Artificial Intelligence:

Representation: Properties and Categories

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Module – 04

Lecture - 03

So we are looking at representation using FOL. So lets focus a little bit on properties. So we have said that we can look at properties as categories so we did that when we looked at the example of for all x Man x implies Mortal x. Where of course Man you can think of as a category or kind of things or a type whereas Mortal is kind of a property of people right or there is a property of human beings that its men who are mortal. But we sort of approximated this by saying that we can think of Mortal also as a set because that's what FOL allows us to do and then we are saying that set of men is contained in the set of mortal entity essentially. But there may be other choices as well. So lets look at this example of The ball is red. How do we express this in FOL essentially. Obviously I could have said All balls are red or something similar just to be similar to the statement we had about man and mortal. But its a choice of predicates that we are talking about here not so much about the quantifiers. So we could have said in a similar sense that we have a set of red objects and lets say we have a set of blue objects as well. And this ball, lets say i am talking about a particular ball called B1 happens to be in this set.

So i could express this as this bag. Likewise I could have said another ball B2 is also red. Third ball B3 is blue which means that B2 belongs to the set of red things and B3 belongs to the set of blue things. And I could express this as follows that Red B1 or Red B2 or Blue B2. This is like saying Mortal Socrates, Mortal Peter, Mortal Suresh and so on. Where we think of the properties Red or Blue and also categories but you are saying that this element falls in that category. There is another way of looking at things.

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supposing I had a set of colours and this set contains things like Red, Blue and so on and so forth. All the colours I should be having and then I could say that we have these three balls B1, B2, B3. So this is my larger domain like this. And now I am saying that there is a relation between B1 and this object Red, B2 and this entity Red, and B3 and this entity Blue.

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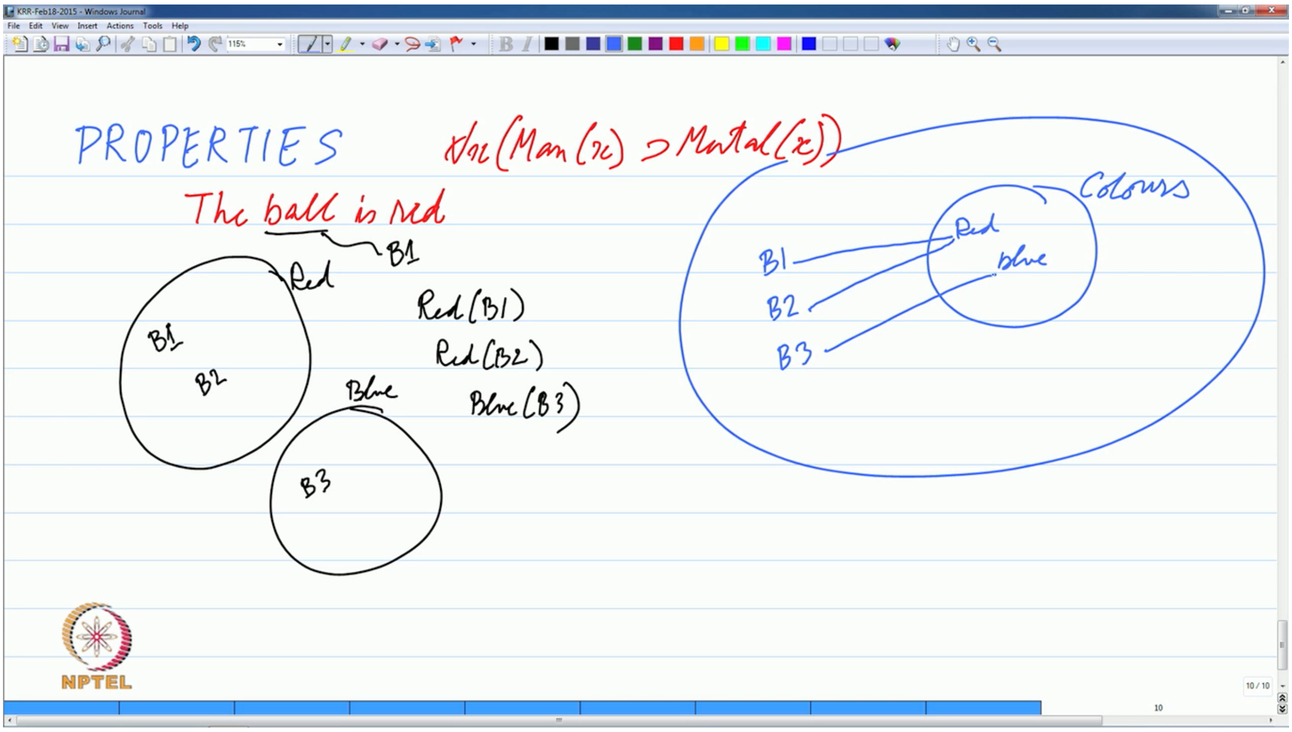

so you see the difference between these two ways of representing things. In the first case Red and Blue are unary predicates and so we give them B1, B2 and B3 as arguments and then we say that B1 belongs to this set of Red things, B2 belongs to this set of Red things. So this notion of Red, as a person who is interested in knowledge representation you may be sort of curious to know is there a right way of representing the notion of Red. What is the notion of Red. Its not so straightforward to define what is Red essentially. What we have done in the second case is we have introduced something to our domain which is a set of colours. So we are saying that colours are a part of our domain. So there is a set of entities which belong to the set of colours, Red and Blue are amongst them.

And if I do that then you can see that the way I will write this sequence is that: Colour B1 Red, Colour B2 Red. This is a totally different way of representation. In this scheme of things Red and Blue are elements in the domain essentially. And we will see that when we talk about representation very often we want to add elements in the domain which may not be tangible which may not be real which may not be really there in the domain but we introduce them to make life simpler for us and we will see that it happens in may many cases almost in all cases essentially. For example if we talk about people, human beings so lets say a person like John has a nose essentially. So if I want to say that lets say John's nose is hurt of something like that then conceptually what do I think of? Do i think of John and nose as two entities in the domain which has this relation of being hurt or something like that. Thats not really the case, the nose is part of John essentially. And in some sense if I think of John as an identity I have no way of thinking about nose essentially. Unless I have a mechanism by which I think of all the components as real entities and putting together of them into a human being as an abstract entity. So John doesn’t really exist in that sense John is the sum of all his parts essentially. Now thats a question we will come back to later again but essentially these are the questions you need to try and answer. What is there in your domain and how are you representing things.

Now if you wanted to make certain statements like Red is similar to orange, lets say as compared to blue. So we want to find the notion of similarity and may be in some domain we wanted to give some numerical value to this similarity. What is the level of similarity, it is very similar or little similar so you might want to give numbers to that. 0.9 stands for highly similar, 0.7 stands for similar but not so similar that kind of stuff essentially. You want to do this sort of a thing, then you can see that obviously the second scheme of things helps better because now we can try ti define a relation between two objects essentially. Whereas when you try to find relations between set red and set blue its not a straightforward thing for us to think about. So these are some issues that we will have to grapple with when we do representation essentially.

Lets take another example lets say Mary gave John a book. How will I represent this statement. Again keep in mind that we are still working with adhoc predicates we are saying okay we will invent predicate name as and when we need them and simply represent things using that. So you can think of a predicate Give for example or Gave if you want. You can already see that there is a problem which is occurring here that especially when we talk about events and we should do that separately. But when we talk of a predicate where essentially transferring things from one person to another then conceptually you are transferring things but you want to use Give as a predicate then what you do is sentence has a word Gave. John said he will gave a book to Mary or John gave a book to Mary or Everyday John gives his dog a piece of bread something like that. All these have different aspect of the word gave essentially and do we define different predicate for give and gave because obviously they are different, gave is in the past tense and give is may in present continuous or something like that. So these are again questions we need to address essentially.

So lets anyway for the time being assume that we have accepted the word Give a the predicate we are using and somehow we will introduce the notion of it being in the past or something later. So we want to say that Mary have something. Now very often in the real world there may be more than one Mary essentially so that's another problem. If we write a sentence like this then which Mary do we refer to. So again we will take a shortcut here as its done in many textbooks you will see. We use some suffix to distinguish one Mary from another. So lets say this Mary is called mary21 and this john is called john2 and she gave him a book. How do I write this a book? Thats the question I want to ask.

So you can imagine your domain has many books essentially and show how you know one book from the domain Mary is giving to john. So i could ofcourse start off by saying that let me just use the term book here. Thats the simplest way of doing this. What about a? Because when i used the constant book it refers to a specific book. Remember that in first order logic you have either constants or variables as terms or functions. In this case there is no function. But we could think of a function here. For example we could think of a skolem function here which says that this is the book Mary gave to john. How do we handle something like a book. Any suggestions? So is book like in the previous example, is z a predicate or is z a term? So remember term denote objects in the domain and we may add objects as and when we please essentially. So is book a predicate or is it a term? What makes more sense. It makes more sense to say that especially because we are saying a book, it makes more sense to say that its a predicate basically because we are saying that there is a set of books and Mary gave one of them to john essentially. And how do we express that now?

We have to say that there exists an x such that x is a Book and lets say we are not really bothered about time at the moment. That is a more faithful representation than by simply using Book as a term.

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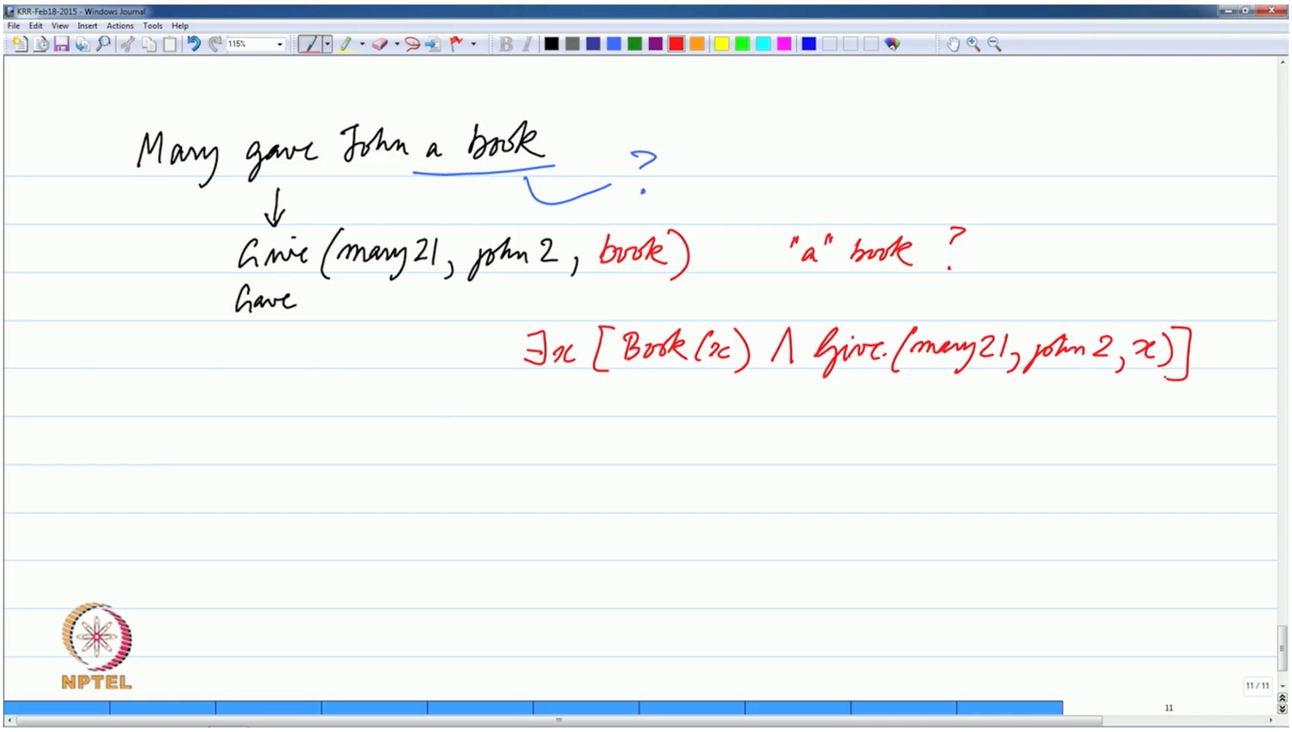

because we are not quite clear as to what does the book stand for. The moment we try to make book as a variable then we would have to somehow say that this variable stands for a book which means its a part of a set of books which means we have to use book as a predicate. Lets look at some more kind of things that we need to worry about. So there are properties like height, weight , length, distance and so on and so forth, you can add for example colour, we have already looked at colour. These are all properties, height of an object, height of a person, height of a tree, weight of an object and so on and so forth. Now with these kinds of properties we often associate values so we may say numerical values, for example Mary is six feet tall. So we are talking about height. You can also talk about qualitative values and we will see at some point sometime we tend to think of them as fuzzy because they are often not well defined. Sometimes they may be, but sometimes they may not be well defined in which case we may say Mary is tall or you might even say Mary is very tall. So we have some properties called height which we can give a value which is numerical in nature. And we can also give a value which is symbolic in nature, which is qualitative or which is fuzzy. We use the term fuzzy because it is not very clear what do you mean by tall. Tall is a set of people then who is tall and who is not tall is not a crisp set essentially because 6 feet could be tall but 5 feet 9 inches is it tall and so on and so forth. At some point you should give a threshold and the moment you give a threshold just below the threshold and just above the threshold they are similar but why are you calling one as tall and one as not. So these are questions which are addressed by people who work in fuzzy logic and fuzzy sets.

You may also want to address questions like Mary is taller than Susy. Again in some sense it addresses the notion of tall. So what I am trying to drive at is we have to find out ways of representing things which can be used in different contexts for different purposes. So if you have a notion of height and weight and distance and so on we can then represent that. So lets look at a idea which we have been hinting at and this idea lets give it a name. This process is called Reification and it basically means introduction of abstract types. Types which don’t really exist in the domain, remember the domain is basically a set, its a set but we want to introduce new things in the set. So we are talking about types here essentially. So lets talk about the type called length. So in some sense length is of a type of an element just like man is a type, cat is a type, apple is a type and so on and so forth, we have now introduced a type called length essentially whose elements are basically denote length in some way.

So once we do that we can kind of define functions like Height lets say mary21 and this a a function which denotes an element of type length. So remember we have introduced a type called length and now we are defining a function called Height Mary, remember that Height is a function, we are talking about a function basically a function is used to denote terms essentially. So we are saying that height of Mary is a term. What kind of term it is. Its a term which belongs to a class of objects which are of type length essentially. So essentially all length we will say that there is an abstract class called length in which everything's length we talk about essentially. Now things like height and length in particular has units essentially. So how do we handle units? We say Mary is 6 feet tall essentially right. Its an example we give. So how do we handle things like feet essentially. Now one way to do that is to think of feet as a function so I could say feet which takes a number as an argument and it also denotes an element of type length essentially. So we are looking at number 6 and we are saying that we are defining a function called feet of 6 and what does feet of 6 is? It basically denotes an object of type length, it basically means that length is a type in which you can compare.

You can say one length is equal to another length for example. And we will see that we can also define a notion of ordering that which length is more than another length, we will do that shortly essentially. But basically length is an abstract entity of some kind and both the functions feet 6 and height mary21, mary21 is basically Mary here are functions and they both return, return is not the right word because this is not functional in that sense. They both point to or they both denote some objects of type length essentially. So you can see that now if you want to say that Mary is 6 feet tall, our task becomes quite easier. All we have to say is that height mary21 which we are calling her is equal to feet 6. now you have to go back to the meaning of this equality. When we defined first order logic with equality the meaning of equality is that both terms, you must realise that both are terms refer to the same object. So we have not defined what that object is .

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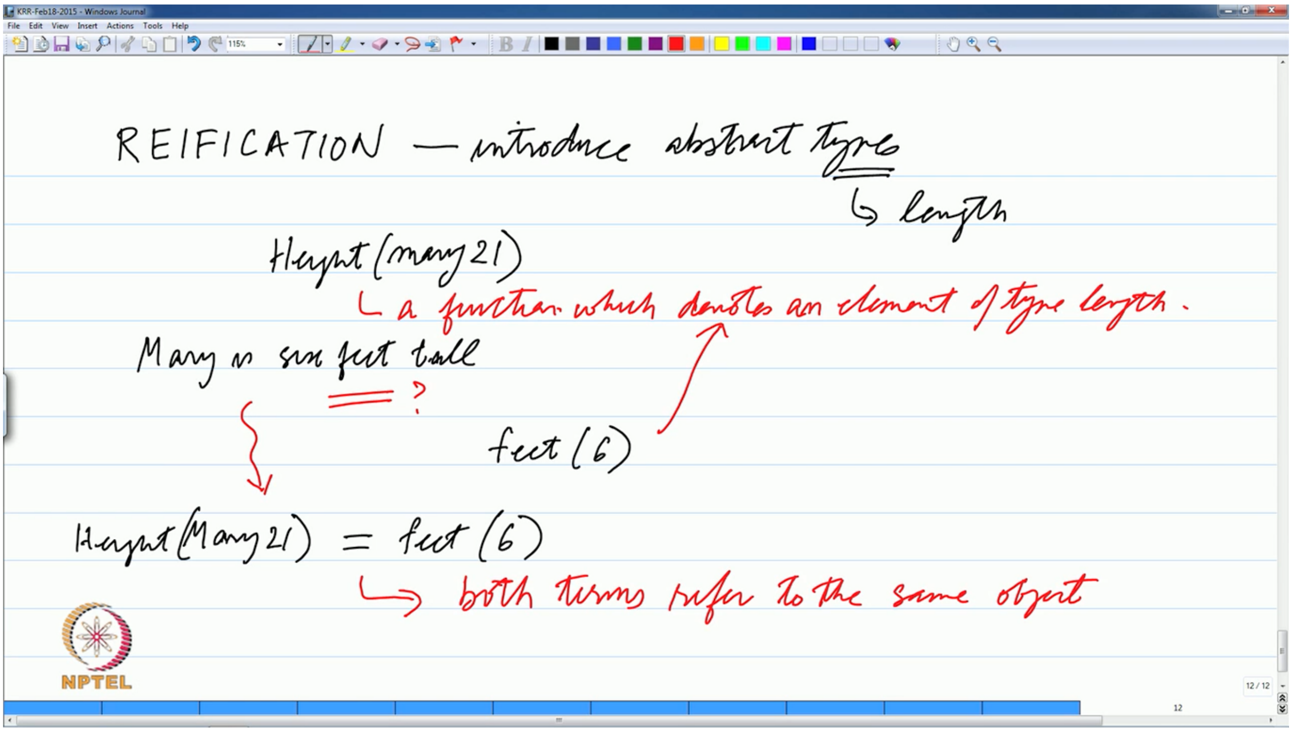

but we are saying that height of Mary denotes an abstract entity of type length. And feet 6 also denotes an abstract entity of the type length essentially. And we are saying that when we say height Mary is equal to feet 6 we are saying that both of them are referring to the same object which is basically we want to say is the meaning of this statement Mary is 6 feet tall. There is a length which when measured in feet has value 6 and Mary’s height is basically that length. Height and length and distance will always be of type length essentially. So that’s one way of saying it. So you can see that feet here is a function from lets say elements to objects of type length. Remember length is an type now, feet is also a function which takes numbers and translates them to objects of type length. So we are in some sense moving everything to type length and then saying that okay we can talk about the length of Mary or height of Mary. We can talk about the length denoted by 6 in terms of feet. And then we can say that its a same object essentially. Or height of Mary is same as 6 feet essentially.

So some questions that you may want to ask is how do you convert? So if you 12 miles is so much in kilometres then how do you write expressions to convert essentially. So i will leave this as a small exercise. Conversion between units. I remember one puzzle with children ask is there is a square and a man walking around the square and for the first leg it takes 80 minutes, for the second leg it takes 80 minutes, for the third leg it takes 80 minutes but for the fourth leg it takes one hour twenty minutes. And people are puzzled you know how can that be and so on so forth. So this is the kind of stuff that you want to figure out when you are talking about conversion essentially.

So lets come to this example of Mary is taller than Suzy, so how do we represent this essentially. So we want to find a consistent way of representation, we want to find a consistent way of talking about length and height and tallness and distance and so on. We want to be able to handle different units, inches, centimetres, miles and kilometres and we should be able to compare things. So you know we should be able to take word problem in some school algebra and arrive at a logical representation which we can solve somehow essentially.
How do we talk about taller than essentially. Of course one simple brute force adhoc way is to say that I have a predicate Taller and I will say Mary taller than Suzy. But its

kind of too simple, two naive and it may not be useful enough to solve problems. Having defined a solution for height we can say that, we can define the notion of Taller x y. Lets say we define this in terms of height essentially. So we can say that Height x and we can define some ordering relation between them which will stand for taller than or longer than and things like that. And obviously we are talking about general terms for all x for all y we say that height of x is more than. So we need this symbol as more than height of this thing. If lets say we are taking in terms of feet but we can have other functions as well. X in feet, not x, some n in feet is greater than m in feet where there is an association between this and this and things and this. So height of x is n and y is m. And when say that height of x is more than height of y if feet of n which is the height of x is in some ordering, we are still talking about some ordering, we are not into numbers yet. Of greater than lets say or if you want to introduce a new symbol here doesn't matter.

This in turn we can define by saying n is greater than m. Where for a set of number we always say that we already have a relation define ordering. We will not go into what is logical foundation of saying that 4 is greater than 3 and 2 and things like that. We will do it if we get time. But there can be a logical foundation based on this and may be at some later point to defining, so what is a number. We have you know used this number very casually 7 8 9 and 10; what are these thing? In logical form how does it come essentially? And philosophers and mathematicians and logicians have grappled with this area for a long time. In fact there is this story of a scientist about 100 years ago. He told somebody in the church, they asked him what do you want to do and McCullum replied that what is a man who may know a number and what is a number than the man may know. You can find this saying somewhere essentially.

So what are numbers essentially so may be we will break off at this time and we will take this up again in the next class and may be we will pay some attention to what do we mean by numbers and you can try to think of this. So you must have heard about different ways of defining a number. And we will also look at one alternate way of representing Mary's height essentially. So here we have said that Mary's height is same as the feet in this, but we could think of feet alternatively as a function from type length to the type number which is a different way of doing things. And we will look at this alternate way in the next class.

So we can think of the function feet as taking a number and returning a type, object of type, this length is an abstract type, we have just defined it, we have just notion of length essentially. And once we have the notion of length we want to define when is one length equal to another length. Or when is one length more than another. Here is what we have tried to define here. But we could also alternatively say that why work with this abstract notion of length. In the end we are comparing length. Why not take length as an type and convert it into number using the function feet in which case you would say something like this, feet height Mary is equal to 6, that's another way of looking.

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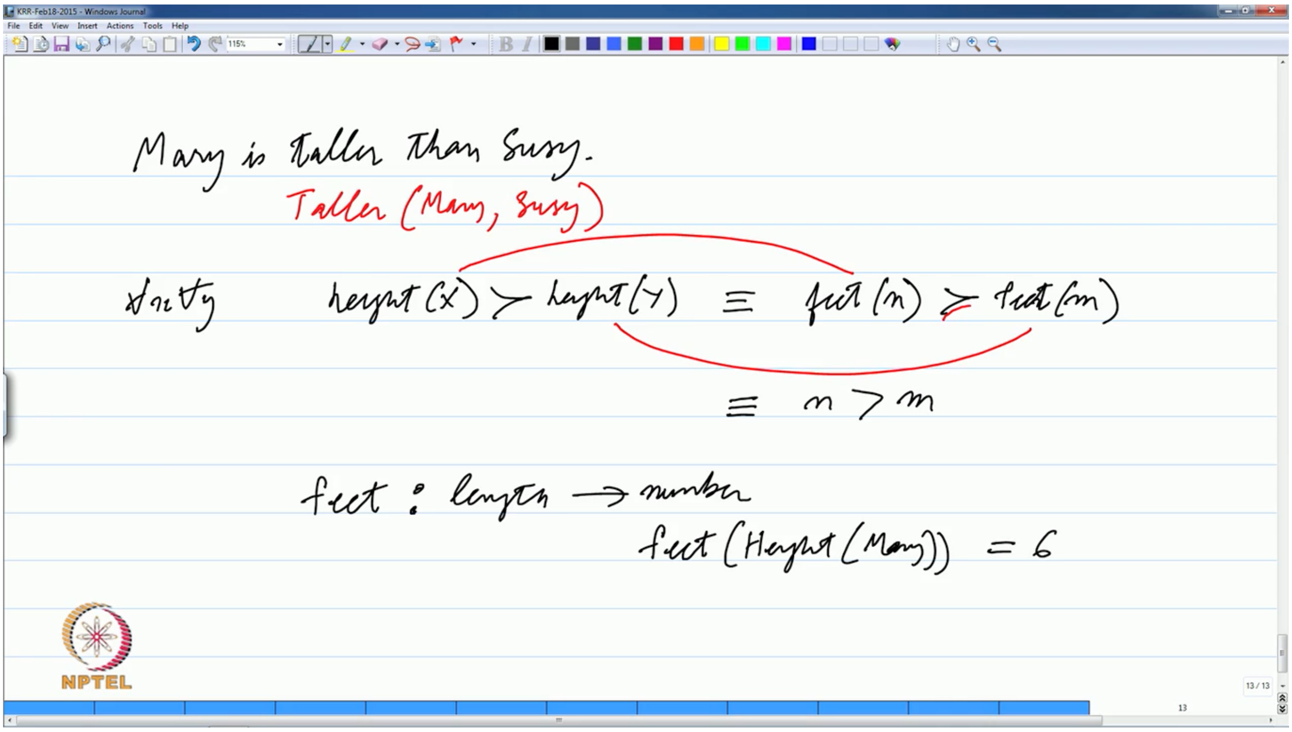

the difference is that feet is a different kind of function. It takes an object of type length and returns a number or points to a number and obviously this will have ways, something will become simpler here something will become simpler in the other thing. We will revisit this problem in the next class.