Discrete Mathematical Structures

Dr. Kamala Krithivasan

Department of Computer Science and Engineering

Indian Institute of Technology, Madras

Lecture # 34

Recurrence Relations (Contd..)

In the last two lectures we saw about recurrence relations and how to solve those recurrence relations or difference equations by finding the homogenous solution and also the particular solution. Depending upon the type of the equation we have to find out homogenous solution then depending upon the right hand side f(r) we have to decide what type of particular solution we can have and then solve it. The solution to the recurrence relation consists of the sum of both the homogenous solution and the particular solution. This is what we have seen in the last lecture and we get an expression for a_r the rth term. We can also get by means of using what is known as generating functions. Earlier we have seen how generating functions can be used for finding the permutations and finding the combinations and so on. We saw both ordinary generating functions and exponential generating functions. Here what we will consider is only ordinary generating functions. How the idea of a generating function can be used to solve the recurrence relation.

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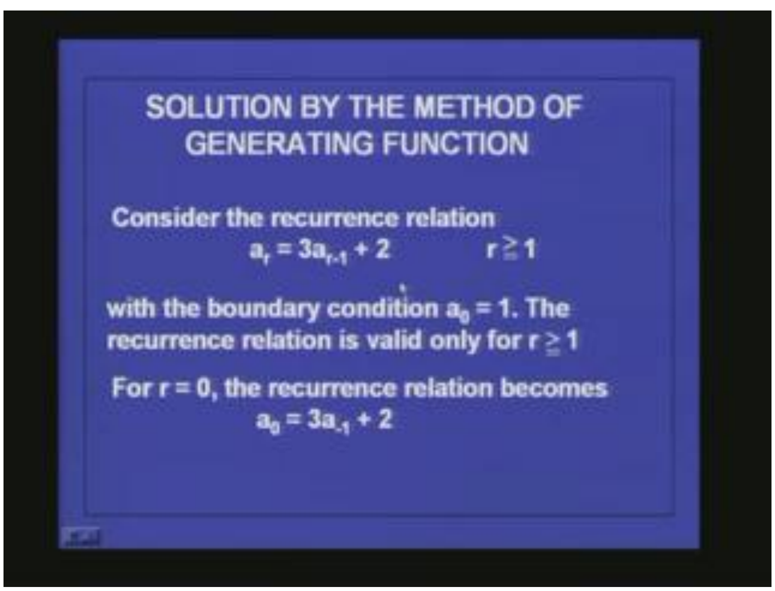

This is what we will see today. Let us take this example, so we want to see the solution by the method of generating function for the recurrence relation. Take this example; consider the recurrence relation a_r is equal to $3a_{r \text{ minus } 1}$ plus 2 this is true for r greater than or equal to 1. So a_1 is equal to $3a_0$ plus 2 and the boundary condition is given as a_0 is equal to 1. This recurrence relation is valid only for r greater than or equal to 1. We can see that if r is equal to 0 the recurrence relation becomes a_0 is equal to $3a_{\text{minus } 1}$ plus 2 and $a_{\text{minus } 1}$ does not have any significance it does not exist. So this particular recurrence relation is valid only for r greater than or equal to 1, why we have to consider this particularly here will become evident in a moment. Now, given this you multiply by z power r both the left hand side and the right hand side, multiplying both sides of the equation by z power r you obtain a_r z power r is equal to $3a_{r \text{ minus } 1}$z power r plus 2z power r.

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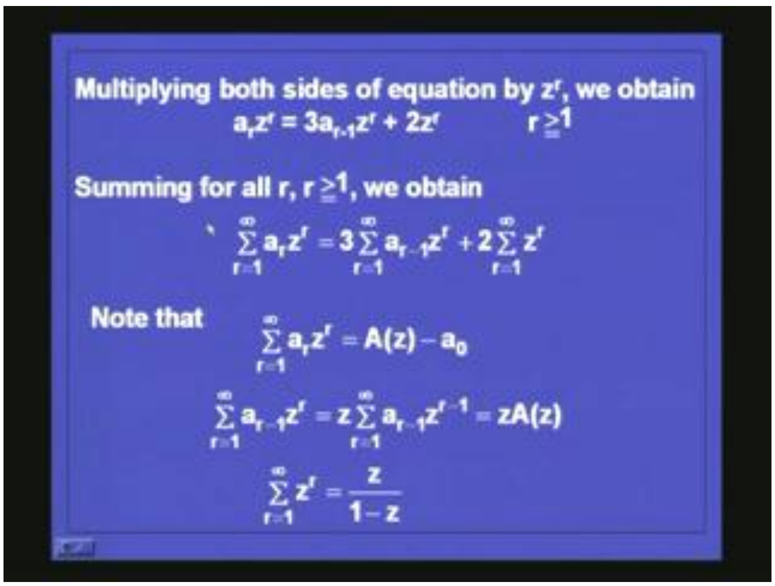

Now, sum it over all r, r greater than or equal to 1, here the reason comes. You cannot sum it for r greater than or equal to because it is not valid for r is equal to 0. So you have to sum for all r greater than or equal to 1 because it is valid for those values of r. So if you sum it for all r you obtain sigma r is equal to 1 to infinity a_r z power r is equal to 3 this 3 you can take out 3 sigma r is equal to 1 to infinity $a_{r \text{ minus } 1}$ z power r plus 2 times sigma r is equal to 1 to infinity z power r.

Now what is this left hand side?
The generating function for a_0 a_1 a_2 a_3 etc is a_0 plus a_1z plus a_2z square plus a_3z cube and so on. So sigma r is equal to 1 to infinity a_r z power r is a_1z plus a_2z square and so on. The first term a_0 is not there so if you denote the generating function by A(z) for this then the first term is not in this so you have to subtract the first term. So this equal to A(z) which represents the generating function for this a_0 a_1 a_2 sequence and because the first term is not there you subtract that. Similarly, look at this portion leave all a_3 later on we can multiply by that 3.

Consider this portion r is equal to 1 to infinity $a_{r \text{ minus } 1}$ zr. Now the generating function is a_1z plus a_2z square and so on so general term should be $a_{r \text{ minus } 1}$ z power r minus 1 but here we are having z power r so you have to take out 1z. So, if you take out 1z from this you will get sigma r is equal to 1 to infinity $a_{r \text{ minus } 1}$ z power r minus 1 which is the general term for the generating function, it starts from r is equal to 1 when you put r is equal to 1 it becomes a_0. So all the terms are there a_0 plus a_1z plus a_2z square and so on. But you have taken 1z out so this becomes z A(z). And, if you look at this term let us consider 2 later and you have to start with r is equal to 1 so it is z plus z squared plus z cubed and so on. But you know that 1 plus z plus z squared etc is 1 by z, here again if you take out 1z out this becomes 1 plus z plus z squared plus z cubed etc and that is 1 by 1 minus z. So sigma r is equal to 1 to infinity is z by 1 minus z. Use these values in this equation then you will get A(z) minus a_0 is equal to 3zA(z) plus 2z by 1 minus z.

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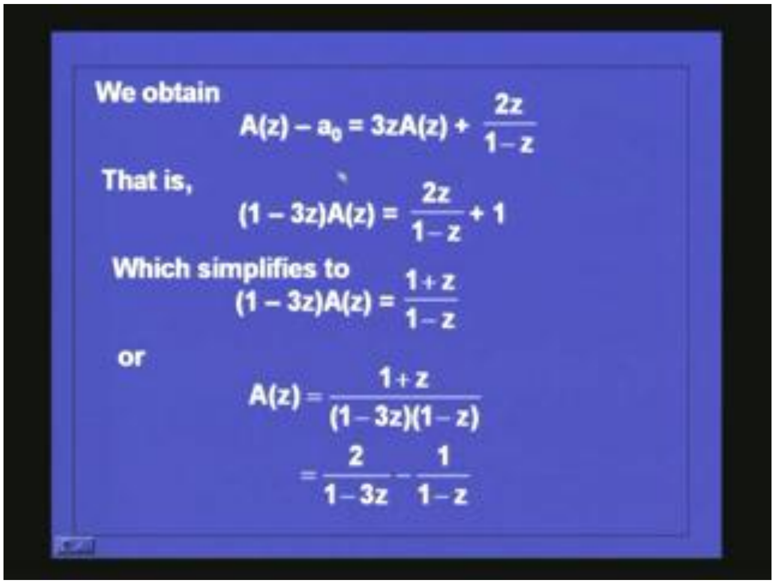

But the boundary condition a_0 is given to be 1 so a_0 you substitute the value of 1 here. So A(z) minus 1 is equal to this so bring the 1 to this side and bringing this 3z A(z) to the left hand side you get (1 minus 3z) A(z) is equal to 2z by 1 minus z plus 1. If you simplify the right hand side it becomes 1 plus z by 1 minus z. So you get (1 minus 3z) A(z) is equal to 1 plus z by 1 minus z. Again bringing this to the right hand side you get A(z) is equal to (1 plus z by 1 minus 3z) (1 minus z). If you resolve this into partial fractions then this can be written as a by (1 minus 3z) plus b by 1 minus z and solve for a and b you get this value. So the general term here will be, this we can see that can be expanded as (1 plus 3z plus 3z) the whole squared and so on so the general term here will be 3 power r z power r. Similarly, here if you expand it is 1 plus z plus z squared etc so the general term is z power r for this.

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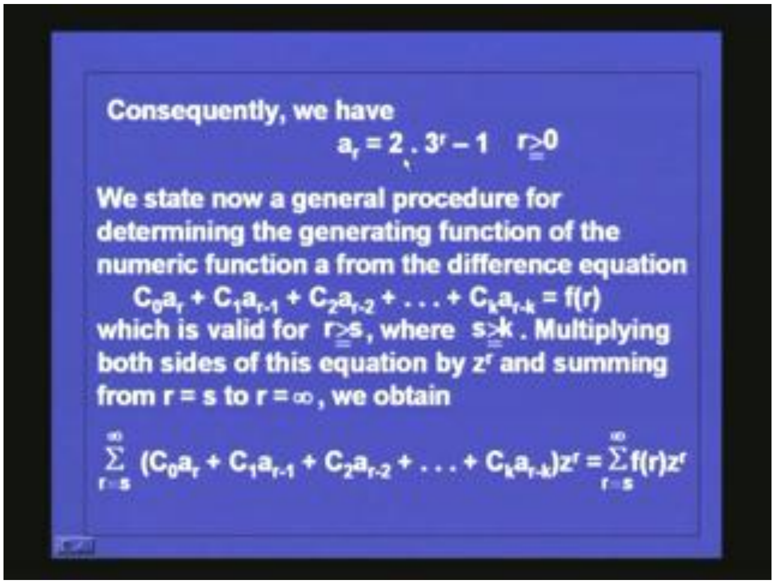

Consequently, you get a_r is equal to 2 into 3 power r and minus 1. Look at this equation which we started with a_r minus 3ar minus 1 is equal to 2 so you can see that 3 is a characteristic root and so a_3 power r will be a homogenous solution and so using the boundary conditions you can find that it is 2 into 3 power r and this is the particular solution this also you can very easily see. But we have obtained the same thing by the method of generating function. And this seems to be a better method because in the case of finding the homogenous solution and the particular solution you guess the value of the particular solution and try to substitute the equation and find the exact particular solution. But when you use generating function there is no guessing or anything you have to straight away consider the equation sum it up and then get the answer.

So, what is the general procedure for determining the generating function of the numeric function a from the difference equation. It is a linear difference equation of the kth order C_0 C_1 C_2 C_k are constants and you have this equation C_0a_r plus C_1a_r minus 1 plus $C_2a_{r \text{ minus } 2}$ and so on is equal to f(r). This is valid for all r greater than or equal to s where obviously s has to be greater than or equal to k otherwise it will not have many. So, if you multiply the whole thing by z power r and then you sum from r is equal to s to infinity because it is valid for r is equal to greater than or equal to s you can only start from s and then sum up to infinity. So we obtain r is equal to s to infinity C_0a_r plus $C_1a_{r \text{ minus } 1}$ the whole expression is multiplied by z power r. Then on the right hand side you have sigma r is equal to s to infinity f(r) z power r. Now, take term by term by term and see how you can rewrite them. The first term here will be C_0a_r z power r and you are summing from r is equal to s to infinity. So you have r is equal to s to infinity C_0a_r z power r.

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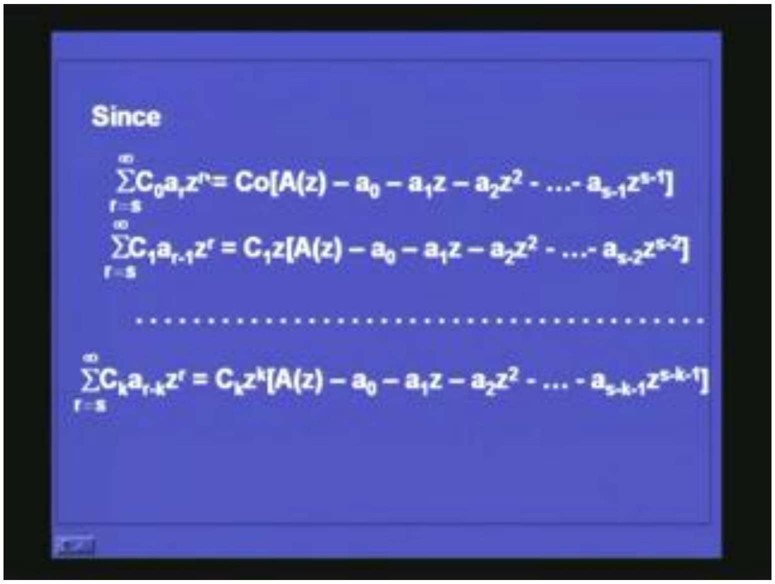

What is this?
a_r z power r is the general term of the generating function a_0 a_1 a_2 etc. If you called the generating function as A(z) here the first a 0 up to as minus 1 they are missing here it starts from as z power s. So subtracting those from the generating function C_0 you can take out so the rest of it will be sigma r is equal to s to infinity and that is the generating function where the first s term starting from a_0 to $a_{s \text{ minus } 1}$ are missing so A(z) minus a_0 minus a_1z minus a_2z squared and so on up to minus $a_{s \text{ minus } 1}$ $z_{s \text{ minus } 1}$.

Similarly, if you look into the second term this is sigma r is equal to s to infinity $C_1 a_r$ minus 1 z power r how can you look into this? r is equal to s to infinity $C_1 a_{r \text{ minus } 1}$ z power r. The general term in the generating function will be a r minus 1 z r minus 1 so take out one z out and C_1 also you can take out so C_1z if you take out you get sigma r is equal to s to infinity a r minus 1 z r minus 1. That is the first r minus 1 terms are missing in the generating function so that will give rise to A(z) minus a_0 a_1z etc up to a s minus 2 z s minus 2.

Similarly, each one of the term you can substitute and you can see that r is equal to s to infinity $C_k a_{r \text{ minus } k}$ z power r, you take the C_k out, you take 1z power k also out so remaining term will be sigma r is equal to s to infinity $a_{r \text{ minus } k}$z power r minus k and there if a few terms in the beginning of the generating function are missing, so it will be A(z) minus a_0 minus a_1z up to minus $a_{s \text{ minus } k \text{ minus } 1}$ z power s minus k minus 1. Having written the left hand side like this and then adding all these things and equating to the right hand side you get A(z) will be 1 by C_0 plus C_1z plus to C_kz power k right hand side you have the sum from r is equal to s to infinity f(r) z power r but these terms which are there when they are brought into the right hand side they will become plus.

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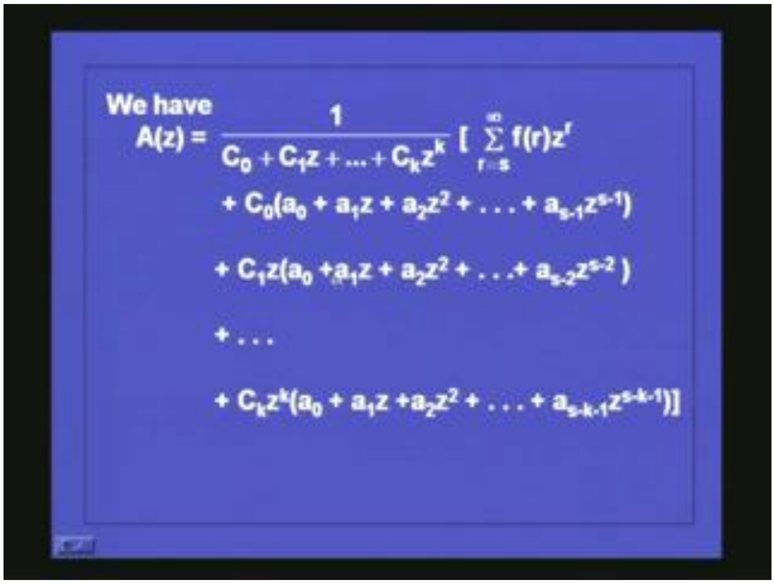

So you get C_0a_0 plus a_1z plus a_2z squared etc then C_1z into a_0 plus a_1z etc and so on up to C_kz power k a_0 plus a_1z plus a_2z so on. So you get a much closed form expression for the generating function. And in general this expression will be such that it will be very easy to find the simple expression for a_r. Let us consider a few more examples and solve them by using the method of generating functions.

Let us consider the same example which we have considered in the last lecture; a_r be the number of regions into which the plane is divided when you draw r ovals each oval cutting every other oval in two points. And this is the recurrence relation we obtained in the last lecture and the solution for this is a_r is equal to r squared minus r plus 2 this is the answer we got by making use of homogenous solution and particular solution. Now, let us see whether we can get this by making use of the idea of a generating function. The boundary condition is a_1 when you just draw one oval it divides the plane into two regions so a_1 is equal to 2, a_2 is equal to when you draw two ovals it divides the plane into four regions so a_2 is equal to 4.

Now, we can check that a_1 is 1 minus 1 plus 2 2, a_2 is 2 square minus 2 plus 2 is 4 and so on. Now, a_0 does not have any meaning because when you do not have any oval it does not make any sense but you choose a_0 so that it satisfies this recurrence relation.

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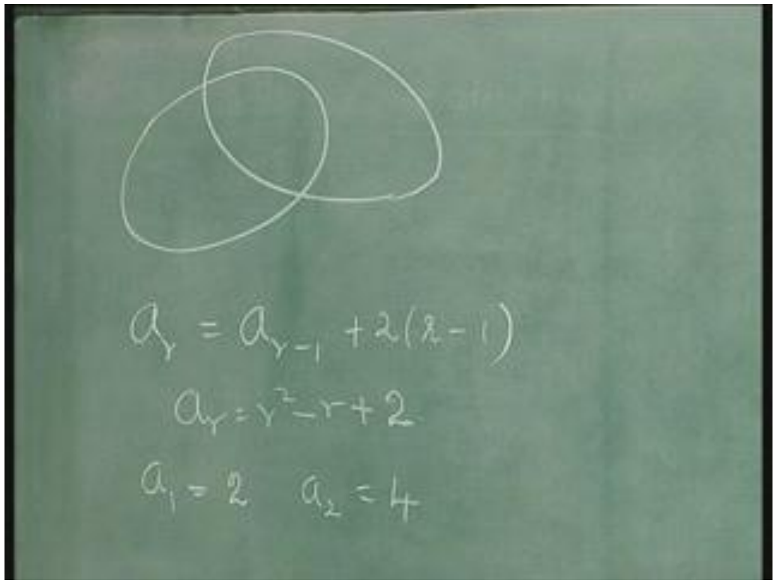

a_1 is a 0 plus 2 into 1 minus 1 so a_0 is equal to a_1 that is equal to 2. a_0 does not have a meaning but to satisfy the recurrence relation you can choose it as 2. Now, look at the recurrence relation, let us find the solution by making use of the generating function concept. a_r is equal to a r minus 1 plus 2 (r minus 1). Now, multiply by z power r then you get a_r z power r is equal to a_r minus 1z power r plus 2 (r minus 1) z power r sum from r is equal to 1 to infinity so r is equal to 1 to infinity a_r z power r is equal to sigma r is equal to 1 to infinity a_r minus 1 z power r plus sigma r is equal to 1 to infinity 2 (r minus 1) z power r.

Now this we have seen is of the form a_1z plus a_2z square so if you represent the generating function for a_0 a_1 as A(z) this is A(z) minus a_0 and this again has in the previous example the general term has to be a r minus 1 z r minus 1 so if you take that z out it will be a_0 plus a_1z etc which is A(z). Then this term is, if you take that 2 z squared out it will be r is equal to (1 to infinity r minus 1) z power r minus 2. This is of the form when r is equal to 1 it is 0 when r is equal to 2 2 minus 1z 2 minus 2 it will be 1 so it will be 1 plus 2z plus 3z square plus 4z power 5 and so on, so it will be like that. This expression will be 1 plus 2z plus 3z square plus 4z cube plus 5z power 4 and so on. This you know is 1 by 1 minus z the whole square. So making use of that here you get A(z) 1 minus z is equal to a_0 plus 2z square by 1 minus z the whole square. But we have seen that a_0 is just 2 and this is 2z square by 1 minus z the whole square.

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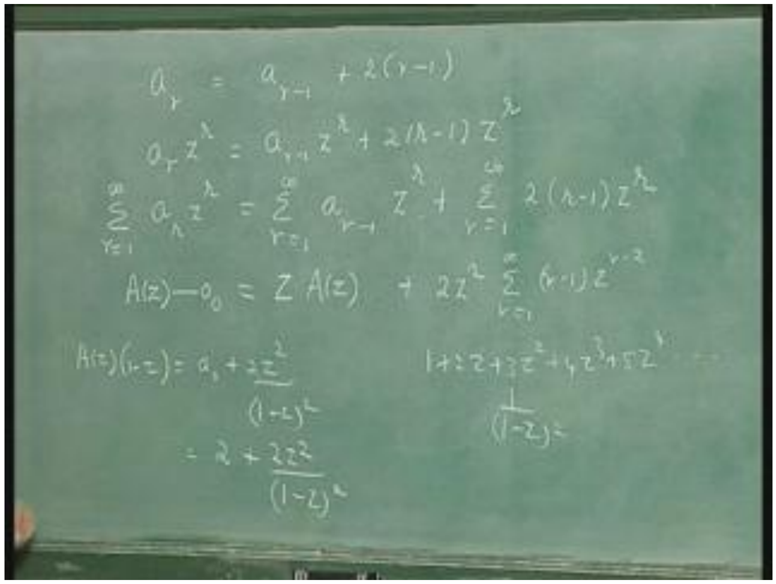

So with this you get the expression for A(z) like that; A(z) is 2 by 1 minus z plus 2z cube by 1 minus z the whole cube. This is the generating function for the sequence a_0 a_1 a_2 a_3 etc. Now the general term is, what will be the general term? z power r that is 2zr it should be like that, here you want to find the term for r that is 2 you have to find the coefficient of z r minus 2 here so that when you multiply by z square you get z power r. But what is 1 by 1 minus z cube? It is 1 plus 3z plus 3 into 4 by 2z square plus 4 into 5 by 2z cube and so on. The general term or the coefficient of z power r minus 2 will be r minus 1 into r by 2 so making use of that here it will be 2 (r minus 1) (r by 2zr) the coefficient of z power r here will be this, this 2 and 2 will get cancelled so coefficient of z power r that is a_r a_r is given by 2 plus r into r minus 1 which is r squared minus r plus 2.

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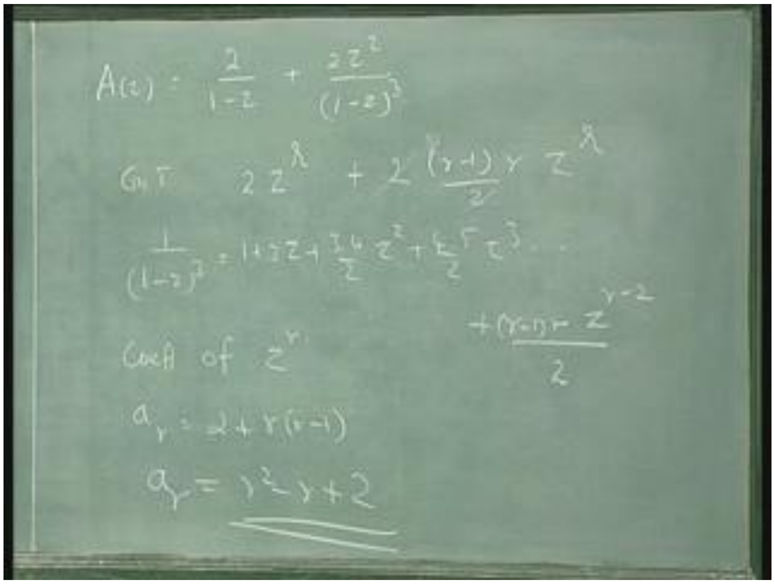

So the expression for a_r is r squared minus r plus 2 which is the same as we got in the last lecture by making use of homogenous solutions and particular solutions. So we see how to solve the same problem by means of the generating function method and also by homogenous solution and particular solution. Let us take one simpler example and solve it in both the ways and see the how the answers agree.

Consider the tower of Hanoi problem, what is the tower of Hanoi problem?
You have three pecks a b c and in one peck you have disks in the decreasing order, the smaller disk is placed above the bigger disk and you want to transfer all of them from this peck a to peck b intermediately you can use peck c. the transfer should be done in a such a way that at no time smaller disk lies below a bigger disk or a bigger disk lies above a smaller disk.

So, if you have a_1 that is just one disc it requires only one transfer but if you have two discs like this first you can put it here then transfer the second one here then again transfer this here so a_2 will be 3. In general when you have n discs you transfer (n minus 1) of them here transfer the nth disc here then transfer again the (n minus 1) disc to this place making use of the remaining peck as an intermediary peck.

Therefore, the expression you get is a_r is equal to 2a r minus 1 plus 1 this is the recurrence relation. And when you have this recurrence relation what is the solution. And you have a_1 is equal to 1 and a_2 is equal to 3 so what will be the value of a_0? a_0 has no disc so it has to be 0 but let us see a_1 is 1, 2 a_0 plus 1 which gives you a_0 is equal to 0. This is also meaningful because when you do not have any discs no transfer needs to take place and the number of steps is 0. So a_0 is 0 in this case.

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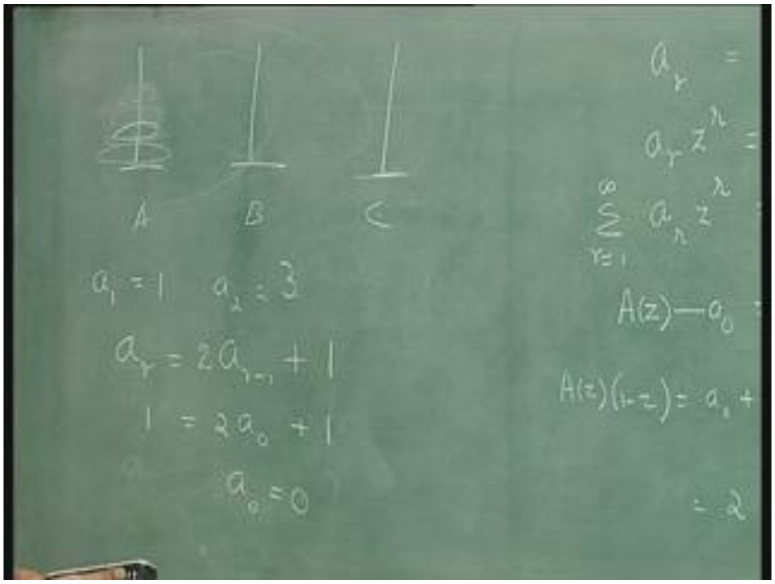

Now, let us see how we make use of the homogenous solution and the particular solution method to solve this. So $a_{r\ \text{minus}\ 2}$ $a_{r\ \text{minus}\ 1}$ is equal to 1 so the characteristic equation is x minus 2 is equal to 0 or x is equal to 2 is a root so homogenous solution is a_r is equal to 2 power r. We have to find the value of a from boundary conditions. Now what is the particular solution the right hand side is a constant and 1 is not a root so you can take the particular solution in the form p a constant so p minus 2p is equal to 1 or p is equal to minus 1. So the total solution art will be homogenous solution a_rh plus a_rp and that will be 2 power r minus 1. So you have the expression for a_r as a_r is equal to a 2 power r minus 1 you know that a1 is 1. So 1 is equal to 2 power 1 minus 1 so 1 is equal to 2A minus 1, 2a is equal to 2 so a is equal to 1. So the expression for a_r is 2 power r minus 1. This is the number of steps required to transfer r discs from peck a to peck b making use of an intermediary peck c such that at no point of time a bigger disc lies above a smaller disc. The expression is given by a_r is equal to 2 power r and minus 1.

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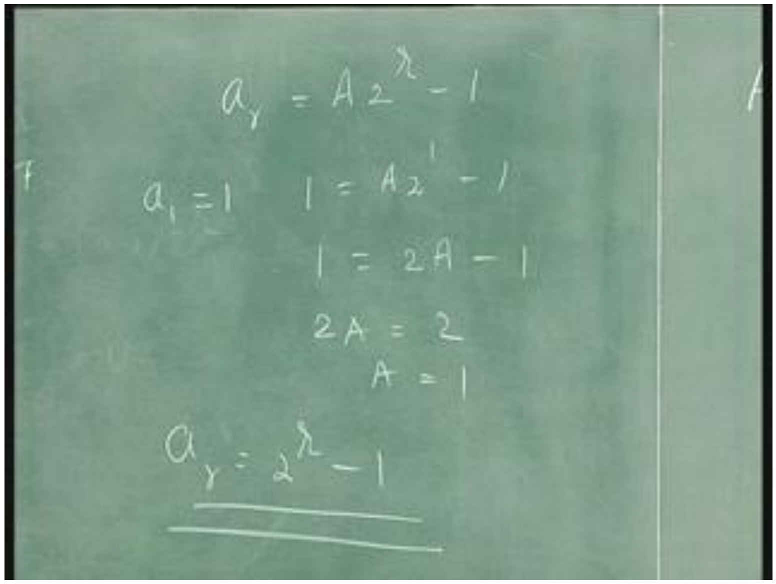

Let us see how the same problem can be solved by means of generating function method. So we have to get the answer a_r is equal to 2 power r minus 1. So we start with this recurrence relation it is valid for r is equal to 1 so this is r minus 1. So multiply by z power r so you get a_r z power r minus 2 a_r minus 1 z power r is equal to z power r. Then sum from r is equal to 1 to infinity so that will be sigma r is equal to 1 to infinity a_r z power r minus sigma r is equal to 1 to infinity 2A r minus 1 z power r is equal to sigma r is equal to 1 to infinity z power r. So, this you know is A(z) minus a_0 and this is 2z A(z) 2 you will take out and then A(z) is equal to this is z by 1 minus z because z you can take out then it will be a sum 1 plus z plus z squared which is 1 minus z.

Now this becomes a_0 is 0 so this becomes A(z) 1 minus 2z is equal to z by 1 minus z or A(z) is equal to z by 1 minus z (1 minus 2z). So resolving into partial fractions let z by 1 minus 2z (1 minus z) is equal to a by 1 minus 2z plus b by 1 minus z. So in this case z is equal to a (1 minus z plus b) (1 minus 2z) equating the constant terms a plus b is equal to 0 equating the coefficient of z you have minus a minus 2b is equal to 1 or a plus 2b is equal to minus 1 a plus b is equal to 0 subtract you get b is equal to minus 1. And because a plus b is equal to 0 a is equal to minus b or 1 a is 1 and b is 0.

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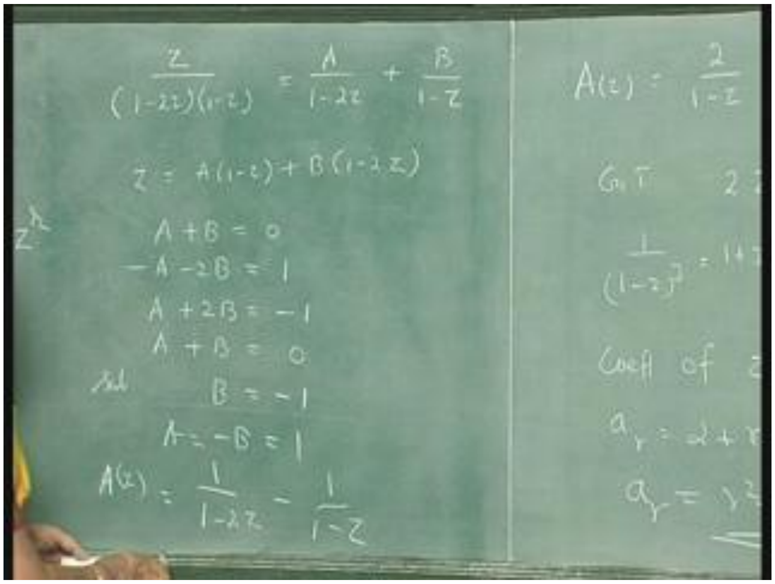

So you get A(z) the generating function as 1 by 1 minus 2z minus 1 by 1 minus z. Now, for the solution you have to find the general term 1 by 1 minus 2z will be 1 plus 2z plus 2z the whole square plus 2z the whole cube and so on. So general term is 2 power r z power r and 1 by 1 minus z is 1 plus z plus z squared and so on so the general term is z power r. So, if you take the coefficient of z power r in A(z) the first term will contribute to 2 power r so a_r that is the coefficient of z power r the first term will contribute to 2 power r and you have a minus here so minus the coefficient of z power r in the second term that is 1.

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So the expression you get is a_r is equal to 2 power r and minus 1 which is the same as we obtained in the earlier case. So the same problem we have solved by making use of homogenous and particular solutions then also we get the same answer. By the method of generating function also we get the same answer. But the generating function method is much more elegant than the earlier one. Sometimes we get generating equations of this form a_r is equal to $3a_r$ minus 1 plus $2b_r$ minus 1 plus b_r is equal to a_r minus 1 plus b_r minus 1. Here there are two variables you have to solve for both a_r and b_r. Here the method of generating function comes in handy, we can solve in a very elegant manner making use of generating functions.

The boundary conditions are given like this as a_0 is equal to 1 and b_0 is equal to 0. Then again use the same method, multiply by z power r so a_r z power r is equal to 3ar minus 1z power r plus $2b_r$ minus 1z power r and summing from r is equal to 1 to infinity this is a_r z power r is equal to sigma r is equal to 1 to infinity 3ar minus 1zr plus sigma r is equal to 1 to infinity $2b_r$ minus 1zr. Now, similarly you will also get sigma r is equal to 1 to infinity b_r z power r is equal to sigma r is equal to 1 to infinity a_r minus 1zr plus sigma r is equal to 1 to infinity b_r minus 1zr.

Rewriting this, If A(z) is the generating function for a_0, a_1 etc and B(z) is the generating function for b_0 b_1 etc then these two equations become A(z) minus a_0 is equal to 3 times z take out that z then you get 3zA(z) plus 2z B(z) and this second equation becomes B(z) minus b_0 is equal to z A(z) plus z B(z), a_0 is 1 and b_0 is 0 by the initial condition. So you have A(z) minus 1 is equal to 3zA(z) plus 2zB(z) and B(z) is equal to z A(z) plus z B(z). Writing it as simultaneous equations you get A(z) (1 minus 3z) minus 2zB(z) is equal to 1 the first equation becomes like this. The second equation becomes z A(z) minus B(z) (1 minus z) is equal to 0 or z A(z) is equal to 1 minus z B(z) or B(z) is equal to z by 1 minus z A(z). Make use of this in the first equation.

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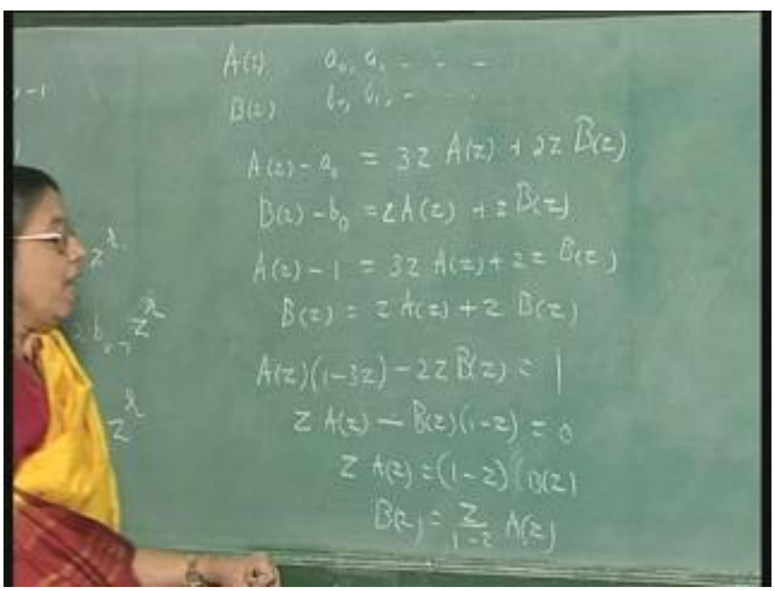

So you get A(z) (1 minus 3z minus 2z) instead of B(z) again you can write this will be z by 1 minus z A(z) is equal to 1 or you get A(z) (1 minus 3z) (1 minus z minus 2z) squared A(z) by 1 minus z is equal to 1 or A(z) (1 minus 3z minus z plus 3z) square minus 2z square is equal to 1 minus z or A(z) is equal to 1 minus z by 1 minus 4z plus z square and B(z) is z by 1 minus z A(z) that is z by 1 minus z into 1 minus z by 1 minus 4z plus z squared this will get cancelled and so you get is equal to z by 1 minus 4z plus z squared. So the expression for A(z) is this.

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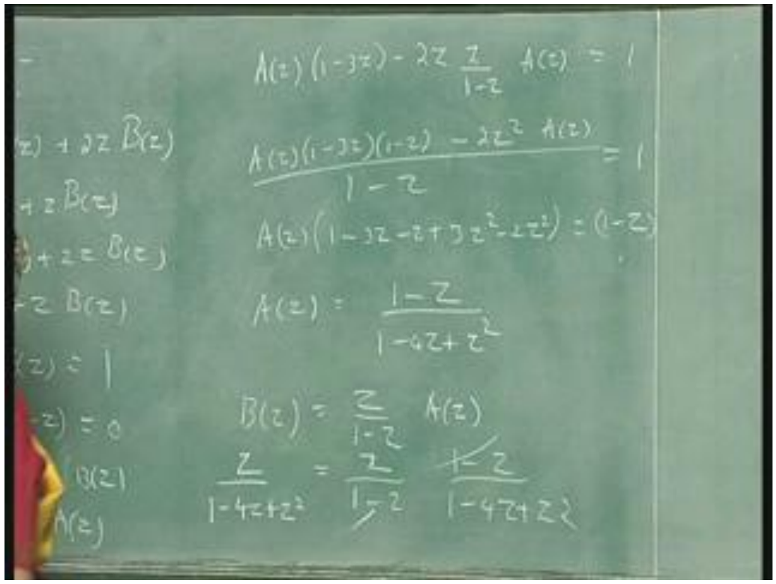

The expression for B(z) is this. Having obtained this can you get the value of a_r and b_r? Now, this is a quadratic expression you have to resolve into partial fractions to get the roots. So 1 minus 4z plus z squared what are the roots? The roots are given by 4 plus or minus square root of 4 squared minus 4 by 2. That is 2 plus or minus square root of 12 by 2 which is 2 plus or minus root 3. So having obtained like this first let us solve for a, A(z) is equal to 1 minus z by 1 minus 4z plus z squared this you can write as a by 1 minus 2 plus root 3z resolving into partial fractions you can write like this plus b (1 minus 2 minus root 3z).

Now you have to find the value of a and b. How do you find that 1 minus z is equal to a (1 minus 2 minus root 3) (z plus b) (1 minus 2 plus root 3) z. So equating the constants a plus b is equal to 1 this is equating the constants. Then equating the coefficient of z you have minus 1 is equal to a into minus 1 (2 minus root 3) that is minus a into this again minus b (2 plus root 3) this is equal to, so 1 is equal to if you remove the minus sign it will be a (2 minus root 3) minus plus b (2 plus root 3).

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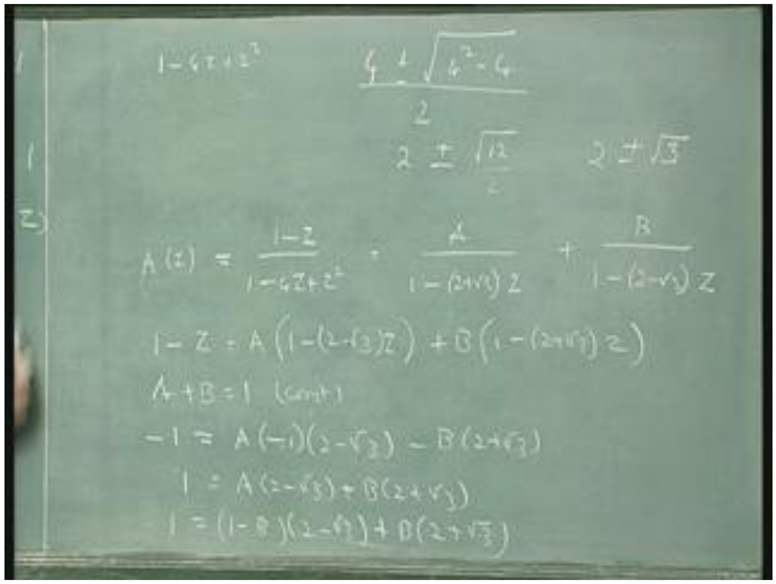

Now making use of this a is 1 minus b so 1 is equal to a is 1 minus b (2 minus root 3 plus b) (2 plus root 3) let us simplify and see what we get. We get 1 is equal to (2 minus root 3 minus b) (2 minus root 3 plus b) (2 plus root 3). If you take the b here B(2 minus root 3 minus 2 minus root 3) is equal to 2 minus root 3 minus 1 that will be B into this will get cancelled minus 2 root 3 is equal to 1 minus root 3. So B will be root 3 minus 1 by 2 root 3 multiplying by root 3 you get 3 minus root 3 by 6 the value of B you get as 3 minus root 3 by 6. The value of A is 1 minus B that is given by 1 minus 3 minus root 3 by 6 is given by 6 minus 3 plus root 3 by 6 is 3 plus root 3 by 6.

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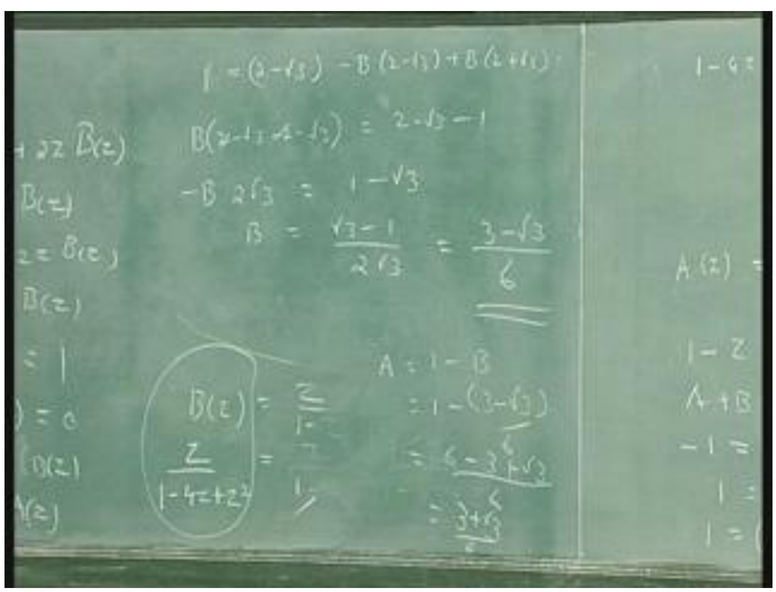

So the expression becomes the value of A(z) where A(z) is given by 3 plus root 3 by 6 by 1 minus 2 plus root 3 (z plus bI) where b is 3 minus root 3 by 6 by 1 so this will be 1 minus 2 minus root 3z. So the general term will be coefficient of z power r will be 3 plus root 3 by 6 and 2 plus root 3 to the power of r plus 3 minus root 3 by 6 and 2 minus root 3 power r this will be the coefficient of z power r. So this is the expression for a_r, a_r is given by this.

Let us check for the value of r is equal to 0 the boundary condition is a_0 is 1, see whether it works out, put r is equal to 0 this is 1 this is 1 so you get a_0 is equal to 3 plus root 3 by 6 plus 3 minus root 3 by 6 is equal to 6 by 6 is equal to 1. a_0 is 1 and that is what we have taken as the boundary condition it verifies here. So similarly you can also find the expression for B(z). Now, B(z) is given by z by 1 minus 4z plus z square. So making use of partial fractions this can be written as a by (1 minus 2 plus root 3z plus b) (1 minus 2 minus root 3) (z) you can write this way. So, that means z is equal to a (1 minus 2 minus root 3) (z plus b) (1 minus 2 plus root 3) o z.

Equating the constants a plus b is equal to 0 these are constants and considering the coefficient of z minus a (2 minus root 3 minus b) (2 plus root 3) is equal to 1. So from the first one a is equal to minus b make use of that here a is minus b so minus a will be b so b (2 minus root 3 minus b) (2 plus root 3) is equal to 1 or b (2 minus root 3 minus 2 minus root 3) is equal to 1 minus 2 root 3 b is equal to 1 or b is equal to 1 by 2 root 3 with the minus sign. And a is minus b that is 1 by 2 root 3. So this you can write as multiply by root 3 and write it as root 3 into 6 multiplying the numerator and denominator by root 3 you get this. Similarly, this is minus root 3 by 6 and this is root 3 by 6.

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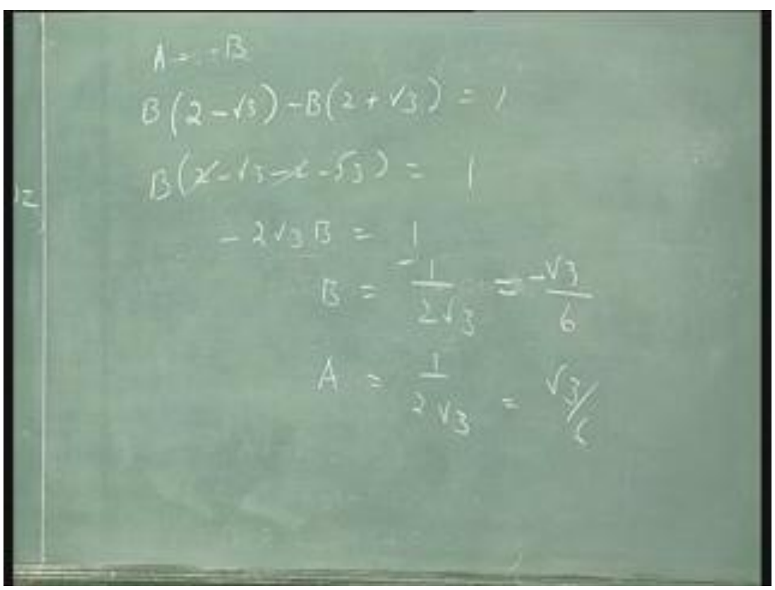

So with this result B(z) is taken to be a by 1 minus root 3z so here a is root 3 by 6 by 1 minus 2 plus root 3z minus in same thing root 3 by 6 by 1 minus 2 minus root 3z this is the generating function for b_r, B(z) is generating for the sequence b_0, b_1, b_2 and so on. The general term here will be root 3 by 6 and 2 plus root 3 power r z power r. And the general term here will be again root 3 by 6 and it is 2 minus root 3 power r z power r. So the coefficient of z power r is the value for a_r so the expression for b_r is given by root 3 by 6 into 2 plus root 3 power r minus root 3 by 6 2 minus root 3 to the power of r, so b_r is this.

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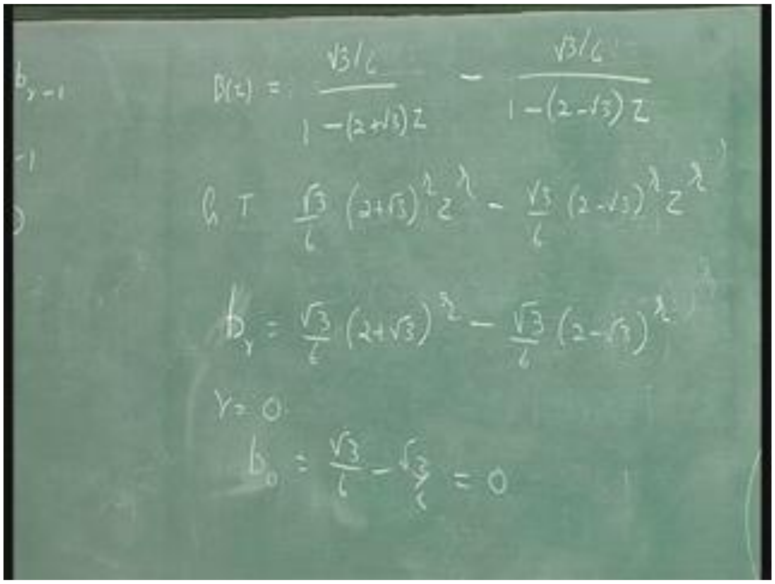

Let us check the value for b_0 when r is equal to 0 b_0 is this is just 1 and this is just 1 so you get root 3 by 6 minus root 3 by 6 it is equal to 0 which verifies with the initial condition that b_0 is 0. So the expression for b_r is this a_r is equal to 3 plus root 3 by 6 2 plus root 3 to the power of r plus (3 minus root 3 by 6) 2 minus 3 power r and b_r is given by this expression. So the method of generating function comes very handy in solving such simultaneous equations in recurrence relations. In general these recurrence relations are very useful in analyzing the complexity of an algorithm.

Most algorithms if you have a size of n it will be divided into two problems of size n by 2 you solve some problems of size n by 2 and then do some other operation to merge it. For example, take merge sort and you divide into two halves then sort each one of the half and then merge it and so on. In such cases if the complexity is given by T(n) when the size is n the time complexity of solving the problem is T(n) then it is divided into two sub problems of size n by 2 and apart from that there will be some more operations to get the whole solution. So that is given by two times T(n) by 2 plus something that could be of order n or order n square and so on. Suppose this is of order n the other operation involving finding the solution for the whole problem given the solution of two sub problems if it is of order n then by usual method you will find that the solution to this will give you that T(n) is of order n log n. that is there will be some constants and we need not worry very much about those constants. The big over notations constants are not really considered.

So, if you have a recurrence relation of this form order n some k times n that k is a constant and n then the solution for such a recurrence relation will give you the T(n) is of order n log n.

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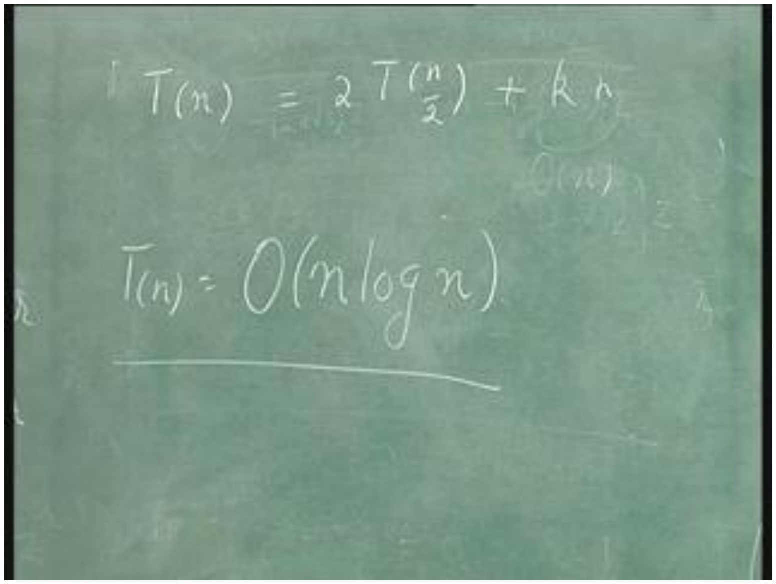

Such things are very useful in analyzing the complexity of an algorithm. So recurrence relations play a very important role in many fields in Computer Science especially in analysis of algorithms and we have seen how recurrence relations can be solved by two methods, one is by the method of homogenous solution and particular solution and adding them up and another is by means of the generating function approach. So next we shall consider some other topics like algebras and so on in the coming few lectures.