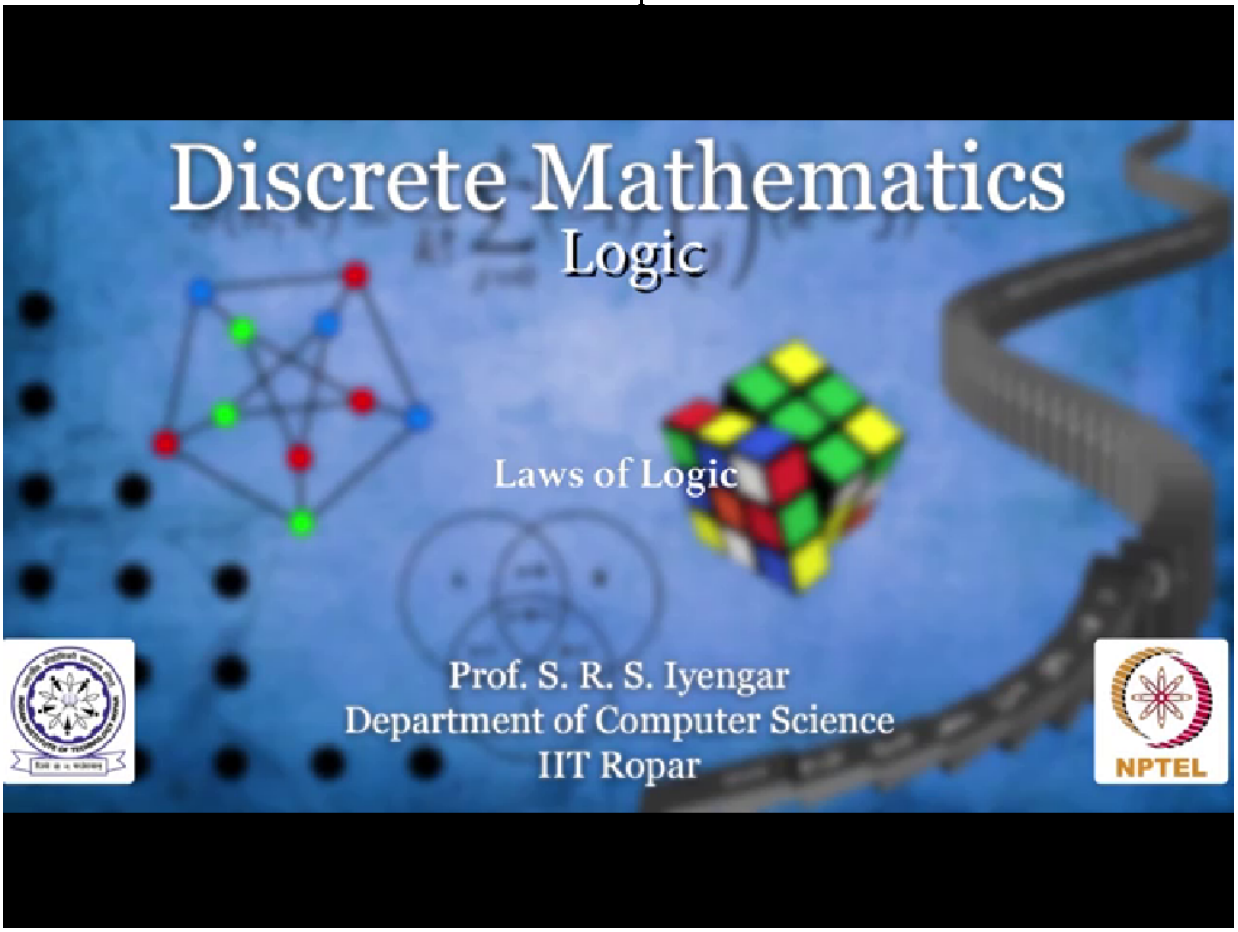
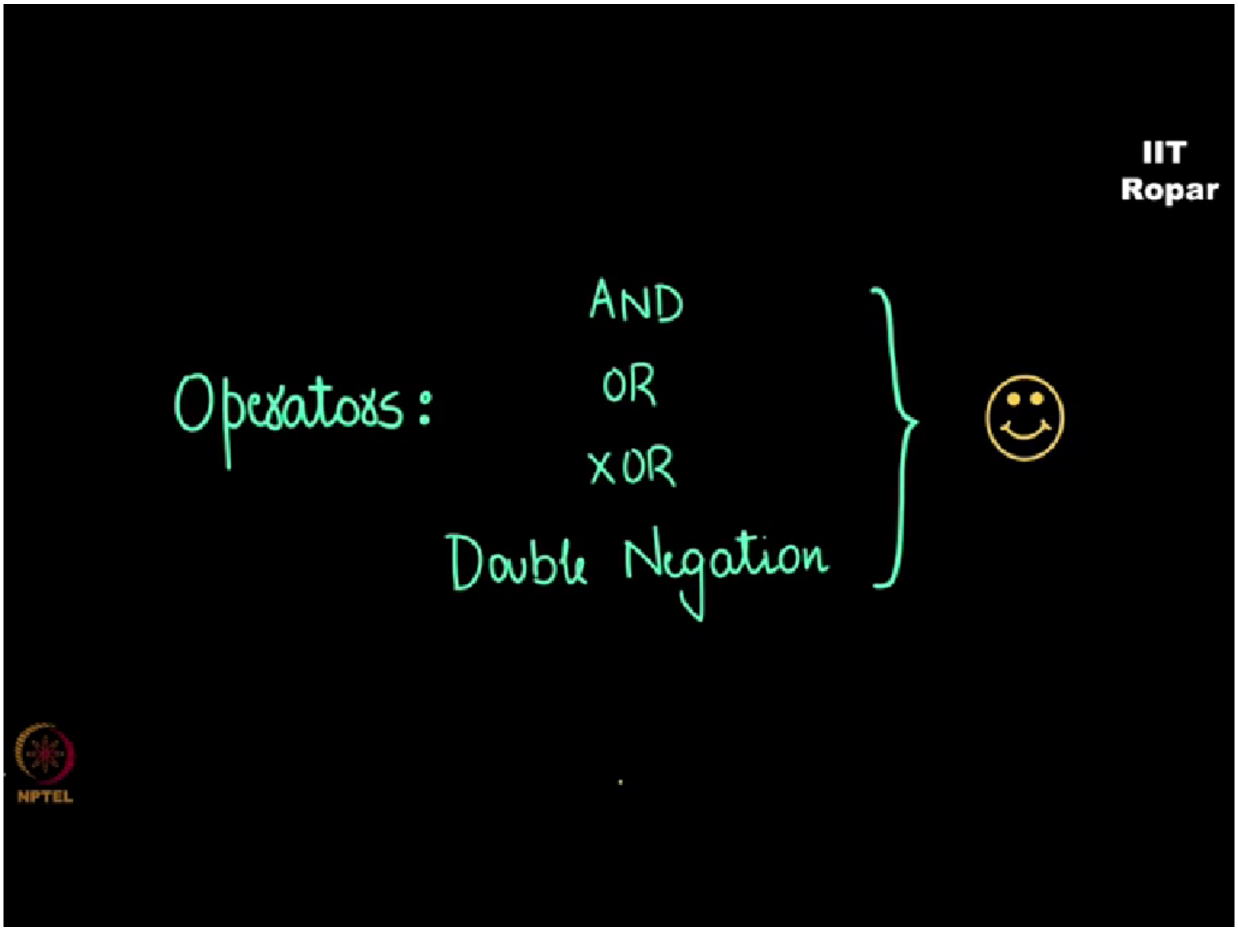
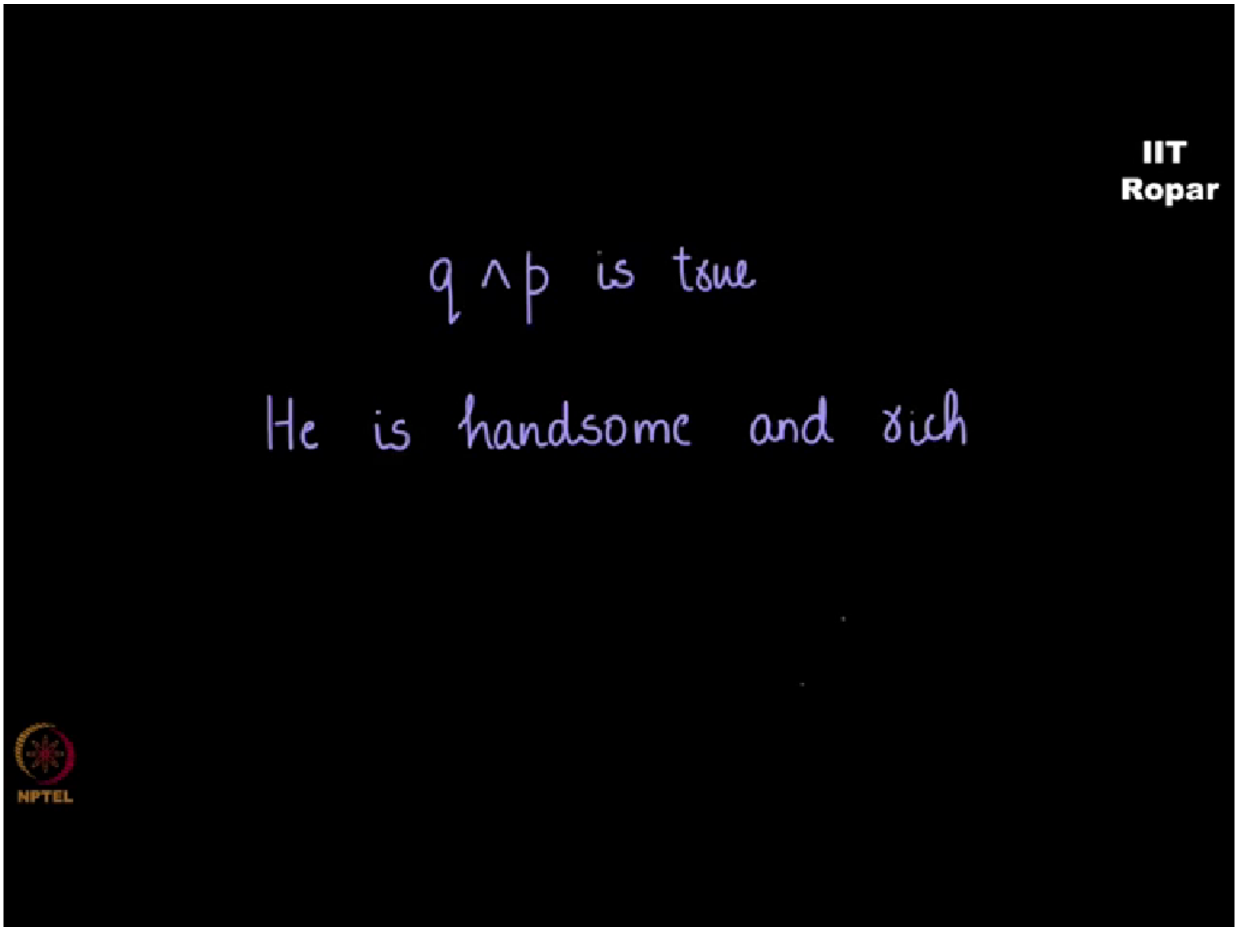
NPTEL
NPTEL ONLINE COURSE
Discrete Mathematics
Logic
Laws of Logic
Prof. S. R. S. Iyengar
Department of Computer Science
IIT Ropar



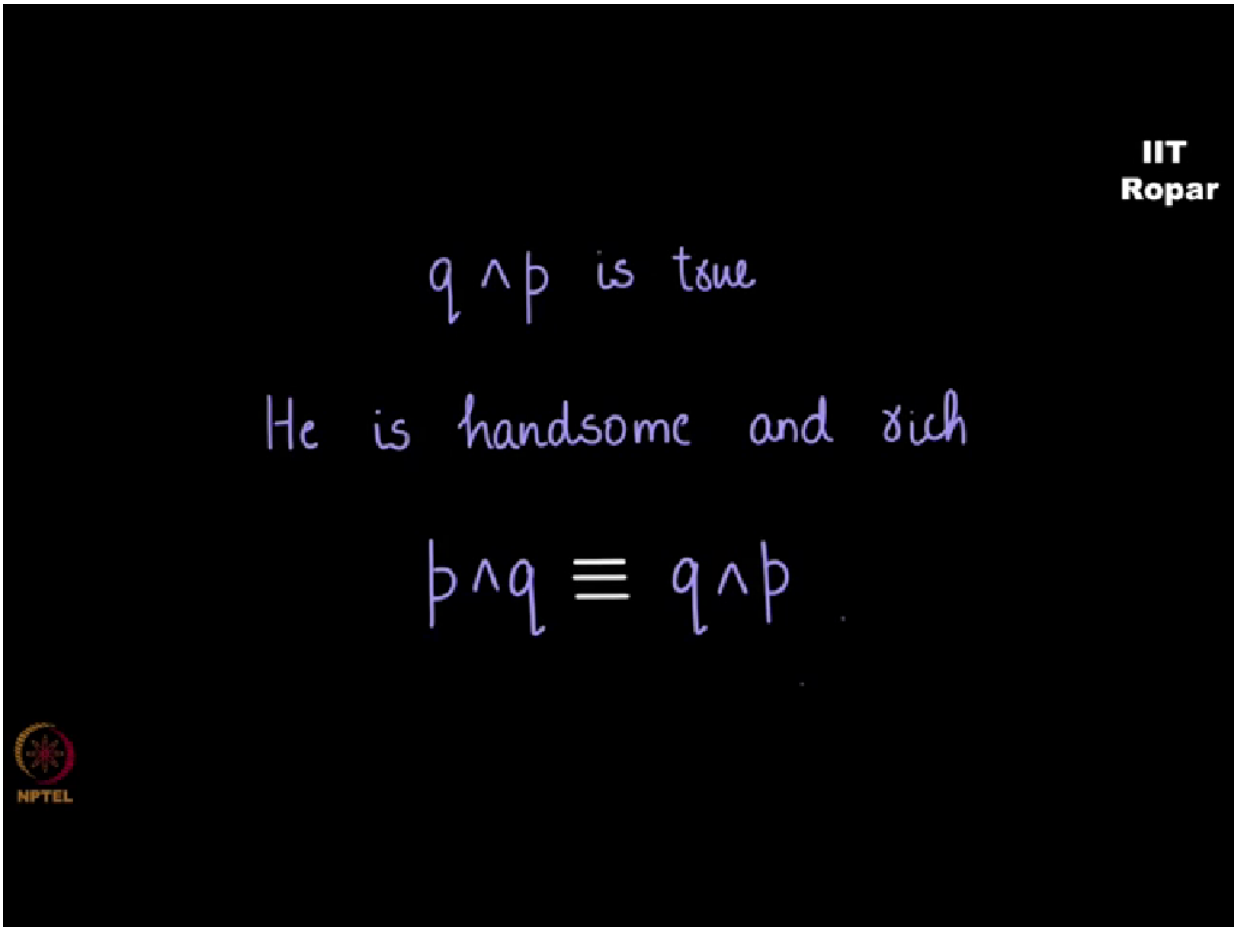
You see what we have seen so far, closely observe that we saw different operators like AND, OR, XOR and things like that, right, double negation and things like that.



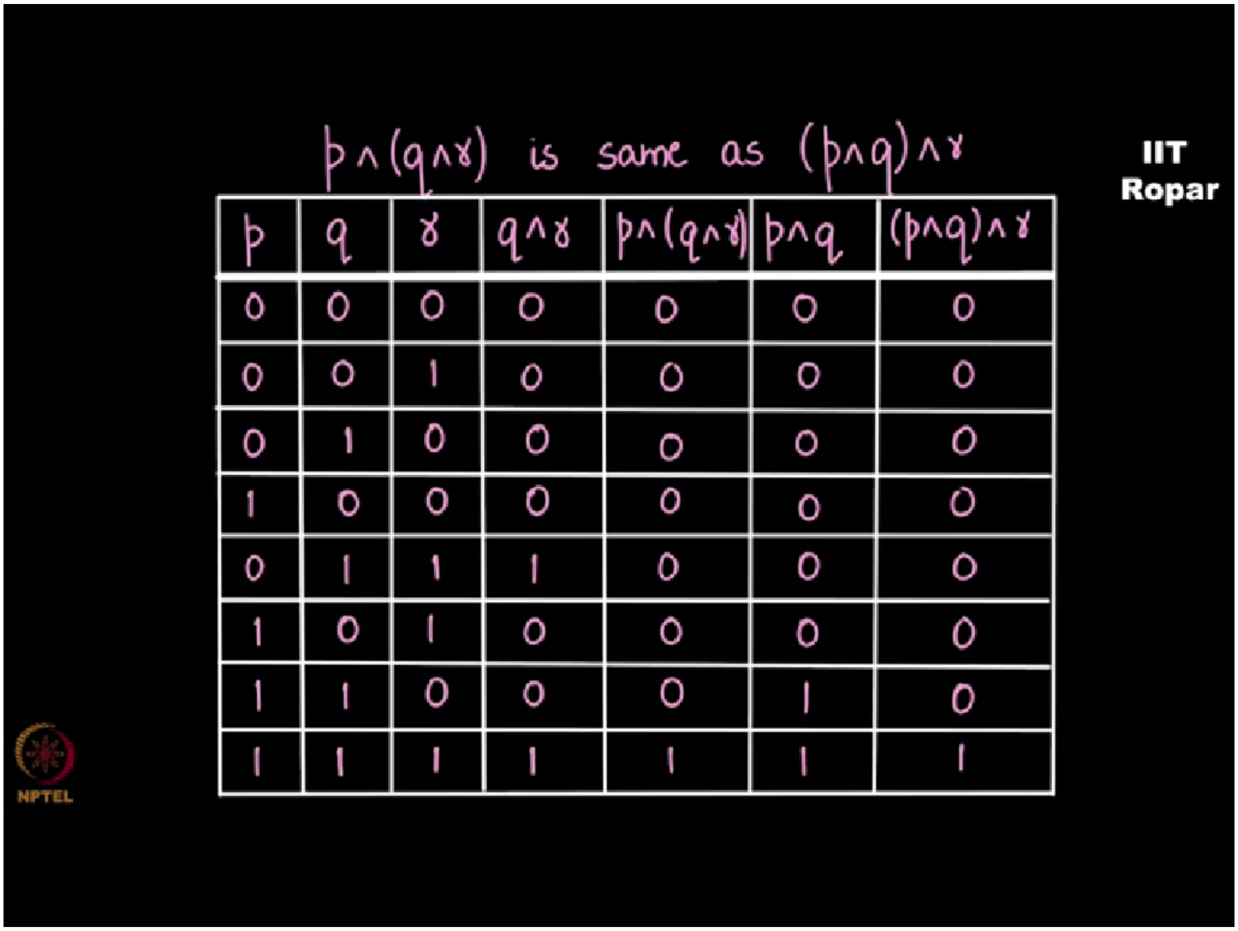
There is something nice about these operators. Although they are very easy to understand, there are some nice properties that they follow. Let us look at those properties.
You see when I say p and q is true, this is same as q and p you see. Correct? He is rich and handsome is same as he is handsome and rich. Correct? Perfect.



Now p and q, q and p, we say are equivalent. It doesn't matter whether you write p first or q first. P and q is same as q and p.

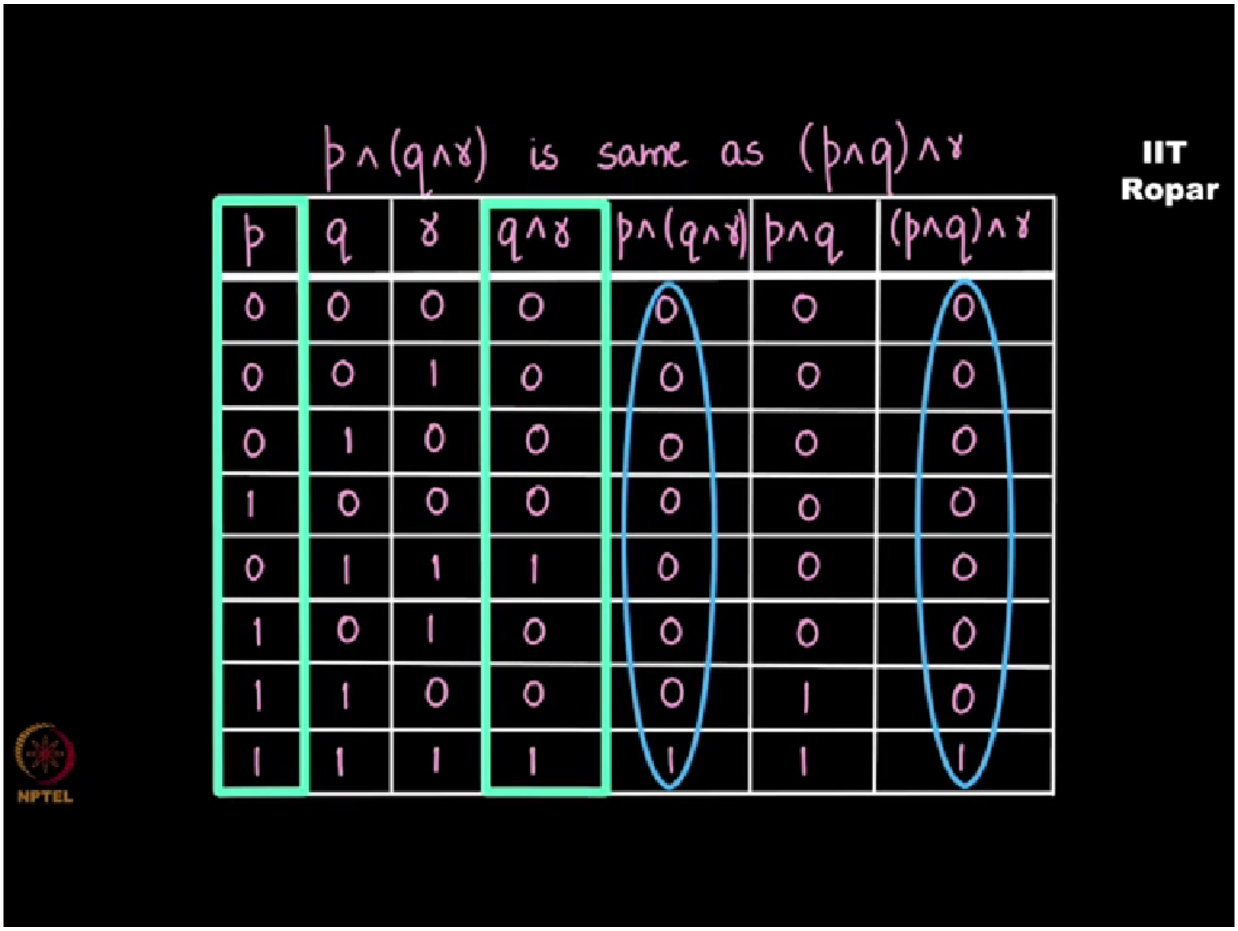


Very soon you will realize what we are doing.
Now look at this p and (q AND r) is same as (p AND q) and r. Why is that?


$p \wedge (q \wedge r)$ is same as $(p \wedge q) \wedge r$

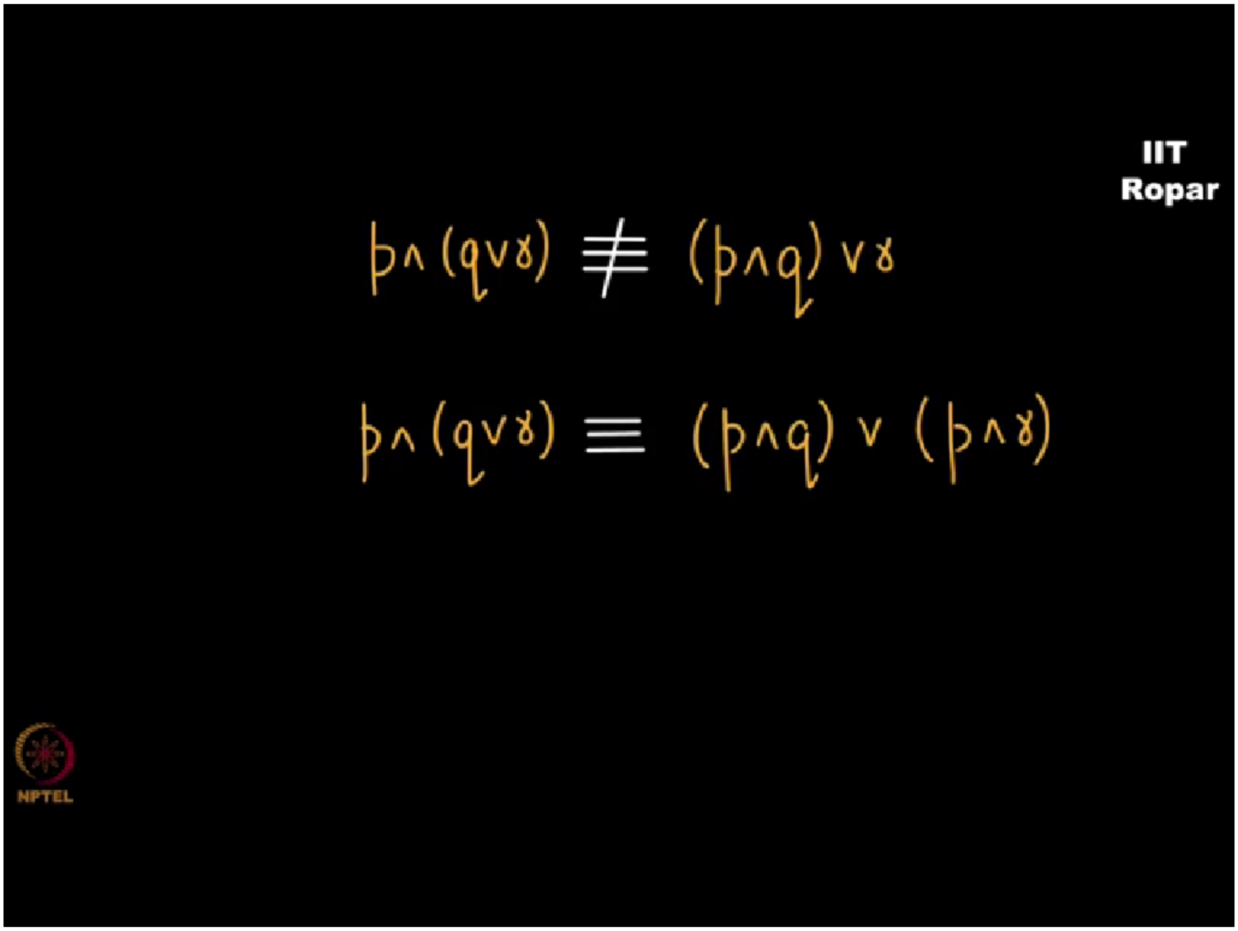
p	q	r	$q \wedge r$	$p \wedge (q \wedge r)$	$p \wedge q$	$(p \wedge q) \wedge r$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
1	0	0	0	0	0	0
0	1	1	1	0	0	0
1	0	1	0	0	0	0
1	1	0	0	0	1	0
1	1	1	1	1	1	1

Look at this truth table. Although sounds very obvious, it's important for you to observe why it is obvious. Right? Look at p AND of (q AND r). First write the values of q AND r and then this particular column you AND it with p. What are the entries that you get? Observe this.
Look at the corresponding entries of (p AND q) AND r. They are the same and hence they are equivalent. Right? Okay.

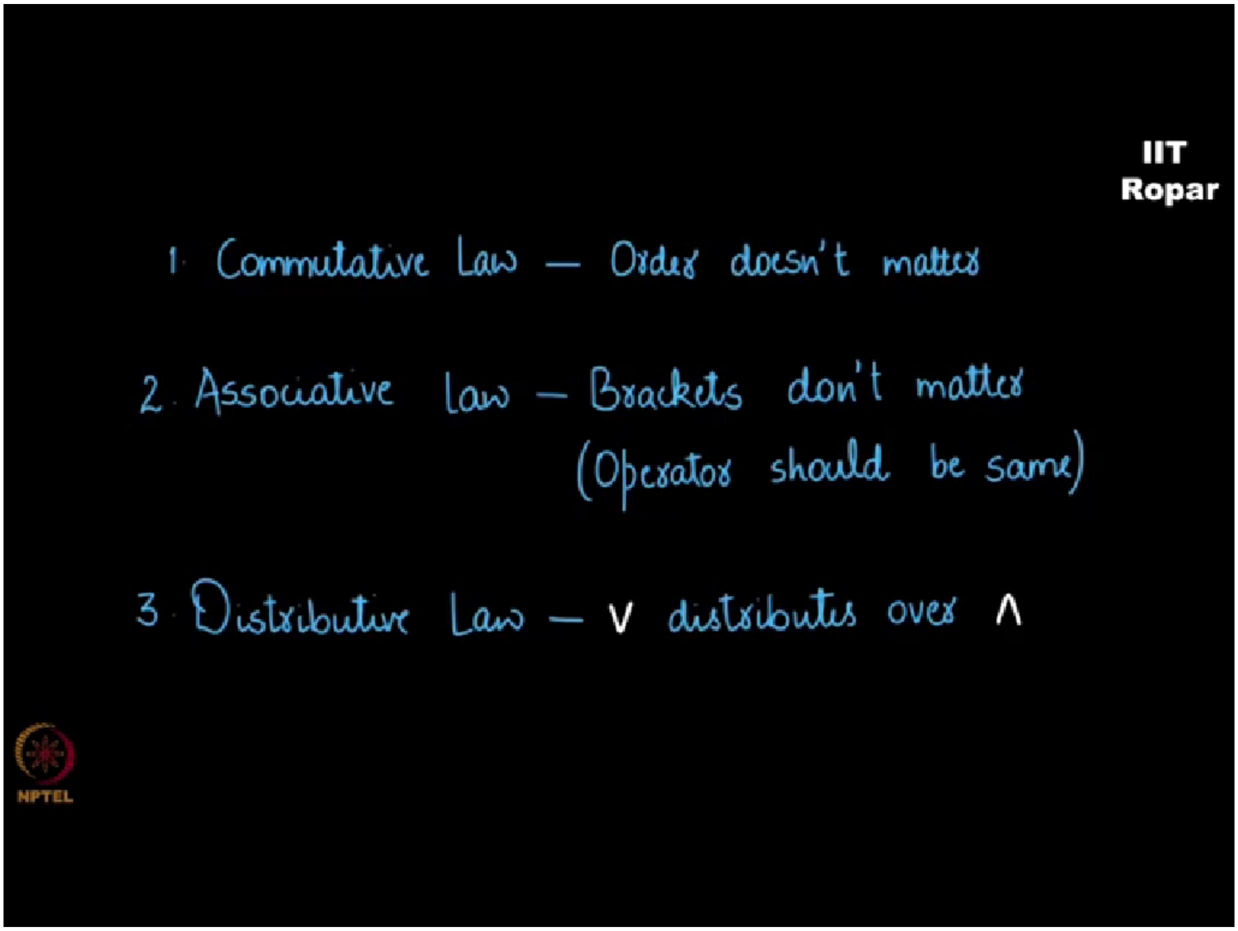


p	q	r	$q \wedge r$	$p \wedge (q \wedge r)$	$p \wedge q$	$(p \wedge q) \wedge r$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
1	0	0	0	0	0	0
0	1	1	1	0	0	0
1	0	1	0	0	0	0
1	1	0	0	0	1	0
1	1	1	1	1	1	1

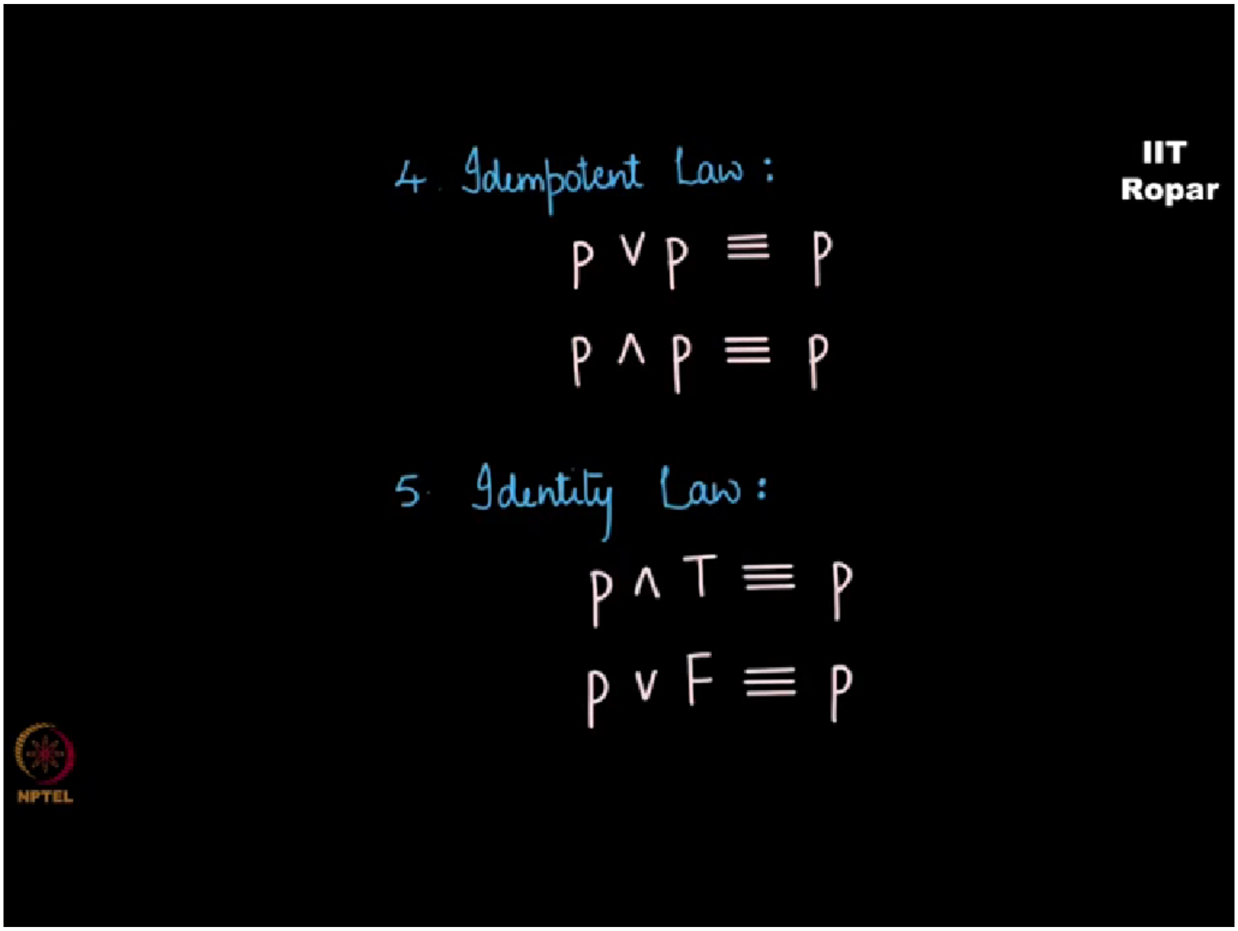
Now what is less obvious is this statement which is p AND of (q OR r). Now this is not the same as (p AND q) OR r. It's not true. Just the way we did before, this is not true. This is something else. This will be (p AND q) OR (p AND r). Okay.



These are some of the very important and interesting laws. They are given some names so that we can refer to them later.



The first one is called the Commutative Law, which means order doesn’t matter. Second one is the Distributive Law, which means brackets don't matter as long as the operator is the same and finally, this is called the Distributive Law where And distributes over OR. A word of caution, you must observe that if you replace AND with OR in the commutative law, it still holds good. Same is true with the other two laws as well.
So note that whatever we are going to say for one operator, it holds good for the other operator also and you must validate and verify it.



Now there are more laws called the Idempotent Law, which is this, Identity Law, which is this, Inverse is this, Domination is this, and Absorption is this.

IIT Ropar

6. Inverse :

$$p \vee \neg p \equiv T$$

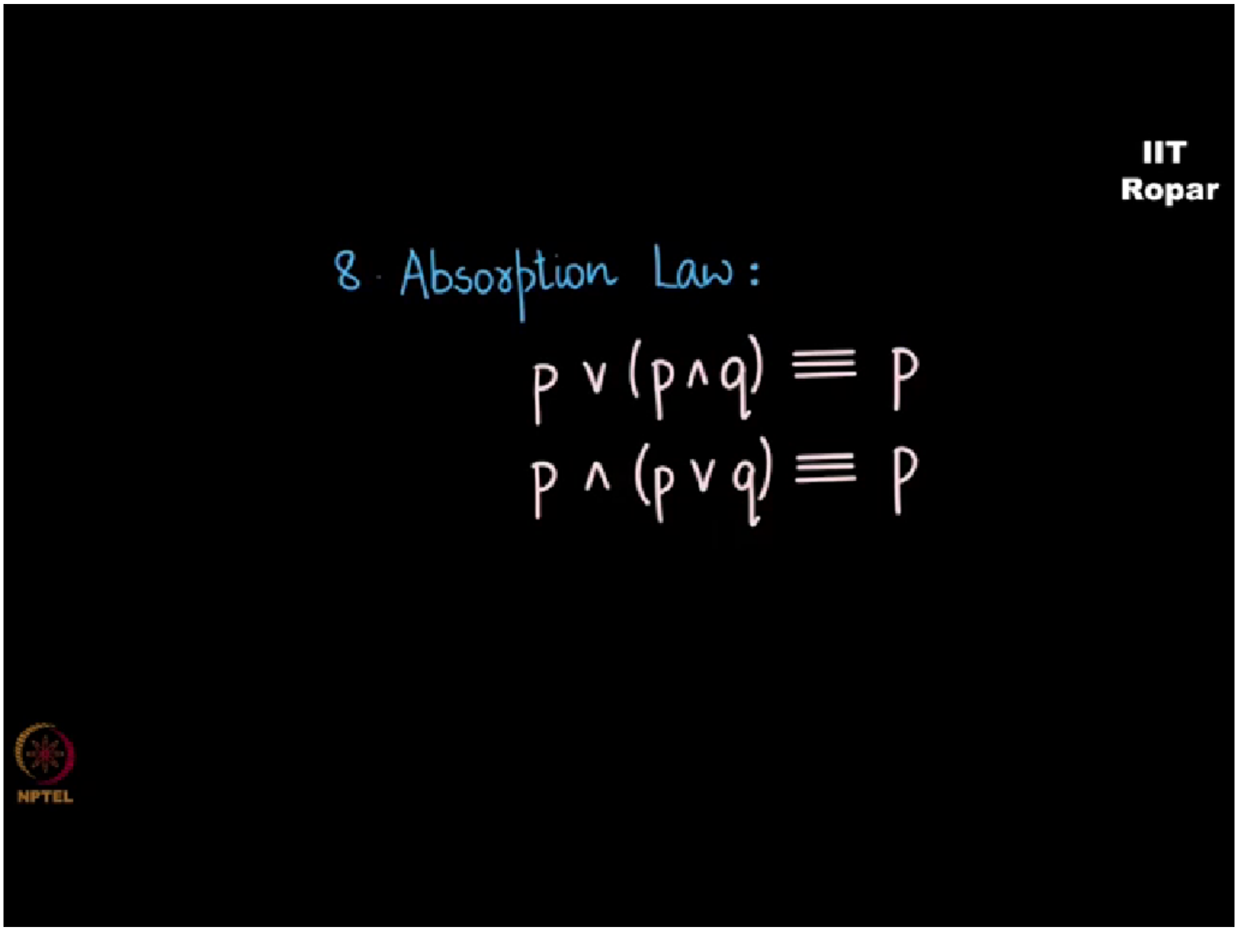
$$p \wedge \neg p \equiv F$$

7. Domination Law :

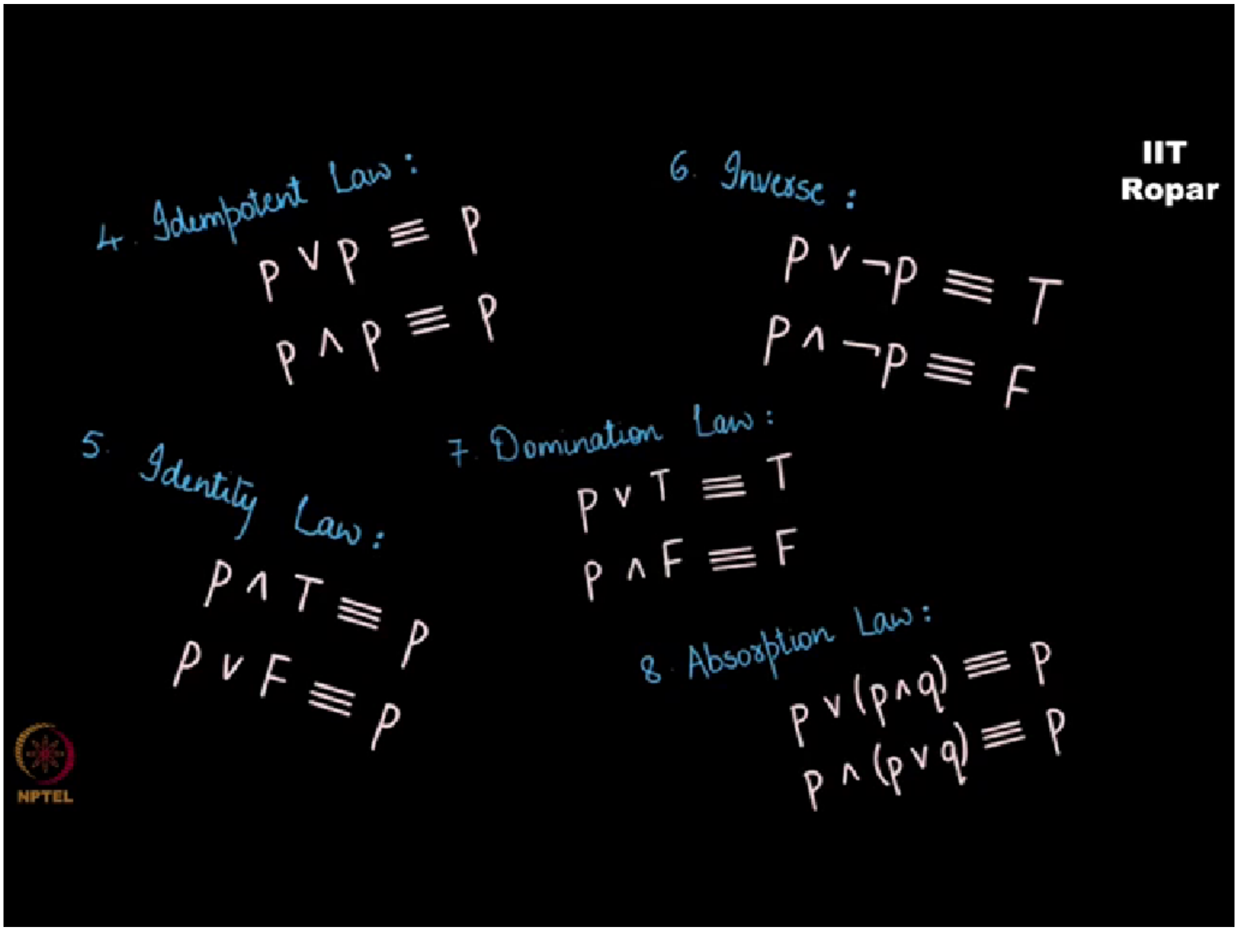
$$p \vee T \equiv T$$

$$p \wedge F \equiv F$$

NPTEL



Looks like a very complicated slide full of a lot of details.

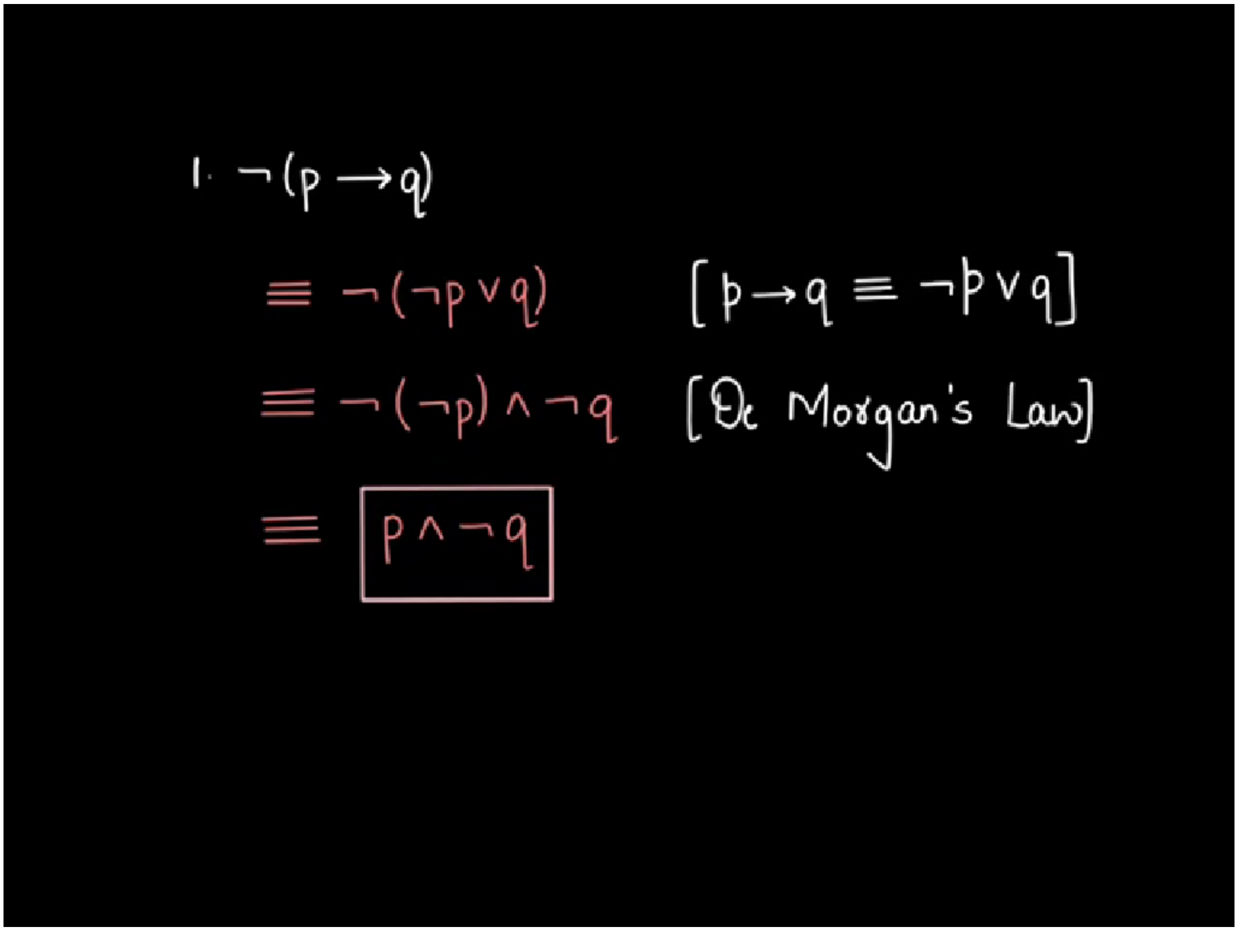


But trust me, if you look at it at one level deep, you will understand that there is nothing in it. It is just like stating a day-to-day English statement, right?

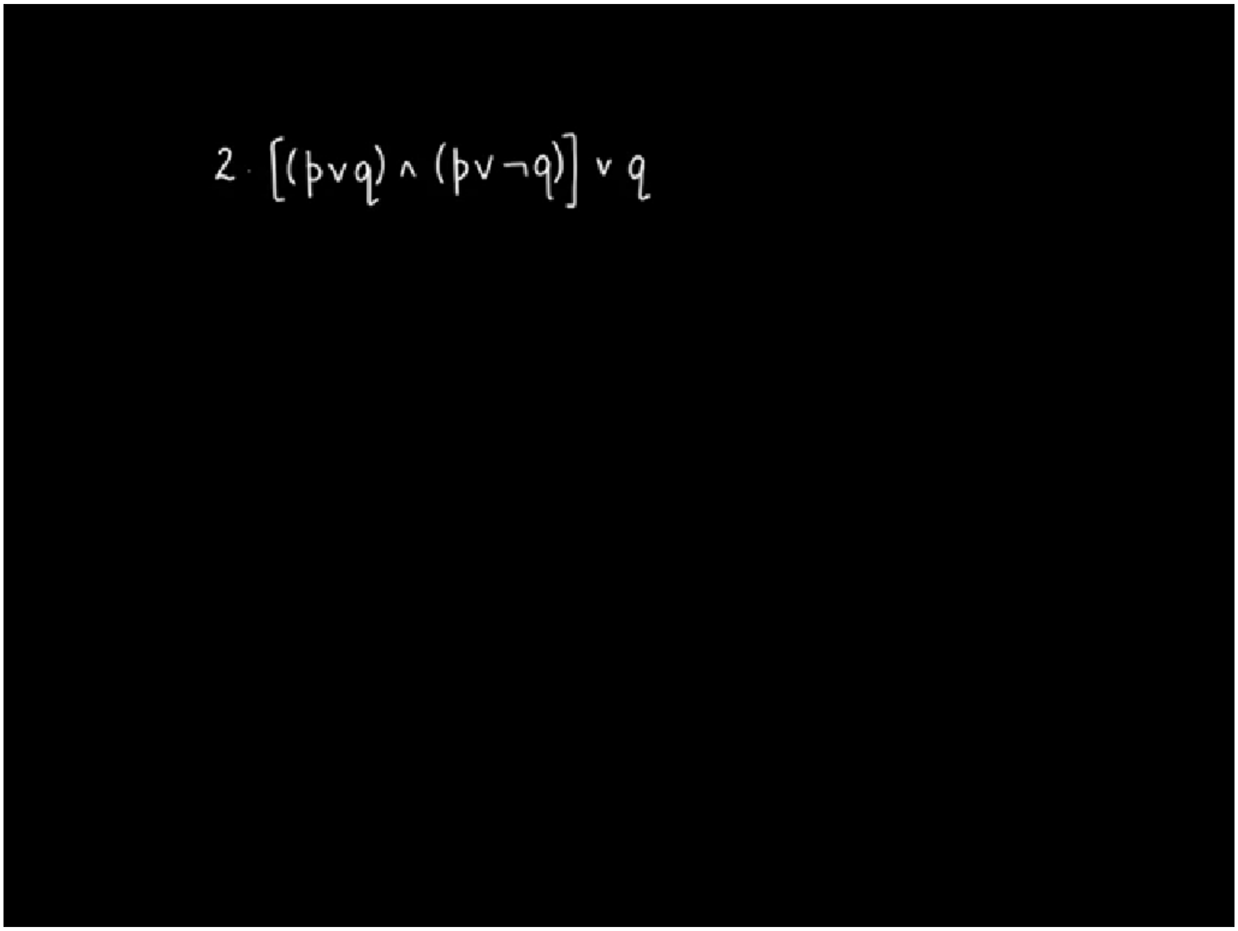
Look at Identity. It is nothing else but your a+0 is equal to a equivalent. Correct? Inverse is like a into 1 by a is 1, that sort of an equivalence, right? So just go through it. These are all very easy and straightforward. If you don't understand, you can always talk to us over the forum and we will be happy to answer, but as of now we have told you the first three laws. The rest we are leaving it to you to understand and write the truth tables and validate the equivalence.

Now that the Professor has taught all the laws of logic, let us now use them one by one and simplify some statements.

The first one, negation of p implies q. This can be written as negation of NOT p or q. I hope you know the reason. P implies q is equivalent to NOT p OR q and hence I wrote this. When I apply De Morgan's laws, I will get it as negation of negation p AND negation q. It's a NOT NOT p and by double negation this becomes p again. Therefore, this is p AND NOT q. So this is done so.



The next one, this statement looks very complicated. Let us simplify it.



So I'll break it into chunks, p OR q and P OR NOT q and a OR for this entire one with q. So what I will do is I will apply the Distributive Law over here and hence we get it as [(p OR q) OR q] AND [(p OR negation q) OR q] as and always you must sit with a pen and paper and do it with us side-by-side. Applying the Associative Law, it becomes [p OR (q OR q)], this was in the bracket earlier. Now I will put q OR q in the bracket by Associative Law. Again, here it is p OR NOT q OR q, the Associative Law again.

And then what is q OR q? Q OR q is q itself. Therefore, this is p OR q and NOT q or q by the inverse or Negation Law, it becomes a true statement. Either q is true or NOT q has to be true, and when we apply an OR operator, we get a true statement. And then I'll denote this by say T_0 just for convenience. T_0 is not something very complicated. I am just denoting a true statement here as T_0. I am applying the inverse or the Negation Law here.

And how did I get q OR q OR q? I applied the Idempotent Law if you remember. So I get it as p OR q and p OR T_0.

Now p OR T_0, any statement with an OR operator with a true statement, it's always true, and hence I will retain this as it is here. So I get it as p OR q AND T_0.

2. $[(p \vee q) \wedge (p \vee \neg q)] \vee q$

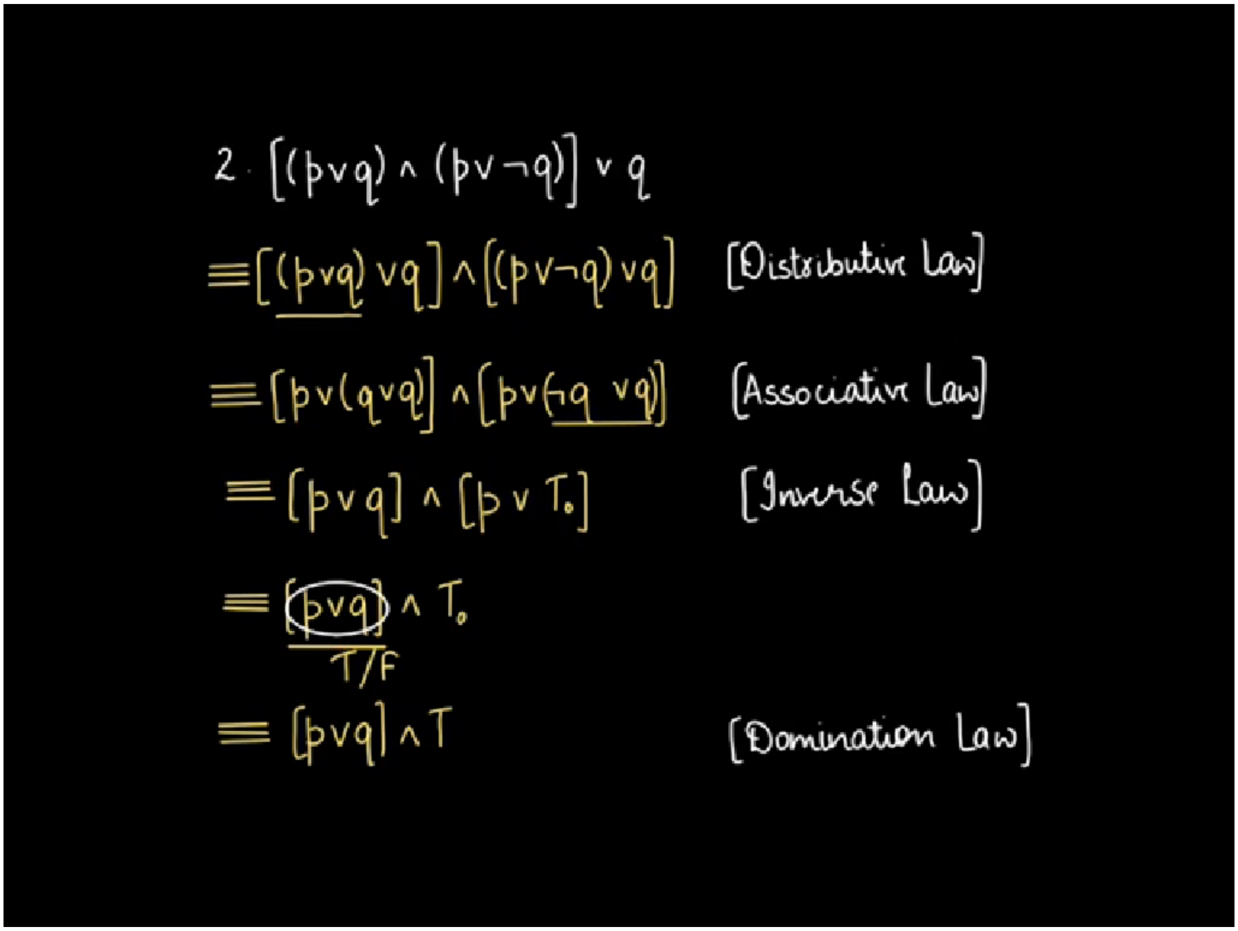
$\equiv [\underline{(p \vee q)} \vee q] \wedge [(p \vee \neg q) \vee q]$ [Distributive Law]

$\equiv [p \vee (q \vee q)] \wedge [p \vee (\underline{\neg q \vee q})]$ [Associative Law]

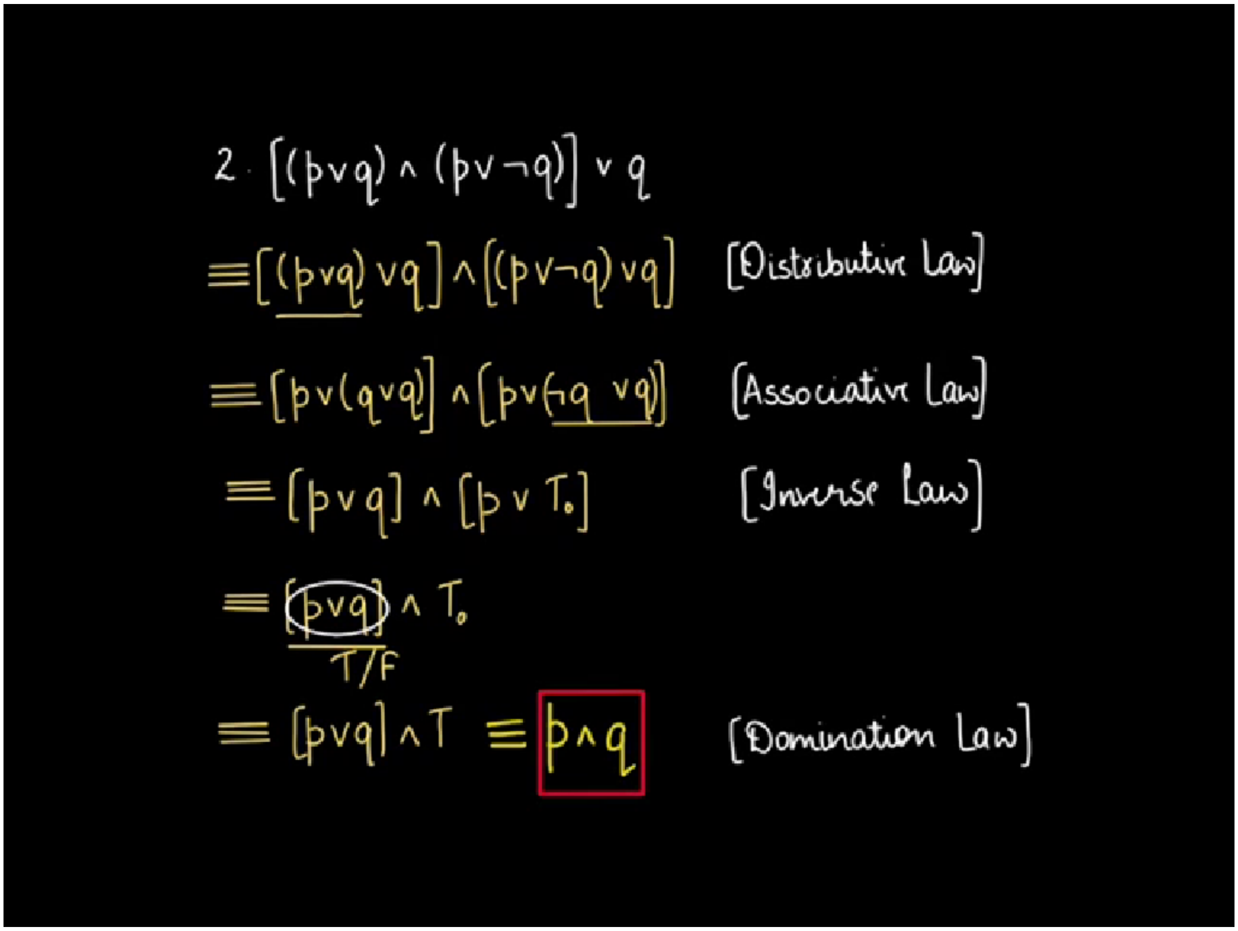
$\equiv [p \vee q] \wedge [p \vee T_0]$ [Inverse Law]

$\equiv [p \vee q] \wedge T_0$

Now how do I compute this? This depends on whether p OR q is true or false. If p OR q is true, then we get it as true itself or the other way. So it depends on what p OR q is. Therefore, p OR q is dominating here by the Domination Law. So I get it as [p OR q] AND T by the Domination Law.



Now, this value depends on what p OR q is. So by Identity Law, this becomes just p OR q. SO this entire statement is simplified to p OR q.



IIT Madras Production
Funded by
Department of Higher Education
Ministry of Human Resource Development
Government of India
www.nptel.ac.in