Discrete Mathematical Structures

Dr. Kamala Krithivasan
Department of Computer Science and Engineering
Indian Institute of Technology, Madras

Lecture - 29

Permutations and Combinations (contd..)

In the last lecture we were considering Permutations and Combinations. What is a Permutation and what is a Combination.

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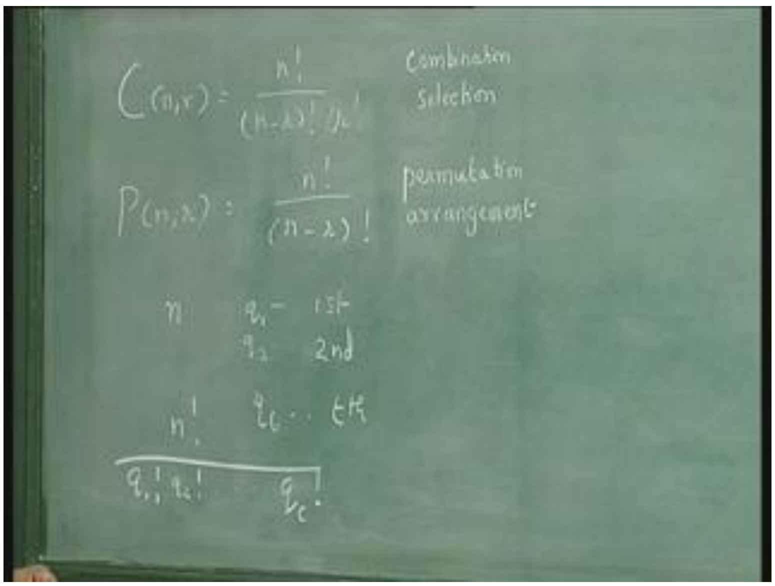

The Combination of r objects out of n objects which is selecting r objects out of n objects is denoted by C(n, r) and the formula for that is n factorial divided by (n minus r) factorial into r factorial, this we have seen in the last lecture. And what is a Permutation or arrangement of r objects out of n objects?

It is denoted as P(n, r) the Permutation of r objects out of n objects and the formula for that is n factorial divided by (n minus r) factorial, this also we have seen in the last lecture.

We have also seen some other formulae for example, if you have n objects out of which q_1 of them are of the first kind q_2 of them are of the second kind and so on and q_t of them are of the tth kind then the number of Permutations are q_1 of them are identical q_2 of them are identical q_t of them are identical and out of this n objects, then the number Permutations we can have is denoted by n factorial divided by q_1 factorial q_2 factorial q_t factorial, this also we have seen and some other formulae also we have seen. So the same sort of idea we are going to consider today in a different manner. We are going to consider the distribution of non distinct objects into distinct cells and we are also going to consider the distribution of distinct objects into distinct cells.

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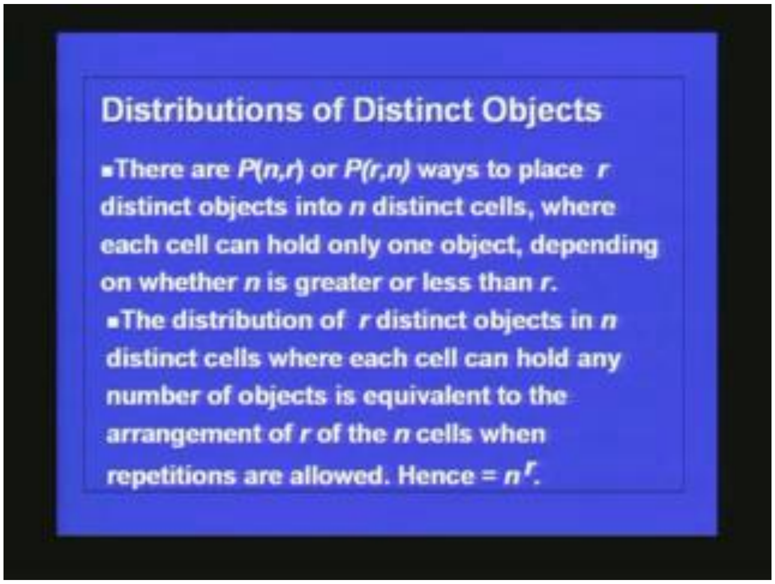

So there are two things we are going to consider today, the distribution of distinct objects into distinct cells and the distribution of non distinct objects into distinct cells.

Let us first consider the distribution of distinct object. You can see that there will be close correspondence between Permutations and Combinations. So, first considering the distribution of distinct objects you are having r distinct object and n distinct cells. Now n may be greater than are equal to r or n may be less than r. So depending on the two cases you have different formula.

The result is like this; in a moment let us see how we get this result. There are P(n, r) or P(r, n) ways to place r distinct objects into n distinct cells where each cell can hold only one object depending on whether n is greater or less than r. So you are having r objects, they are all distinct say a_1, a_2, a_r and you are having n cells they are also distinct these objects are all distinct cells are also distinct. So in how many ways can you distribute them into these cells so that one cell can hold only one object. There are two possibilities; n may be greater than or equal to r or n may be less than r or equal to r so in these two cases what are the results.

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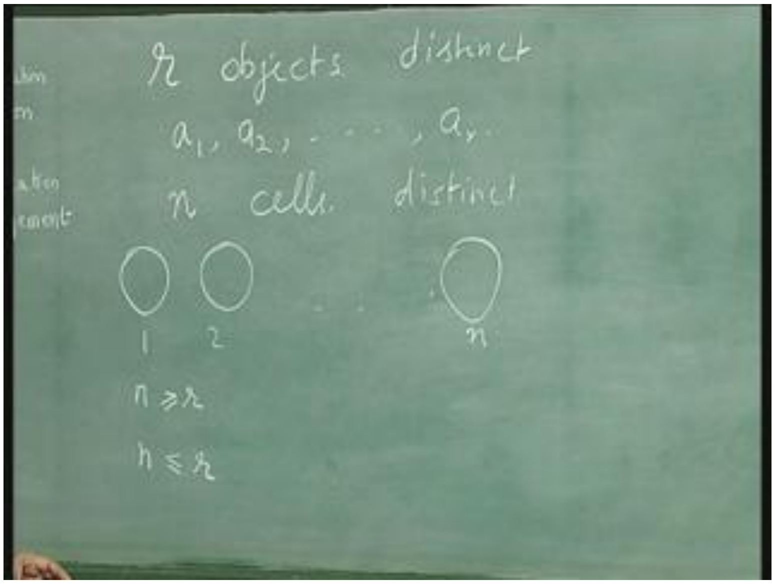

Now when n is greater than or equal to r you take the first object you choose one cell and put it in that, that it can be done in n ways. One cell can contain only one object so after selecting one cell you should not consider that cell again so after that take the second object there are (n minus 1) cells remaining so the second one you can put in any one of the (n minus 1) cells so that is (n minus 1). Then take the third object that can be put in any one of the remaining (n minus 2) cells.

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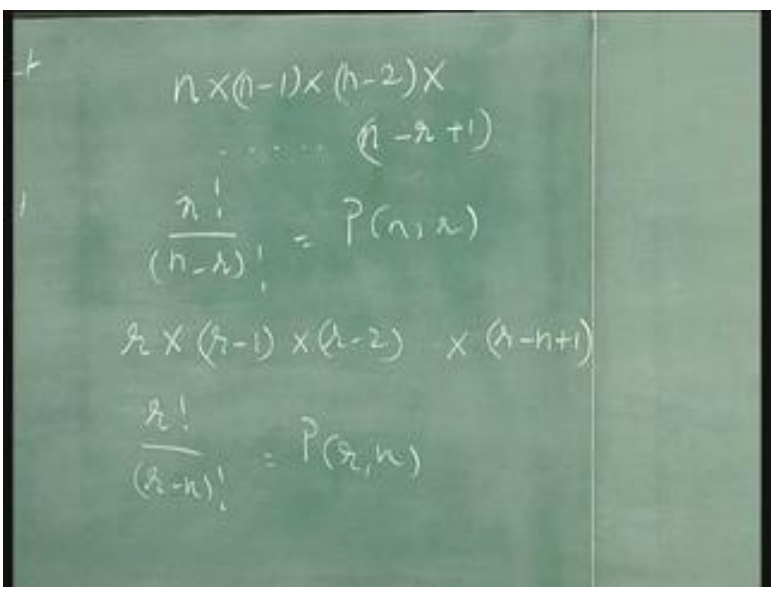

We are considering the case when n is greater than or equal to r so that will be (n minus 2) and so on until the rth object when you take it there will be (n minus r plus 1) cell remaining you can put it in any one of them we are considering the case when n is greater or equal to r so the last object you can put in any one of the (n minus r plus 1) cells. So this is the product you have and we know that this is nothing but n factorial divided by (n minus r) factorial and that is just P(n, r).

Now consider the case when r is greater than or equal to n. in this case we get the result as P(r, n) let us see how we get this. There are r objects now and n distinct cells where objects are distinct and cells are also distinct and how do you fill, one cell can contain only one object. So take the first sum, you can put one of the r objects into this. Select one of them and put it in the first cell that can be done in r different ways.

After doing that you are left with (r minus 1) objects pick one of them and put it in the second cell and that can be done in (r minus 1) ways. And after doing that you are left with (r minus 2) objects take the third cell and pick one of them and put it in the third cell that will be (r minus 2) and the last object or the nth cell can be filled in (r minus n plus 1) ways by selecting one of the (r minus n plus 1) objects and this reduces to r factorial divided by r minus n factorial which is nothing but P(r, n). So you see the connection between the Permutations of r objects out of n objects or in the other case n objects out of r objects and see how this is related to the way of filling n distinct cells with r distinct objects.

And we have considered the two cases where n may be greater than or equal to r or less than or equal to r. So this is one result. We are not allowing any repetition that is one cell can contain only one object. When one cell is allowed to have more than one object what happens?
The distribution of r distinct objects in n distinct cell where each cell can hold any number of objects is equivalent to the arrangement of r of the n cells when reputations are allowed.

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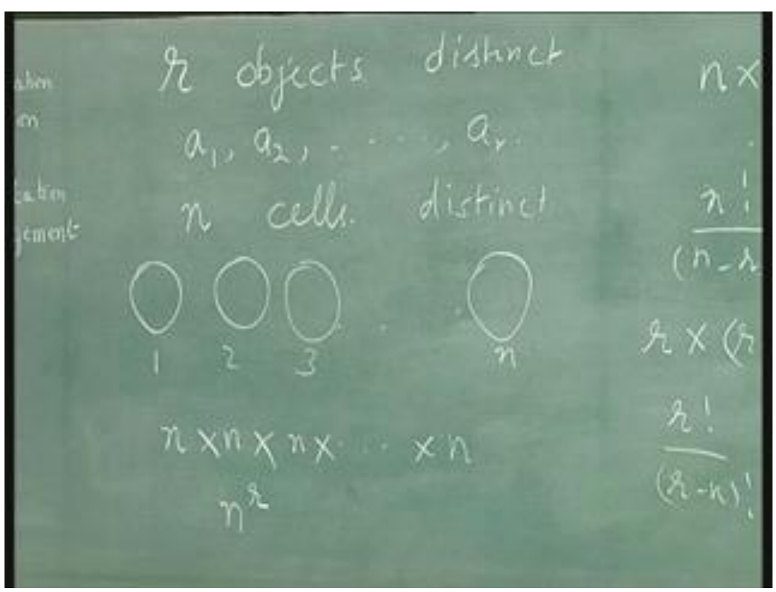

How do you get this? You are having r distinct object n distinct objects now each cell can contain any number of objects you are not putting any restriction on that, in that case what happens? Take the first object, you can select any one of the n cells and put it there, take the second object you can select any one of the n cells put it there, take the third object you can select any one of them and put it there because one cell can contain more than one object.

Like that the rth object also you can select any one of the n cells and put it there. So this becomes equal to n power r. We are not saying anything about n greater than or equal to r or less than or equal to r, in both cases the result is the same, it is equal to n power r. When the order of the objects in a cell is also considered what is the number of ways of distribution.

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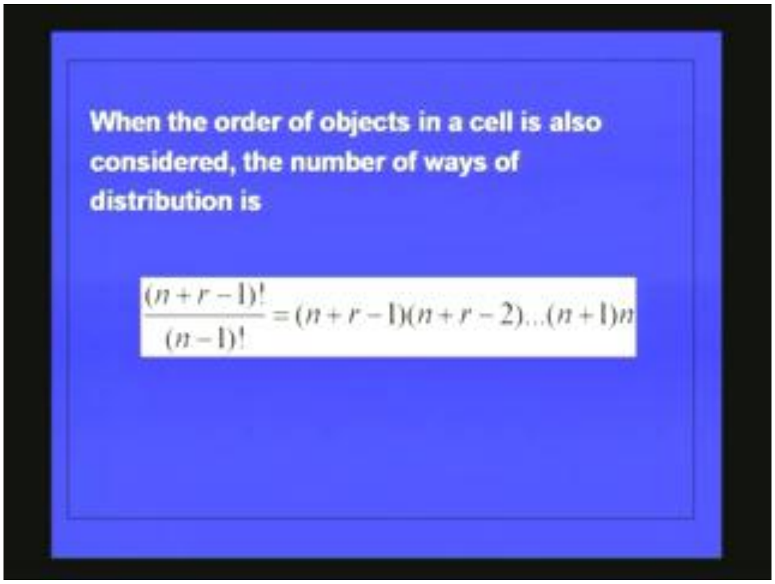

For example; again in this case we are having r distinct objects and n distinct cells but now each cell can contain more than one object and the order in which you are putting the object in a cell is important.

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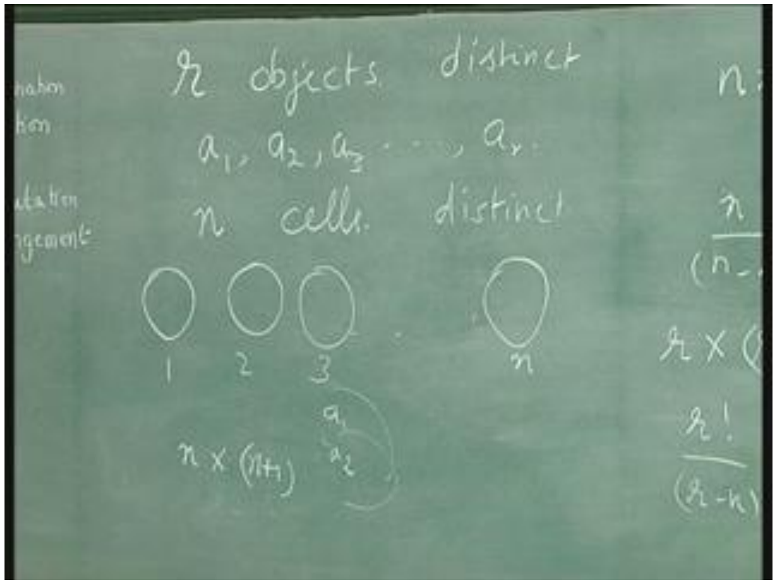

For example, suppose I select a_1 and put it here and then I select a_2 and put it here then I come to a_3 I want to put it in this cell but I can put it like this or I can put it like this so the order becomes important. In that way what is the answer? You see, this is the number of ways of selecting r objects out of n objects where repetitions are allowed so the answer will be like this; it will be (n plus r minus 1) factorial by (n minus 1) factorial and given by this, how do you get this?
It is like this; take the first object you can put it in any one of the n cells in n different ways.

Suppose I put it in this take the second object I can put it in any one of the remaining (n minus 1) cells or if I want to put it in this cell it can be prior to this or after this. So actually there are (n plus 1) ways of putting the second object in the n cells. Having done that take the third object suppose I put it here then there are (n minus 2) ways of putting in the vacant cells and here there are two ways here there are two ways, if I want to put it here I can put it here or here or I can put it here or here. So that will amount to (n minus 2 plus 2 plus 2) that is (n plus 2) ways of filling it.

Supposing a_1 and a_2 have been put in the same cell then the third object when I take I can put it in any one of the (n minus 1) cells or if you want to put it in the this cell I can put it here I can put it here I can put it here, there are three ways of putting it in the same cell that will be (n minus 1 plus 3) which is (n plus 2) again.

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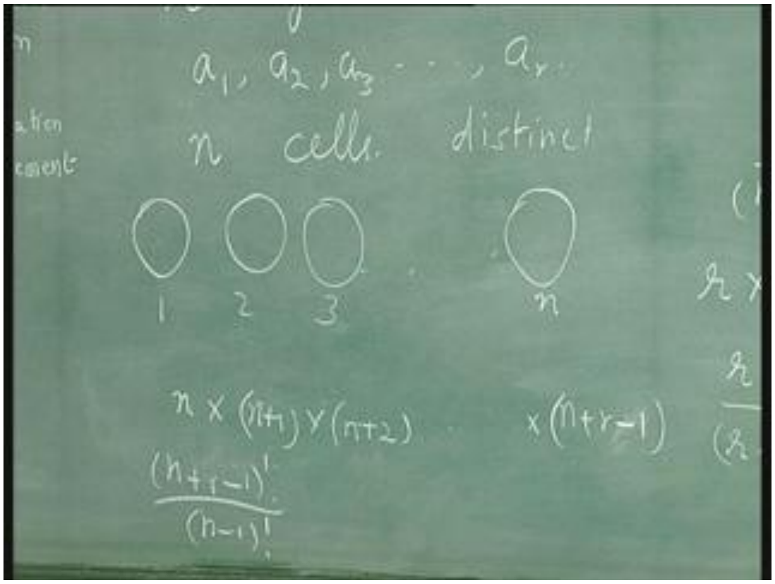

So, when I take the sec third object I can put it in (n plus 2) ways in any one of the n cells where each cell can contain more than one object and also the order of the object in the cell is taken into consideration. Proceeding this way the rth object can be put in (n plus r minus 1) the rth object can be put in (n plus r minus 1) ways so that will become equal to (n plus r minus 1) factorial by (n minus 1) factorial. So this is how we get this result. Immediately you consider a problem.

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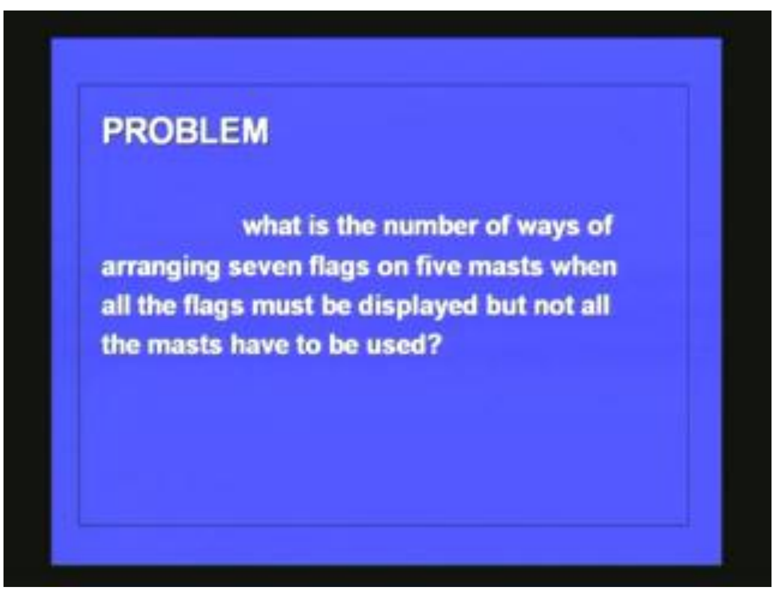

What is the number of ways of arranging seven flags on five masts when all the flags must be displayed but not all the masts have to be used?

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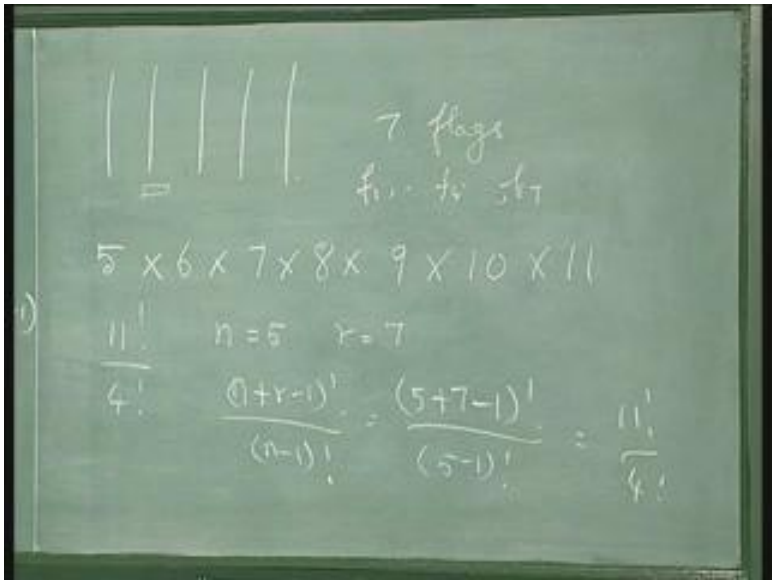

You are having 5 masts and you are having 7 flags; in how many ways can you display them where you need not have to use all the 5 masts but all the 7 flags you have to display. Take the first one I will call them as f1, f2, f7 take the first flag I can put it in any one of the 5 masts in five different ways. Having done that take the second flag suppose I have put the first flag here take the second one I can put it in any one of the four masts or if I want to put it here I should put it before this or after this, there are two possibilities so that can be done in 6 ways.

And the third flag I can put in 7 ways then the fourth one I can put in 8 ways, the fifth one in 9 ways, the sixth one in 10 ways, the seventh one in 11 ways. So these seven flags can be put in the 5 flags in 5 into 6 into 7 etc up to 11 that is equal to 11 factorial divided by 4 factorial or here what is n here?
In this case n is 5, r is 7 so what is the formula? The formula is (n plus r minus 1) factorial divided by (n minus 1) factorial. So the formula is (n plus r minus 1) factorial divided by (n minus 1) factorial that is 5 plus 7 minus 1 factorial divided by 5 minus 1 factorial which is the same as 11 factorial divided by 4 factorial and we have got the result like this. So this example exactly illustrates what we considered earlier.

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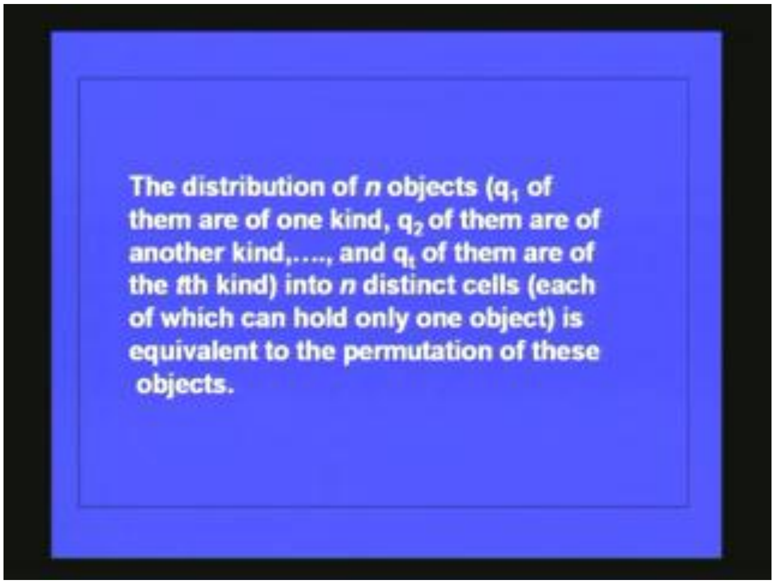

Now, the distribution of n objects q_1 of them are of one kind q_2 of them are of one kind and so on, what is the formula for this?
You want to distribute n objects into n cells where q_1 of them are of one kind q_2 of them are of one kind and qt of them are of one kind into n distinct cells. Here please note that the number of objects and number of cells are the same. So you are having n objects and n cells.

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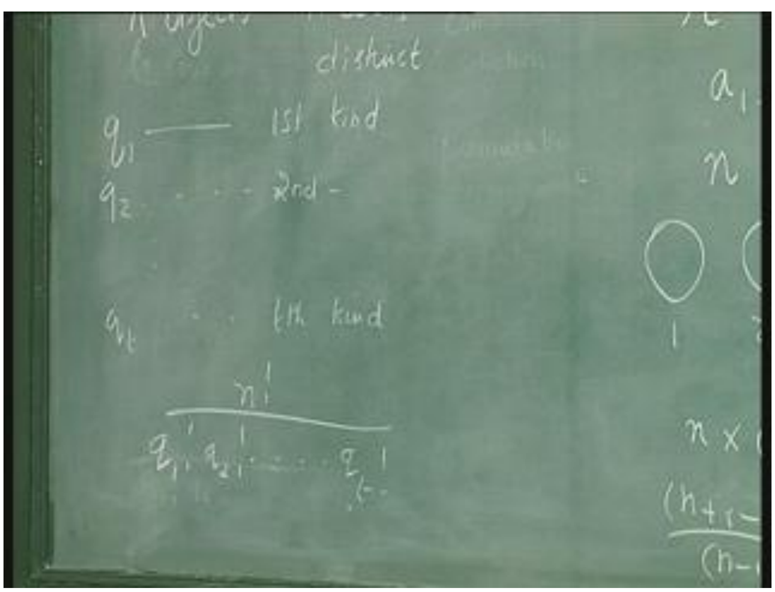

Cells are distinct but objects q_1 of them are of the first kind q_2 of them are of the second kind and so on q_t of them are of the tth kind. That is in essence what you are considering is you are permuting n objects out of which q_1 of them are identical q_2 of them are identical and q_3 of them are identical and so on. This distribution in essence means that you are permuting n objects you are arranging n objects where q_1 of them are identical q_2 of them are identical and qt of them are identical. The formula for that is you know is n factorial divided by q_1 factorial q_2 factorial q_t factorial this is the formula for that which we have seen in the last lecture.

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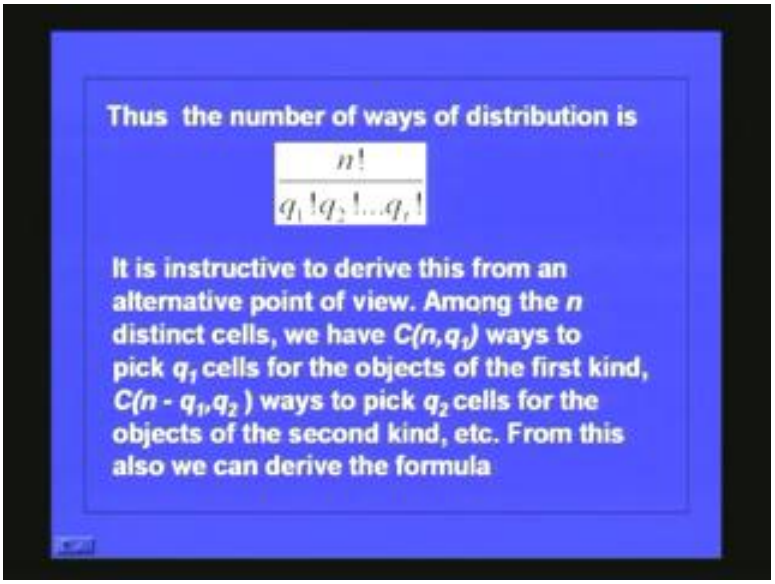

Thus the number of ways of distribution is n factorial divided by q_1 factorial q_2 factorial q_t factorial. You can also derive the same formula in a slightly different way, how do you do that?

Among the n distinct cells you have to select q_1 of them we have $C(n, q_1)$ ways to pick q_1 cells for the objects of the first kind. After having done that you are remaining with (n minus q_1) cells out of which you have to pick q_2 cells and that can be done in C(n minus q_1, q_2) ways to pick q_2 cells for the objects of the second kind and so on. In that case what happens, let us see what happens.

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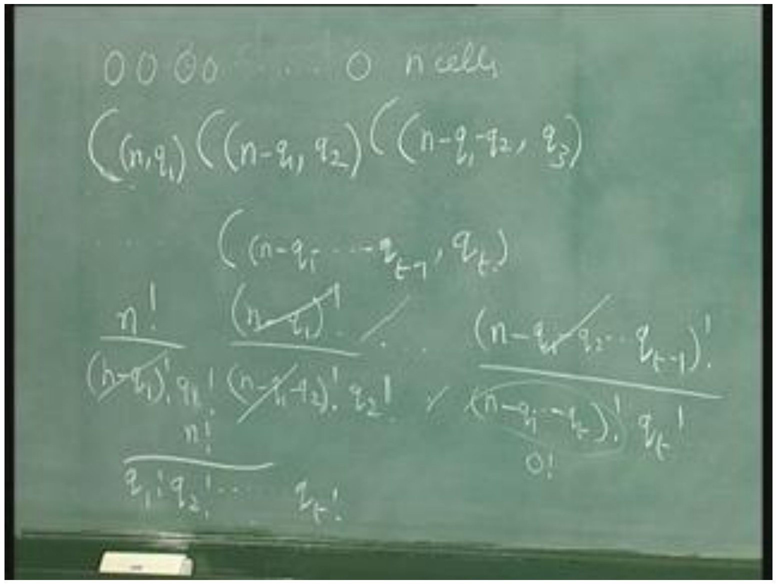

You are having n objects and n cells. So we want to select q_1 cells out of these n cells and put the objects of the first kind and that can be done in C(n, q_1) ways. After having selected q_1 cells we are left with (n minus q_1) cells out of which we select q_2 of them to put the second type of objects. After having done that we are left with (n minus q_1 minus q_2) cells out of which we select q_3 of them to put objects of the third kind and so on. And finally you are left with (n minus q_1 minus q_t minus 1) cells. In which you put the objects of the tth kind q. But this is nothing but q that you must remember.

So having done that let us expand and see this will come to n factorial divided by (n minus q_1) factorial into q_2 factorial this will be n minus q_1 factorial divided by (n minus q_1 minus q_2) factorial into q_2 factorial and so on. And the last one will be (n minus q_1 minus q_2 minus q_t minus 1) factorial divided by (n minus q_1 minus q_1 up to q_t) factorial into q_t factorial this will be the formula. Now you can see that this will cancel out with this one, this will cancel out with this one and so on and this will cancel out with the previous term and this is nothing but 0 factorial which is taken as 1. So the answer you get is n factorial divided by q_1 q_2 factorial and so on up to q_t factorial.

Now in this case we have taken n objects and n cells the n cells are distinct but the objects are of this type. Now instead of taking n objects if you have taken r objects what will be the formula and that is given by this.

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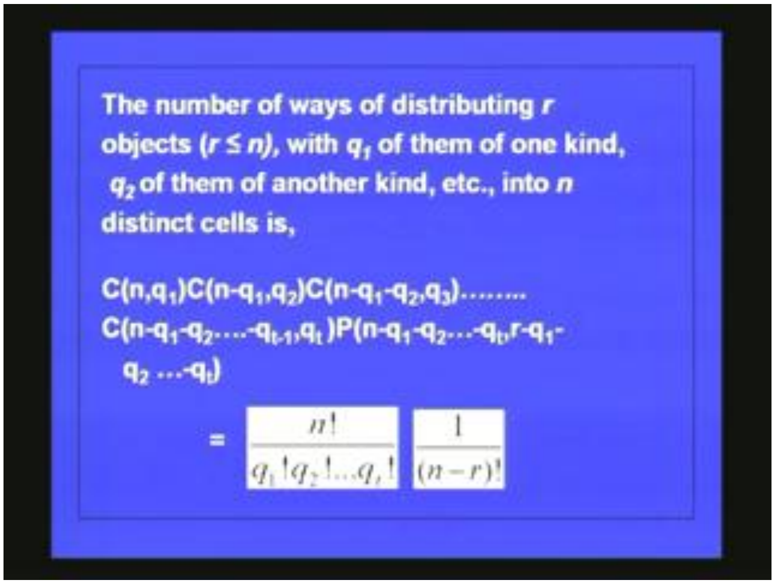

So the number of ways of distributing r objects where n is greater than or equal to r with q_1 of them of one kind q_2 of them of another kind etc into n distinct cells is C(n, q_1) C(n q_1, q_2) the same argument holds here also, but in the last case you have to argue the same way, in the last case we have taken this as 0 factorial but that will be a 0 factorial q_1 plus q_2 plus q_3 up to q_t is equal to r.

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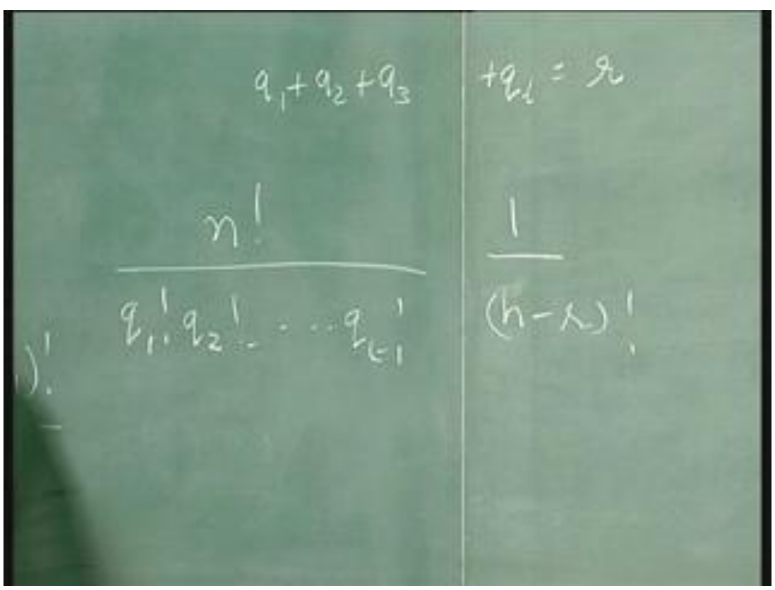

There are totally r objects and q_1 of them are of one kind and q_2 of them are of a one kind and so on. So q_1 plus q_2 plus q_3 up to q_t is equal to r. So instead of this you will get (n minus r). So the answer in this case will be n factorial divided by q_1 factorial q_2 factorial q_t factorial into 1 divided by (n minus r) factorial.

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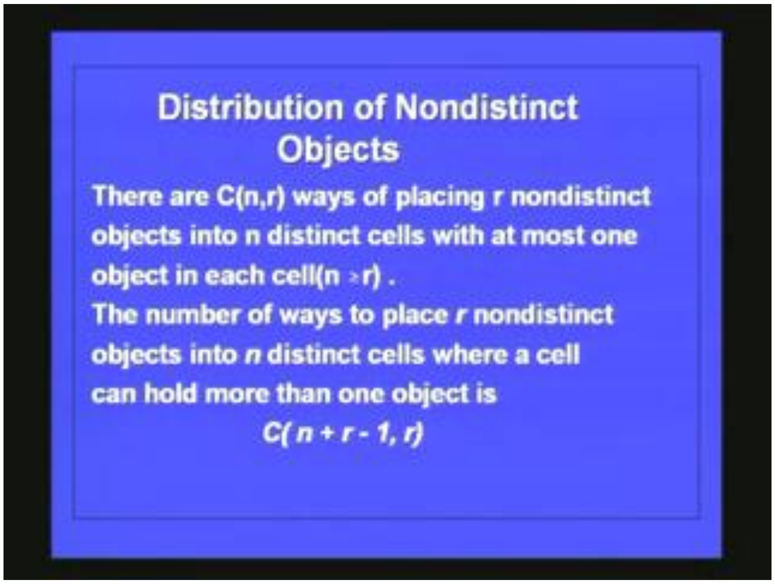

So far we considered distribution of distinct objects into distinct cell. We considered r distinct objects and n distinct cells and we considered the case where each cell can contain only one object and we also considered the case where each cell can contain more than one object and so on. And we also considered the case where all the r objects are not different but q_1 of them are one kind, q_2 of them are of another kind and so on. Next let us consider non distinct objects and distinct cells. Let us see what the result is. So we want to distribute non distinct objects into distinct cells. So what you are having is you are having n distinct cells and r non distinct objects.

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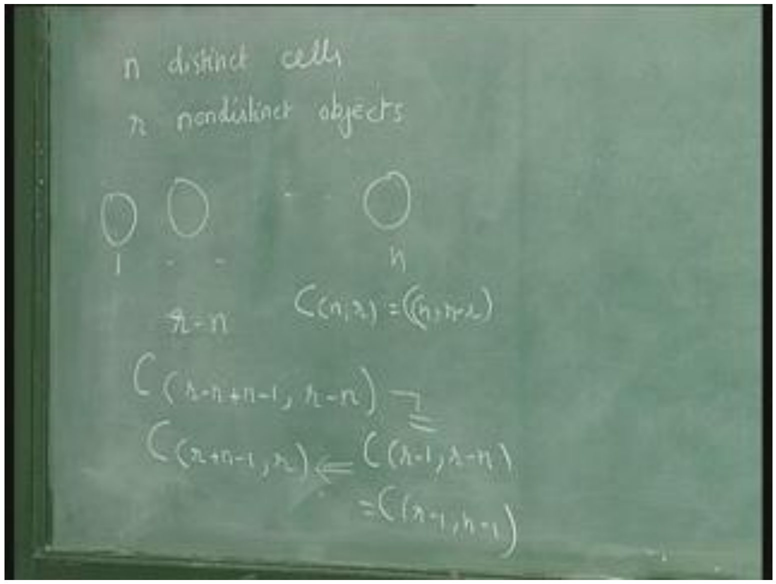

You are having n cells n distinct cells and you are having identically r object. And we take n to be greater than or equal to r, n is grater than or equal to r. Then there are C(n, r) ways of placing r non distinct objects into n distinct cells with at most one object in each cell. So you can put only one object in each cell so with that restriction if you see you will realize that out of the n cells you have to select r of them and put the objects in those cells. So you are selecting from the n cells r cells. That is equivalent to saying C(n, r) number of ways of selecting r object out of n objects.

Here you are selecting r cells out of n cells and that can be done in C(n, r) ways. So there are C n r ways of placing r non distinct objects into n distinct cells with at most one object in each cell. Now, if the cell can contain more than one object what is the formula? The number of ways to place r non distinct objects into n distinct cells where a cell can hold more than one object is given by this, how you get this?
When two or more objects are put in one cell the order does not matter here because all the objects are identical so the order in which you put them in one cell does not matter. So you have to really select r cells out of the n cells but here repetitions are allowed. This is equivalent to saying the number of ways to place r non distinct object into n distinct cells is equivalent to saying or this is equivalent to the number of ways of selecting r cells out of the n cells where you allow for repetition. And from the earlier lecture we know that the number of ways to select r objects out of n objects where you allow for repetition is given by this formula C(n plus r minus 1, r). Because of this formula the number of ways to place r non distinct objects into n distinct cells is given by this.

Now, here you are also allowing the case when a cell can be empty or you are not selecting a cell for putting an object. But if you do not allow that, you allow that each cell should contain at least one object.

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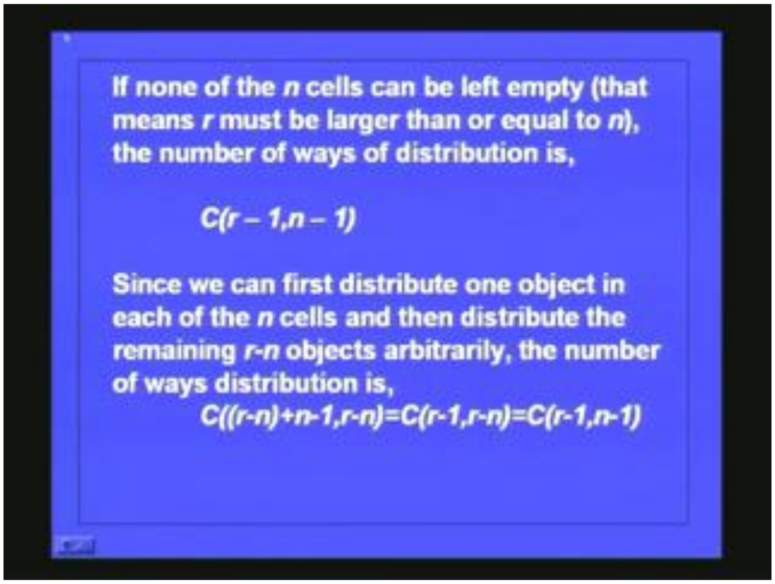

So you put the restriction that none of the cells can be empty. So you put this condition if none of the n cells can be left empty then what are the number of ways of distributing r objects into n distinct cells where the objects are all identical. Here, if you do not want any one of the n cells to be empty that means r must be larger than or equal to n. So in this case r is greater than or equal to n. Here the number of ways of distribution is given by this formula C(r minus 1, n minus 1).

How do you get this?
Now you are having r objects and n cells. First you select n of them they are all identical so you just pick n of them and put one each then you will be left with (r minus n) object. First you are selecting n objects and then putting one in each one of the cells then you are left with (r minus n) objects. The objects are identical. Now you want to put them in the n distinct cells and by the formula this will be equal to C((r minus n) plus (n minus 1)), r minus n). Because the formula is, you know that, C(r minus r plus n minus 1), this is the formula.

Now instead of r you are having (r minus n). So in this place you are having (r minus 1) and in this place also instead of r this is the original formula and because you taken out n objects and put in each one of the cells you are left with (r minus n) objects so in this formula you have to replace r divided by (r minus n) and (r minus n) here also. So this will reduce to C(r minus 1, r minus n) this is equal to this and C you know this formula C(n, r) is equal to C(n, n minus r) why? The number of ways of selecting r objects out of n objects is equivalent to the number of ways of rejecting (n minus r) objects out of n objects. So this is equal to C(r minus 1, r minus 1) minus this will be (n minus 1) so you get this formula.

That is what this portion says. We can first distribute one object in each one of the n cells and then and then distribute the remaining (r minus n) objects arbitrarily. The number of ways of distribution is given by this which is equal to C(r minus 1 r minus 1, n) which is is equal to C(r minus 1, n minus 1).

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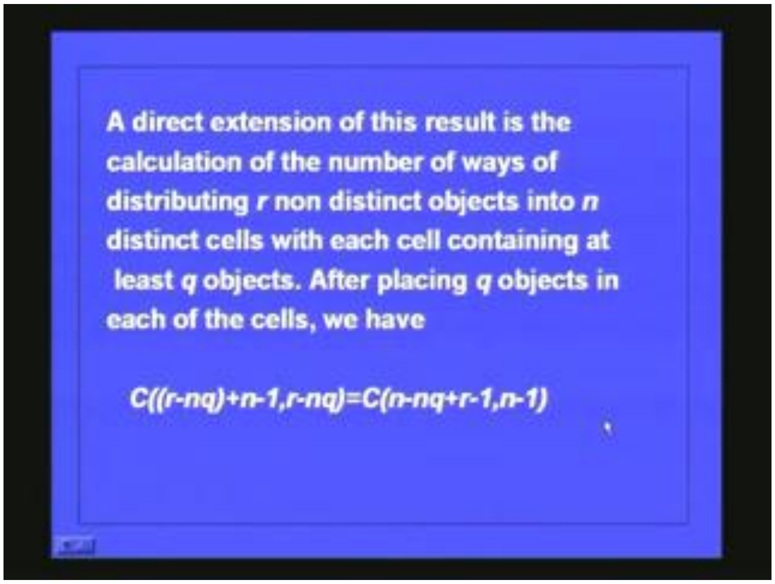

Now, a direct extension of this result is instead of putting the restriction that each cell should contain one object suppose I put the restriction that each cell should contain q objects then what happens?

So we want to put the restriction that each cell should contain q objects. So, first you pick out q objects they are all identical so put q of them here, q of them here, q of them here. So you have taken out (r minus n_q) objects and put them in q in each one of the cells so you are you are left with (r minus n_q) objects.

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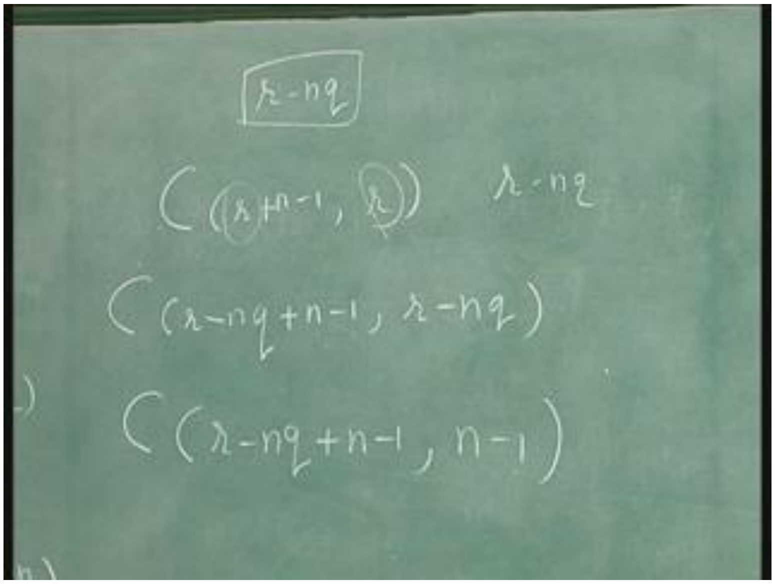

You have taken out n into q objects from this and put q in this q in this and so on so you are left with (r minus n_q) objects. This (r minus n_q) object you can distribute among the n cells in an arbitrary manner. So in the formula C(r plus n minus 1, r) instead of r you have to put r minus nq. So the formula will be C(r minus nq plus n minus 1, r minus nq) and that will be is equal to C(r minus n q plus n minus 1, n minus 1) because of this result C(n, r) is equal to C(n, n minus r) so you get this result. The number of ways of doing that will be C(r minus n q plus n minus 1, r minus nq) and instead of this you can write (n minus 1). So, we have considered distribution of r non distinct objects into n distinct cells. Now let us consider a problem immediately after this.

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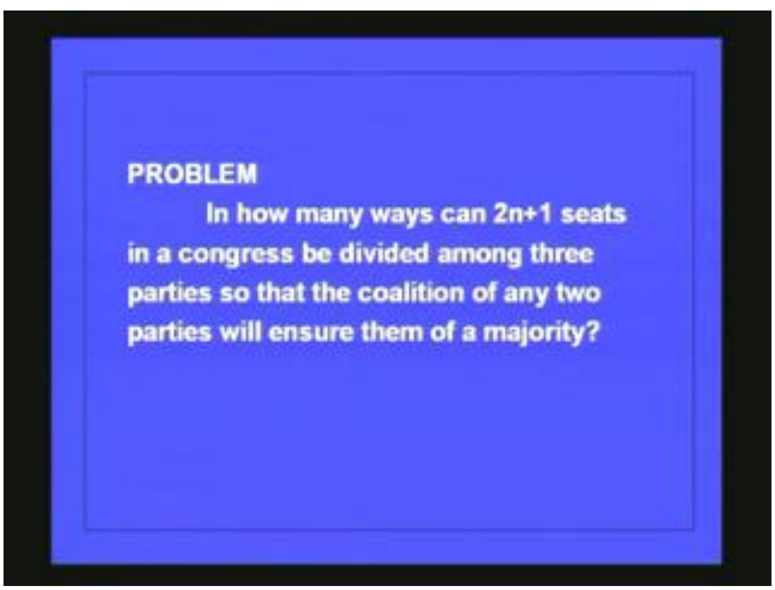

In how many ways can (2n plus 1) seats in Congress are divided among three parties so that the coalition of any two parties will ensure them of a majority. So you are having assembly seats or parliament seats something like that, you are having (2n plus 1) seats and three parties are contesting for them.

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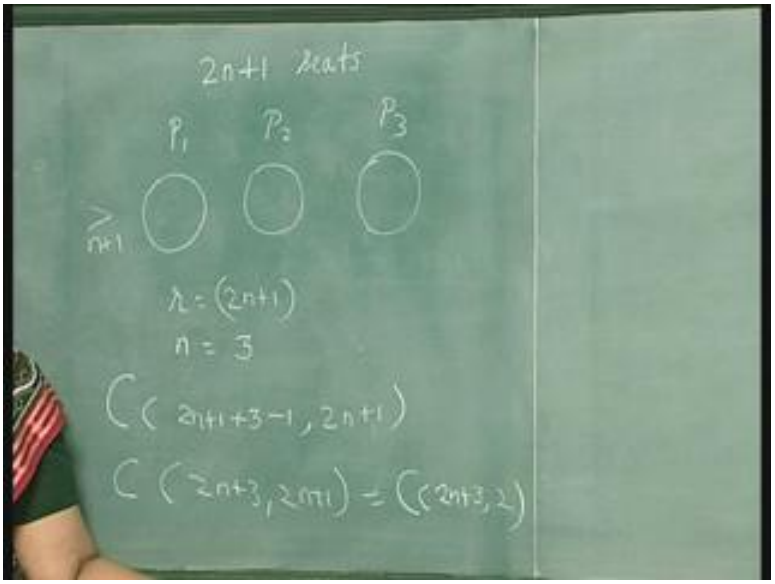

Each party may win some seats and for formation of the government the number of seats should such that it should be possible for any two parties combining together coming together and form a government. So suppose the three parties are P_1 P_2 P_3 it is like you are having three distinct cells, three distinct parties are there. The seats there is no distinction so it is identical objects you are having (2n plus 1) identical object and you want to distribute them among these. So in how many ways can you distribute these (2n plus 1) seats between three parties?
Here r is equal to (2n plus 1) and n is equal to 3. So the formula is C(r plus n minus 1) that is (2n plus 1 plus 3 minus 1, r) that is (2n plus 1) that is equal to C 2n plus 3, (2n plus 1) or this is also equivalent to C(2n plus 3, 2). This is the number of ways in which the (2n plus 1) seats can be distributed between the three parties. But there is one more condition here that is any two parties combining together should be able to get a majority and the majority will be (n plus 1) or more. Suppose this gets more than (n plus 1) seats in that case these two together will get less than n seats and in that case these two together will not be able to form a government.

The condition is, the distribution of the seats between the three parties should be in such a way that any two parties combining together should be able to form a government. So, out of the general number of distributions which is given by this you have to subtract those distributions where one party gets more than or equal to (n plus 1) seats, so what is that?

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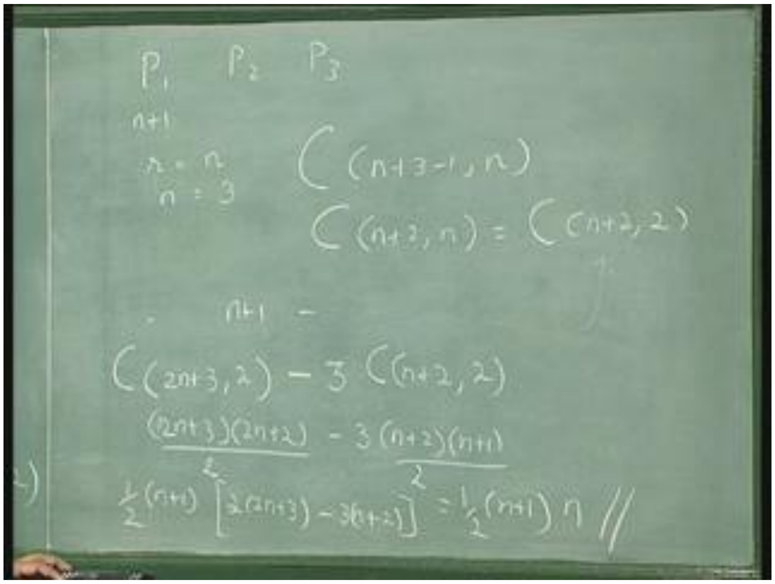

Suppose I have P_1 P_2 P_3 like this, (n plus 1) seats are already given to P_1, the remaining n seats are distributed between P_1 P_2 P_3, in how many ways can you do it?

You can do it in C into the number of seats is now r so r is equal to n now n is equal to 3 so what is formula? The formula is C(n plus 3 minus 1), n and that is 3 C(n plus 2, n) which is equal to C(n plus 2, 2). So the number of ways in which P1 gets (n plus 1) seats or more is given by this formula. This should be subtracted from the total number of distributions available and similarly P_2 can get (n plus 1) or more seats that should also be avoided. So that is again the number of ways in which P_2 gets (n plus 1) or more seats and the remaining seats are distributed between P_3 is again the same number and a similar case holds for P_3 also. So the total number of ways in which the seats can be distributed among parties such that any two of them will be able to combine and form a government is given by C(2n plus 3, 2 minus 3) times (n plus 2, 2) and what is this?

This is (2n plus 3) (2n plus 2) divided by 2 and this is (n plus 2) (n plus 1) divided by 2. If you simplify this will be 1 divided by 2 (n plus 1) into 2 into 2n plus 3 minus 3n plus 2 and that will be 1 divided by 2(n plus 1). This if 4n plus 6 this is minus 3n plus 6 will be just n. So the answer is half of n into (n plus 1) or n into (n plus 1) divided by 2.

These are the number of ways in which you distribute the seats or the parties can be given the seats such that it is possible for any two of them joining together and form a government getting a majority of (n plus 1) or more. Now what will happen if the same argument is given but instead of (2n plus 1) seats, see when you have an odd number of seats it is very easy, you have a majority (n plus 1) or more is a majority. Now the same if you have an even number of seats 2n seats you are having majority will be (n plus 1) or more you must get. So what will happen?

You have to use the same formula if you have 2n seats the number of ways of distributing the 2n seats among the three parties you have two n seats you have to distribute them among the three parties.

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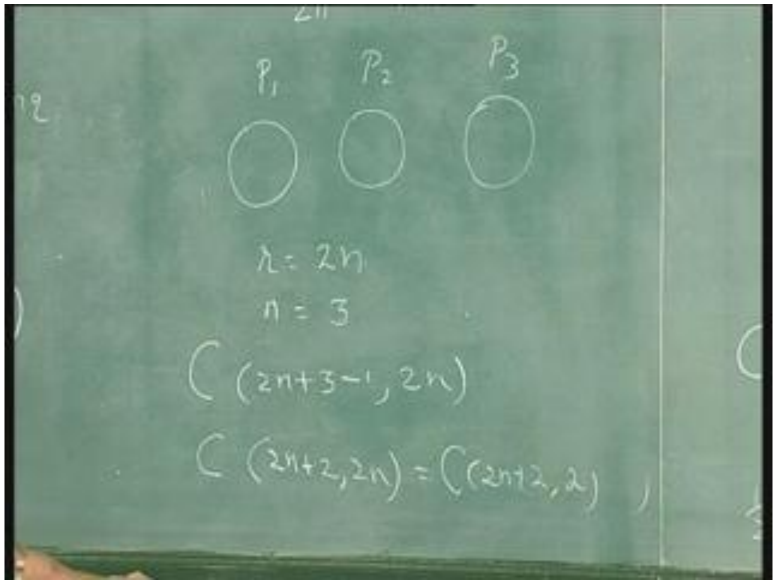

In this case the formula if you apply r will be 2n and n will be 3 so you will get (2n plus 3 minus 1), and that will be (2n plus 2, 2n) or that will be C(2n plus 2, 2). Now, you have to avoid cases where P_1 gets n. If the remaining gets then also they cannot combine and form a government. So when P_1 gets n and the remaining are distributed among the three such a distribution will not help to form a government by P_2 and P_3. So using the same formula supposing P_1 gets more than or equal to n seats what is the number of ways of distribution? In this case the remaining n can be distributed among the three. So what is n? n is 3, what is r? r is n. So the formula is C(n plus 3 minus 1, n) which C(n plus 2, n) or C(n plus 2, 2). So the formula in this case again the same thing you have to consider for P_2 you have to consider for P_3. So the formula here is the total number of ways of distribution is given by C(2n plus 2, 2 minus 3) times this is the formula where P_1 gets n or more seats similarly for P_2 and P_3. So you get three Times C(n plus 2 minus 2 plus 3).

Why do I get plus 3 in this case?
In the last example when I considered we did not consider this plus 3 but here we have to consider plus 3. The reason is, I am considering this formula when P_1 gets n or more objects when P_1 gets n objects and P_2 gets n objects and P_3 gets 0 objects this will be accounted for once in this formula. It will also be accounted for when you consider the formula for P_2 getting n or more objects. So this one will be accounted for more than once, it will be accounted for twice.

Similarly, the distribution for P_1 P_2 P_3 this distribution and this distribution and this distribution each one of them will be accounted for two times in this.

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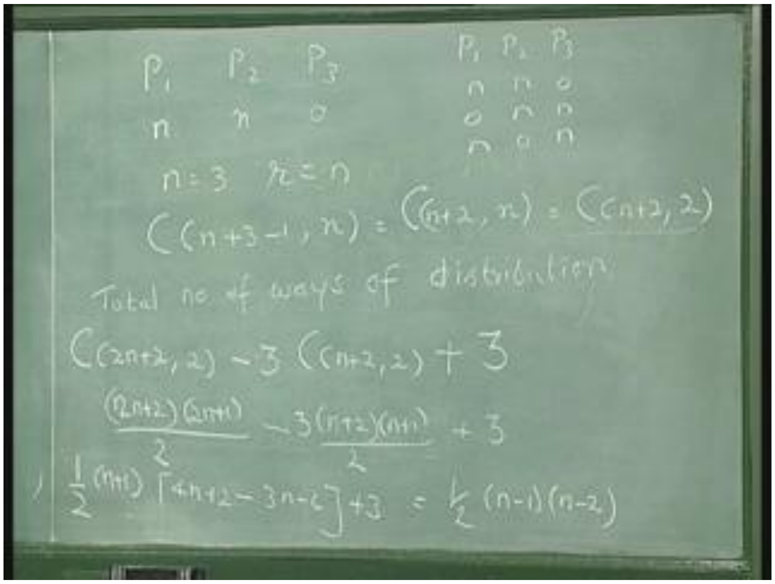

You should not subtract twice and that is why we are adding 3 and this will give you the answer. This will be (2n plus 2) (2n plus 1) divided by (2 minus 3) (n plus 2) (n plus 1) divided by (2 plus 3) and if you simplify this will be 1 divided by 2(n plus 1) if you take out this will be [4n plus 2 minus 3n minus 6 plus 3]. If you simplify, this will be equal to 1 divided by 2(n minus 1) (n minus 2). So we have seen the number of ways of distributing r distinct objects into n distinct cells and also r non distinct objects into n distinct cells and how this concept can be used to solve some problems.

Let us consider one or two more problems in Permutations and Combinations. Let us consider this problem: Prove the identity 1 into 1 factorial plus 2 into 2 factorial plus n into n factorial is equal to (n plus 1) factorial minus 1, discuss the combinatorial significance. Now you know that 1 into 1 factorial is equal to (1 plus 1) (1 factorial minus 1 factorial), 2(1 factorial minus 1 factorial) that is 2 factorial minus 1 factorial 1 into 1 factorial, 2 into 2 factorial is like that you can say it will be (2 plus 1) 2 factorial. Instead of 2 I am writing it as 3 minus 1, so this is (2 plus 1) 2 factorial minus 2 factorial that is equal to saying 3 factorial minus 2 factorial.

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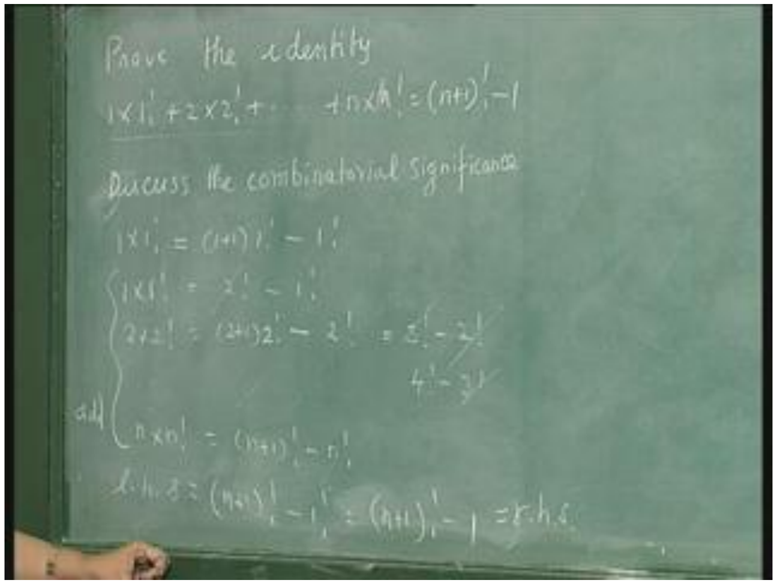

Proceeding like this n into n factorial will be (n plus 1) factorial minus n factorial. So add all these things you will get the left hand side so if you add all you get the left hand side of this. 1 into 1 factorial plus 2 into 2 factorial etc up to n into n factorial, but you see that this minus 2 factorial will cancel with this, the next term will be 4 factorial minus 3 factorial so this will cancel with this and so on and this n factorial will cancel with the previous terms. So this will reduce to (n plus 1) factorial minus 1 factorial that is equal to (n plus 1) factorial minus 1 which is the right hand side. So you have proved the inequality. Here we have not used any combinatorial argument we have just use the formula and shown.

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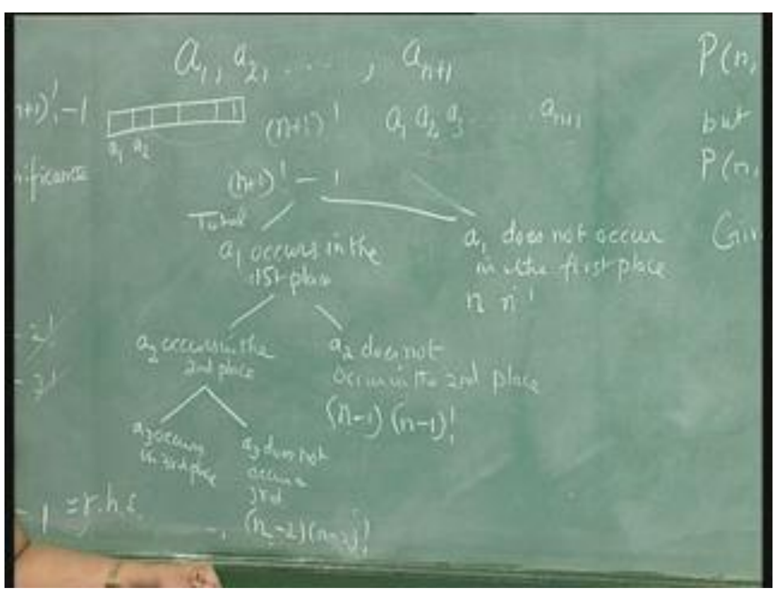

Now what is the combinatorial significance of this?

Consider (n plus 1) objects a_1, a_2, a_n plus 1 you are having (n plus 1) objects and you are permuting you are arranging them in all possible ways.

What is the number of ways of arranging? That will be (n plus 1) factorial out of which you remove the Permutation where a_1 occurs in the first place, a_2 occurs in the second place, a_3 occurs in the third place and (a_n plus 1) occurs in the n plus 1th place. You remove this Permutation that will be (n plus 1) factorial minus 1 that is the right hand side. How do you get it in a different way?

Among the set of Permutations you consider the Permutations where a_1 occurs in the first place and a_1 does not occur in the first place. You can divide the total number of set of Permutations into two parts like this a1 occurs in the first place and a_1 does not occur in the first place. If a one does not occur in the first place the first place can be filled in n ways. After filling the first place with other things than a1 you will put with n objects and the rest of the n places you can fill in n factorial ways. So that is equivalent to n into n factorial.

Now consider the case where now a_1 occurs in the first place, in this you can split into two ways such as a_2 occurs in the second place and a_2 does not occur in the second place.

You have filled the first place so if I consider the (n plus 1) places like this you have filled the first one now with a1, the second one you should not have a_2. After leaving out a_2 you are having (n minus 1) objects and you can fill with any one of them with this place that can be done in (n minus 1) place. After doing that you have filled these two places with two objects so you are left with (n minus 1) objects the remaining (n minus 1) places is filled in (n minus 1) factorial ways. So the number of ways in which a_1 occurs in the first place and a_2 does not occur in the first place is given by this.

Now, you have the Combination or you have the Permutations where a_1 occurs in the first place and a_2 occurs in the second place. This again after putting a_1 here and a_2 here the remaining (n minus 1) places can be filled in different ways that you can split like this, a_3 occurs in third place a_3 does not occur in third place. And if a_3 does not occur in the third place this can be filled in (n minus 2) ways remaining (n minus 2) factorial (n minus 2) (n minus 2) factorial. So you see that proceeding like this you get the left hand side n into n factorial plus (n minus 1) (n minus 1) factorial plus (n minus 2) (n minus 1) factorial and so on. So the total number is equivalent to the number of ways in which you can arrange all the objects but in which you are removing from that the Permutation where a_1 occurs in the first place a_2 occurs in the second place a_3 occurs in the third place and (a_n plus 1) occurs in the nth place. And in a similar manner you can give a very simple explanation for this problem also.

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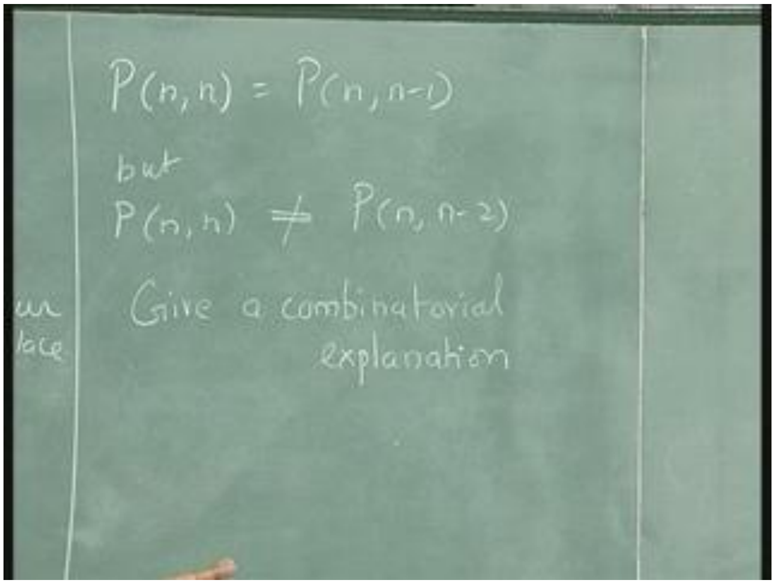

Look at this; this can be also proven in a very simple manner. For all these calculations when you use the computer you have to use or you have to calculate n factorial. It is very difficult and Time: consuming it is Time: consuming to calculate n factorial in a manner 1 into 2 into 3 and so on rather you use in approximation which is called Stirling's Formula.

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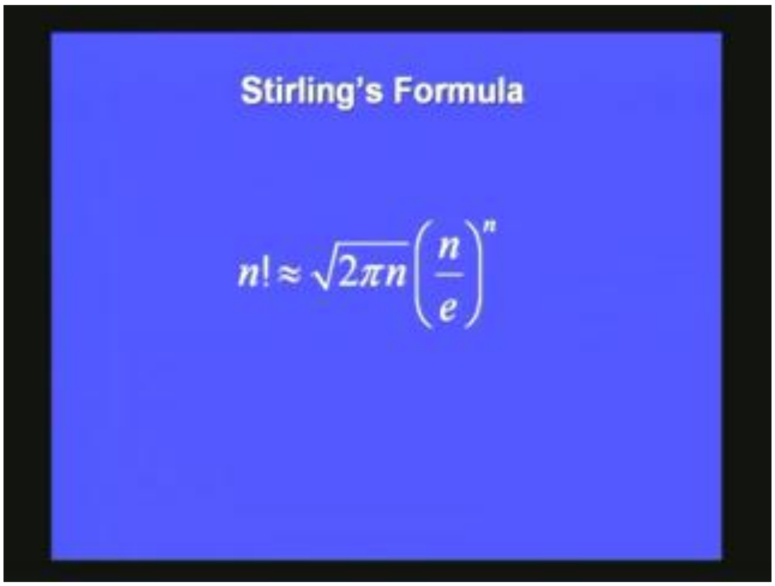

The Stirling's Formula is; n factorial is given by square root of 2pi n into n divided by e to the power of n. You use this formula for calculating n factorial when you want to use the computer. So in this lecture we have seen distribution of r objects into n cells. Cells are distinct but the objects could be distinct or non distinct we have considered the formulae for them. We have also seen the correspondence between them and Permutations and Combinations. In the next lecture we shall see about generating functions, a concept which comes very handy in finding out the number of Combinations, number of Permutations etc.