Introduction to Wireless and Cellular Communication

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Lecture - 48
CDMA Receivers
CDMA system Capacity

So, the SINR is given by the expression here.

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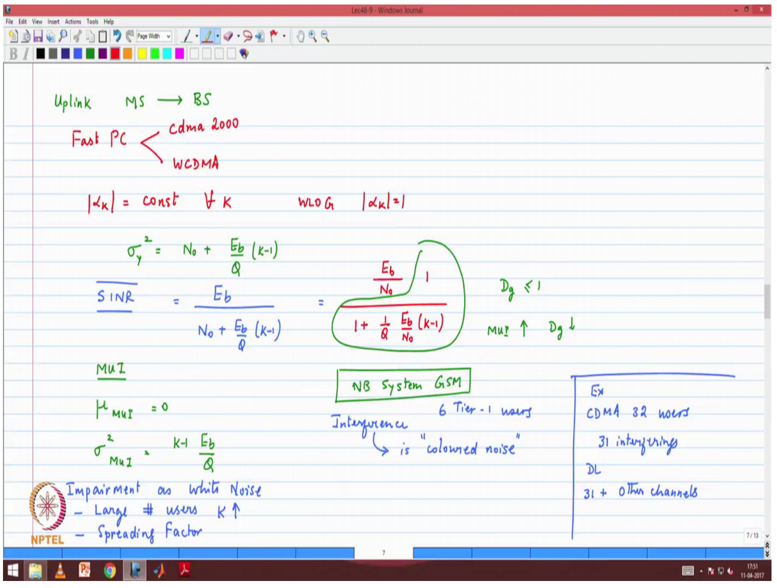

We compared it with narrow band systems such as GSM, where the impairment would be colored noise. Whereas, in this case it starts to with the based on the centre limit theorem, large number of users, the impairment looks Gaussian.

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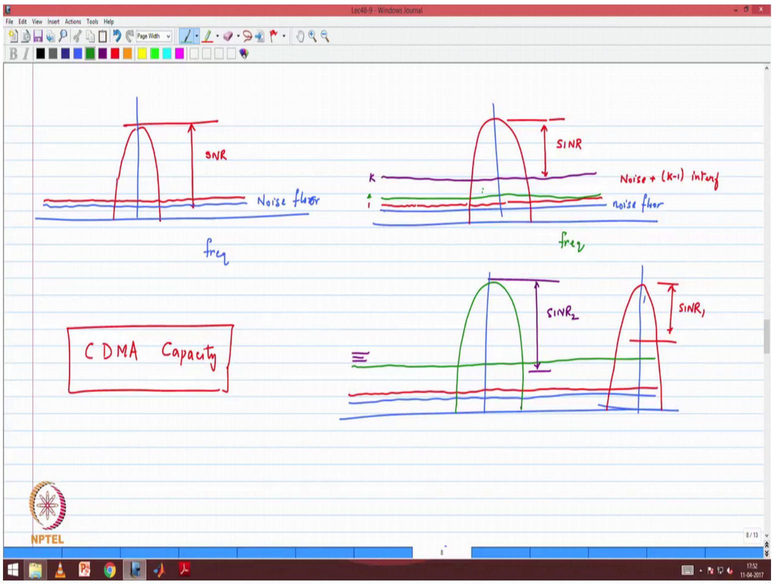

Then we sort of gave an intuitive view of what happens if one of the users is transmitting with more power than they need to. So, one of the question during the break was what are this lines mean.

So, basically if you look at it as a function of frequency, what is your power spectral density? White means it is white flat it is present there, because of your large spreading factors all of your users signal also looks like white noise, but they add to the noise. So, basically as you add more and more users the noise floor seems like it is rising. That is the way to visualize it. Now if one of the users is transmitting with more power they would raise the noise floor by a much higher amount than the other users.

Now where will it affect you when you start dispreading the users. When you dispread the user whose transmitting with more power it does not affect that user at all. So, basically you can see that the green user is fine, but when you detect the other users you will pay the penalty. And that is exactly what we find in the near far problem as well. So, basically this is a manifestation of the near far problem and it comes from the fact that the users with the transmitting with more power than they need to actually raise the interference floor.

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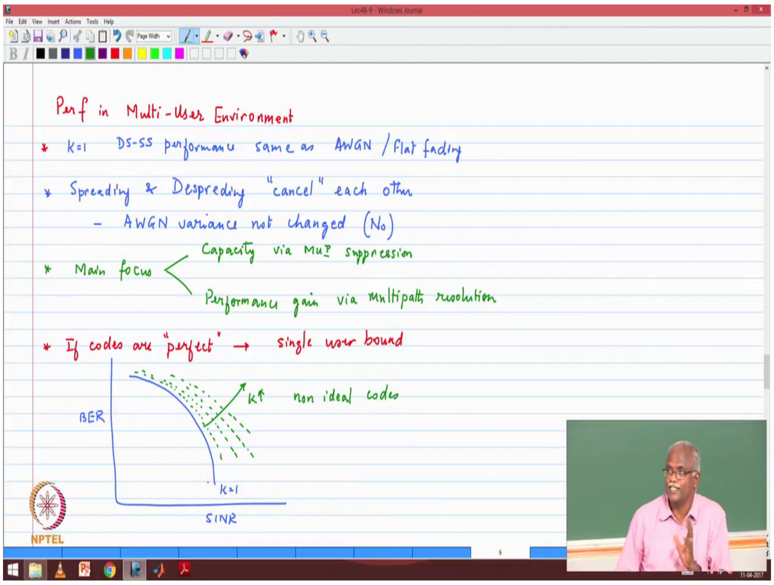

We mention that the best scenario, best performance that we can hope to achieve is the single user bound. And usually as the number of users increases you will find that the BER degrades, of course you want to make it as close to the single user bound as possible.

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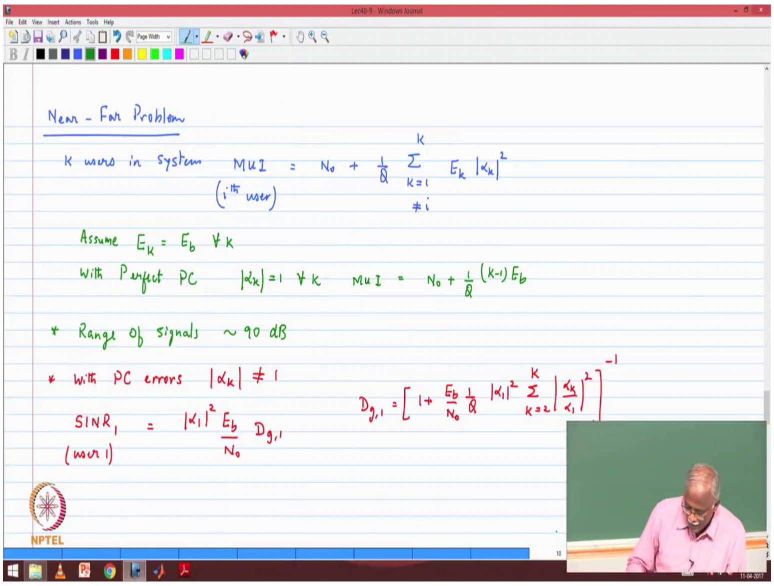

Near far problem: we can related in terms of the SINR and the degradation that the you are going to see for that particular user.

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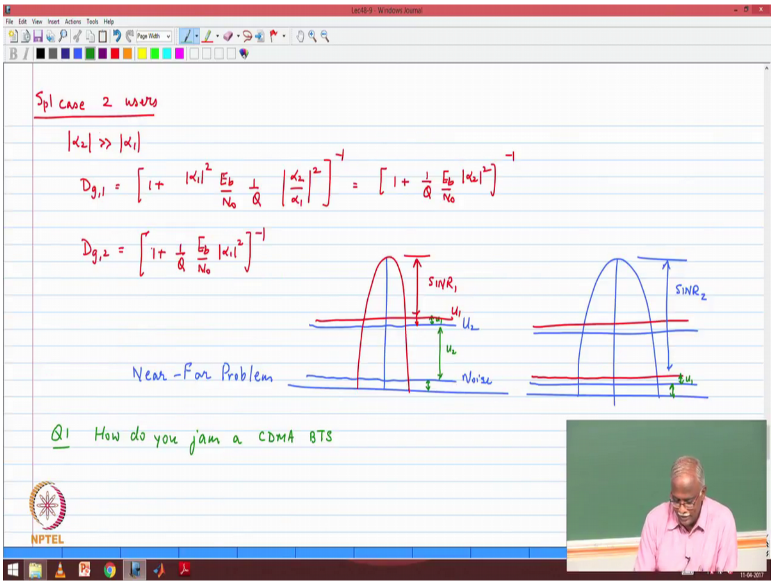

And we showed with the case of two users if one of the users is much stronger than the other then that is an undesirable situation- for us in a CDMA system. And what you will end up doing is alpha user 2 will get detected without any problem; user 1 will get affected because of the presence of user 2. And of course, that that told you how to jam a CDMA base station.

Now we move into a very interesting component which is called CDMA capacity.

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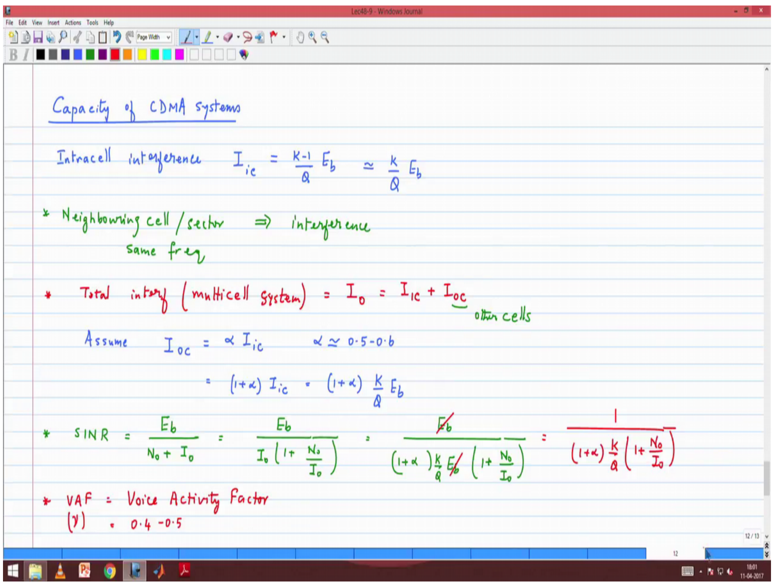

When CDMA and TDMA were sort of completing with each other, the arguments always was which system had more capacity and always argument was CDMA systems have more capacity, and what we are some of the ways in which they were looking at the capacity of a CDMA system. The following is a way capacity of CDMA systems, very very interesting discussion, very very interesting perspective.

So, we recognize that in a CDMA system interference can come from within your own cell and from outside. So, more dominant one is the interference coming from your own cell-we call that as intra cell interference. We will use the term ic subscript intra cell. And this is approximately equal to K minus 1 by Q times E b, assuming ideal power control. And we are going to make an approximation this K minus 1 is small compared to Q, so this is the same as K by Q times E b- that is just easy for us to work with minus sign this.

So, we also recognize that that is intra cell interference. Neighbouring cell, neighbouring sector all of them are using the same frequency. Neighbouring cell, neighbouring sector all are using the same frequency, so which means using the same frequency which means they are all generating interference. Ultimately all of this is going to affect the performance of my system. So, the total interference which is the cellular system which is a multi cell system you have to take into account what interference being generated by the other cells also. This is a multi cell system.

So, I naught is the total interference, this will be I intra cell plus I other cell oc other cells. Indicate that oc stands for other cells. Now in order for us to put a greater arms around the interference environment we make the following assumption. Assume that the total interference coming from other cells can be some fraction of the intra cell interference. And if you have a good system design should be somewhere between 0.5 to 0.6.

Your own cell is creating a large number of amount of interference, because every user is there this power control issues all of them are present. So, therefore, there is a certain amount of interference that is coming from your own cell. All the other cells put together create an interference which is also significant, but not as much as your own cell interference. So, the total interference can be classified as 1 plus alpha times I intra cell interference. And this is nothing but 1 plus alpha times K by Q times E b. That starting that is from equation 1, I have a substituted that the expression, ok.

In a CDMA system we are always interested in SINR of the user. So, SINR of the user is given by E b signal component N naught AWGN component by plus I naught; I naught is a total interference that is present in the system- I naught. And several interesting ways in which we can interpret this result, so I will do a series of expressions each of them will add some more insight into our understand. I am going to do the first of those expressions, I am going to factor out I naught in the denominator: I naught then becomes 1 plus N naught divided by I naught that is equal to E b and I naught is given by 1 plus alpha K by Q E b 1 plus N naught by I naught.

All these are equivalent expressions for SINR. Interesting, E b E b will cancel, so this will give me 1 divided by 1 plus alpha K by Q 1 plus N naught by I naught. Something seems wrong in this expression. How can I get SINR without E b in the picture? I naught has got the picture, some the denominator E b did not go away its still there do not worry now I would explain.

Now comes another very very interesting unique feature of a CDMA system. And if you have worked with practical systems you will really appreciate this. Something called voice activity factor. Those of you who have friends who are working in speech or maybe you yourself are working in speech you know what is voice activity factor. When a is talking to be if you monitor the conversation at any given point only one person is speaking, the other side is silent. And actually if you look at studies there is a lot of times and neither of them is speaking, I think if both of them are thinking.

So, what is an x o? There is actually there are idle times. So, then in a GSM system what happens? A is talking, b is not talking, but there is a link provided for b to a; what does that link do? Does not (Refer Time: 09:23) is just idle, is just basically transmitting some maybe some background noise. So, voice activity factor is a very interesting phenomenon. Now in a CDMA system if you do not have anything to transmit what will you do? You can transmit with low power, because you do not have anything to transmit. So, basically you will transmit very low data rate which you will spread by a large spreading factor and you will transmit at low power.

So, voice activity factor typically is in the range of 0.4 to 0.5, we use the Greek symbol nu for that, this is typical for speech. And if there is no transmission or no need for either you transmit very low rate or you do not transmit anything at all either way what will happen your signal is not causing as much interference to others right, because your signal is power level has gone down.

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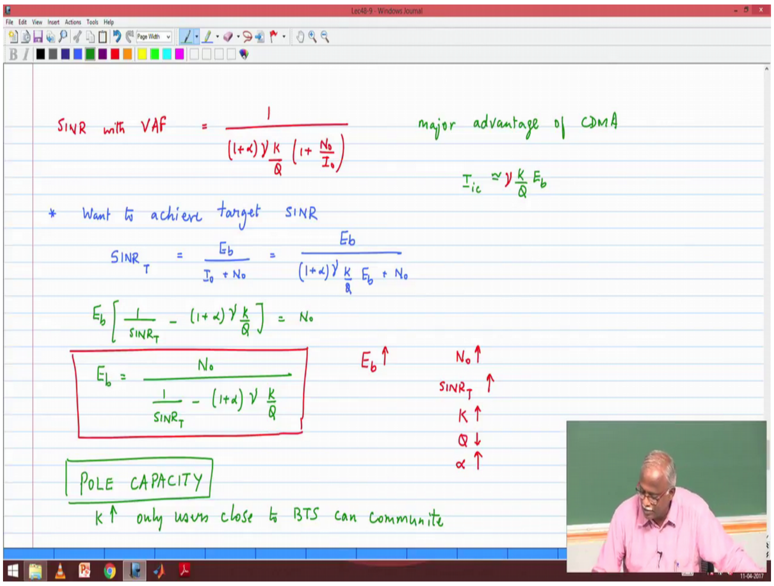

So, the SINR with voice activity factor, this is very unique to CDMA system. In a GSM system whether you have a voice activity factor or not, does not any difference your signal is not affect or capacity is not affected. Here the capacity will get affected. 1 plus alpha, that voice activity factor because if you only 0.4 of the time is present; that means, K users 0.4 of the time right, K times nu. So, nu times K divided by Q directly it is affecting your into 1 plus N naught by I naught. This is a major advantage of a CDMA system and it is not there for the narrow band systems.

Again, if you want to go back and look at the expression; basically what we are saying is intra cell interference is said was approximately K by Q times E b, if voice activity factor is incorporated becomes nu so which means your inter intra cell interference went down almost by factor of half. So, in initiative terms you can load your system double, because only half the users are active at any given time. So, if you have capacity of K you can actually load it up to 2 K, because only half the users are going to be activate any given time. So again, this voice activity factor plays a very important role in our system.

So, typically in any CDMA system the role of power control and system design is you want to achieve a certain target SINR. And the goal is every user must be able to have the same SINR so you can get a good performance for each each of the users. So, the target SINR, maybe want to write a T or there something for target is given by E b by I naught plus N naught, this is the same as E b divided by 1 plus alpha times nu times K by Q E b plus N naught. Group the E b terms together you will get E b into 1 by SINR target minus 1 plus alpha nu K by Q this is equal to N naught. The energy level with which you must transmit that particular user signal is given by N naught your noise variance that is concept that known to you divided by 1 by SINR T minus 1 plus alpha nu K by Q. This is a very very important equation that we work with.

So, the power with which you must transmit a particular user signal under the assumption of ideal power control is given by this expression. Now very intuitive to understand what is the links between these two. If N naught increases; that means your noise floor, your receiver noise signal is bad so N naught goes up E b has to go up. If you say that my SINR target has to go up, so you will find that your E b has to go up. The only way you can meet that.

If the number of users goes up then your denominator become smaller. So therefore, E b has to go up. If your spreading factor becomes smaller then E b has to go up to compensate for the reduction spreading factor. If the interference from other cells goes up, again E b has to go up because you have to compensate it in your own cell. So, you can see that all the factors that are around you and see how it links to E b. Very important, this is a one of the key concept that (Refer Time: 15:21).

We are now going to introduce another unique concept; just like voice activity factor there is another very unique concept. I will write that concept name later after we have describe the concept itself.

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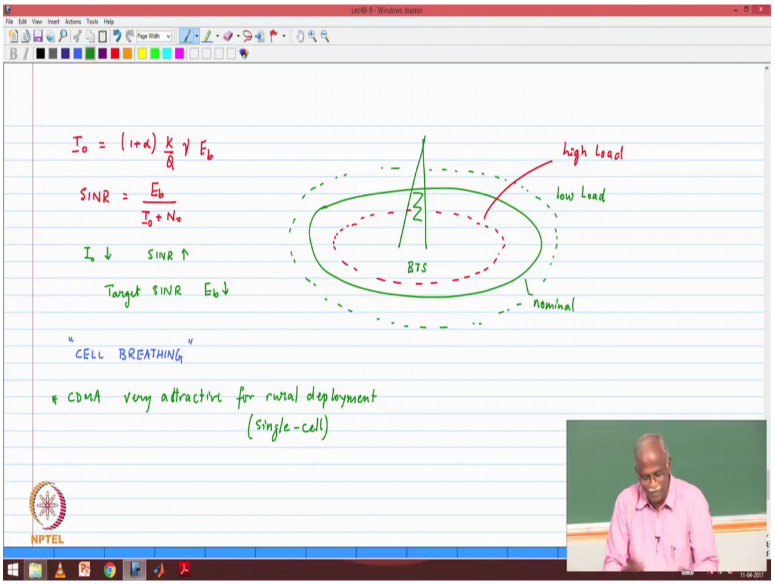

The total interference in a CDMA system is given by 1 plus alpha K by Q nu times E b. 1 plus alpha, alpha is the other cell interference, K is the number of uses, Q is spreading factor voice activity factor. SINR is always E b divided by I naught plus N naught. Maybe sort of trivial observation but it is important for us, let me just write it down. If I naught goes down what happens to your SINR; SINR goes up: two things you can do one is of course if you want maintain a target SINR. So, for target SINR you can reduce your E b that is 1. Another thing you can do what is that? If your I naught goes down. So, think of it in terms of a system this is my base station, I have a nominal capacity or nominal coverage radius and this is my nominal boundary, if I transmit at this from this place I will reach the base station under the assumption of the interference.

Now, I naught has come down then what happens, I can either at the boundary I can transmit with less power or if I am willing to transmit with a same power level what is it I can go further away from the base station. That is very very important. So, there is a boundary which is larger than my nominal boundary when the interference reduces. So, this is the boundary under low interference or low load; low load meaning a few number of user are there in the system. The corollary to that if I have more users than I have plan for my boundary is to go shorter- is going to look like only these people can communicate with my base station. So, this is the boundary under high load.

So, this is another very unique feature of CDMA system which is called cell breathing. So, when you breath your lungs go in and out, so just like that if interference increases the coverage decreases, and when the interference reduces coverage increases. So, this is the concept of cell breathing. So, you cannot have a fixed boundary for a CDMA system, it is something that is going to very based on the load. And its very important concept that is we can take advantage of.

Now this is the reason why CDMA is very very attractive for rural deployment; very attractive for rural deployment why is that? One is there not too many cells, so other cell interference is not much is present. And if the number of users small you can have a very large cell. So, one base station will cover you know several kilometer, area square kilo meter of; so attractive for rural type applications, rural deployments also if you have a single cell deployment. Rural or a single cell deployment it has very very attractive properties.

Because, see if were a GSM system, if I were a boundary whether there is other users present or not I have to transmit the same power level. Whereas, here if other user are not there in a system I am the only one in the system, I can transmit much lower power I can conserve battery so which means that there are certain advantages of a CDMA system in terms of the battery life and power all of that becomes an important element.

Now comes the asymptotic case; asymptotic case always very interesting. What happens if I keep on adding more and more users into the system, what will happen? That red circle will keep shrinking, shrinking, srinking eventually what does that mean you cannot be outside of the red circle you have to be inside the red circle. If the red circle keep shrinking, shrinking, shrinking, eventually what happens all of you around the base station; everybody has to be on the base station that is called pole capacity.

So, pole capacity is the asymptotic capacity that you can actually in a CDMA system. It is no use, because everybody is standing next to the base station but it is asymptotic limit. So, basically it is the as K increases only users closer to the base station; only users who are close to BTS can communicate with a BTS. That means, that red circle is shrinking BTS can communicate. And in the limit it will be all the users who are around the pole that is called pole capacity.

Pole capacity tells you that you cannot expect pole capacity that is the maximum you can achieve in a CDMA system. So, there is a number for the pole capacity, and we will write that down.

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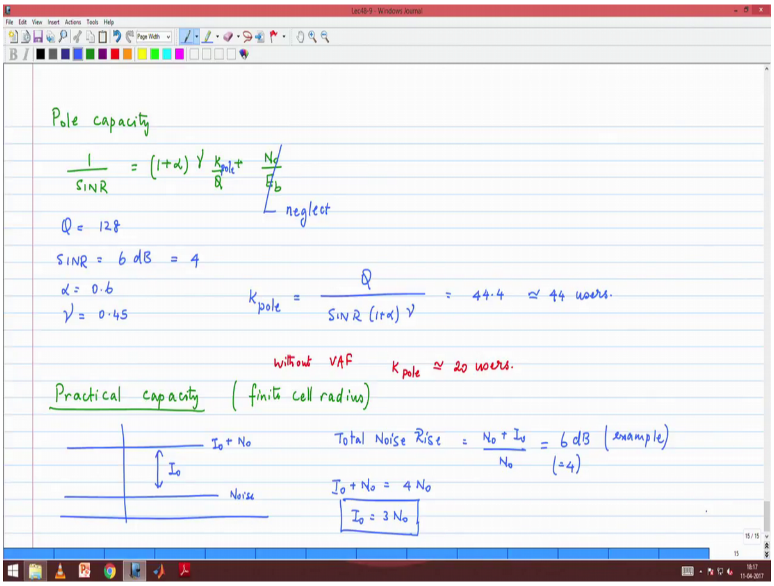

Pole capacity useful for us to write it down in terms of 1 by SINR, I mean easy for us to visualize this. 1 plus alpha times nu K by Q plus N naught by E b, ok. And 1 by SINR; that means usually SINR is a reasonable amount you design your system to have 1 by SINR this nothing but 1 by E b by N naught so this is a small quantity.

So, we will say that we will neglect this in the asymptotic case, only the interference is going to dominate your noise is not going to dominate. So, what is this one say that the pole capacity; so may be we should write K subscript pole. Under pole capacity noise is not an issue all we have to worry about is the interference that is been generated by the system. So, let us see what this number is going to be. If Q is equal to 128 that is my spreading factor my target SINR I want to achieve a 6 dB, so that is equal to approximately 4. Other cell interference is 0.6 factor, the voice activity factor is 0.45. I want to estimate what is K pole.

Basically substitute: it will be Q divided by SINR into 1 plus alpha times nu this will come out to be 44.4 which is the same as 44 users, interesting. What are the assumptions? Q equal to 128 a spreading factor. I probably have 128 spreading course available large number of spreading course, but I cannot achieve 128 that uses in my system why; because there is non-ideality is in the system. I also have other cell interference, all of that is going to play an important part in my system.

Now, another very interesting observation: if I had not exploited voice activity factor. So, like system like GSM it cannot exploit voice activity factor incapacity, without voice activity factor the pole capacity would have been 20 users. So, voice activity factor plays a huge role in a CDMA system, almost doubles the capacity of a CDMA system. But at the end of the day we are all good engineers, we ask the question is what is the practical capacity. Of course, in some it is not 44 it is something less than 44, but how much less than 44.

So, this is where our understanding of the CDMA interference comes into play in a very very useful manner. So, practical capacity of a CDMA; practical CDMA system. So, the first assumption that you say; they do not give me the pole capacity business with no use, that means I have to have infinite number of cells no use, no coverage at all everybody is at the base station. So, I need to have a finite cell radius.

So, in other words I cannot let interference completely dominate the show, I have to limit it. So, one way that we can limit it is go back to our drawing of the power spectral density if this is noise level do not let interference keep on raising, we just say there is an upper limit on the noise. This is the upper limit at which the I naught; see this I naught this level will be N naught plus I naught. So, basically you put a upper limit to which then noise the interference. So, this is called the noise rise.

So, there is the AWGN and then the interference goes on increasing the noise. So, that is called the noise rise. So, the total noise rise; noise is actually interference what we refer to it as a rise in the noise floor this is given by N naught plus I naught divided by N naught. So, we say that this can be for example; this is an example 6 dB. This is just example, we could have chosen at to be 5 dB we could have chosen at to be 10 dB, but each of them has got implication. If you say 10 dB that means noise interference will be more interference will be allowed into the system your cell radius is going to shrink. So, you have to play that you have to be careful about that.

So, you will be say 6 dB is usually a very good number for a practical system, so this is an example. So, 6 dB is nothing unique about 6 dB, something that you will have to choose as a designer. So, this is nothing but 4, this is equal to 4 6 dB is nothing by 4.

Says I naught plus N naught is equal to 4 times N naught or I naught is equal to 3 times N naught. So, the total interference that will allow in your system is going to be bounded by 3 times N naught, you are not going to arbitrarily allow the interference to increase.

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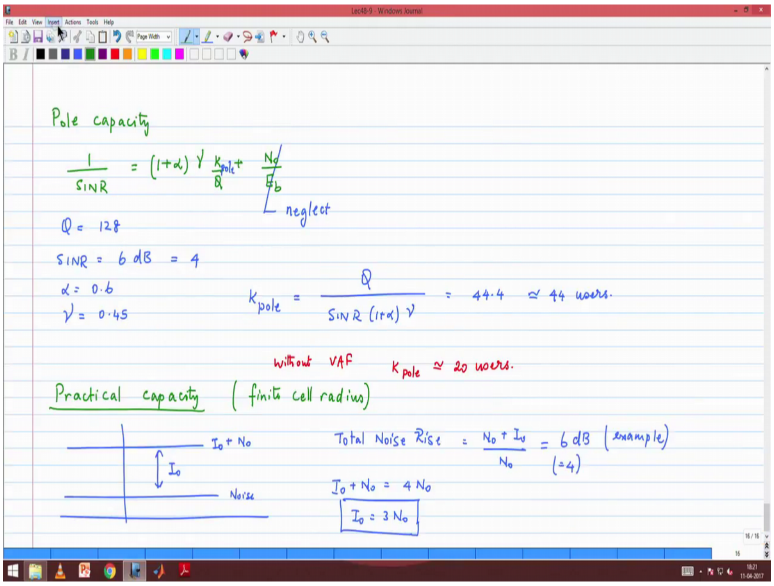

So, under this assumption the capacity will now be calculated. So, SINR is equal to E b by N naught plus I naught; N naught plus I naught is 4 times N naught E b by 4 times N naught or in other words E b can be written as 4 N naught times SINR. And we will this see in a moment why that is a useful for us.

So, I naught the total interference is given by E b into 1 plus alpha nu divided by Q into K. That is the expression for the total interference. I naught is equal to 3 times N naught E b is equal to 4 times N naught from; I call this equation 1. So, from 1 is equal to 3 times N naught times SINR 1 plus alpha nu divided by q into K. N naught will cancel on both sides, K is equal to 3 by 4 Q divided by 1 plus alpha nu by SINR, interesting interpretation. What is this quantity? That is pole capacity. So, the practical capacity is a fraction of the pole capacity, it is a fraction that is less than 1 and what determinant this fraction, the noise rise.

So, this is a fraction, fraction less than 1 that is determined by the noise rise. If I allow more noise rise I will have a larger fraction; that means capacity will increase but my radius of coverage will decrease. Therefore, we will be. So, in the previous example that we calculated if I achieve, if I set a 6 dB noise rise then this will result in practical capacity. Practical capacity is 3 by 4 into 44 which is 33; this 3 by 4 into 44 which is 33 users. This is still a very good system because able to support a large number of users in a multi user environment and achieve very good capacity and all the benefits that we have claimed for CDMA, ok.

Any questions on capacity calculations; how do we estimate practical capacity, how do we estimate the pole capacity, and what are the impacts of these quantities. Any questions?

Now comes another question that I want you to think about an answer.

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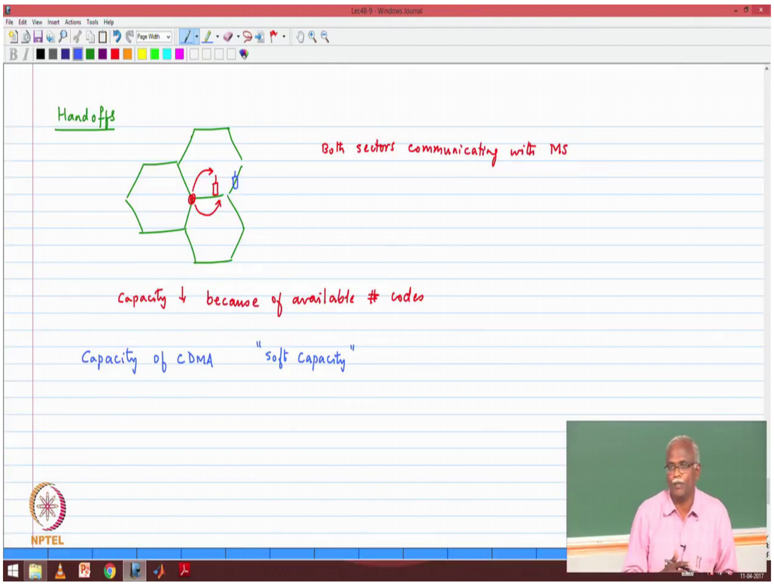

When we talked about handoffs in a CDMA system we said there can be something called soft handoff. Do you recall what soft handoffs? One mobile talking to two base stations correct. I just want to spend 1 minute on, so soft or softer does not matter basically both of them have the similar.

At that time we made a statement which said that soft handoffs is not good for capacity So since, we have already talked about capacity we need to justify the statement. So, if this is my base station and my mobile is on the border of two sectors, I can talk to the mobile through sector 1 and I can talk to the mobile through sector 2. So, both sectors are communicating so this is actually softer handoff that does not. Both sectors are communicating with the mobile station.

Now since two base stations are listening to the mobile, the mobile can actually transmit to less power. I can transmit with less power. Now if I transmit with less power capacity should increase, why is capacity decreasing. You agree know transmit with less power it is going to overall interferences going to go down, capacity is going to increase. So, basically this is a scenario of macro diversity. Right, two base stations are listening to the bare to the mobile both are transmitting, so therefore the mobile gets good signal, the base stations are the mobile can transmit with less power.

Why is the capacity going down? The capacity is going down because you are using a code in sector 1 and a code in sector 2. The capacity is not going down because of the interference your generating, its capacity is going down because of the number of available codes. So capacity, you can of course achieve certain capacity in each of those sectors, but if your codes are not present; basically used up your codes so capacity is reduced because of available codes- available number of codes.

Because, what happens is each of these mobiles there will be several of these mobiles that are on the border of the cell of coverage, so which means that all of them will want to go into soft handoff. The minute you have large number of mobiles going into soft handoff; yes they are not generating much interference, but they are actually using up codes in your system. So, that is where capacity impact comes from.

So, soft handoff is very good in terms of robustness, it is very good in terms of signal quality, but not so good in terms of the number of codes that its use, because each of these it is using one code from each of the cells that is talking to. And of course, you can imagine that if the mobile moves to a boundary of another slightly different location it may actually be in soft handoff to 3 sectors. So, which means you are you are using resources from three sectors. So, again mobiles love it, but the system does not like it so much. So, you say no, always limits the number of soft handoff system.

So, given this scenario the capacity of a CDMA system; you can visualize it as noise rise, you can think of it as codes available. Capacity of a CDMA system very different from a TDMA system. TDMA system is very simple to calculate capacity I have 8 times slots was the capacity 8 users there is nothing. I cannot go to 9 so basically it is fixed. But in a CDMA system there is no fixed number why, you can say that the voice activity factor instead of 0.4 it is 0.39; what will happened your capacity will increase. If you slight tweaking of the parameters will change your capacity.

So, it is one of those systems where we say that the capacity of a CDMA system actually has the property of soft capacity. It is not a hard number, it is something which depends on number of factors how much interference you are getting from other cell, how good is your power control, how good is your voice activity factor exploitation all of that if you assume ideal then you get a number which is very very large. Then you start to look at practical assumptions and then you put a limit and then you compare with the capacity. So, at the end of the day you still have one more parameter to to vary and that parameter is your size of yourself because of cell breathing. So, again the number of capacity of a CDMA system can never be quantified easily, it is a soft capacity. You can always increase, you can add one more user to the CDMA system with a slight penalty for everybody. In a GSM system that is not possible. You reached 8 users that is it the no more ninth user is blocked, whereas in a CDMA system you have basically an understanding of soft capacity. Any questions on capacity, because this is a very very important concept it tells you what is the difference between a CDMA system and a TDMA system or any other any other system.

So basically, the last part of our discussion on CDMA systems; again I wanted to make sure that whatever you have studied in digital communications kind of ties in together.

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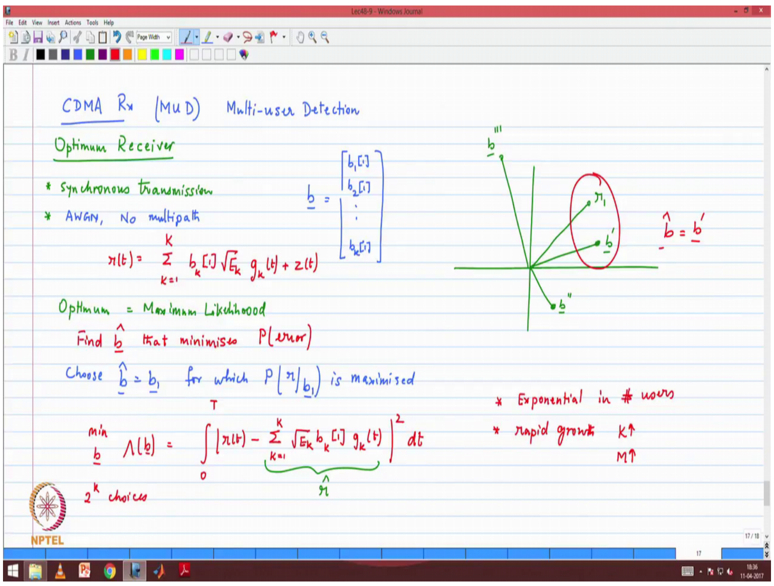

So, we will look at CDMA receivers more closely; CDMA receivers. So, basically multiuser detection receivers; where do you use where do you need multiuser detection it is at the base station. So, multiuser detection what are our options in terms of the different receivers that we know from digital communications, how do we apply them, and what is the complexity that we are talking about. Because as I mentioned to you CDMA systems are far from trivial: one is because you have got all these sequences, the long code, short code all of that.

But even if you have overcome all of those hardware challenges at the end of the day to actually be able to detect the signal is non-trivial. So, the first thing that we would like to look at is; what is the optimum receiver for multi user detection. Again we will look at a simplified problem, because the full complex problem is harder, but this gives us enough of a insight into the system. So, the multi user figured that we have drawn earlier in the in the lecture please keep that picture in mind. So, it is synchronous transmission uplink-synchronous transmission and we will assume that the channel is AWGN channel so no multipath; it is an AWGN no multipath made it as simple as possible. And even for this the receiver is so complex that we would be hard for us to implement. Let me justify that statement.

So, at a given time each of the K users will transmit a bit user 1 transmits b 1 b subscript 1; the 1 with the 1 within brackets is the time index the subscript is the user index, b 2 1 b k 1. Since all of them are transmitted at the same time I need to be making a decision on each of those b 1 through b k. So, the received signal r of t is given by summation K equal to 1 to uppercase K b K of 1 root E k that is the level the signal consolation scaling times g K of t plus z of t. Right, g K of t is a spreading sequence spreading waveform.

And very quickly let me remind you that the optimum receiver for us is the maximum likelihood receiver. And the maximum likelihood is described as follows: it is that receiver that will find the sequence or the vector b hat is the estimate that minimizes the probability of error; so the most likely sequence that was transmitted that minimizes the probability of error- that is the criterion that we are using.

So, in other words to say to you know more formal manner it is to choose that b hat equal to some vector b 1 for which the probability of r given b 1 is maximized- b 1 is a vector that is the statement. So, you basically want to find out that vector of values which gives you the most likely probability of receiving the vector r the or the received signal r that you have opting. It is not a complex statement, basically it says if in a two dimensional case if this was r and if one combination of vectors; let me call that as some b dash says that the if you use b dash this is where you will this is what your r vectors will be another things is this is b double dash and b triple dash, ok.

So, each of these three vectors or combinations says this is what r would have been had you transmitted b hat. So, you look at these three combinations and say well you know this is the one that is closest in Euclidean distance, and therefore my b hat I am going to declared to b equal to b dash. That is maximum likelihood detection. So, you look at all combinations of vectors that are possible, for each of them you construct r, and then see which is the one that is most likely to have occurred.

So, here is the problem statement. You have to minimize over all possible combinations of the vector b; vector b if this is there are K and its binary. There are two possible K choices that are possible. You have to look over all two power K choices that is the minimization step. And you have to look at the following the objective function.

So, objective function is minimized overall b such that the integral from 0 to t r of t that is the received signal minus the constructed signal for you have the combination of transmitted bits; K equal to 1 to uppercase K root E k b K no b 1 sorry b 1 of; I have to be careful with my notation here b I have used K- b K of 1 g K of t magnitude squared d t. So, for every choice of b I have took; this is like saying this is what r hat will b if I use this particular transmitted vector of bits. So, you are looking at the Euclidean distance between r of t and r hat of t and its not strictly Euclidean distance it is actually the distance squared an integrated. So, basically you are looking at the squared distance from the received vector to the reconstructed vector, ok.

You have to consider all two power K choices. So, some observations about the m l is that it is exponential in terms of the number of users; that itself is a source of complexity. Now if you do non binary it is even worse because then it becomes not two to the power k, but it becomes m to the power k. So, there is a very rapid growth in complexity; rapid growth if K increases or if you are constellations size increases both of them will cause exponential growth in capacity.

So obviously, this is a system that is complex, is the best but it is going to be very complex. So, let us see how to get a little bit more insight into the system.

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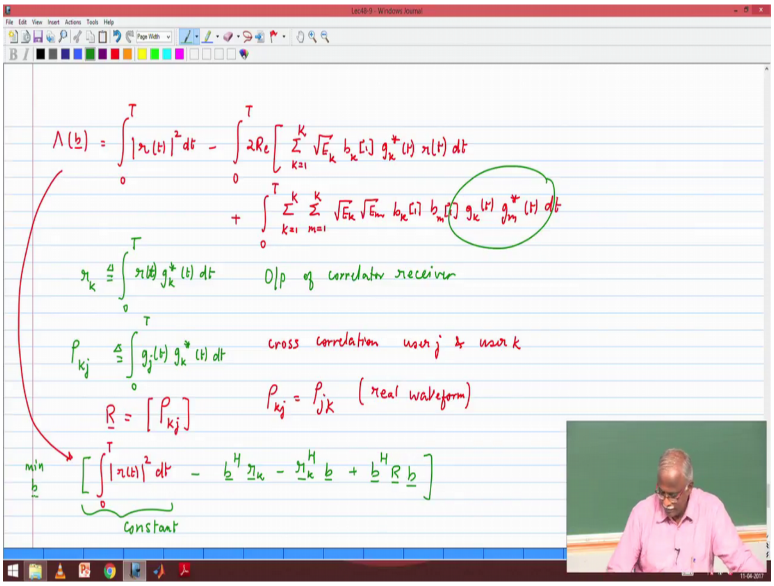

So, the objective function gamma of a particular vector b this is given by integral 0 to T, r of t basically I am expanding the term within this. This will become r of t magnitude squared dt minus integral 0 to T, two times real part of summation K equal to 1 to uppercase K root E k into b K of 1 g K star of t times r of t.

R of t dt and then there is a third term which is integral 0 to T basically the term and its conjugate say becomes a double summation; K equal to 1 to upper case K summation m is equal to 1 to upper case K square root E k square root E m b k of 1 b m of 1 g K of t g m star of t dt. So, basically whatever was within the modules square I have expanded it out. Again this may look messy, but insides are there in just one step. Basically I am going to define I am going to make certain observations about this.

So, when I take r of t the received signal and passed through a wave form correlated with a wave form of user k dt. I am going to call this as r subscript k; that means, received signal correlated with the k- th users spreading wave form. So, this can be thought of as the output of a correlator receiver. Then another one comes from this term, where it is a integral between 0 to T g j of t g K start of t dt. I am going to define this as a correlation coefficient between the waveform of user j and user k; K is the one that I am using to detect. So, first subscript is the waveform using in the detector and j is the uses spreading wave form.

So, this is a cross correlation; cross correlation between the spreading wave forms of user 1 and user j and user k; user j and user k at the wave form level. And I can write down cross correlation matrix which is basically these correlation terms and it can also verify that row kj is the same as row jk, because we are using real wave forms so because of the real wave forms. So, I basically get a correlation matrix which is symmetric which has certain symmetric properties.

So, this expression can now be re written as follows: the first one I re written as 0 to T mod r of t whole square dt, making the observation that no matter what be I choose this term does not change, this is always a constant. And no point in keeping this constant I can going to drop this constant in the next step. But before that this term, this middle term can be re return in terms of the quantitities that we just now defined. This can be written as b hermitian times r k minus r k hermitian times b when you have both those terms you get a real part and again if you make sure that sure that you look at the expressions you should be able to get.

This one last time can be written as b hermitian R b. I am going to drop the first term and written only the terms that are remaining.

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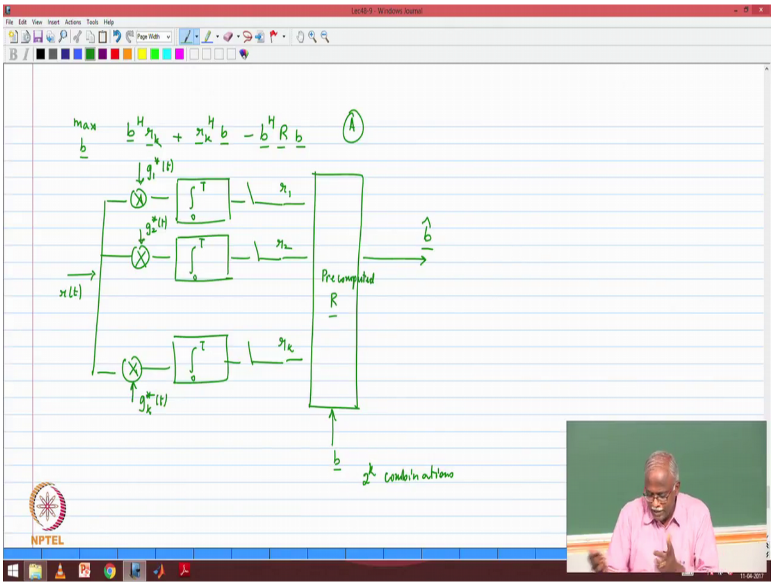

So, basically what am I trying to do I am trying to minimise over all possible b. The same problem can be re written as leaving out the first time. It can be written as maximization overall b of the fallowing of b hermitian times r r k minus sorry plus r k hermitian times b minus b hermitian R b. So, basically it is leaving out the first term whatever is left with them I took at the minus sign, so minimization is became a maximization. So, this is what we have to work with. This is this is optimum receiver you are not made any change.

So, it is very interesting to see what is it that we need to what we need to implement this. So, you take r of t; it is very intuitive very the result is interesting. So, I correlate with g 1 star of t and followed by and integration from 0 to T and then sample it and this output will be r 1. Same r of t I multiply by g 2 star of t followed by an integration from 0 to T this becomes r 2. And the last of the correlates g subscript K star of t followed by an integration sampling this becomes r k.

So, this is how you would generate all the r k's. So, you take it into a block for which you have fed in all the r k's, all the different combination of b's you feed into this one and what does this term already need to know it should know what r is basically they crossed correlation. That is a pre computed r is already present inside. So, based on this you can then say which is your best b had by applying this equation; the applying a.

So, you for any r you calculate r 1 into r k you have to try all 2 power K combinations; 2 power K combinations you have to try. And decide which of them gives you the maximum value. So, let me given example for you to try and then we will close to that.

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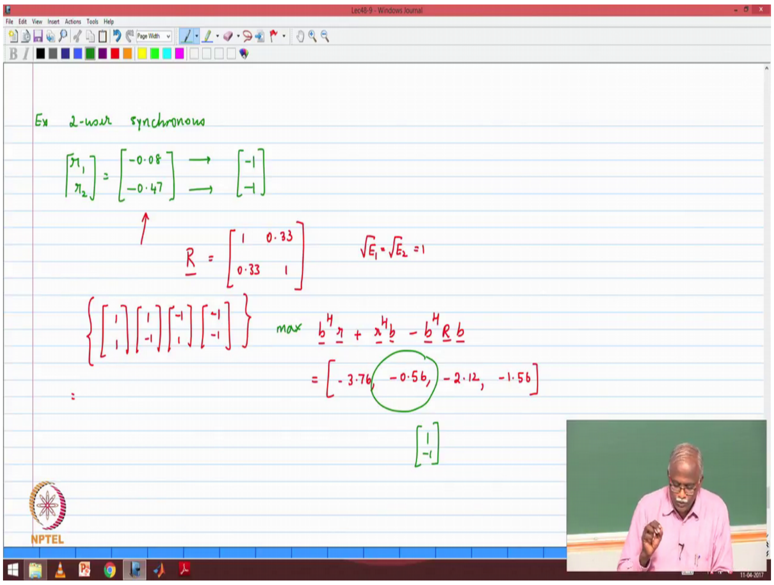

The example that I want to try is a simple two user case; two user synchronous CDMA case and this is the following information that we have. The receive vector from the correlation of the two. So, basically r 1 and r 2 this comes out to be minus 0.08 minus 0.47. So, in the absence of any other decision mechanism what you would do you take r of t you correlate with the wave form for user 1, then you take r of t correlate with user 2 and you get these two values.

If you now say make a decision on this the simplest ways if I got minus 0.08 the decision should be in favor of a minus 1. So, this one I will say that the transmitter symbol was a minus 1, this one also transmitted symbol was a minus 1, because what did I; I took the received signal I correlated as r 1 the wave form of user 1 assuming that it has suppressed user 2 I got a value minus 0.08 which I have to map to minus 1.

Similarly, I do this. Now the optimal receiver says- this is good you got this information, but you totally forgot about the cross correlation between the wave forms. They were not ideal. So, the cross correlation was is given to us as follows: 1 0.33, 0.33 and 1 is symmetric. And also said you can assume E 1 is equal to root E 2 equal to 1 does not difference in terms of their constellations.

The possible sequences are 1 1; 1 minus 1; minus 1 1; and minus 1 minus 1. Four possible combinations are there, you would have to try all of those. So, for each of them you try b hermitian times r plus r hermitian times b, basically you get scalar values for each of those then you have b hermitian r b. So, you know what r is, you know what b is are you can substitute and you can verify. So, these values for the four vectors that we have we have used come out to be minus 3.76, it is request you to verify that minus 0.56 minus 2.12 and minus 1.56. Please use this value and go, ok.

And I am trying to find the maximum and I am interested in doing the maximum. In this vector the maximum is minus 0.56, others are more negative. And this corresponds to 1 minus 1. So, actually your original decision was not correct, the maximum likelihood receiver says its 1 minus 1. And why did you make a mistake in the original. In the original case you did not take into account the interaction between the spreading sequences, you assume that the spreading sequences were perfect and did the separation perfectly which it did not.

So therefore, a maximum likelihood decision is going to give in some cases a very non intuitive decision, but does based on the interaction between the spreading codes. So, I just shown the seed for what is the optimal receiver, it is definitely not what you would do intuitively, because intuitively just says correlate user 1 a way form put a threshold and says to r the plus 1 or minus 1.

Now, you have to take into account the interaction between the spreading codes and that is going to make. Now the question that we then ask is well this is too complex; even for two user case its non trivial. So, how do we make it into reasonable complexity? Now from a maximum likely hood type of scenario, the next lower complexity but reasonably good performance is something called zero forcing receivers? We will look at what is a zero forcing recover in the contest of a CDMA.

Zero forcing receivers have got a drawback in that they do not care about the noise. The noise may get boosted in the process of getting a zero forcing. So, what is the best which does not boost the noise MMSE receiver, so we have ml zero forcing MMSE; what all that you have studied in (Refer Time: 58:23) communications sort of comes together that to say- oh this is the complete way of handling CDMA receiver.

So, the ideal receiver for us in the MMSE receivers which is good in terms of complexity, but in order to get appreciate the MMSE receiver its good for us to have started with the optimal receiver to talk about the zero forcing type will; in the case of

CDMA its not call zero forcing it is called a decorrelating receiver, but essentially zero forcing and the other term other type is the MMSE.

So, just part of the tomorrows next lecture will be the decorrelating receiver, a part will be the MMSE receiver. With that we complete our discussion on CDMA, we will move in to the mimo framework.

Thank you.