Introduction to Wireless and Cellular Communication

Prof. David Koilpillai
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 11
Cellular System Design, Capacity, Handoff, and Outage
Cellular System Capacity, Trunking

Good evening, we will begin the lecture 11. In today's class, we would like to look at a couple of examples, which highlights for us our understanding of the cellular system design using hexagonal cells and the tessellation. Also like to introduce you to looking at the aspects of signal quality as we mentioned cellular systems are interference limited, I would like you to get a feel for what are some of the matrix that are used, and how to interpret that.

We move from designing a cellular system to understanding the capacity of a cellular system that takes us into the discussion on trunking, again it is a term that is probably some of you are familiar with, but we will define it and associated with the trunking concept the grade of service. We will spend a fair amount of time on a case study, a very interesting case study of a particular city, which is where three operators have deployed each of them has taken a different strategy would like you to analyse and think about what are the methods.

To complete our basic understanding of the design of a cellular system, the last component will be the aspect of handoff, which is necessitated by the mobility of a user. As you move away from the coverage of one base station, move into the coverage of another base station, the mechanisms that must have been placed for you to have a continuous call to be processed, so that is the plan for today's lecture. Again I hope you have hand copy of the outline of the course of the lecture which was uploaded before the start.

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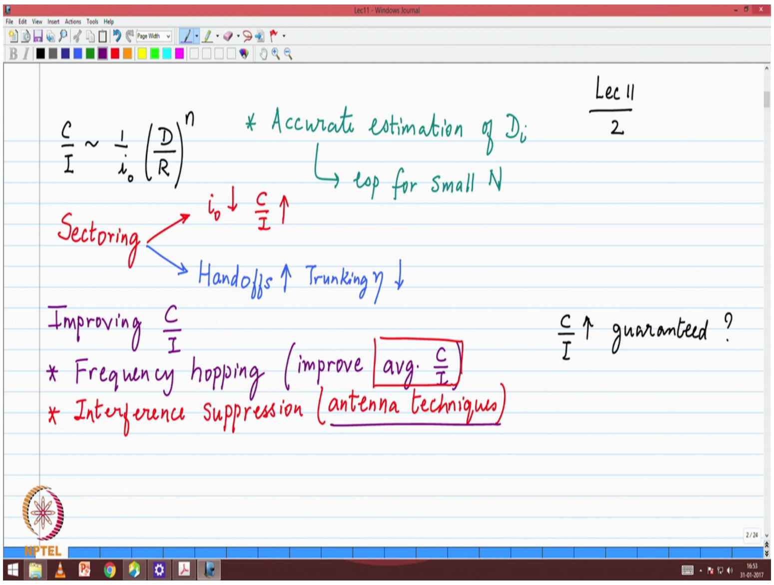

So, the basic problem or the task of design is for us to look at the aspects of C over I, C over I estimation and we can look at it in terms of the approximation that we have 1 over i naught D by R raised to the power n, n is our path loss exponent. Again if you are talking about small cluster sizes, we saw that the impact of the distance between the interfering base station and the mobile is an important one. Sectoring as we saw gave us ability to reduce the number of tier one and interference automatically that means, C over I would go up. The drawback of sectoring was that you would have more handoffs, but more than the handoffs, you are actually going to affect yourself in terms of the trunking efficiency as we will see in today's lecture

Other ways of improving C over I, one we said of frequency hopping. Basically each of the users in the system are changing their frequency on a using a pseudo random frequency pattern. Frequency pattern is the centre frequencies come in a pseudo random pattern. Am I always guaranteed that the C over I in a frequency hopping system is better than a system that does not do frequency hopping, is am I always guaranteed that, so basically is C over I improvement is it guaranteed for every hop, yes or no? The answer is no, because there maybe another user using the same spectrum and maybe somebody who is much closer to you, therefore, there are some hops where you are going to have possibly worse C over I than a system that you did not hop.

But the most important thing to note is that you have improved the average C over I on average you will not get stuck with the bad interferer who is constantly bothering you, you will average out the interferers. And therefore of all the interferers in the system if you could average your interference across them, it is going to be better than the worst case. So, again this is for to take advantage of to remove the worst case interference scenario. And of course, we have not we did mention it in the context of the introduction that mimo systems have multiple antennas at the base station and at the mobile and those are techniques that we will exploit to use to do interference suppression. So, again that would help us improve the C to I calculations.

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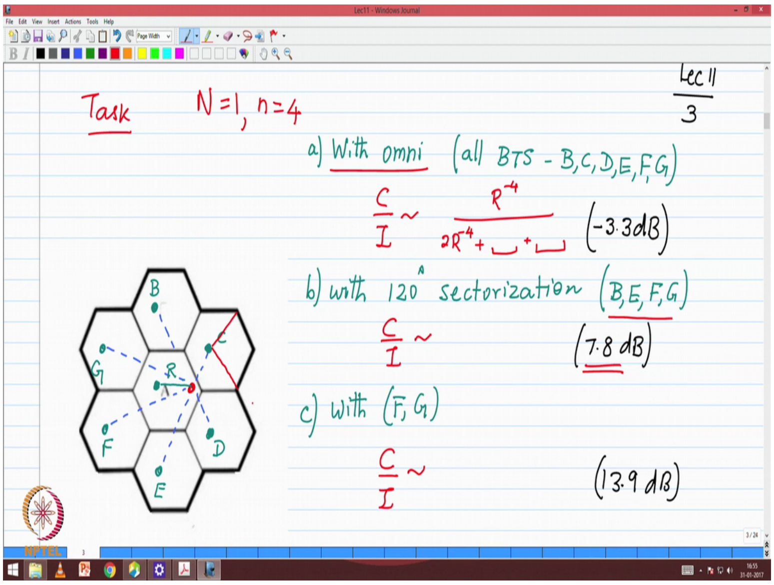

Now, first example or task that I would like to you to consider is a system, which uses one cell reuse pattern. So, basically I have taken the first tier of the interferers, the six cells around it. And looked at the interference scenario, basically the interferers are all the base stations around you, your target base station my figures seems to more slightly offset. Assume that all the base stations are in the centres, and this red dot is at the vertex of that hexagon. So, basically you have to compute the distances from each of the interfering base stations.

Now, let us take the first case. When you have the scenario of Omni all the base stations are going to interfere six of them B, C, D, E, F, G. C over I, if you calculate basically it would be the sum of the interferences. So, it would be R raised to the power minus four divided by two times R raised to the power minus 4. Notice that there are two base stations C and D, which are at a distance of R from there plus there are two more terms which you can compute based on the geometry. But again given that your desired signal is being over powered by two interfering signals at the same distance, you expect your C over I to take a very significant hit and which is what it does minus 3.3 dB, and again please go through the exercise. So, useful illustrated exercise.

Now, if I introduce 120 degree sectorization, so basically we will assume that we are going to do 120 degree sectorization facing the eastward direction. So, basically all of these base stations which are using the same frequency are going to be radiating in the eastward direction. So, basically do the sectorization for all of them. Notice that C and D immediately are removed from the picture, because they are radiating a wave from the mobile. But the remaining four are still present, but they are not at the same distance. So, therefore, you do see some advantage, you see that sectorization actually has bought you a lot for a n equal to 1 system gives you 7.8 dB.

If you did something clever to make sure that you could eliminate the B and E as well maybe if the mobile moved in a little bit something of that form then C over I actually improves to 13.9 dB. So, you can actually get fairly good signal to noise ratio provided you are you understand the impact of sectorization and of interference. Again, if there are any questions let me know I am assuming these are fairly basic calculations, you will be able to do them fairly easily.

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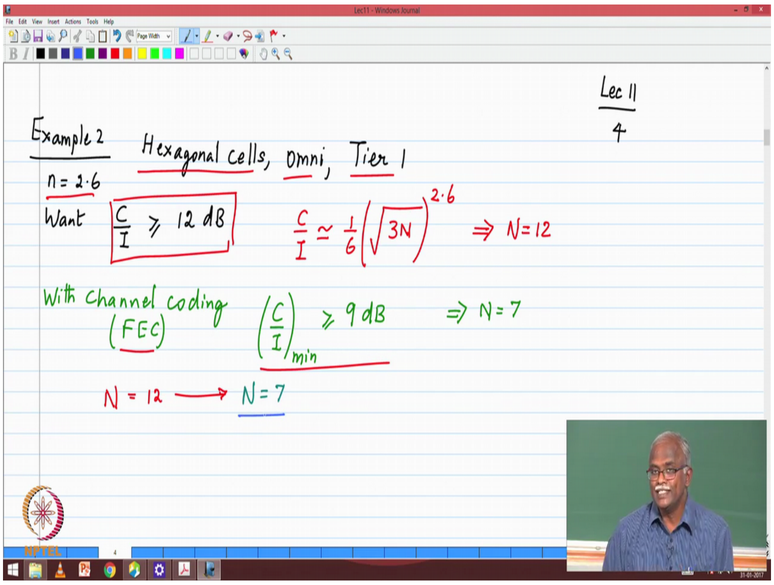

So, the typical problem that you would encounter in a scenario like this, you would be given condition such as hexagonal cells, Omni directional tier 1 maybe sometimes tier 1 and tier 2 you are supposed to take. Path loss exponent is given to you. And you want to calculate the cluster size. Given that you have to satisfy a certain C over I condition. Again this is already something that we have done the purpose of this example is not to repeat that.

So, this calculation resulted in a cluster size of 12. Now, given that you are working in a environment where the channel will introduce a lot of errors, the introduction of error correcting codes or forward error corrections or channel coding is very advantageous. So, assume that you have introduced channel coding into your system which means that you have to compress your source a little bit more; the little extra head room that you have you use it for error protection. But given that you have error protection, you are able to achieve the target performance at C over I of 9 dB. If that is possible if you can design your error correcting codes to work at a target of 9 dB then you redo a calculation. Basically go back and plug in saying that your targets (Refer Time: 08:53) only 9 dB, and you find out that now we have a n equal to 7 as the as the new system. Now this is a correct way a valid way of reducing the cluster size.

So, again one of the ways you already have one of the tools that you already have is the use of coding to achieve the desired reduction in the cluster size. And again the benefits of reduction in cluster size I do not have to tell you, you have already looked at it several times. So, many ways of either reducing the interference or making a system more robust all of these are come into picture when we want to design the entire system. Let us move on. Now I would like to you to consider a real life example and basically put all the pieces that we have discussed so far.

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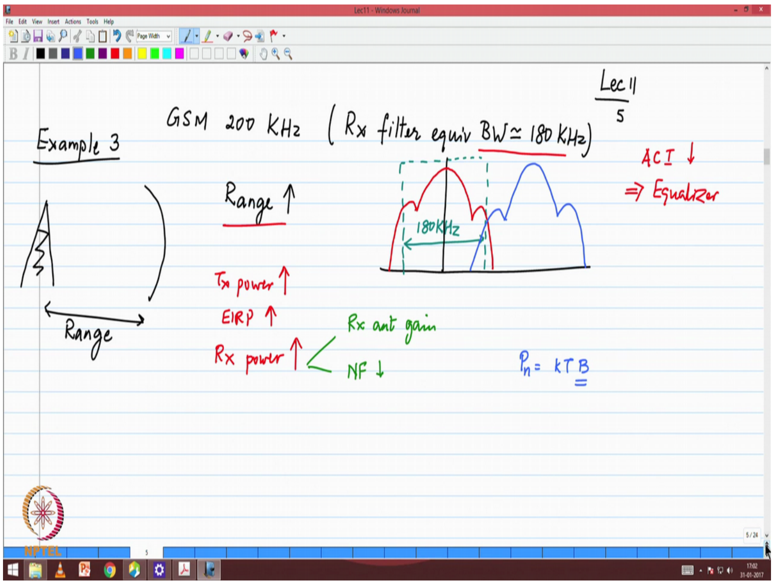

GSM uses 200 kilohertz channelization and the spectrum looks somewhat like what I have drawn. Basically the spectrum has a shape that has got a bump and then the there is a falloff rapid falloff. So, the neighbouring channel if I place it at 200 kilohertz spacing you can see that there is a fair amount of overlap that is your adjacent channel. Now, in GSM, it is customary that your receiver will have a receive a bandwidth of 180 kilohertz I just want you to why is it 180 kilohertz, why not 200, what are the benefits. I want to reduce adjacent channel interference I want to protect myself against adjacent channel interference.

Now, what is the price that I pay loss of signal is that the only price is the only the loss of signal. What happens to the noise, what noise go into the receiver because you have your signal and you have actually filtered your noise, so you will get coloured noise. Colourization of noise or colouration, the channel basically is introducing memory into these into the system, so which means that you have to have an equaliser. So, this would basically mean that you would have to have an equaliser. You have gained in terms of the adjacent channel interference, but the price that you have paid is that your receiver will now become more complex.

Now, why not make it 160 because there you will start to lose more of the signal and you will have higher complexity and by the way reducing it to 160 may not buy you much in terms of adjacent channel. So, keep that picture that there is one reason why we choose the receive filter. So, the example that we now want to look at is I have designed a system, I have basically laid out my system and I find that I have achieved a certain range. But now the goal is I want to improve the range, I want to increase the range, what are the tools that are available to me, what are different ways very quickly, what are the what are the ways in which I can improve the range.

Student: Transmit power.

Transmit power ok, I can increase transmit power I would say it as EIRP which means that you could increase transmit power, you could increase transmitter gain you can reduce feeder losses you can do many things. So, basically transmit power can be raised or EIRP effectively you can increase the EIRP. What else can I do, what do I do if I want to improve my receive signal power? R x signal power if I want to increase my receive signal power what are the ways in which I can do that? I have to increase my R x antenna gain that is one way I have use a low noise figure. So, I can reduce my noise figure that will effectively give me a lower noise floor which will give me an improvement in terms of that.

And possibly I can think of the R x filter and one way, but the reduction and the receive filter gives me a benefit is that if you remember we do the P n calculation, P n is equal to K T B that is the receive filter. So, basically by reducing the C filter I will give a little bit of boost to my signal to noise ratio because the bandwidth has come down, but I pay a price for that. So, I would be very, very careful in terms of the or change in the bandwidth of receive filter, but that would be yet another of doing that. And of course, frequency hopping to reduce the interference, but that would not affect my range. So, basically it would mean that I have to either increase my signal or reduce my path losses. Again path loss, we do not have much control over. So, this were the some of the ways in which we would look at the task. So, those are the three quick examples.

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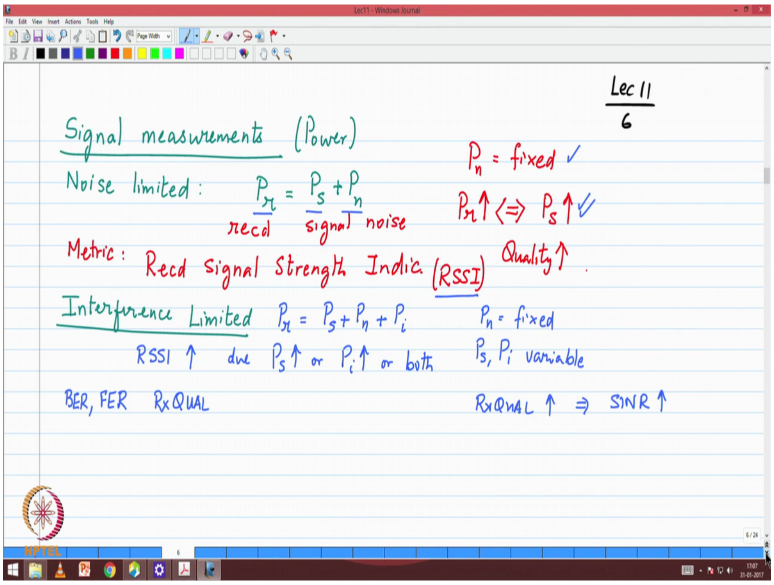

So, now we move on to looking at signal quality. And I want you to start thinking along the lines of what we have studied so far and apply that. So, let us first take a look at the noise limited case. Noise limited case, if I put a power meter at the after the receive filter, basically it will be signal plus whatever noise is present. So, the received signal power is the signal power plus the noise power, whatever comes after the receive filter, so that would be my total received power, the signal component plus noise component. And in this P n is fixed because I know once I receive a bandwidth, I know the P n is fixed. So, if I find a an increase, if I find a way to increase my receive signal power or for some reason my signal power goes up then I am very happy because that tells me that is because my signal power went up because the noise power has not changed, the noise power remains.

So, one of the matrix that we use in noise limited systems or in any wireless system is a measurement called the RSSI. I am sure you would have come across the term. It stands for received signal strength indicator. So, basically whatever is the received signal you take its signal strength or the power of the measure the power of the signal, it is a combination of signal plus noise you know that the noise is fixed and therefore, it gives an estimate of the how much signal is present. Now, I want to move from there to a interference limited scenario. So, in interference limited scenario, if I do P r, I am going to find P s it will be a signal component plus there will be a noise component plus there is a interference component. So, like before P n is fixed I cannot do much about this, but P s and P i are things that can change because of the environment. So, these are variable.

So, if I now look at an scenario where RSSI is increased for through some mechanism, I find that I have received should I immediately celebrate? No, because it could be signal or it could be interference, in both case in mean in one case I am very fortunate in another case I am actually quite unhappy. So, this could be due to P s increasing or P i increasing that would also increase or both. Now, if both increase does not help me because signal and power both have gone up C over I have not changed, but if P i goes up I am really in trouble.

Now, how do I distinguish between these different scenarios; and for that, the only way we can do it is by looking at the bit error rate or the frame error rate. So, there are certain fields in which we can do those measurements. And looking at that we get a metric called R x QUAL received strength signal quality basically that tells you what is the bit error rate or frame error rate and again different cellular systems will define it. But this basically will tell you that the signal is good or bad which means that you know if R x QUAL is high that means, your signal interference plus noise ratio is high.

So, R x QUAL is good implies SINR is good. So, one hand you have RSSI then you have R x QUAL and you have a combination of these two which are a very useful mechanism for us. So, is everyone clear about the signal quality measurement in an interference limited environment and the interpretation of that, because now we are going to try our understanding of this in a very very interesting example.

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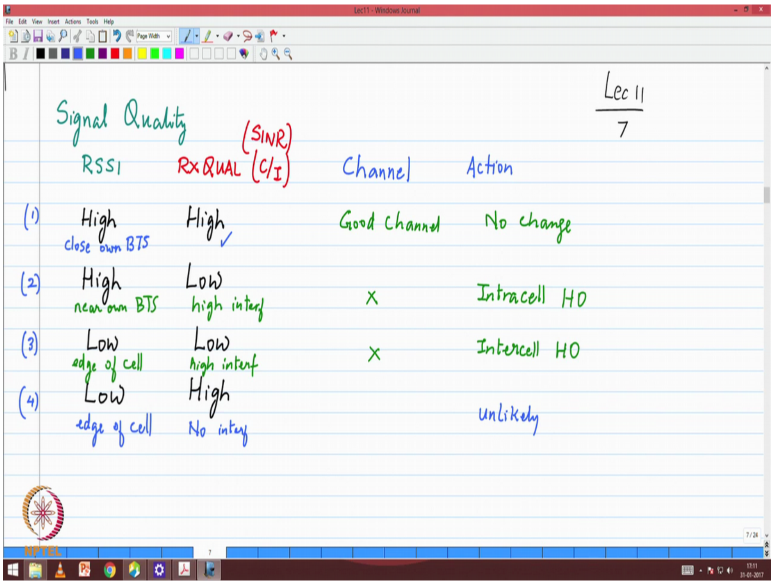

So, I am doing measurements on my receiver. I have a scenario case one where RSSI is high and R x QUAL is also high. So, what type of channel is this, the scenario if RSSI is high that means, I am probably close to my base station. So, again I am interpreting this. So, close to own base station which is a good thing if I am close to my own base station I am far away from the interferer. So, this is also a good scenario. So, the assessment is that this is a good channel, very good, keep the channel do not do anything no action required no change, keep it as is. Second scenario my RSSI is high, RSSI is high and my C over I is low is that possible? Give me a scenario that such a situation happens

Student: (Refer Time: 19:04).

If you are living in a around the (Refer Time: 19:08) basically you are on the fifth or sixth floor you have a very good path to your channel, but you also have path to the interfering channel. So, basically this is a situation where you are not far from your own base station you are near on BTS, but there is high interference not a good situation. Do I need to change to another base station? No, because actually I am close to my base station; all I need to do is change my frequency, so this is a bad channel I need to do an intracell hand over to the same base station, but you ask for another resource another frequency because then you will see probably a better scenario.

Now, let us move to the third case. So, the second case was a case where I am close to the base station, but I am seeing a lot of interferences. Third scenario RSSI is low. RSSI low means P s is low, P i is low everything is low. So, basically that means, you are far from the base station far. So, basically we will call it as edge of cell, edge of cell. Now, your C over I is low. What does that tell you?

Student: (Refer Time: 20:23).

You are closer to the interfering base station. So, this is a scenario of high interference, but there is no point changing to another frequency within my own cell because that is not going to help me. So, this is a bad channel. So, in this case, I better ask for an intercell handover. I will go to another base station and maybe change because I am too far from my own base station. And then the last scenario, my RSSI is low; that means, its possibly edge of cell and I have high C over I. What happened, magic can this channel happen?

Student: Yeah.

Or the what conditions?

Student: Transmit power (Refer Time: 21:13).

So, basically this is this has there is no interference right or interference is very low. So, again you are not happy because your own signal is not very strong, but this is a highly unlikely scenario. Because this basically tells you that you are in a noise limited environment, because basically you are far away from your cell, but you are still not seeing interference that means, your that means, the other base stations are not creating any interference or basically interference. So, this is a unlikely scenario in a cellular system that is heavily loaded the such a scenario rarely happens. So, both very quickly you can look at it and say unless both RSSI and R x QUAL are high, you have to do something about it, so quickly look at which way you want to do and respond to the situation that will help you handle the system. Any questions?

Student: What is the action required?

Which one?

Student: Low RSSI, high RSSI.

So, what would you do if it is a noise limited system, you are far away from your own base station, you would ask for handover, intercell handover. The intercell handover, but again being a interference limited system to say oh, there is no interferences a sort of contradiction, but if you are fortunate to have that then very good just ask for a intercell handover and to a closer base station, but it is a scenario that is probably somewhat unlikely to happen.

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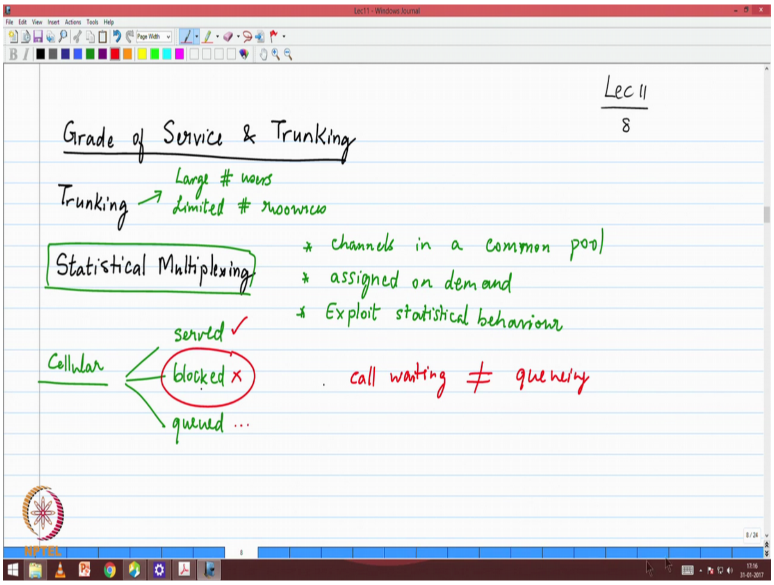

Now, this sets the stage for us to move into our discussion on the capacity of the system. We would now design the system, we know cluster size, we know all of the elements that are associated with it. Now, want to understand the ability to work with these systems. Trunking, the term that is used comes from the earlier telephone system. So, basically this means that I have a large number of users, large number of users, and there is a limited number of resources. So, what I do is I pool my resources remember that the ticket counter example, I treat the users as one large pool, and I try to service them. So, basically using the mechanism of pooling the users, pooling the resources that that way of thinking about it is the notion of trunking.

In today’s terminology we do not use the notion of trunking it is referred to as statistical multiplexing. And again we will try to establish the link between the tool. So, our preliminary understanding of trunking since that the channels are in a common pool all the channels in a common pool. Now, when there is a demand for a channel that is assigned. So, channels assigned on demand and then released once the requirement is end upon termination it goes back to the pool. Now, what is it that the whole notion of trunking is the whole notion of trunking says that all my users hopefully are not going to ask for resource at the same time, so that is notion say there are large number of users and their usage patterns are similar, but hopefully they are not going to ask. So, the notion that not everybody is going to ask for at the same time, so basically we want to exploit statistical behaviour of the traffic.

Now, we use this notion of statistical multiplexing very heavily in internet traffic, because we have a large number of users who are connected to the internet. And we want each of them may ask for download of a large file, but the statistical multiplexing says is as long as they all do not ask for it each of them will see good data rate. So, basically exploit statistical nature or statistical behaviour, it can be behaviour; if you talk about the users statistical nature when you talk about the traffic. So, basically this is the notion of these system.

Now, let me just define the context and then we move forward quickly. So, we have a cellular system which fits into this mechanism large number of users, limited number of resources. Now, typically in this type of scenario when a user is asking for resources, if the resources are available, the service the user will be served. Either you are served or you are blocked or the third scenario is that you are queued; you are in a line waiting for the service to happen. Served means that means the resources were available you are readily given. Blocks means you are said, no come back at a later time, this means wait. Now, is the cellular system a queued system or is it a blocked system, a system that does blocking or queuing.

Student: Blocking.

Blocking, but what about call waiting.

Student: (Refer Time: 26:51).

Is not that what queuing is supposed to be you say wait till think about it.

Student: Source, have being allocated to that (Refer Time: 26:59).

Excellent, excellent point the resource has already been given to you. So, it basically you are not blocked, the only reason the call did not complete is because the user was not. So, call waiting is not a queuing system. So, call waiting is not same as queuing. Our system that we have for cellular is a system that will block. If it does not have resources, it will block, the block the user keep that picture in mind well, let us just quickly define our measure of capacity and develop a quick intuition for the system.

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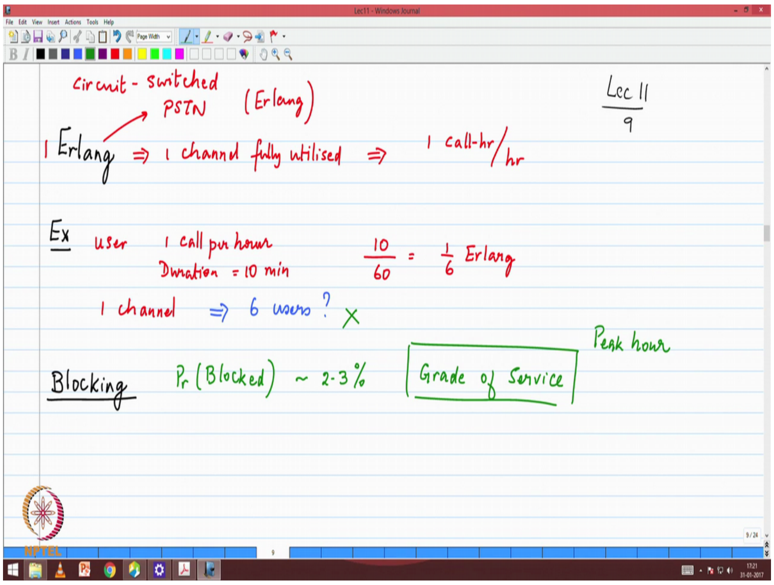

Now, this goes back to circuit switch context we will discuss our capacity using a circuit switch context, but the principles that we learn are very much valid for the packet switched context also and so the tools are very relevant. So, you know keep in mind. Basically this comes from the PSTN era where all of our networks were switched landline telephone networks and a Danish engineer by the name a Erlang actually did the capacity calculations and that is why we call it as the Erlang formula or the Erlang when you talk about traffic. So, the first point that I want to make is that the definition of one Erlang which is basically a fundamental unit of traffic one Erlang implies one channel that is fully utilized one channel it is fully utilized that means, there is no idle time for this particular channel.

So, basically if you think of your unit of time as 1 hour, you have basically use this channel for 1 call hour that means, the entire hour the call was on for an hour per hour one call hour per hour means there is no idle time the channel has been fully utilized. So, let us make a quick example for this, and then build on that. Supposing I have a user, this user makes one call every hour one call per hour and the and the duration of the call is ten minutes duration equal to 10 minutes. So, now, the if I look at it the utilization is 10 minutes out of 60 minutes. So, basically its one-sixth of an Erlang of what this user has the is the traffic that this user has provided he did not provide one Erlang and again one user cannot provide more than one Erlang that is the upper limit most of the time you give something less than one Erlang.

So, now the question that arises is I have one channel which can carry the traffic of one Erlang and I have user which generates one-sixth Erlang. Can I say I can now support six users? So, six users question mark implies six users question mark? The answer is yes and no. Yes, if all of them are perfectly multiplexed statistically, when one finishes somebody else starts using that channel, but if they are random user they are basically will make their calls this will not work. Basically this is a scenario that will end up in a lot of block situation, so that is the problem that Erlang analyzed.

Now, given a particular number of channels given a certain number of traffic that is being generated by these users, how much statistical multiplexing can I assume therefore, to give reasonable amounts of performance. So, the probability of blocking is that the when you ask when a particular user asks for a channel, the channel is not available. So, probability of these of a user is getting blocked. Now will become a percentage, what is the probability is it a you know 2 percent probability or 3 percent probability. So, usually typically it is in the 2 to 3 percent is probability of blocking this is what we referred to as grade of service. In the landline system, you are supposed to have guaranteed service. So, even a one percent grade of service was you know some times considered as not good enough, but in a cellular system sometimes you are very happy if you can do 5 percent grade of service, because there are times when there are peak hours is.

So, typically the blocking probability of grade of service has to be measured during peak hour. And there are certain scenarios like for example like if let us say there is a been a train accident then of course, the system will get completely jammed, because everybody is trying to make a call those are exception situation. But under normal conditions, if you have big traffic that is the under that conditions what is the blocking probability and that is what we are going to be measuring.

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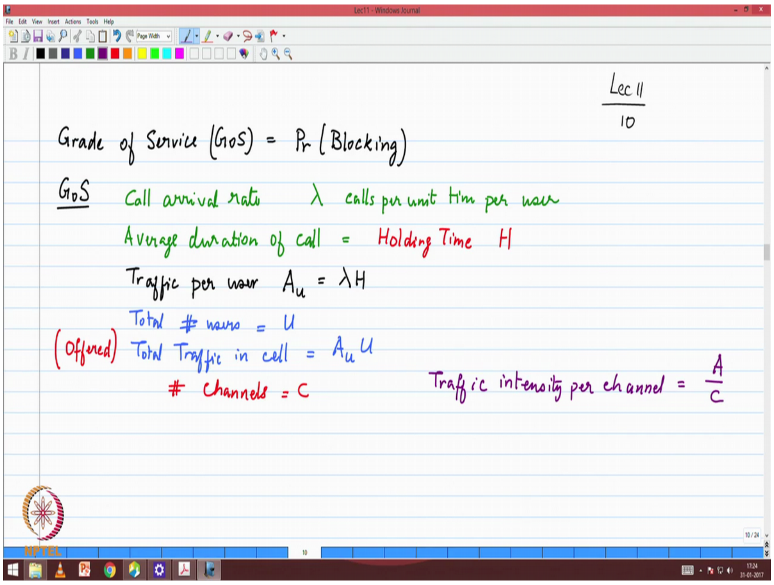

So, now, very quickly, let us put into place the mechanism for measuring the grade of service or the blocking probability. So, here are the basic parameters that Erlang had used in his analysis. So, there is a certain call arrival rate and this is let us call it as lambda, lambda calls per unit time per user. So, this could be for example, two calls per hour per user or something if my unit time is per hour or one call per 10 minutes. Whatever is your unit of time the basically this is the arrival rate, the average duration of a call average duration of a call that is called the holding time that is the time for which a you are using the channel or holding the channel. Let us call this, this is called holding time or H, and whatever is your unit time, you should use the same unit here as well. If you used minutes you use minutes here also this is holding time and that is denoted as H.

Now, the traffic intensity per user or the traffic offered per user, so basically the traffic per user traffic per user is A u, A stands for traffic, subscript u is the per user will be lambda calls per unit time and each time you are going to hold the channel for lambda H. So, basically if its two calls of a 10 minutes that is your loading onto the channel. So, lambda H gives you the traffic and if the total number of users in your cell total number of users is equal to u then the total traffic within the cell total traffic in the cell will be A u time that is per user times U the number of users.

There is an important distinction that is made this called the offered traffic; that means, this is the traffic that the users are offering to the system. Now, if it is 2 percent blocking then you would have to multiply this by 0.98 to get the carried traffic that is what the system usually carry. So, this is the difference between what the users offer and what the system carries, it is not assigned because of the blocking probability. We also make the following assumption that there are number of channels available to us number of channels equal to C. And this basically leads us to the most important parameter traffic intensity per channel that is the key metric that in our discussion that will be A divided by C.

Now, as you can see the given traffic, if I keep increasing my number of channels; obviously, blocking probability should reduce because you know that is the basic understanding that the more resources are available. And eventually if I have as many channels of number of users there will not be any blocking. But the idea is not to reach that point just to find out what is that number that will give you good grade of service without asking for the number of channels equal to number of users, so that was the primary contribution of a Erlang.

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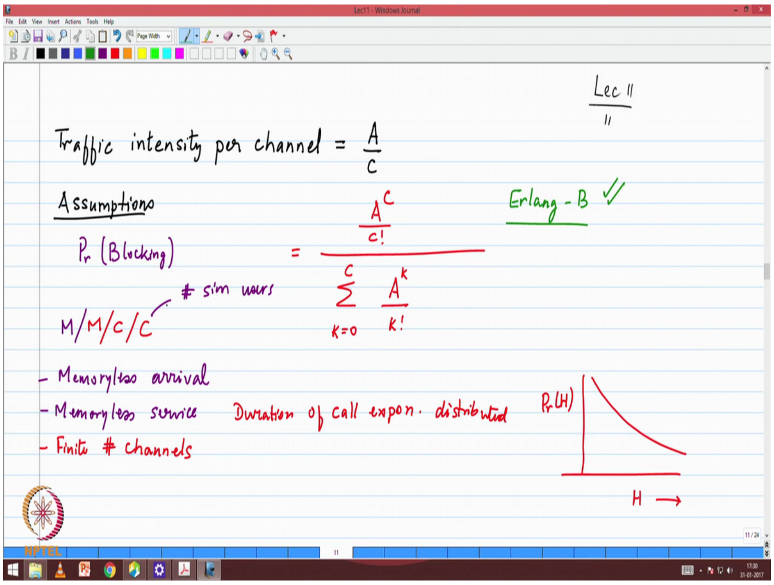

So, now given this frame work would now jump ahead and give you the results that Erlang had derived. And here are the basic assumptions. The basic assumptions are that I have a system that that it has C channels and this C channels are being offered a certain amount of traffic, and this traffic my desire is to find out what is the blocking probability of this system. So, the formula that Erlang had derived is that the probability of blocking.

Now, this assumption of probability is based on a certain mechanism that is arrived basically he said that we will assume that there are the arrivals the rate at which the calls are coming in are memory less, so memory less arrivals that is the first assumption. So, it does not matter you do not make any assumptions about who else is has made a call or not, so that is the first assumption, so memory less arrivals.

Second one is that memory less service. So, this also tells us that how much time you hold the call does not affected by how much time somebody else is holding the call. So, what was the assumption that he made that the duration of the call is exponentially distributed, duration of call is exponentially distributed. What that means is that this is if you call this as the probability, probability of H, and this is H - holding time. This probability actually is a decaying expression that likelihood of very long calls is less than shorter calls that was the assumptions that Erlang made that we not valid anymore because people do talk for long amounts of time. But at least his assumption was that you know the shorter calls were more likely to happen, and most important that there is a finite number of channels.

So, in the notation of queuing theory which many of you will be familiar this would be called as an M M queuing system memory less arrivals memory less holding time then there are C channels and C users, wait, it should be C is number of channels and C is the number of simultaneous users. Let me just clarify that this is the number of simultaneous users that you can support. The number of users is much larger, number of simultaneous users you can support. So, in other words why am I is not it obvious that c and C are the same in some systems which we will talk about if you have C channels you may reserve one or two channels for emergency use, and therefore it will be C minus 2 or C minus 1. So, the number of simultaneous users can be less than the number of channels available if you have to put any restrictions on the usage of the channels. So, memory less arrivals, memory less service C channels and number of maximum simultaneous users is also given to us.

So, according to this the blocking probability derived by Erlang is a power C divided by c factorial divided by summation K equal to 0 through C upper case is the number of channels A power k by K factorial. Now this is a result that we would like to use because our intension is not to prove Erlang’s hypothesis, but know that it is been verified in circuit switched system and therefore, our interest is to understand the traffic. So, basically assuming that the blocking probability is given by this, this is the formula that you will have studied it has Erlang-B, Erlang formula for blocking there is another formula called Erlang-C for queuing again ours is not a queuing system. So, therefore, we deal explosively with the Erlang-B; I do not even mentioned the Erlang C a part of it.

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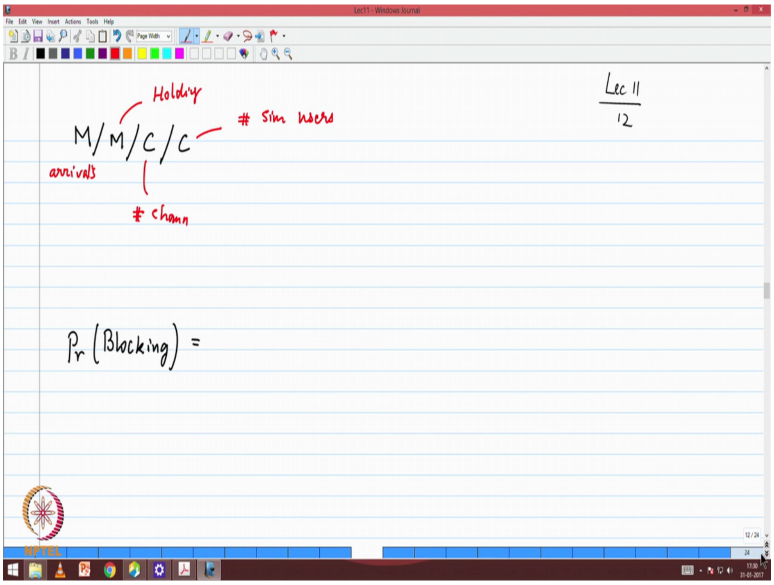

Now, what I would like to do is look at look a little bit closer at Erlang's contribution because there is some very, very valuable observations. So, basically quickly write down this is memory less arrivals, memory less holding, holding time. This is the number of channels; this is the number of simultaneous users. So, please be clear on the notion that is being used. And blocking probability formula we have already written I would not do that.

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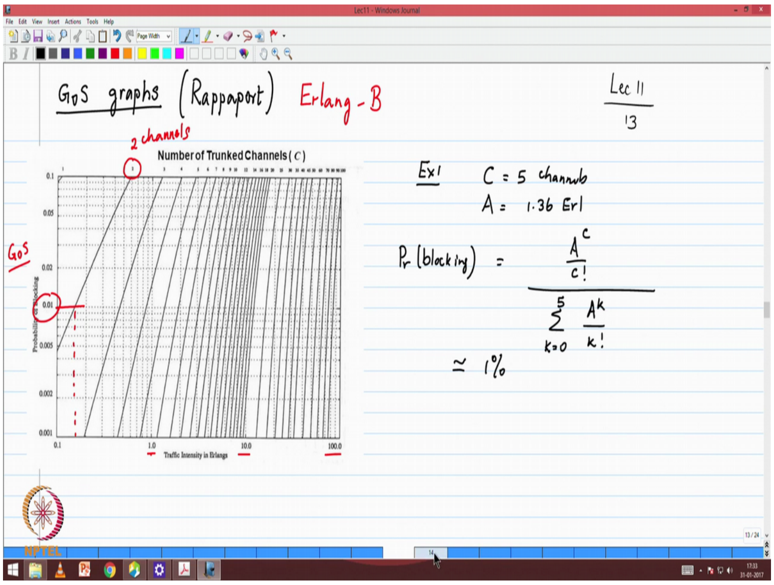

So, basically Erlang calculated these blocking probabilities and drew then in the form of graphs the y-axis is blocking probability that is grade of service. So, this you could also label it as grade of service. And on this axis is Erlang. So, eventually all the systems as traffic keeps increasing does not matter eventually you will get two blocking probability of one, once traffic reaches on certain level you will reach. So, there is a upper limit of blocking probability of one, but that is not the range of interest most of the time the range of time interest is around one percent blocking probability. So, notice that it is a system that is plotted logarithmically on a log scale on the y-axis, it is also log on the x-axis notice this is 1, 10, 100 that is the offered traffic in Erlang. Each of these graphs corresponds to a certain number of channels this says two channels the print is little bit small, but hopefully you can see it on the figures that you have on the phone.

So, if I have two channels and I want to have a blocking probability of 1 percent then I look 1 percent line then I will map it down and then I tell you the traffic has to be 0.17 erlangs that is all I can carry if I have only two channels. So, basically this is a very useful chart; however, keep in mind this has very steep graphs. So, you have to get your reading fairly accurate do not do go approximation because then you will go your answers will be quite inaccurate.

So, let us quickly apply this. Let us take a couple of examples and apply it. Again I am assuming these are very straight forward for you to do, but it is good as an exercise.

Supposing, I have C equal 5 channels, and I have a as 1.36 Erlangs the total of a traffic I want to know what is the blocking probability, probability of blocking. So, this is not a graph problem it is a problem for which you have to do the calculation. So, basically A power C divided by c factorial divided by summation k equal to 0 to 5 A power k by k factorial and you know it is not difficult just actually do the calculation you will find that this comes out to be approximately 1 percent. So, good system for a traffic of 1.3 Erlangs if I have five channels then I get a blocking probability of approximately 1 percent that is a good design.

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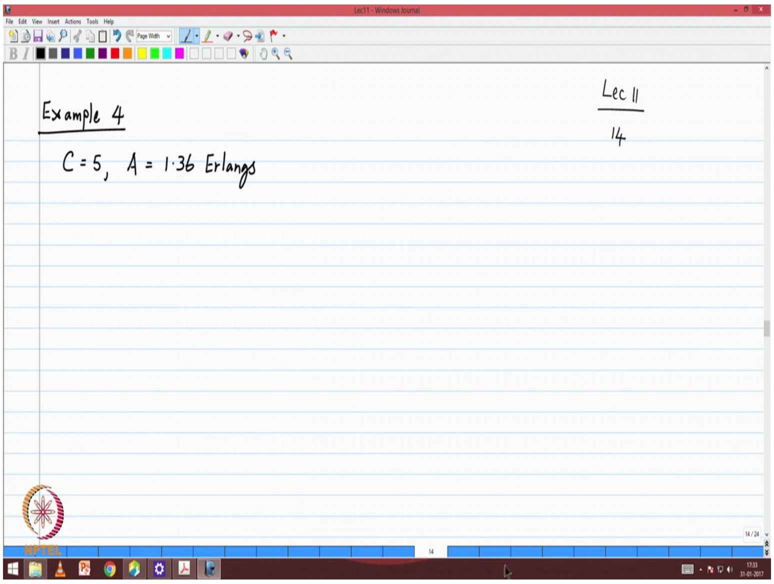

So, basically please go through and calculate this example that will be useful exercise.

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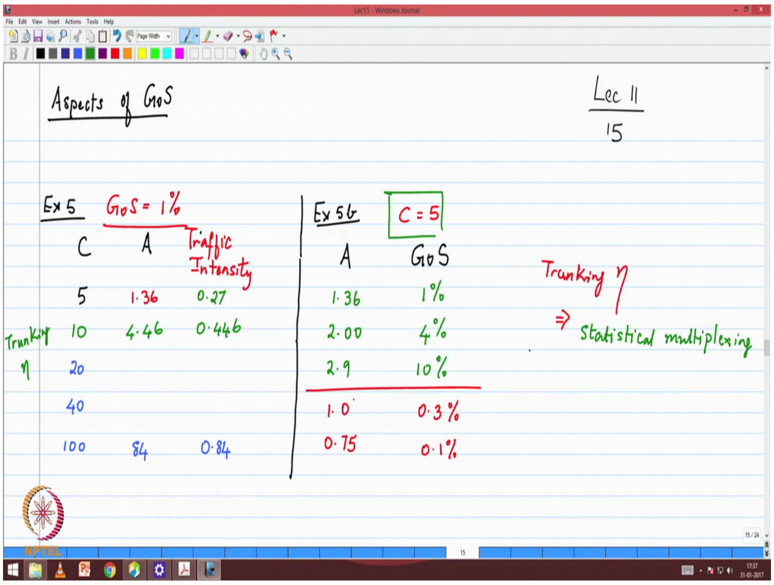

Now, here come the variation of this particular example which are very, very helpful for us. So, we have the reference point. If I have 5 channels and a traffic of 1.36, I will get a blocking grade of service of blocking probability of 1 percent and this corresponds to a traffic intensity traffic intensity would be A by C a by C of 0.27 that is the traffic intensity that I have. Now, if I go to 10 channels and I still want to have one percent blocking probability, I go to the graphs get it one percent blocking probability and I find that this corresponds to 4.46 Erlangs can be carried. So, basically this is 0.446 is the traffic intensity.

Notice that now I am able to put a lot more traffic intensity onto my system with 10 channels I am able to support a lot more traffic. So, keep in mind that because of the non-linear relationship between the number of channels now a very quickly let us try some I mean I would like you to try out some of the other examples 20, 40 and may be jump all the way to 100, please fill in the intermediate values. You will find that for one percent blocking probability with 100 channels, you can support 84 users. So, which means that the loading or the traffic intensity that you can carry is almost 0.84 that means, your channels will be heavily utilized and the more they are utilized that means, you are able to carry more traffic. So, this is a very good scenario for us to have. So, keep in mind that it is a non-linear relationship, there is a huge advantage when you have more number of channels, this is what we referred to as trunking efficiency.

Now, if you remember we said do not do too much of sectorization, because you will lose trunking efficiency. Probably now it becomes clear because if my channel originally had originally had 60 channel, my cell had 60 channels, I divided into three sectors each of them will become 20. I noticed that it is a non liner relationship the net traffic that I now will carry in the same cell for the same 60 channels will be less than if I had a single cell of the 60 channels, so again that is the trunking efficiency.

Let us look at this result this particular from a slightly different angle. Let me fix the number of channels as 5. If the offered traffic is 1.36 Erlangs, I know that I will achieve a grade of service of 1 percent; increase this slightly to 2 Erlangs the blocking probability jumps up to 4 percent; go to 2.9 this goes to 10 percent. So, notice that you are nowhere close to the channel the number of channels is only 2.9, but there is a non-linear growth of the blocking probability. So, again so this is on the one direction what happens if I reduce if I go down 1.36 where I have one percent blocking probability to 1 Erlang, it goes down to 0.3 percent and then just the slight reduction between beyond that goes down to 0.1 percent.

So, there are huge advantages keeping a large number of channels in the pool because that is when you get the trunking efficiency. So, trunking efficiency very, very important, this is leveraging what we know as statistical multiplexing. Any questions? Hopefully this gives you very good handle on understanding why we said do not do so much sectorization or try to make your cluster size as small as possible, so that you can get as many channels in each cell as possible. So, those are the thinking behind and what we are asked to.

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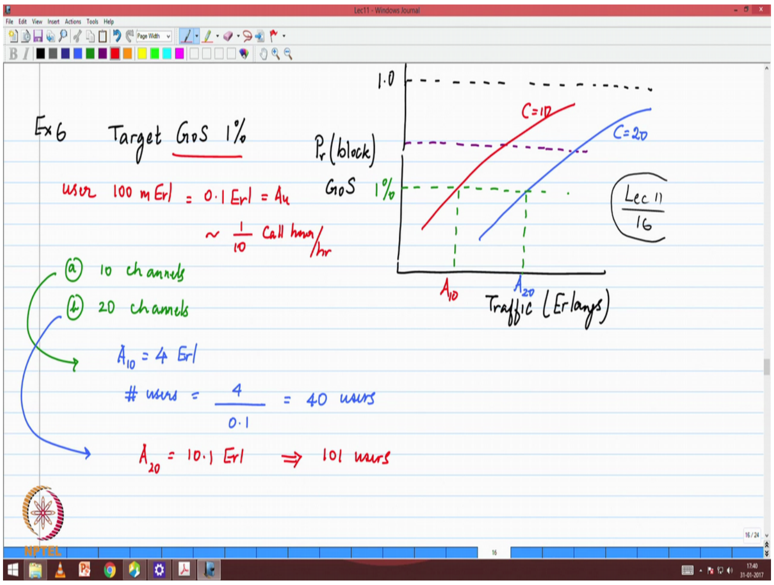

Let me close with one more very simple example. So, this is a situation where my target grade of service is 1 percent. And if you want to draw go look at the graphs. There are two scenarios I have a each user generates 100 mille Erlangs that is the same as 01. erlang or in other words one by tenth of a call hour per hour. Let us you can interpret this as each user makes one call basically for one-tenth of the hour that is may be 6 minutes and that is all the user makes. So, this is the traffic per user. Now, I have two scenarios. I have one scenario where I have 10 channels and another scenario where I have 20 channels. I have to satisfy the same grade of service and each of those cases I want to know how many users I can support.

So, basically I go to the to the graph the blocking probability versus Erlang traffic this is the Erlang-B graphs. I take the 1 percent line take for C equal to 10, I drop the x-axis, I know that is the total traffic that I can carry with 10 channels with the blocking probability of 1 percent, so that comes out to be 4 Erlangs. So, for this case A 10, A 10 corresponds to 4 Erlangs 10 channels blocking probability of one. So, the number of users I can support will be 4 Erlangs divided by 0.1 because that is the number of Erlangs per user. So, this tells me that we can support a total of 40 users in the system.

Now, I double the number of channels and I find that A 20; A 20 again from the graph not done anything, no calculations is 10.1 Erlangs. So, this would imply that I would have satisfy one over one users. So, doubling the capacity of number of channels in more than doubles the number of users that I can, this is another way of visualizing the trunking efficiency of the system.

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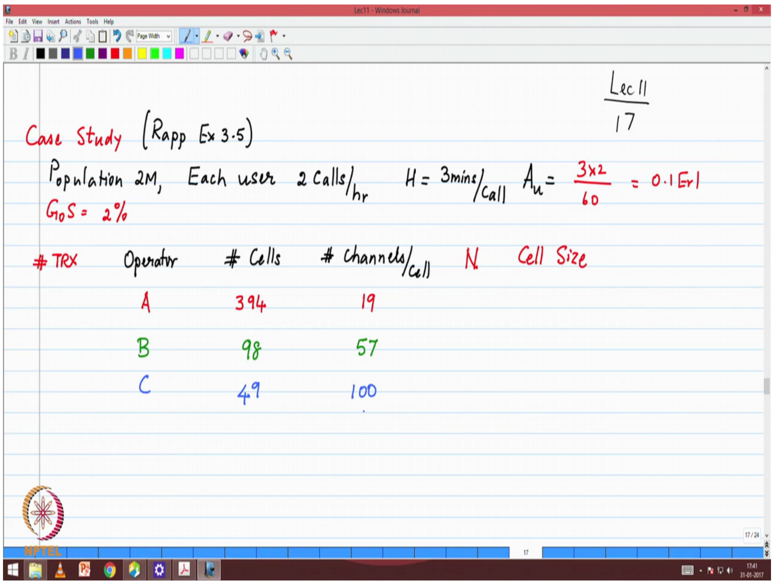

Now, may be just to get you thinking about the case study, let me just give you the data and then we will pick it up from there. City with the population of 2 million, each user makes two calls per hour, 3 minutes per call. So, this is the traffic per user will be 3 times 2 divided by 60. So, basically this is the same case as what we had previously 0.1 Erlang is my, I want to have a grade of service of 2 percent.

Now, operator A has designed a full system has made 394 cells to cover the system and is getting 19 channels per cell. Operator B is created a coverage with 98 cell basically all of them have covered they all have met the 2 percent grade of service and he has 57 channels per cell. And then the third one is operator C he has created 49 cells he has created 100 channels per cell and has covered the cells.

Now, what is the strategy of operator A, B and C obviously, they have designed it very differently some of them will have large cluster size some of them have got large cell size, somebody is got small, somebody is optimized for calls, somebody has optimized for capacity. So, take a look at it and try to see if you get an feel for A, B and C who is the smartest of these three operators and you know if you had do it which way you will do it, so something for us to think about. Any questions on this problem? Basically this is a problem in I would like you to look think about and then we will discuss it in detail at the starting of next lecture.

Thank you.