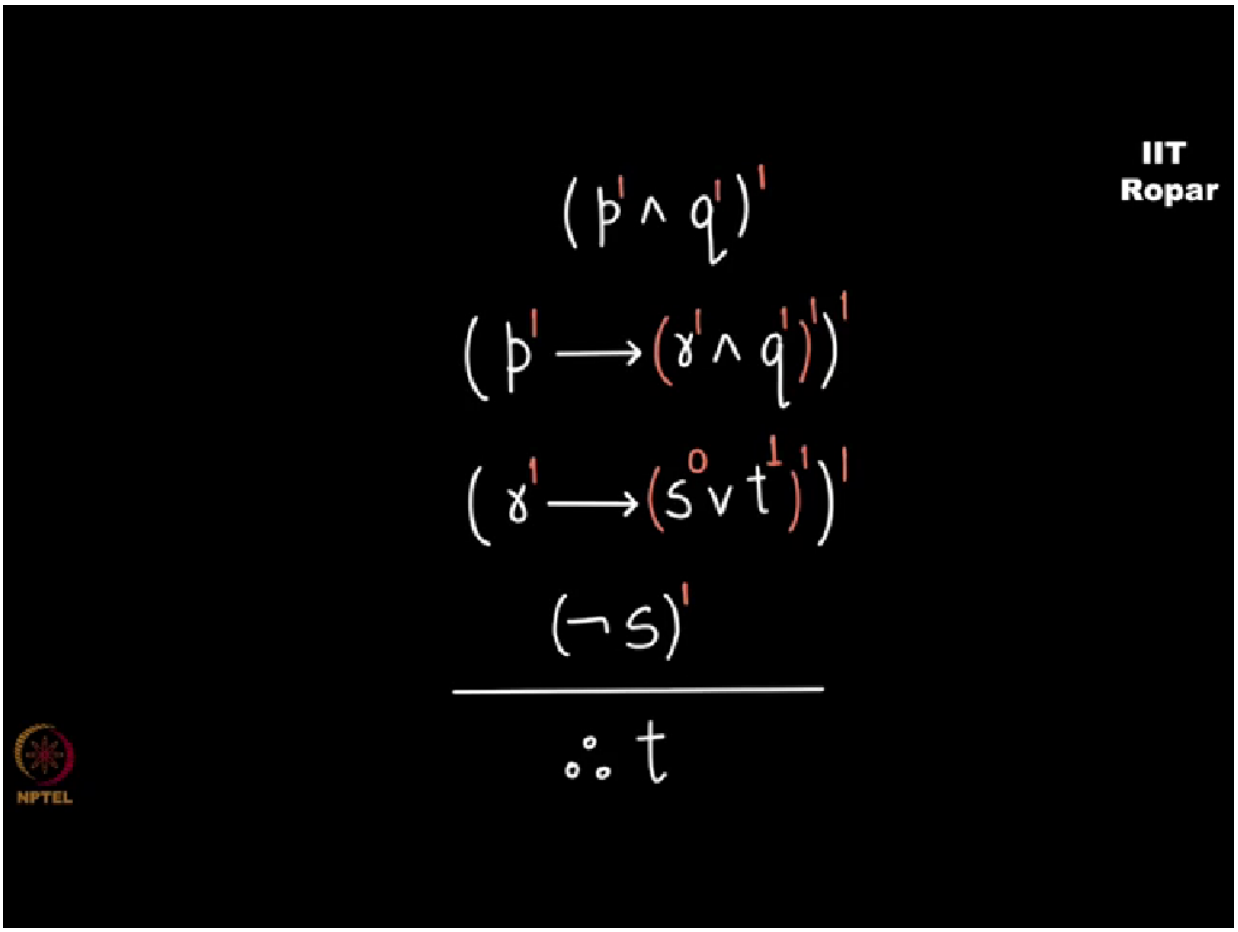

I'm sure you're trying to get a hang of this problem. If you are not still do not worry. We are going to solve more problems. You will soon get the hang of it. A trick in understanding some of the mathematical questions is to see the exercise problem repeatedly, okay, is to see the same problem again and again. Do not be in this temptation to solve more problems without understanding the first problem. The idea is to understand the first problem properly and repeatedly observe it, solve it again and again, see the internals of it and then go ahead to the next problem and solve more problems.

It is given that p AND q is true, and p implies (r AND q) is true. R implies (s OR t) is true and s is known to be not true. In other words, NOT s is true. What can you conclude? Let's see.

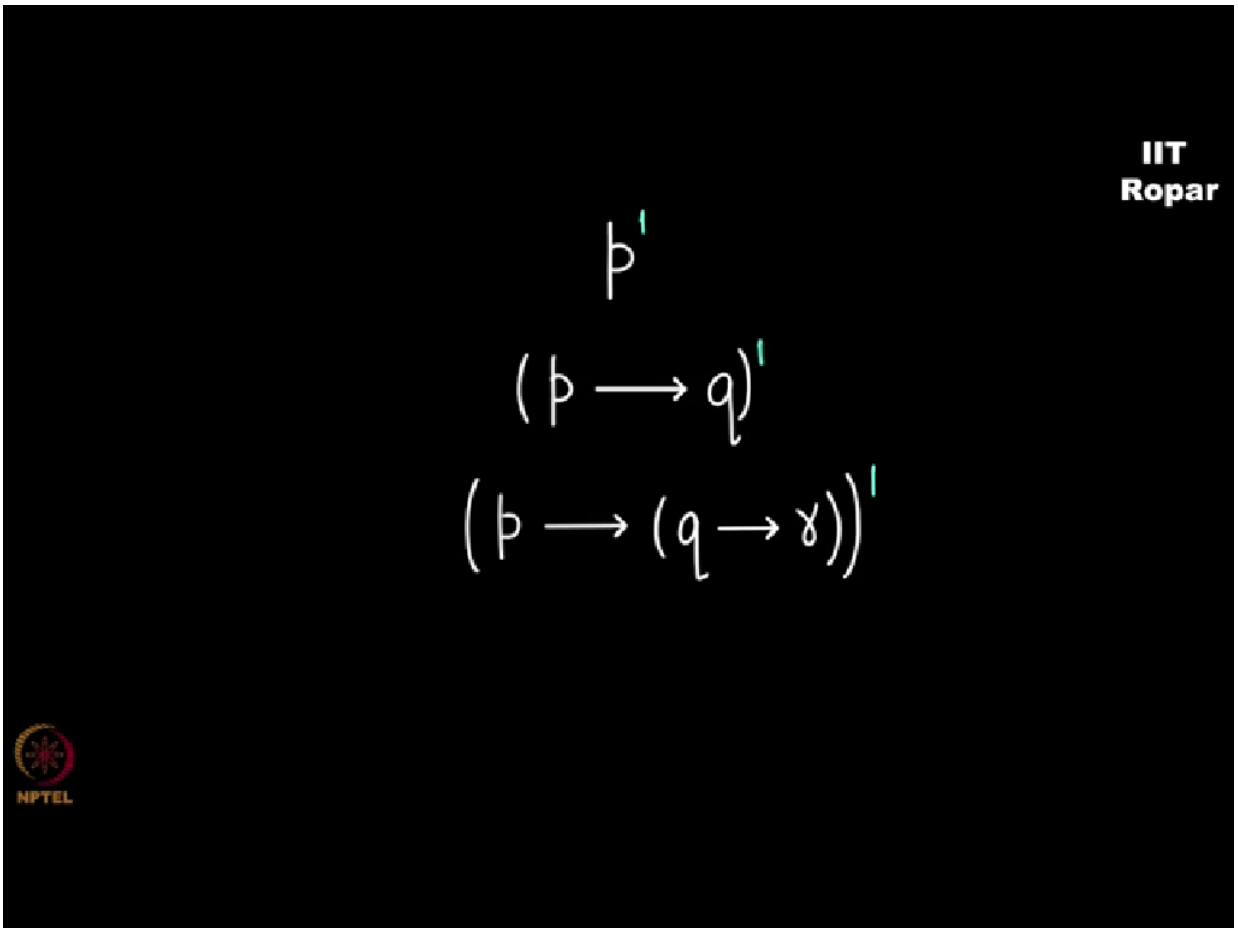



So p and q is a nice way to say that p is true and q is true. Okay. Given that p is true, let me write 1 on top of p. P is true. P is implying something. This something has to be true when p is true. This entire thing is true. When the entire thing is true and the entire thing is AND of several variables, then individually, these variables must be to true. Isn't it? Only if this is 1 and this is 1 can the AND of two things be 1. So r is true, q is true. So let me write that down. R is 1, and then q is not – q, q, of course, was true here you see, p and q both were true.

Now r is true. R is implying something, which means this something should be true so which means s is true, OR t is true. I don't know which one is true, but then I have some information sitting here. Do you see? NOT s means s is not true. So put s equal 0, which makes this -- which forces that t must be true. Why? S, r, t is true. S is false and hence t must be true. Therefore, t is how we write the answer. Right?



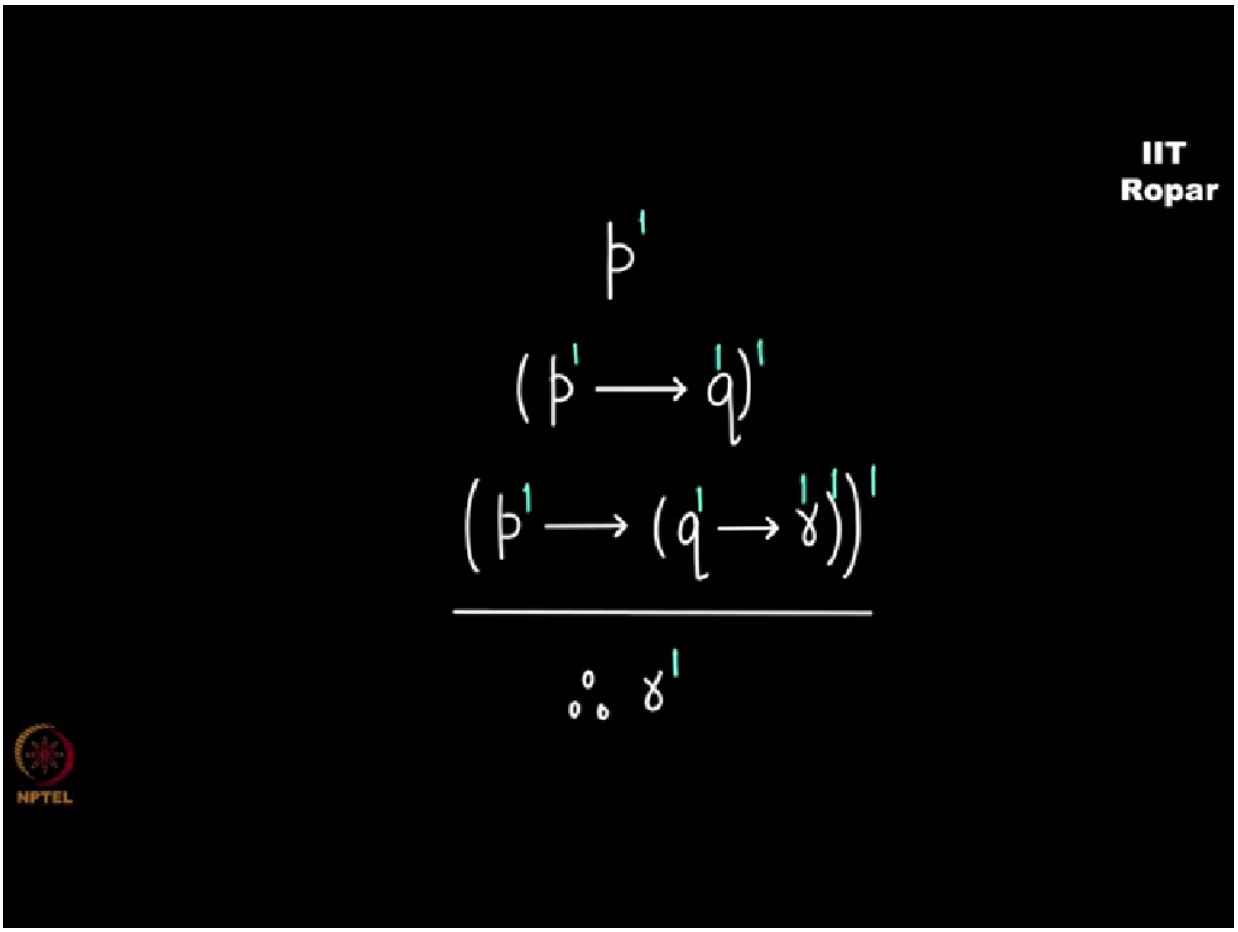

I say that p is true. I also say that p is implying q is true. I also say that p implies (q implies r) is true. Here is a typical example of where thinking English-wise language-wise can become complicated. You have a nested implication here. P is implying some other implication.
Now let's see what we can conclude from this. It is so nontrivial at least I am now stumped. I have no clue how to proceed with this problem at the first look of it. Let me go slowly.




P is given to be true. Let me write 1 on top of p. P is implying q. So p is given to be true. P implies q is true, which means q has to be true. I put 1 on top of q.

So I repeat, the way we are arguing is always this. Whenever an implication is given to be true, here the question is, these three statements are true. These three logical statements are true. Okay. These three Boolean expressions are true. P is given to be true. P implies q is true. We cannot question what if p implies q is false. That can never happen. That is given to you. Everything above this horizontal line is given to you. P is true. P implies q is true. When p is true and p implies q is true, q should be true is all that I am saying. That's all is a point in all these problems. Okay. Fine, let's go ahead.

So p was true. Q was given to be – we inferred that q should be true if p is true and p implies q. Now p is implying something. P is true. So this something has to be true. This something has to be true and q is 1. See if q is 0 and you say something has to be true, r can be 0 or 1. Right? Zero implies 0 is true. Zero implies 1 is true, but given that q is 1 and q implies r is 1, we know for sure that r is indeed 1. So I say, therefore, r is 1, which means, therefore, r is true and this is my inference.

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