Discrete Mathematical Structures
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Lecture # 8
Normal Forms

Today we shall consider about normal forms. Already we considered a little bit about what is meant by a Disjunctive Normal Form and Conjunctive Normal Form. And we took the Conjunctive Normal Form as a clause form and we used it in resolution principle. But again we shall go through those definitions and also see what is meant by principal Disjunctive Normal Form and principal Conjunctive Normal Form. We will also see what is meant by prenex normal form in the first order logic. So let us recall some definitions which we learnt earlier. A product of the variables and their negations in a formula is called an elementary product.

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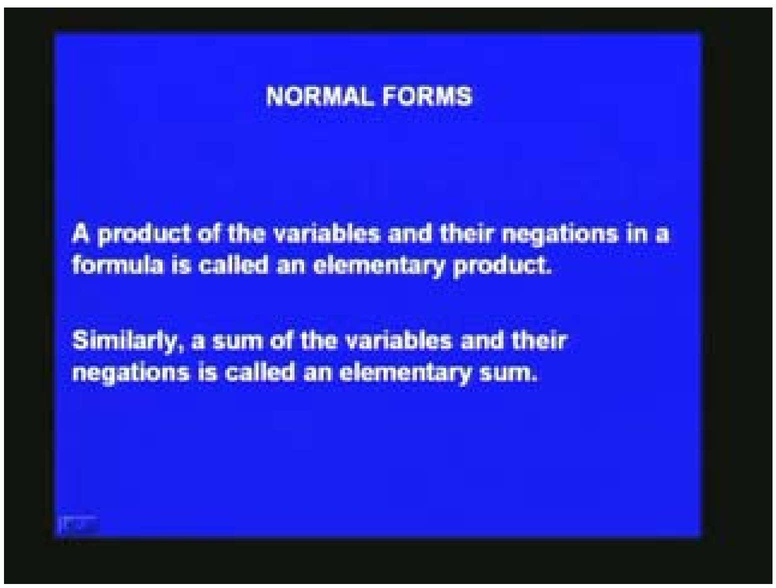

For example, something like P AND Q AND NOT R something like this is called an elementary product. Similarly, a sum of the variables and their negations is called an elementary sum. For example, something like P OR NOT P OR Q OR NOT R something like this is called an elementary sum. Now you can see that a necessary and sufficient condition for an elementary product to be identically false is that it contains at least one pair of factors in which one is the negation of the other.

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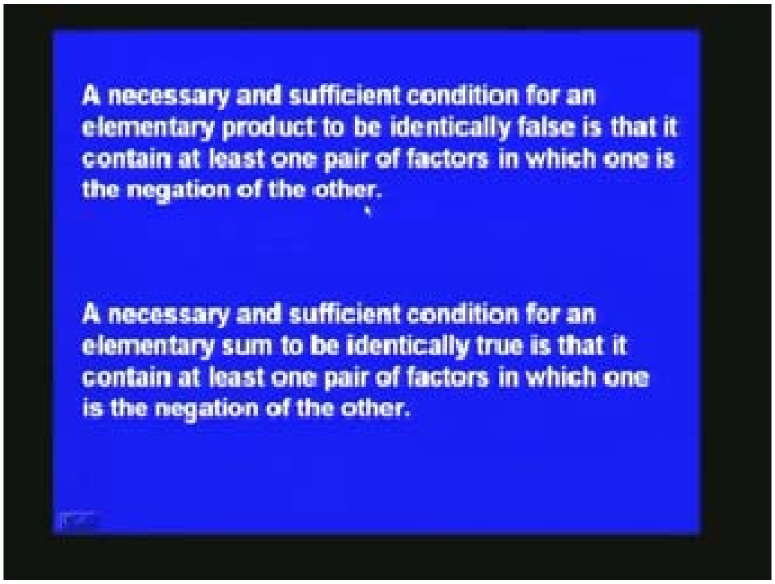

Suppose I have an elementary product like this, if it contains one pair of variables or one pair of letters where one is the negation of the other you have Q AND NOT Q then this becomes identically false because you know that Q AND NOT Q is 0 that is false and something and 0 is 0. So an elementary product is identically false if it contains two literals where one is the negation of the other. And the other way round, a necessary and sufficient condition for an elementary sum to be identically true is it contains at least one pair of factors in which one is the negation of the other.

So suppose I have something like this P OR NOT P Q OR NOT R something like that then you have one literal and the negation of the variable in another literal. So you have one variable and also the negation of that. This we know that it is a tautology P OR NOT P is 1 so 1 or something will be 1. Now any formula involving these connectives using the propositional variables is called a wff or well formed formula of propositional logic.

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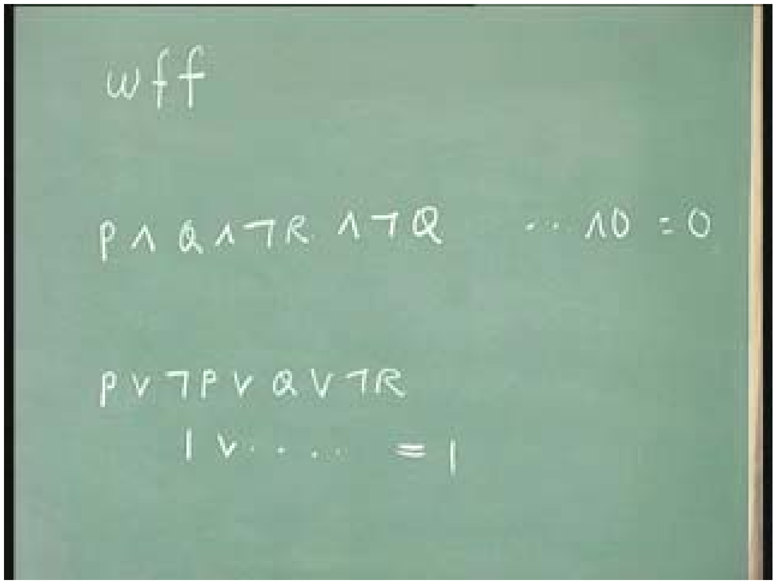

Now you can bring any well formed formula of propositional logic into Conjunctive Normal Form and Disjunctive Normal Form. First let us take Disjunctive Normal Forms: A formula that is a well formed formula which is equivalent to a given formula and which contains of a sum of elementary products is called a Disjunctive Normal Form of the given formula.

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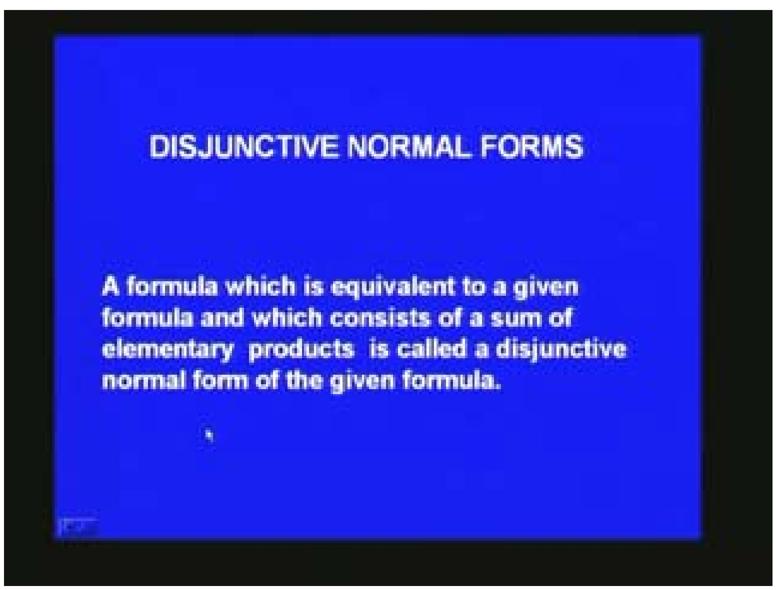

So it should be of this form something like it should be the sum of elementary products. So a sum of elementary products is called a Disjunctive Normal Form, it will be something like this or again something. Now we can bring any well formed formula into Disjunctive Normal Form.

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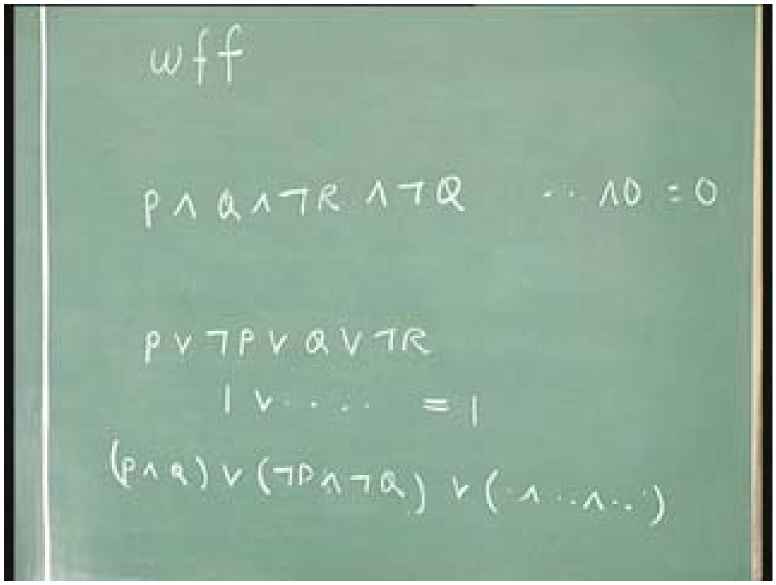

Let us take some examples; first let us take this, we want to bring this well formed formula into Disjunctive Normal Form. How do you go about doing that? First of all you can convert implication into NOT so this will become NOT NOT of P OR NOT of Q, you can write P implies Q as NOT P OR Q so this can be written as NOT of this OR P is equivalent to NOT Q.

Now P is equivalent to NOT Q you can write as suppose you have P is equivalent Q this is equivalent to saying P AND Q OR NOT P AND NOT Q. Either both of them must be true or both of them must be false, this we know. So using this rule here you will get, this portion is as it is, this we can write as P AND NOT Q OR NOT P and NOT P and NOT of NOT Q.

So using De Morgan's laws this will become P and Q because this will become NOT of NOT of P AND NOT of NOT of Q. May be I will write that step also NOT of NOT of P AND NOT of NOT of Q using De Morgan's laws and OR will be P AND NOT Q and this should be NOT P AND Q and must be in brackets here also. So this is equivalent to saying P AND Q OR P AND NOT Q OR NOT P AND Q. So this is an elementary product, this is an elementary product and this is an elementary product. And you have a sum of elementary products so this is a Disjunctive Normal Form. Let us take one more example, let us convert this into again Disjunctive Normal Form.

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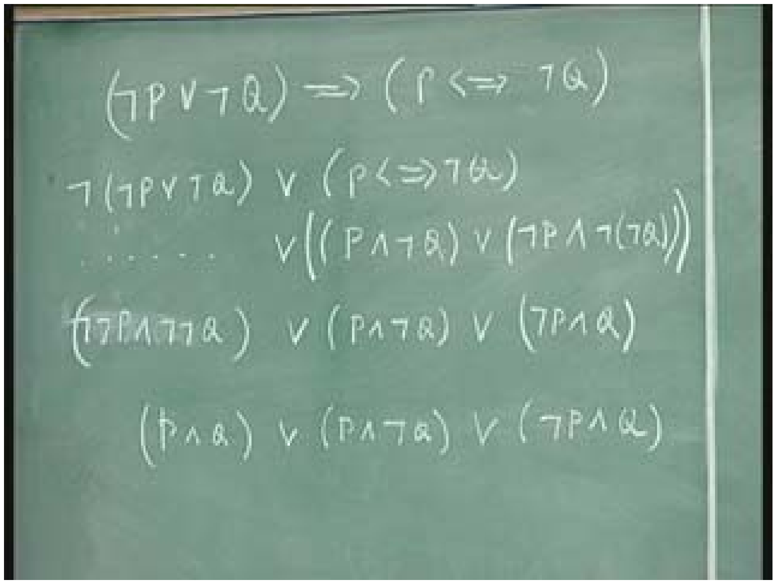

How do you go about doing this? P implies Q AND R so you can convert implication into NOT P OR Q so this will be NOT P OR Q AND R AND. Again you can write this one like this; NOT of NOT of P OR NOT Q AND NOT R. So this again you can write as NOT P OR Q AND R AND P OR NOT Q AND NOT R. Now you can use distributive laws, you can write this as NOT P OR Q AND R AND P OR NOT P OR Q AND R AND NOT Q AND NOT R. Again here you can use distributive laws and this will be NOT P AND P OR Q AND R AND P OR, here again you can use distributive laws and this will become NOT P AND NOT Q AND NOT R OR Q AND R AND NOT Q AND NOT R. We can remove unnecessary parenthesis and write it in the proper form. So here it is an elementary product, elementary product, here again elementary product, elementary product.

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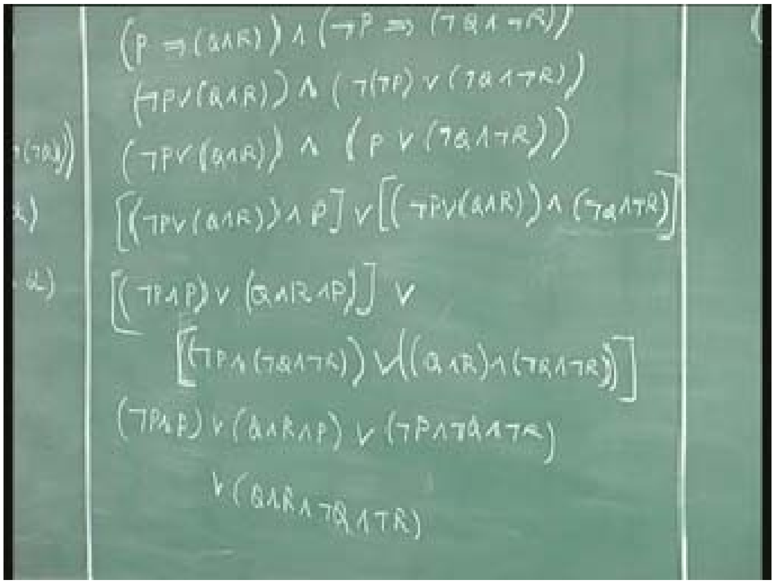

So if you rewrite this it will become NOT P AND P OR Q AND R AND P OR this one will be NOT P AND NOT Q AND NOT R and this will be Q AND R AND NOT Q AND NOT R. So this is the disjunctive normal form. Again I want to mention that disjunctive normal form or Conjunctive Normal Form is not unique. For example, you know that you have Q and NOT Q so this whole term is 0 so it can be omitted. So it is not unique but you can bring any formula to the disjunction of elementary products.

Next we shall consider Conjunctive Normal Form or CNF. That is the one we used earlier. What is a conjunctive normal form? A formula which is equivalent to a given formula and which consists of a product of elementary sum is called a Conjunctive Normal Form of the given formula.

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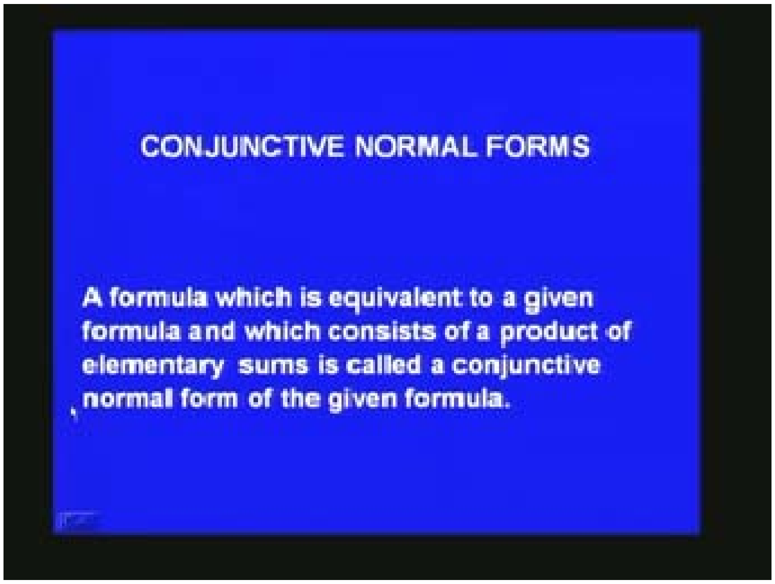

So it will be like this; be something like P OR Q AND something OR OR this is called Conjunctive Normal Form. It is the product of elementary sum. Each one is an elementary sum and you have AND here product of elementary sums. Again you can bring any well formed formula of the propositional logic to Conjunctive Normal Form. Let us take one more example here, so this is the formula and we want to bring it to Conjunctive Normal Form.

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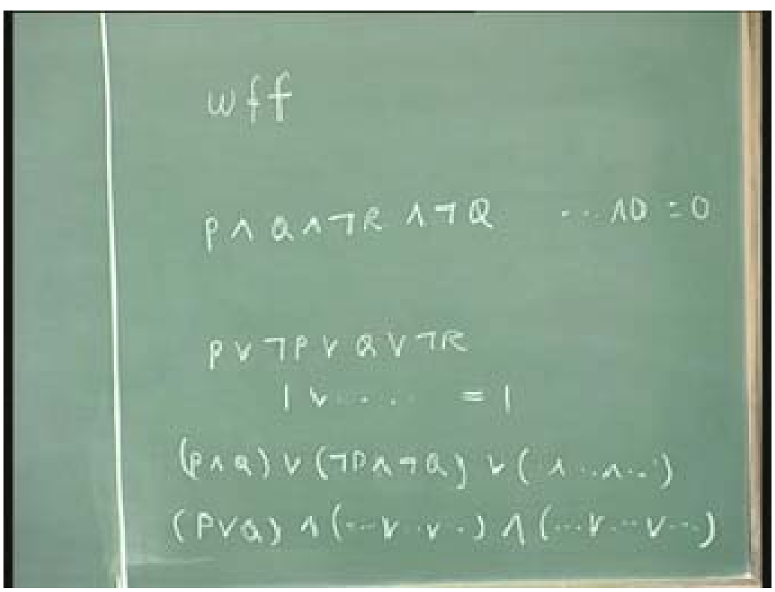

So this you can write as NOT of NOT of P OR NOT of Q using implication rule for implication P implies Q can be written as NOT P OR Q. So NOT of this OR this one you can write as P implies NOT Q AND P implies Q and Q implies P so NOT Q implies P. Using De Morgan's laws this will become P AND Q OR NOT P OR NOT Q AND NOT of NOT of Q OR P.

Here, again I am using the rule NOT P implies Q is NOT P OR Q and the same rule here. So what will you get? You get P AND Q OR NOT P OR NOT Q AND this will become NOT of NOT of Q is Q so you will get P OR Q. We can use distributive laws now so this will become P OR NOT P OR NOT Q AND P OR Q AND Q NOT P OR NOT Q AND P OR Q. So if you expand this using distributive laws you will get P OR NOT P OR NOT Q AND P OR P OR Q and this one will become AND there is a OR here, Q OR NOT P OR NOT Q AND Q OR P OR Q. So this is an elementary sum, this is an elementary sum, this is an elementary sum, this is an elementary sum and you have the product of elementary sums and this is the Conjunctive Normal Form.

Now you can see that because you have P OR NOT P this will reduce to 1 and because you have Q OR NOT Q this will reduce to 1 and this will reduce to P OR Q and because P OR P is P and Q OR Q is Q so ultimately the whole thing will reduce to P OR Q. If you want to simplify this then this will reduce to this. But we are not at present interested in that, this is the Conjunctive Normal Form.

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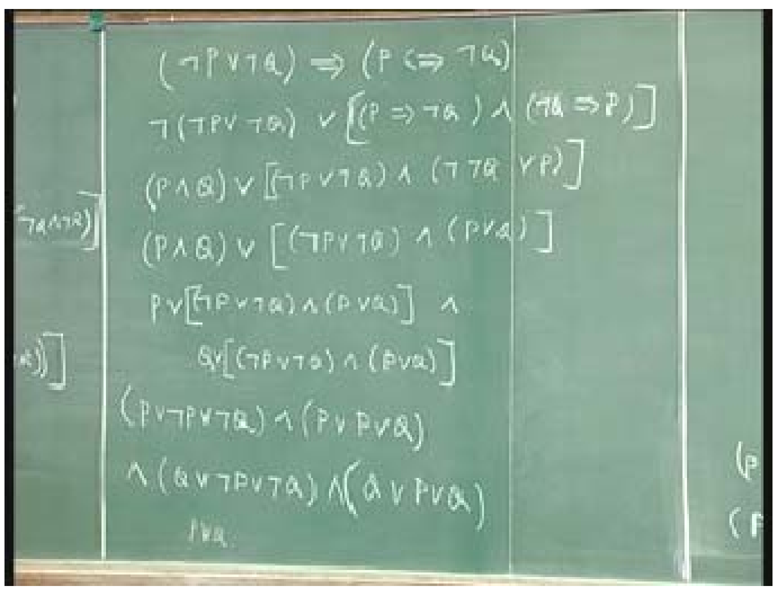

So any well formed formula of the propositional logic can be brought into either Disjunctive Normal Form or Conjunctive Normal Form. Now we will consider principal Disjunctive Normal Forms. Let us take two propositional variables P and Q. Now consider the terms P and Q or P AND NOT Q or NOT P AND Q or NOT P AND NOT Q. They are called minterms. There are four minterms here; P and Q is a minterm, P AND NOT Q is a minterm, NOT P AND Q is a minterm and NOT P AND NOT Q is a minterm. In a minterm every variable will be present only once either in the negated form or in the non-negated form.

If you have two variables it is the conjunction of two literals they can be negated or non-negated. What is the principal Disjunctive Normal Form? For a given formula an equivalent formula consisting of disjunction of minterms is known as its principal Disjunctive Normal Form. Such a normal form is also called the sum of products canonical form.

So you should have disjunction of minterms. Earlier we had disjunction of elementary products here we have disjunction of minterms. Let us see how to convert a given formula into principal Disjunctive Normal Form. Let us take some example.

Consider this example; P OR NOT P implies NOT Q implies R you can write this as P OR NOT P implies by changing this one into the form you can write it as NOT of NOT of Q is Q OR R. So this you can write as P OR again NOT of NOT of P P OR Q OR R so this will become P OR P OR Q OR R it is nothing but P OR Q OR R. Now this is the sum of elementary products but each one should be a minterm this is not a minterm so we have to convert this into a minterm for which you have to add the variables Q and R.

Now you know that you can write P like this: P is P AND Q OR NOT Q because Q OR NOT Q is one and P AND one is P so you can write like this AND R OR NOT R. And then if you expand using distributive laws this will become P AND Q AND R OR P AND Q AND NOT R OR P AND NOT Q AND R OR P AND NOT Q AND NOT R. So you will see that each is a minterm this is the minterm, this is the minterm, this is the minterm, this is the minterm so P is equivalent to these four minterms the sum of the four minterms.

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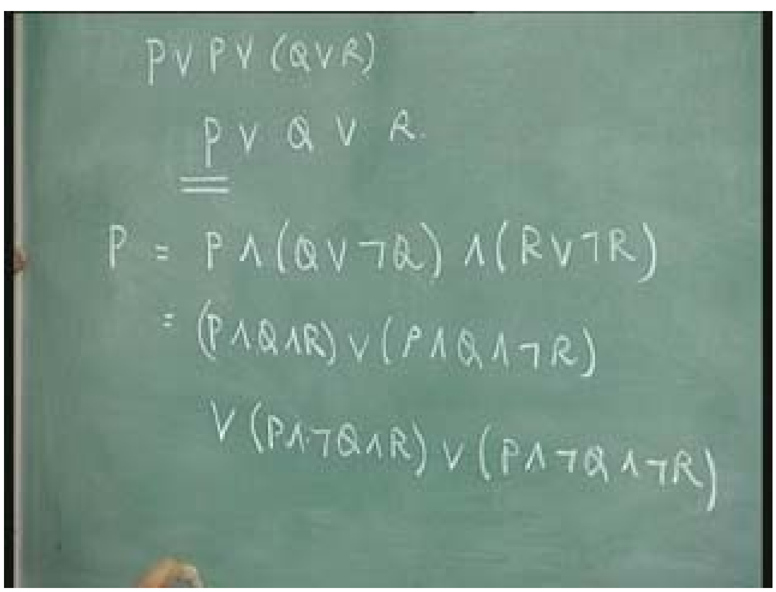

Similarly, you can write for Q also. Q will be equal to P AND Q AND R OR NOT P AND Q AND R OR P AND Q AND NOT R OR NOT P AND Q AND NOT R. So you have to provide the missing variables. Here you have only Q P and R are missing so you have to provide them in all possibilities. So you have P R NOT P R P NOT R NOT P NOT R like that you have to provide.

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Similarly, R also you can write, ultimately the whole formula will be equivalent to, you can omit repetition P and Q and R is there here also it is there the whole formula will become P AND Q AND R, OR P AND NOT Q AND R, OR P AND Q AND NOT R, OR P AND NOT Q AND NOT R, OR NOT P AND Q AND NOT R, NOT P AND Q AND R, OR NOT P AND NOT Q AND R.

So out of the eight possibilities of combining them each variable should be present in the minterm either in the negated or in the non-negated form. So with three variables you can possibly have eight minterms and in this formula out of the eight seven are present. So this is the way you convert a given well formed formula into principal Disjunctive Normal Form.

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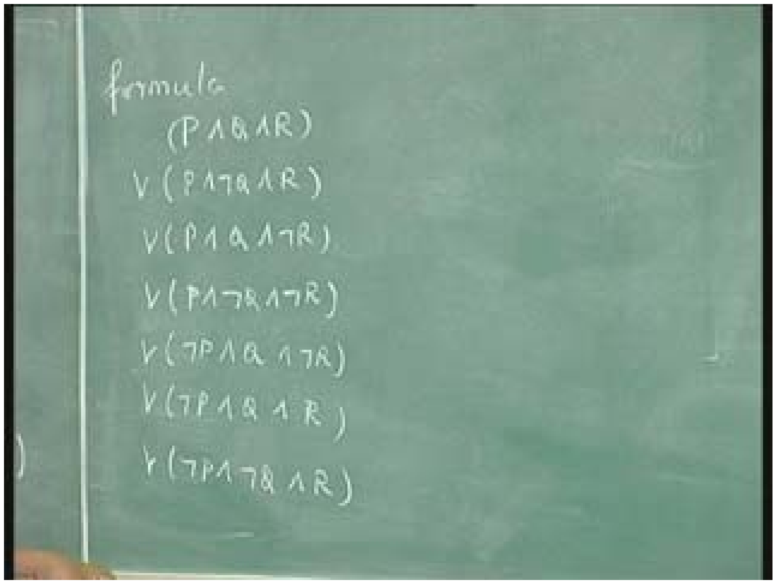

This is also called sum of products each is a product and you have the sum of them. Now there is a way of writing this, usually you write NOT P AND NOT Q AND NOT OR as 0 0 0 NOT representing 0.

For example, P P AND Q AND R as 1 1 1, P AND NOT Q AND R, P NOT Q AND R this you can write as 1 0 1 like that. So the eight possible minterms you can represent by binary numbers from 0 to 7. So in this case what will this represent? This will represent 1 1 1 that is 7 and this will represent 1 0 1 that is 5 and this will represent 1 1 0 that is 6 and this will represent 1 0 0 that is 4 and this will represent 0 1 0 that is 2 and this will represent 0 1 1 that is 3 and this will represent 0 0 1 that is 1. So, if you allow the minterms to be represented from 0 to 7 in this case in general if you have n variables you can represent the minterms by integers from 0 to 2 to the power n minus 1.

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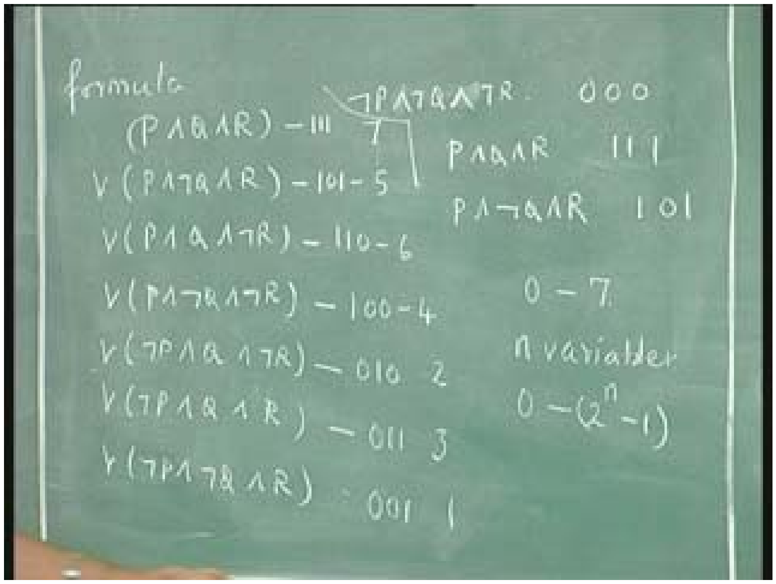

So if you do that you represent the principal Disjunctive Normal Form as sigma for example, you can represent this one as sigma 1, 2, 3, 4, 5, 6, 7. With two variables suppose I have two minterms P AND Q NOT P AND NOT Q and a principal Disjunctive Normal Form like this how will you represent this? There are only two variables they can be represented from 0 to 3 and this will represent 3, 1 1 that is 3 and this will represent 0 0 that is 0 so you can write this whole expression as sigma 0, 3.

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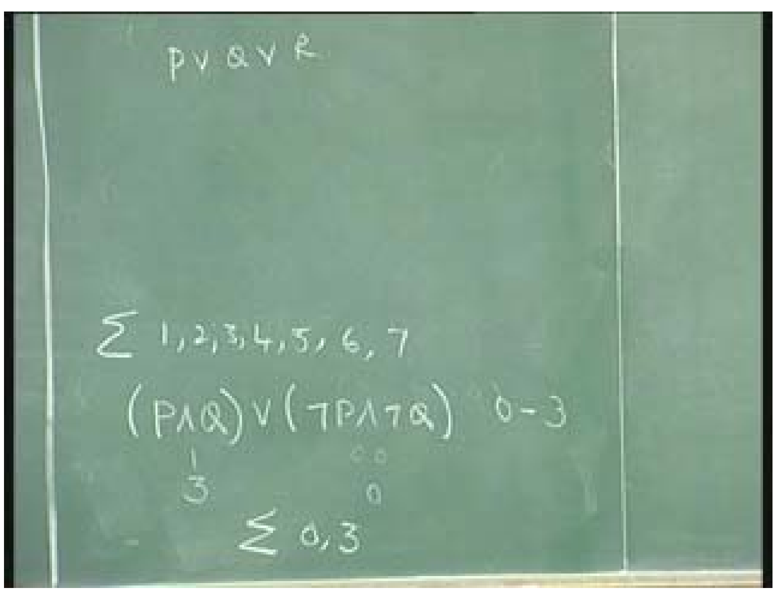

Next we shall consider principal Conjunctive Normal Forms. We shall first define what is meant by a maxterm.

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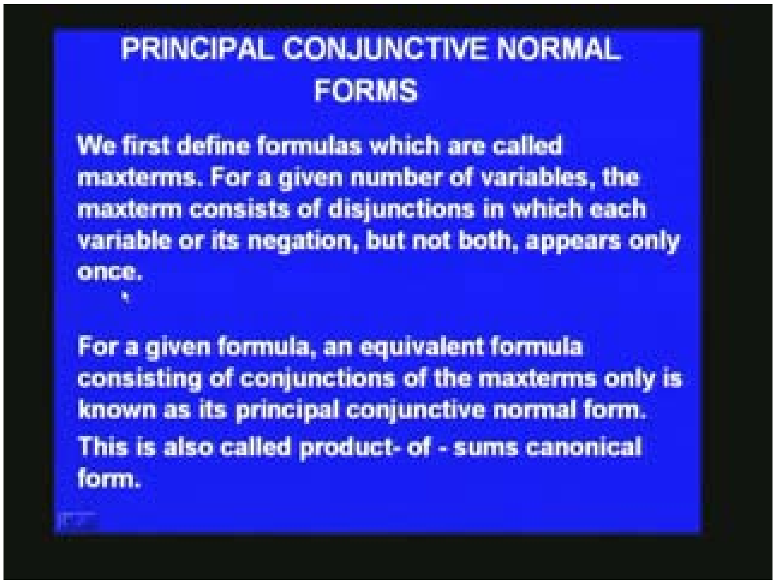

For a given number of variables the maxterm consists of disjunctions in which each variable or its negation, but not both, appears only once. So suppose you are having variables P and Q with P and Q what will be the maxterms? P OR Q, NOT P OR Q, P OR NOT Q and NOT P OR NOT Q these are called maxterms.

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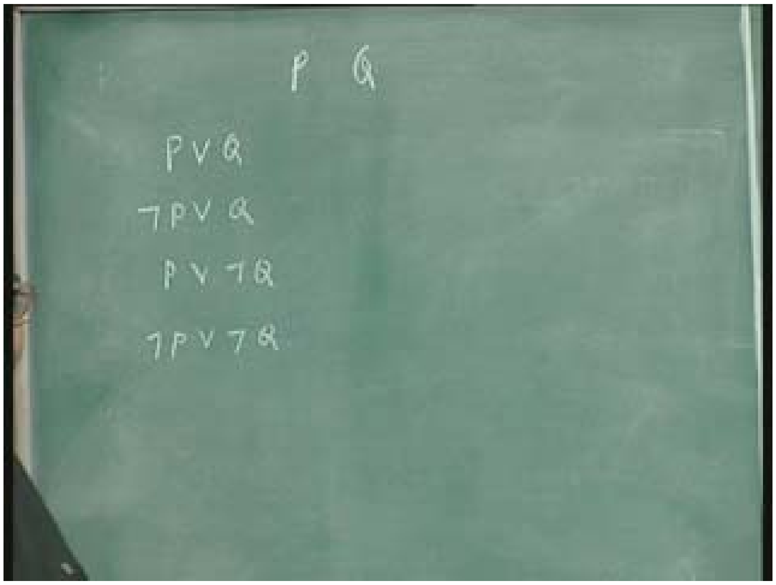

They are disjunction of literals where each variable is present in the negated or non-negated form but not in both forms. So you can bring any formula into principal Conjunctive Normal Form. What is principal conjunctive normal form?

For a given formula an equivalent formula consisting of conjunctions of maxterms only is known as principal Conjunctive Normal Form. And this is also called product of sums canonical form. And any formula can be brought into principal Conjunctive Normal Form let us see how we can do that. The earlier example which we considered this one reduced to this then we added extra variables to make it to principal Disjunctive Normal Form.

Now look at this, this is a maxterm with three variables this is a maxterm so the whole thing reduces to just one maxterm in this example.

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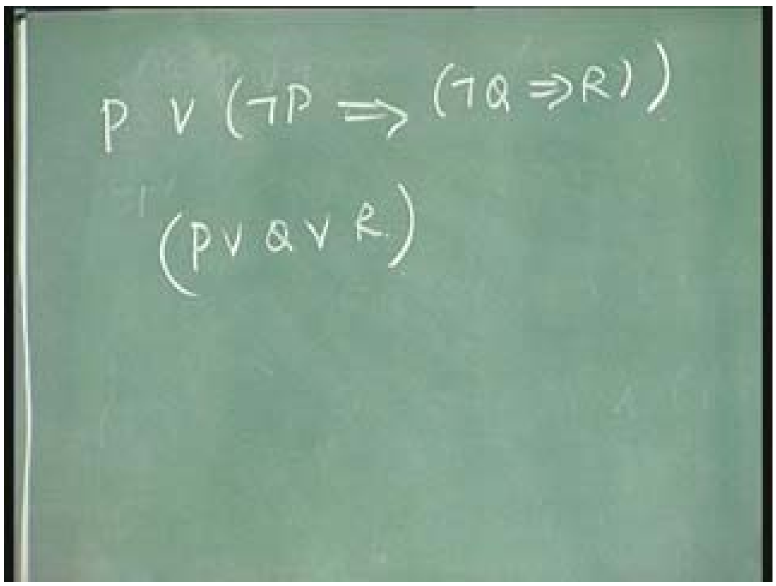

Let us consider one more example; Q implies P AND NOT P AND Q this you can write as NOT Q AND NOT P AND Q. So this is NOT Q OR P this is a maxterm AND NOT P AND Q. Now you have to convert them into maxterms so NOT P you can write as NOT P OR Q AND NOT Q this is 0 so NOT P OR 0 is NOT P. And this one using distributive laws will become NOT P OR Q AND NOT P OR NOT Q.

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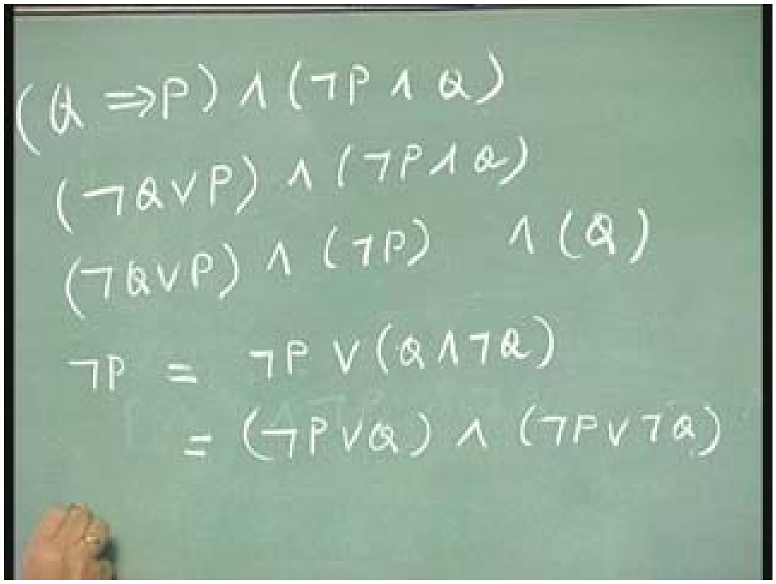

Similarly, Q can be written as P AND NOT P OR Q you have to provide the missing variable like this. That is equal to using distributive laws P OR Q AND NOT P so the whole formula reduces to NOT Q OR P that I will write as P OR NOT Q using commutative laws NOT P. This term becomes like this P OR NOT Q and instead of NOT P you have to write these two terms NOT P OR Q AND NOT P OR NOT Q. And instead of Q you have to write these two terms P OR Q AND NOT P OR Q.

Now you can see that NOT P OR Q is repeated twice, it is not necessary to write twice. So probably you can remove this, this is repeated. So you have the conjunction of maxterms. Each is a maxterm and you have the conjunction of maxterm. This is the principal Conjunctive Normal Form.

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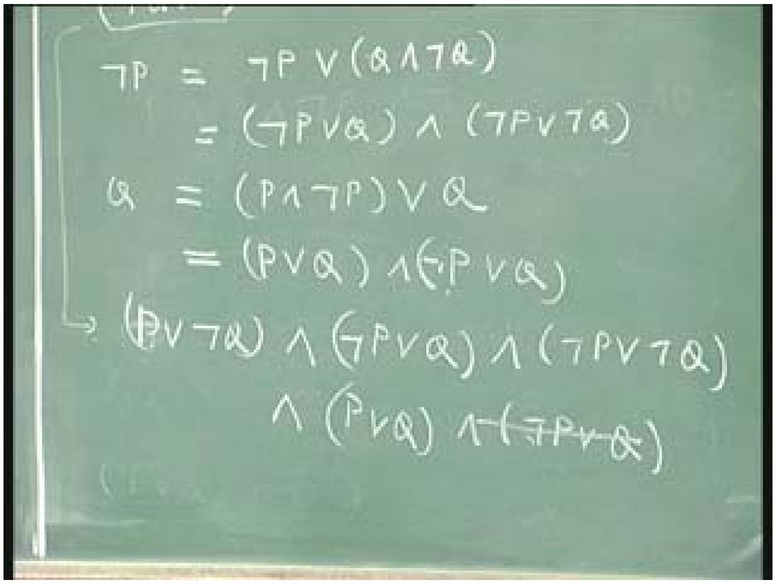

So, principal Conjunctive Normal Form is a product of maxterms. So with two variables I have the maxterms like NOT P OR NOT Q, NOT P OR Q, P OR NOT Q, P OR Q and this you can represent in binary like 0 0, 0 1, 1 0, 1, 1 representing the numbers from 0 to 3.

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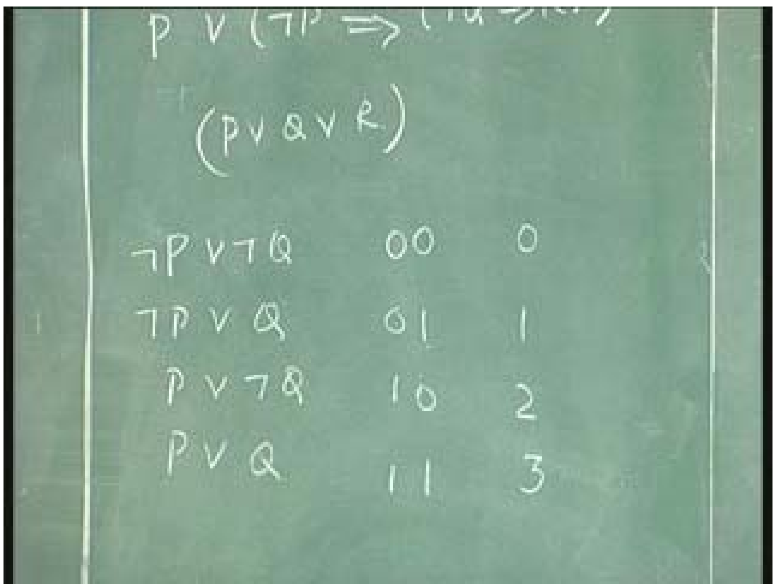

Thus, if you look at the last one which we considered all the four maxterms are present here. So this can be represented as product 0, 1, 2, 3 so this is the notation used.

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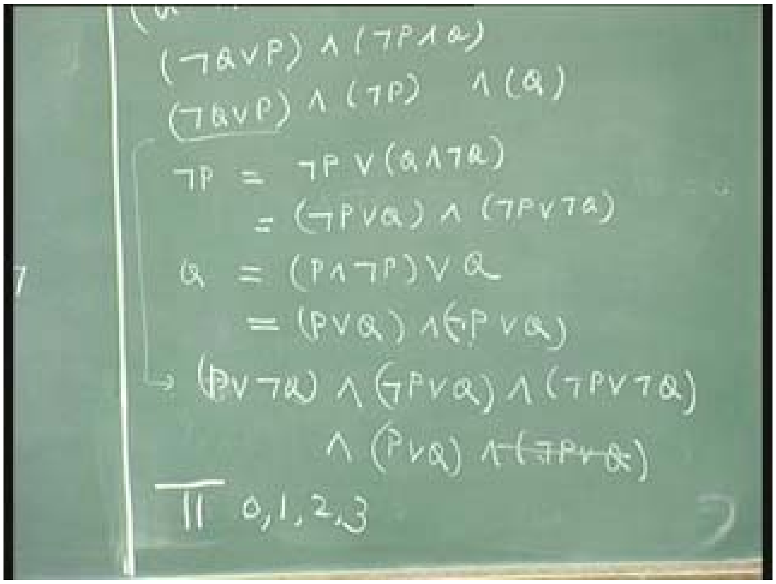

Now with three variables again you can have the maxterm represented by 0 to 7. And if you look at this one this represents the maxterm 1 1 1 or 7 so the whole formula i can be represented by pi 7. Suppose I have P OR Q OR NOT R with three variables AND NOT P OR Q OR R AND NOT P OR NOT R NOT Q OR R suppose a formula consists of these three maxterms this will represent 1 1 0 and this will represent 0 1 1 and this will represent 0 0 1. So the whole thing can be written as pi of 1, 3, 6 you can write the formula as pi 1, 3, 6. This is another notation for writing the principal Conjunctive Normal Form as we consider for principal Disjunctive Normal Forms.

Next we have to consider prenex normal form. So far we have considered only propositional logic we did not use quantifiers at all. Now if you use quantifiers how will you write it in a normal form? What is the normal form for first order logic?

For first order logic the normal form is called prenex normal form and the definition is like this: a formula F in the first order logic is said to be in a prenex normal form if and only if the formula F is of the form Q_1 x_1 Q_2 x_n (M) where x_1 x_2 x_n are the individual variables so every Q_i x_i is of the form for all of x_i or there exist xi. Q_1 represents a quantifier for x_1 Q_2 represents a quantifier for x_2 and so on. It can be the universal quantifier for all or the existential quantifier there exist. And the rest of the formula is given in M and M is a formula containing no quantifiers. Now the Q_1 x_1 Q_2 x_2 etc this is called the prefix of the formula and M is called the matrix of the formula.

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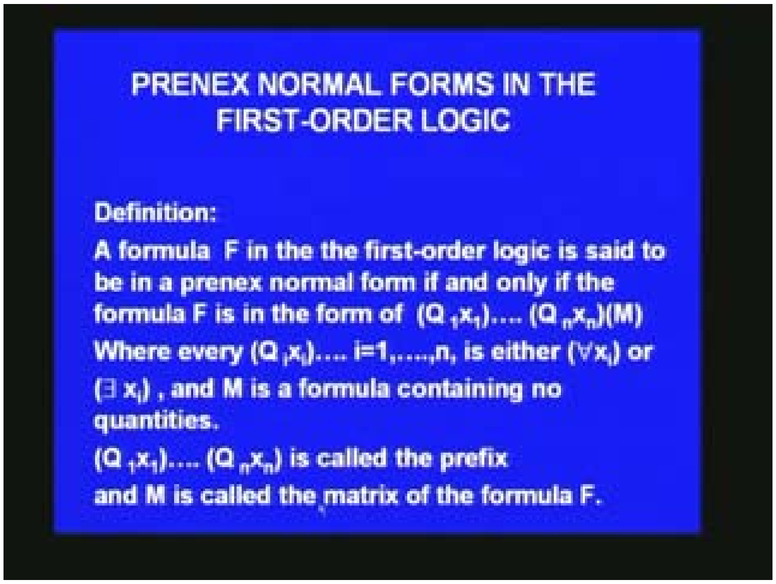

Now you can bring any well formed formula of first order logic into prenex normal form. Let us see how to go about that.

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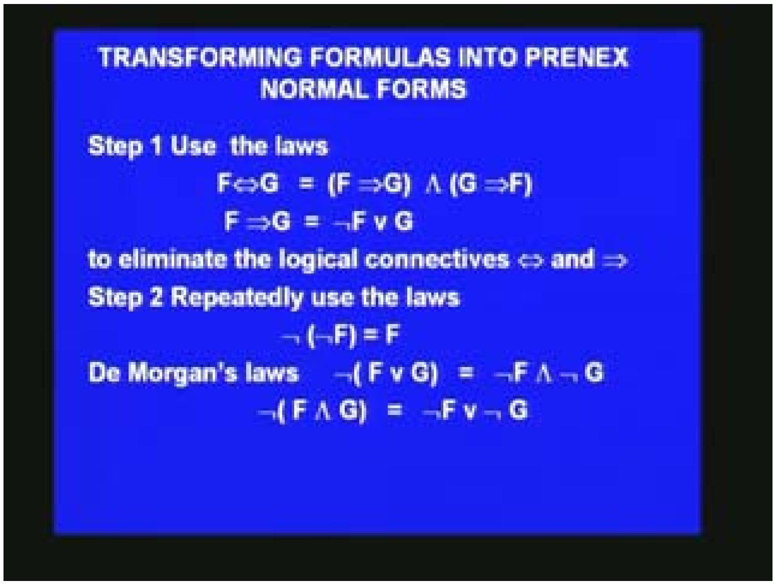

The steps you have to go through are given like this: if you want to bring any well formed formula of the first order logic how to transform into prenex normal form. First step will be, use the laws that is if F is equivalent to G you have to replace it by F implies G AND G implies F and if you have F implies G you have to replace it by NOT F OR G. Then by this you will be eliminating the logical connective equivalence and implication.

The second step will be repeatedly use NOT of NOT of F is equal to F. And De Morgan's laws NOT of F OR G is NOT of F AND NOT of G and NOT of F AND G is NOT of F OR NOT of G. And also the laws NOT of for all of F F(x) by bringing the NOT inside for all will change to there exist. We have learnt these things earlier. So this is equivalent to saying there exist x NOT of F(x). So you are bringing the NOT inside. Similarly, here if you have NOT there exist x F(x) brings the NOT inside and you will get for all of x NOT of F(x). So the second step is; bring the negation signs immediately before atoms.

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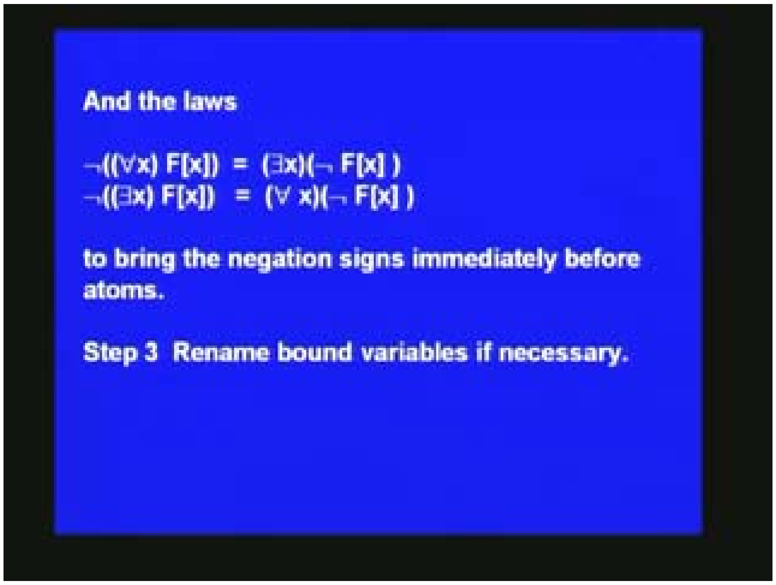

The third step is, rename bound variables if necessary. We shall see what it is taking by some examples. And step four will be make use of the laws. Now if you have Q(x) F(x) OR G where G does not contain x you can write it like this:

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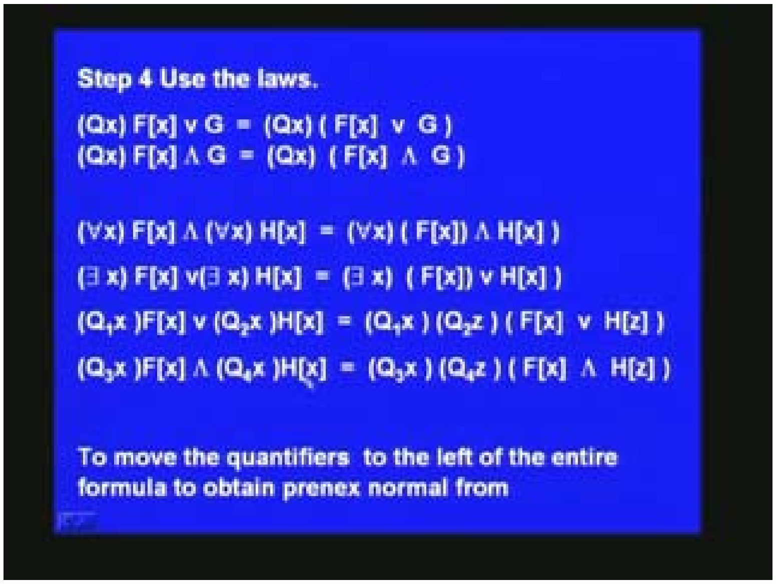

Similarly, if you have Q(x) F(x) AND G where G does not contain x you can write it like this. And for all distributes over AND so this one can be replaced by taking for all of x out and writing this way. It is for all of x F(x) AND for all of x H(x) you can write as for all of x F(x) AND H(x). And there exist x distributes over OR so in a similar manner if you have there exist x F(x) OR there exist x H(x) you can take out there exist x and write it as there exist x F(x) OR H(x). But if you have for all AND OR you cannot use that and similarly there exist will not distribute over and this you have to remember.

Now if you have Q_1x F(x) OR Q_2x H(x) then how can you bring out the quantifiers Q_1 Q_2 can be for all or there exist how can you take them out? Now this x or this quantifier is binding this portion and this quantifier is binding this portion. So the best thing for us will be rename one of the variables so keep x here and instead of x you write it as z. In that case this will not have x and this will not have z so you can take the quantifiers out this is renaming of variables. So keep it as it is and this one you rename as z and then take out the quantifiers. This you will get as Q_1x Q_2x F(x) OR H(z) you are renaming the variable here. This is using OR, this is using AND then here again you have the same thing. If you have Q_3x and Q_4x this is binding this and this is binding this so it is better to rename this variable as z and so by taking out the quantifiers you will get this. That is to move to quantifiers to the left of the entire formula to obtain prenex normal form.

Let us consider one or two examples. So let us consider some examples to see how to convert a well formed formula of first order logic into prenex normal form. We have to bring all the quantifiers to the front and the rest of the formula should be after the quantifiers. The first one is called the prefix and the later one is called the matrix of the formula. Now take this example: for all of x P(x) implies there exist x Q(x). And we have seen the steps we have to use in converting a well formed formula of the first order logic to prenex normal form.

First we have to replace equivalence and implication. So let us try to replace this implication, this you can write as NOT for all of x P(x) OR there exist x Q(x). This given formula is equivalent to this, we are using the rule P implies Q is equivalent to NOT P OR Q. Now the next step will be to bring the NOT nearer to the atom P of x and Q(x) they are the atoms, so how will you bring the NOT inside? When you bring for all of x will become there exist x so you have there exist x NOT(P(x)) OR there exist x Q(x).

We know that for all distributes over AND and there exist distributes over OR. So we can use distributive law in the reverse and take the there exist out so you can take out there exist x and write it as NOT of P(x) OR Q(x). This is in prenex normal form, this is the prefix and this is the matrix.

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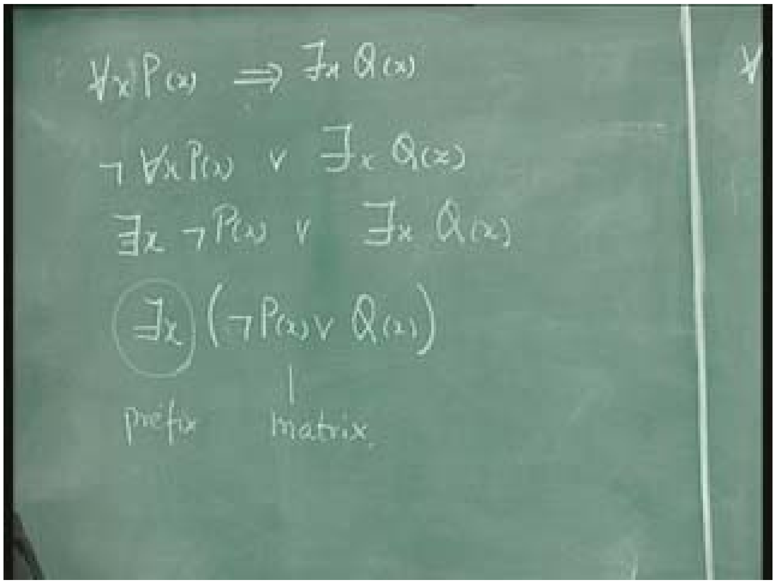

Let us take one more example: this has four variables, this is the well formed formula involving four individual variables x, y, z, u. For all of x for all of y there exist z P(x, z) AND P(y, z) implies there exist u Q(x), y, u.

Here P is a predicate variable with two variables and Q is a predicate variable with three variables, it is a ternary predicate. The scope of x for all of x and for all of y will be the whole thing, the scope of there exist z is this portion, the scope of there exist u is this portion. So we must be very careful about the scope of each quantifier.

Now fortunately in this example the two quantifiers for all x and for all of y are outside the rest of the portion we need not have to do anything about them. Only the remaining thing we have to change. So this can be written in this form for all of y and this one again the first step is to replace implication by using P implies Q is equivalent to NOT P OR Q. So this can be written in the form NOT there exist z P(x, z) AND P(y, z) OR there exist u Q(x) y u. Now for all of x for all of y remains as it is.

The second step will be to bring the NOT inside so in that case there exist will become for all so for all of z NOT(P(x, z)) AND P(y, z) for all z is the whole thing AND there exist u Q(x), y, u. Now for all of x for all of y this you can write as for all of z, use De Morgan's laws this will become NOT(P(x, z)) OR NOT of P(y, z) and the scope of z is this portion OR there exist u Q x, y, u.

Now you can see that the scope of z is this portion and z is not occurring in this portion at all. And similarly for there exist u the scope is this and u is not appearing in this portion. That way this portion is free of u and that portion is free of z. So using for all z here is not going to affect it in anyway because it does not contain z. And using there exist here there exist u here is not going to affect because this portion does not contain u at all. And for all of x for all of y the scope is the whole thing and they are already outside so we need not bother about that.

Therefore, without affecting the formula we can take out for all of z and there exist u outside. This is possible because this does not contain z and this does not contain u. Supposing it contains something we have to renamed the variable some things like that. So in this particular example there is no necessity for renaming of variables so what you get is for all of x for all of y for all of z there exist u then you get NOT(P(x, z)) OR NOT(P(y, z)) OR Q(x), y, u. So we have brought the given formula into prenex normal form. This is the prefix which contains all the quantifiers and this is the matrix portion which does not contain any quantifiers.

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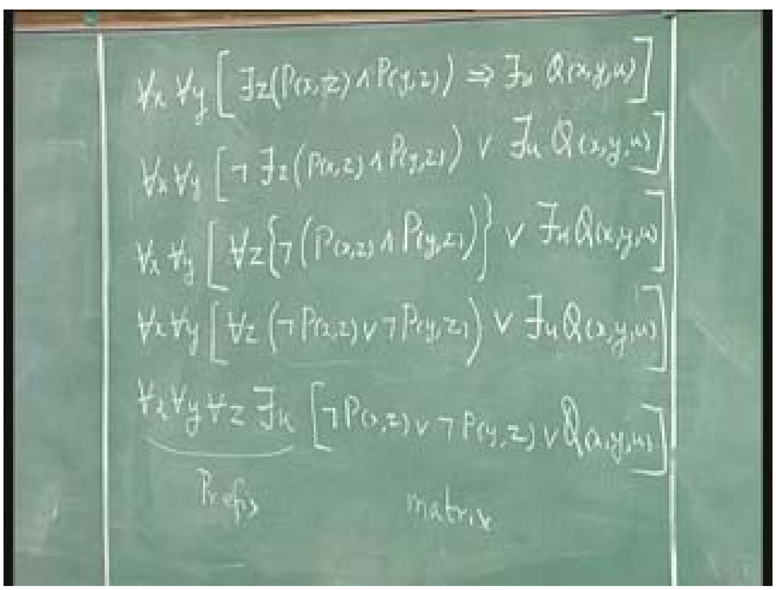

Like that we can bring any formula of first order logic into prenex normal form. Propositional logic also you can look at as Boolean algebra which has got application to switching circuits. I am not going into the detail of these circuits but you must have heard about AND gate OR gate and so on. An AND gate is represented like this; it has got two inputs and the output is P AND Q. P and Q can be 0 or 1 and output will be 1 when P and

Q are both 1 otherwise it will be 0. This is known as the AND gate in switching circuits. And an OR gate is represented like this; there will be two inputs P and Q and output will be P OR Q it will be 0 when both P and Q are 0 when one of or both of P and Q are 1 it will be 1. It represents the AND connective in the propositional logic. The NOT gate in switching circuits is represented like this; there is one only one input and output represents NOT P and this is called the NOT gate.

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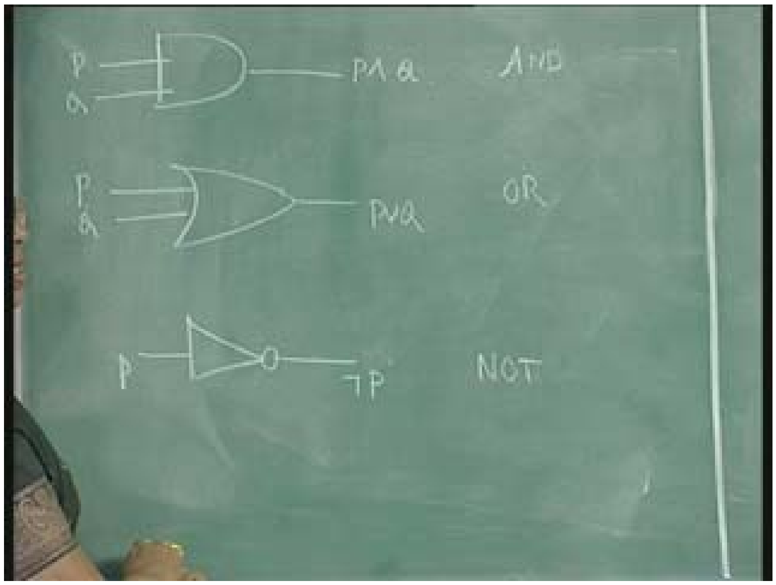

In switching circuits we also have two more gates known as the NAND gate and the NOR gate. NAND which is actually NOT AND complimentary to AND and the truth table for that will be like this: P Q, the NAND operator is usually denoted like this pierce arrow. When this is 0, this is 0, this is 0, 1 you have to consider the four possibilities 1 0 1 1. In the case of AND AND of P and Q will be one in this case so NOT AND will be 0 and in the other cases it will be 1. So it is complimentary to the AND table. P AND Q will be like this so it is complimentary. This is represented by a gate of this form OR.

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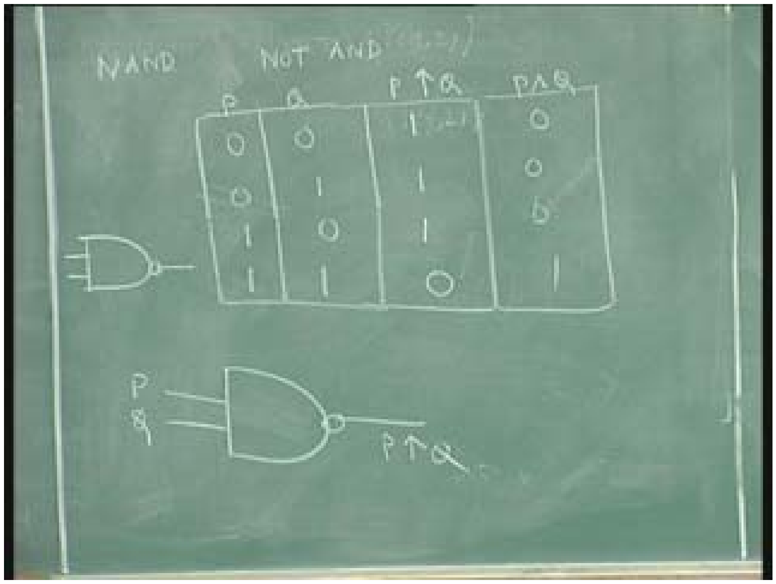

We also have another gate NOR which represents NOT OR. The truth table for that will be like this: P Q this is sometimes represented by this notation. Again we have possibilities 0 0 0 1 and 1 0 1 1. What will be the table for P OR Q? P OR Q will be 0 in this case in the other three cases it will be 1. So, it is NOT of OR so it will be 1 0 0 0.

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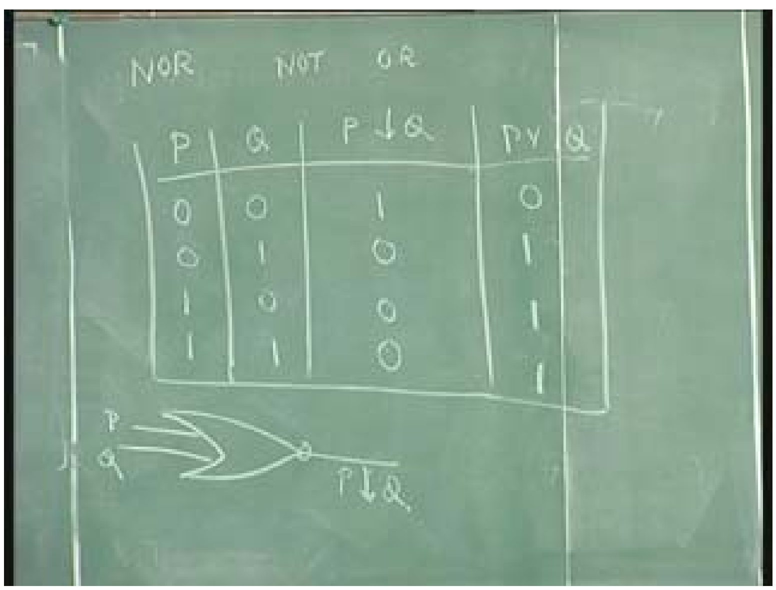

The truth table for NOR gate is given by this: it is represented by a diagram like this with two inputs P Q P. Now any Boolean function can be represented by a Karnaugh Map and you can represent it as a canonical sum of products or disjunction of minterms.

For example, involving two variables P OR Q OR R if you represent it, I again do not want to go into the details of what is meant by Karnaugh Map and all that, those who are familiar can connect it up this.

Suppose I have three variables P Q and R this is P this is Q and R then 0 0 0 is this 0 0 1 is this, better use P Q and R here so 0 0 will be this 0 0 1 will be like this 0 1 0 3 4 5 6 7 cells can be represented like this. P will be represented by this, Q will be represented by this and R represented by this which includes all the seven cells and this will be written in the form sigma of 1, 2, 3 4 5 6 7 and this we have already seen. This is to show the connection between Boolean algebra or propositional logic and Boolean algebra and switching circuits.

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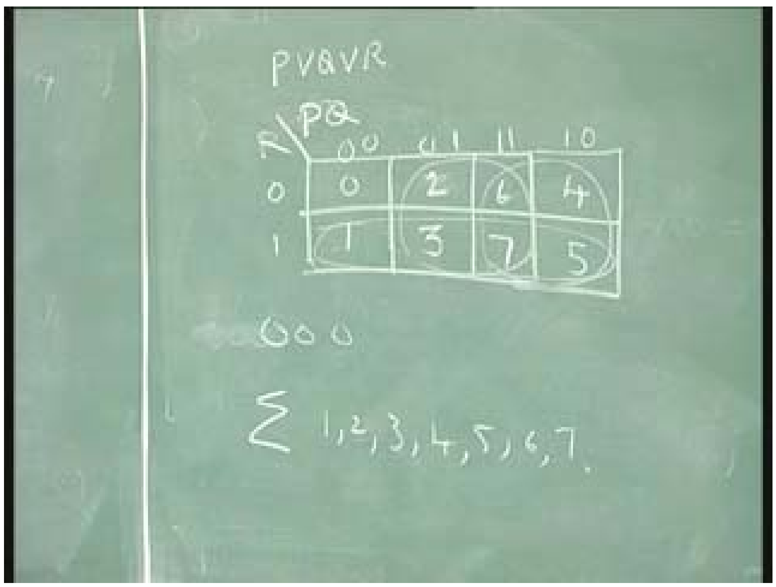

We already know what is meant by AND OR NOT. Two more gates are there in switching circuits NOR and NAND. And any Boolean function can be represented like this as a principal Disjunctive Normal Form.