Discrete Mathematical Structures
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Lecture # 27
Pigeonhole Principle

In the next few lectures we shall consider some topics in combinatorics. The first one we shall consider is the Pigeonhole Principle. It is a very simple concept and we shall consider the concept and then some problems under the principle.

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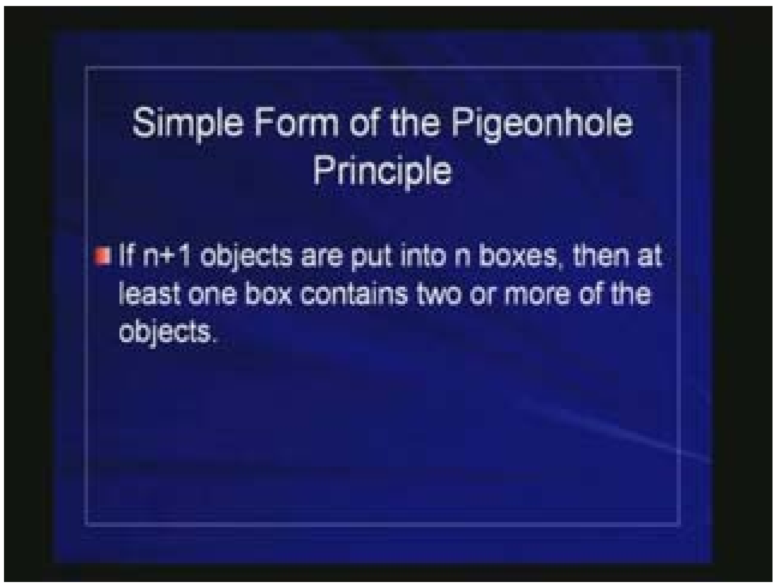

The simple form of the Pigeonhole Principle is like this: If n plus 1 objects are put into n boxes, then at least one box contains two or more of the objects. It is a very simple thing, very easily understandable and so on. In fact something like this is said if n plus pigeons are put into n pigeon holes at least one pigeon hole will contain two or more pigeons. That is why this concept is called Pigeonhole Principle. It is a very simple idea. Let us see how this idea can be used to solve some problems.

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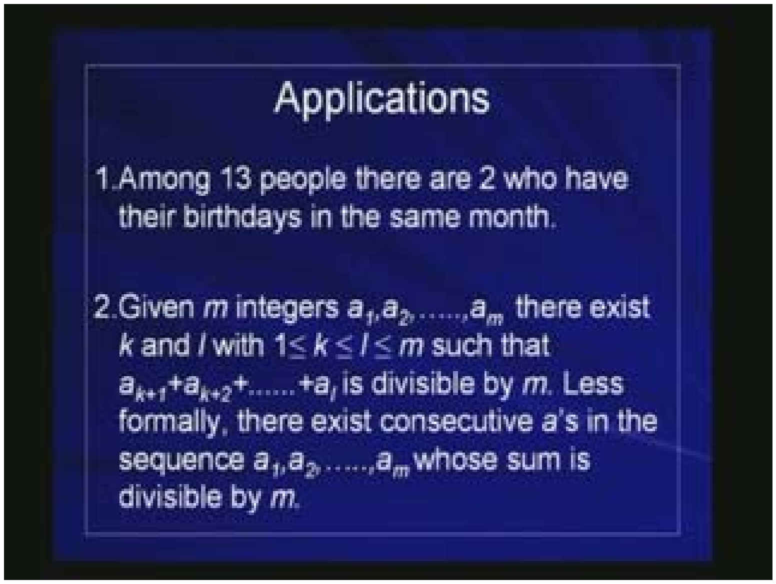

Some applications are as follows: Among 1three people there are 2 who have their birthdays in the same month because there are only 12 months there are 1three people 2 of them will have their birthday in the same month, in a very simple concept. The next one is, given m integers a_1, a_2, am there exist k and l where k and l are between 1 and m such that $a_{k \text{ plus } 1}$ plus $a_{k \text{ plus } 2}$ plus a_l is divisible by m. That is if you take the successive numbers and sum that will be divisible by m. If you put it less formally there exist consecutive a's in the sequence a_1, a_2, am whose sum is divisible by m.

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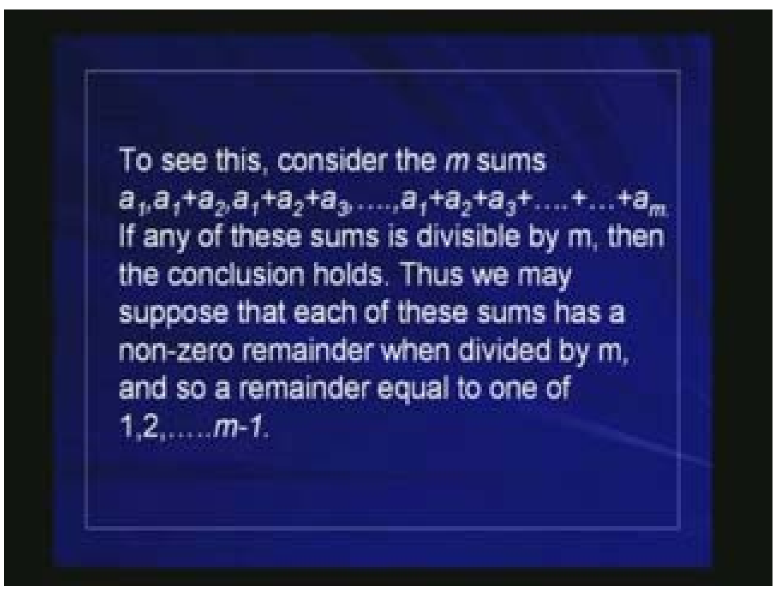

How do you use Pigeonhole Principle to prove this?

To see this consider the m sums a_1, a_1 plus a_2, a_1 plus a_2 plus a_3. Consider a_1, a_1 plus a_2, a_1 plus a_2 plus a_3 and so on. There are m such sums. If any of these sums is divisible by m then obviously the conclusion holds. The result follows: so thus we may assume that each of these sums has a non zero remainder when divided by m. That is, none of these sums are divisible by m. So when divided by m they will leave some remainder and that is non zero so it will be from 1 to m minus 1 one of the integers from 1 to m minus 1. The remainder will be equal to one of the integers 1, 2, 3 up to m minus 1.

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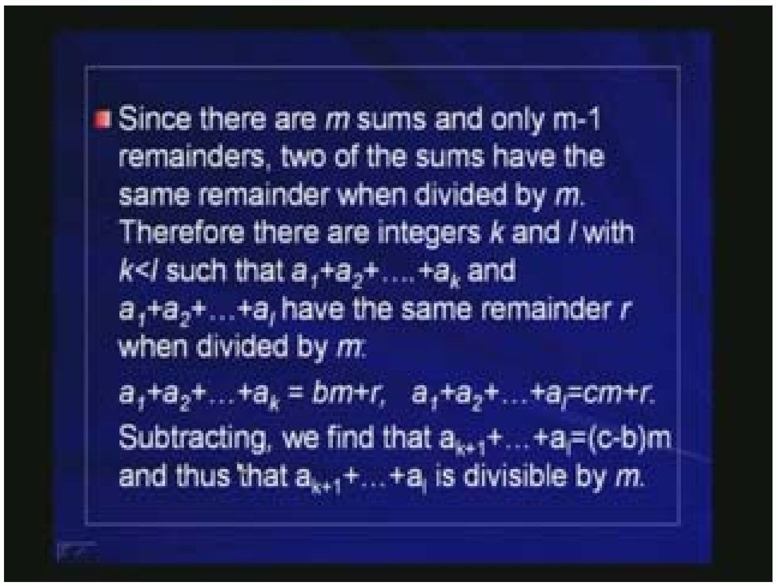

Now, we have m sums and only m minus 1 remainder m minus 1 possible values for the remainders two of the sums will have the same remainder when divided by m. Therefore there are two integers k and l and let us assume that l is greater than k such that a_1 plus a_2 plus a_k leaves the remainder r when divided by m. And a_1 plus a_2 plus a_3 up to a_l also leaves the same remainder r when divided by m. That means a_1 plus a_2 plus a_k is equal to sum b cross m plus r where b is an integer. And a_1 plus a_2 is equal to a_3 up to al is cm plus r where c is an integer.

Now l is greater than k so subtract this from this you will find that $a_{k \text{ plus } 1}$ up to a_l that is the remaining elements from here after a_k that is equal to c minus b cross m that is where c is an integer and b is an integer so c minus b is an integer and this sum $a_{k \text{ plus } 1}$ up to a l is divisible by m. Thus we prove that $a_{k \text{ plus } 1}$ $a_{k \text{ plus } 2}$ up to a_l is divisible by m, this is another simple application. There are strong forms of the Pigeonhole Principle let us consider them.

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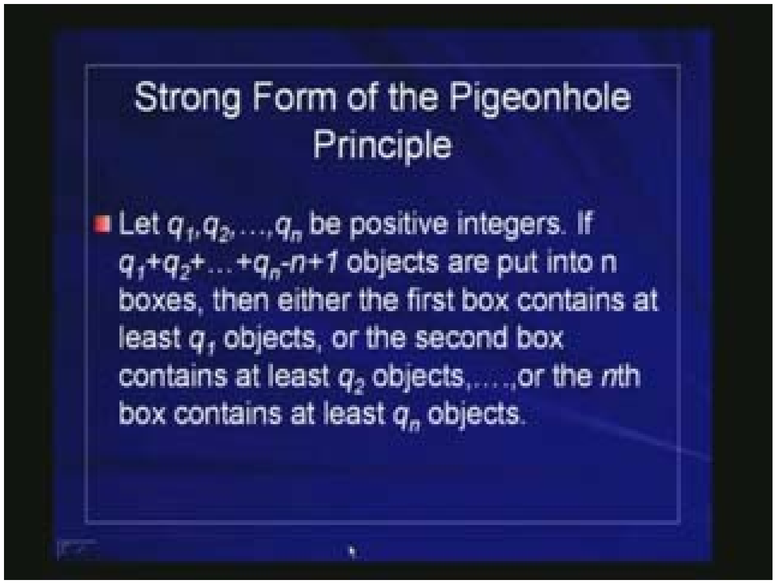

The same idea but extended in a slightly different way. Let q_1, q_2, q_n be positive integers. If q_1 plus q_2 plus q_3 plus q_n minus n plus one objects are put into n boxes, then either the first box contains at least q_1 objects or the second box contains at least q_2 objects and so on or the nth box contains at least q_n objects. This is again very simple. Let us see, there are n boxes 1 2 3 n and there are q_1 plus q_2 plus q_n minus n plus 1 objects.

Suppose the first box contains less than q_1 objects maximum it can have is q minus 1. And if it contains less than q_2 objects the maximum it can contain is q_2 minus 1. And proceeding like that the maximum number this can have is q_n minus 1 if the nth box contains less than q_n minus 1 elements. So the total number of elements you can have in all the boxes will be q_1 plus q_2 plus q_n minus n in this case. So if you have q_1 plus q_2 plus q_n minus n plus 1 objects at least one of the boxes will contain more than the supposed element. That is, either the first box will contain q_1 objects or more or second box will contain q_2 objects or more or the third box will contain q_3 or more objects or the nth box will contain q_n or more objects.

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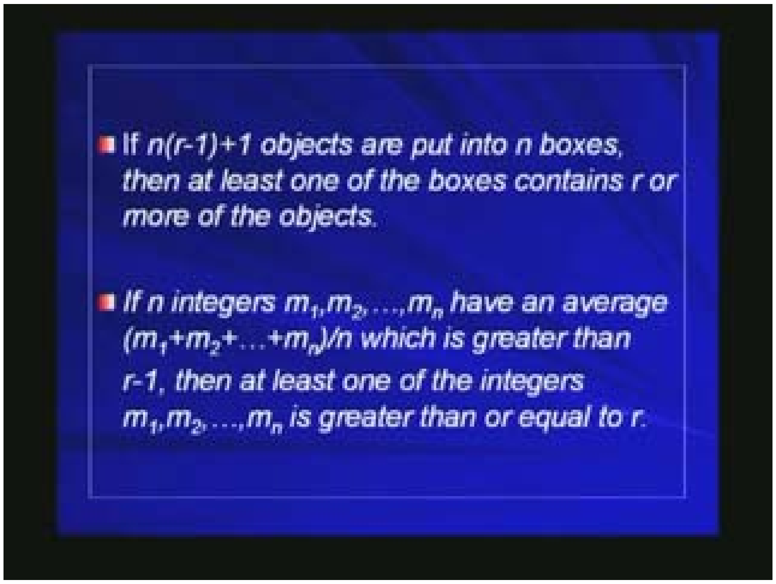

Putting it in another way: suppose q_1 is equal to q_2 is equal to q_3 etc. Suppose n cross r minus 1 plus 1 objects are put into n boxes then at least one of the boxes contains r or more objects. If each box contains less than r object the maximum it can contain is r minus 1 so totally in n boxes you can have only n cross r minus 1 objects. But now there is n cross r minus 1 plus 1 objects. So at least one of the boxes will contain r or more objects. Putting it in another way, this is another form of the statement.

The n integers m_1, m_2, m_n have an average m_1 plus m_2 plus m_n by n because you have n integers the average is given by this number. And if the average is greater than r minus 1 then at least one of the integers m_1 plus m_2, m_n is greater than or equal to r because otherwise the average will not be greater than r minus 1. So these are the different ways of putting the Pigeonhole Principle, depending upon the application we take the proper form. The idea is very simple but when we use it to solve problems we have to think and apply it in a proper manner. Let us consider some problems.

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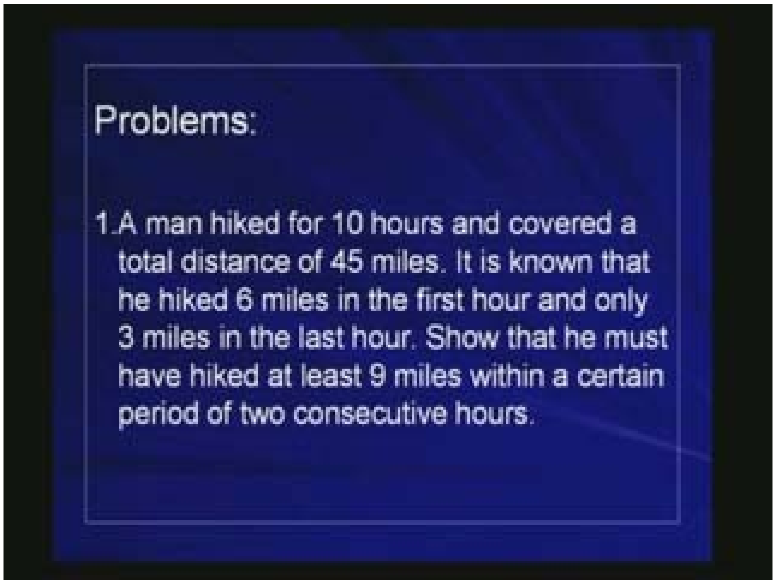

A man hiked for 10 hours and covered a total distance of 45 miles. It is known that he hiked 6 miles in the first hour and only 3 miles in the last hour. Show that he must have hiked at least 9 miles within a certain period of two consecutive hours. This is the problem.

A man hiked for 10 hours. So let a_1, a_2, a_{10} be the number of miles he hiked in the ith hour. So a_i is the number of miles he hiked in the ith hour. Of course i will vary from 1 to 10. Now, we know that a_1 is 6 and a_{10} is 3. And it is said that we have to prove that consecutive two hours he would have hiked at least 9 miles. Suppose he has not hiked 9 miles or more in any two consecutive hours then what does that mean a_1 plus a_2 is less than 9, a_2 plus a_3 is less than 9 and so on, a_9 plus a_9 plus a_{10} is less than 9.

Therefore, adding you will get a_1 plus two times a_2 up to a_9 plus a_{10} is less than 9 times 9 so 81. Add one more a_1 and a_{10} but we know that a_1 and a_{10} are 6 and 3 respectively. So that is equivalent to adding 9, you get that two times sigma a_i is less than 90 or sigma a_i is less than 45 which would mean it is contradicting the factor that he has hiked 45 miles in 10 hours. But we know that the man hiked for 10 hours and covered a total distance of 45 miles. So, if you assume that he has not hiked 9 miles or more in any two consecutive hours we come to the conclusion that the total distance covered by him is less than 45 which is not true. So, we are arriving at a contradiction and that would mean that he has hiked 9 miles or more at least in some consecutive hour.

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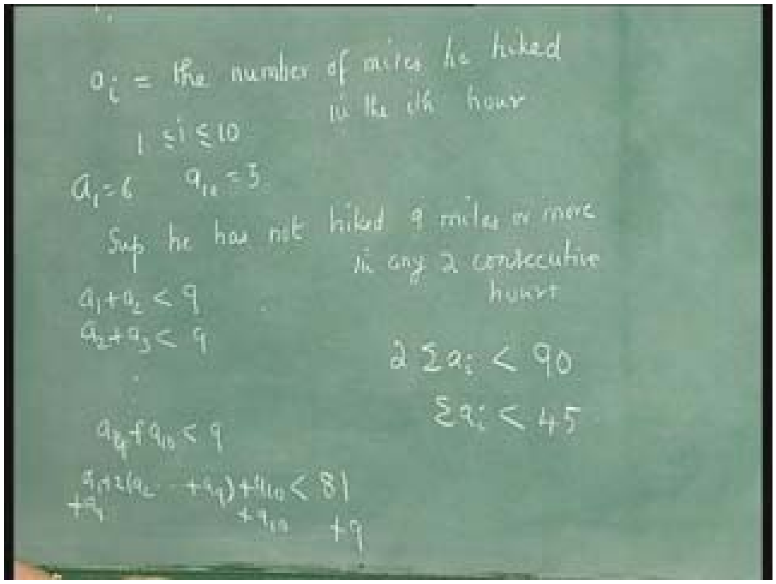

Let us take some more examples.

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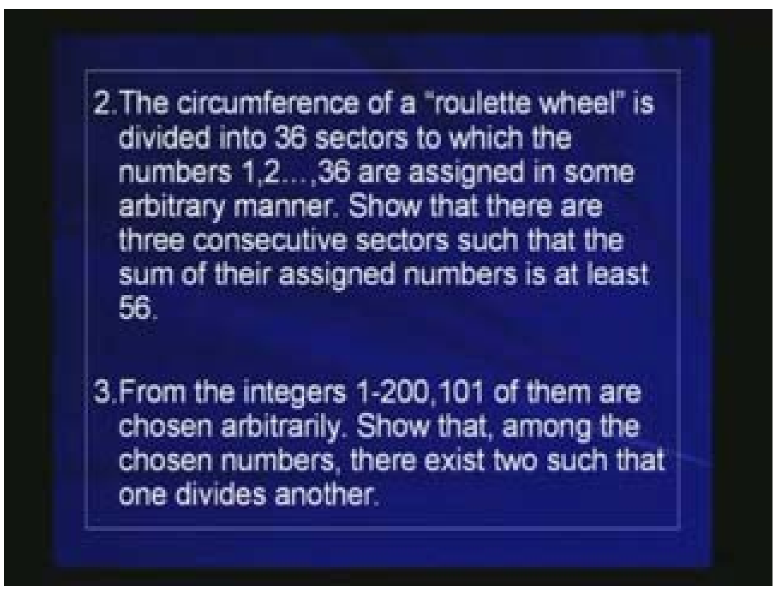

The circumference of a roulette wheel is divided into 36 sectors to which the numbers 1, 2, to 36 are assigned in some arbitrary manner. Show that there are three consecutive sectors such that the sum of their assigned numbers is at least 56. Again this problem is very similar to this one. You have a circumference circle which is divided into 36 possible sectors a_1 a_2 a_3 and so on, a_{36} and each of them is assigned a number between 1 to 36 and the problem says that we have show that at least in one place three consecutive sectors the sum will add up to 56 or more.

Suppose not in the same idea if a_1 a_2 a_3 are assigned, suppose the conclusion is not true then you have a_1 plus a_2 plus a_3 is less than or equal to 55, if it is not 56 or more it has to be less than or equal to 55. Similarly, a_2 plus a_3 plus a_4 is less than or equal to 55 and proceeding like that a_{35} plus a_{36} plus a_1 is less than or equal to 55 and a_{36} plus a_1 plus a_2 is less than or equal to 55. Now add up this you will get a_1 is occurring in three places a_2 is occurring in three places so the sum will be three times sigma a_i that will be less than or equal to 55 cross 36 there are 36 inequalities adding the right hand side of the inequalities you get this.

But what is sigma i?
Each a_i is between 1 and 36, the numbers between 1 and 36 are assigned to a_1 a_2 a_3. So this will be 3 cross sigma a_i will be 36 cross 37 by 2 because the sum of the first ten integers is given by n cross n plus 1 by 2, here the sum of the first 36 integers is given by 36 cross 37 by 2 that is less than or equal to 55 36 cross 37 by 2, 36 canceling 36 on both sides you will get 3 cross 37 is 111 and 111 is less than or equal to taking the 2 on the other side you will get 110. So you get the result 111 is less than or equal to 110 which is not correct so you are arriving at a contradiction. So the result follows that is there are three consecutive sectors such that the sum of their assigned numbers is at least 56.

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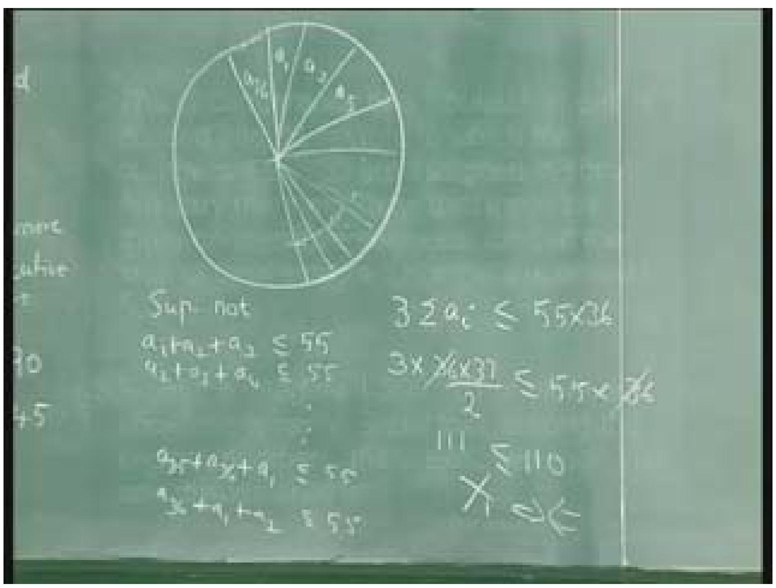

I can see the similarity between this problem and the previous problem and you can also see the application of the Pigeonhole Principle. The third one is like this; from the integers 1 - 200 101 of them are chosen arbitrarily. Show that among the chosen numbers there exist two such that one divides another. So, the numbers are between 1 and 200 that is 1, 2, to 200 and you are selecting 101 of them. Show that there are at least two where one will divide the other. Now this can be solved in this manner, you have 100 holes or 100 holes or boxes one for each for the odd numbers 1 3 5 7 and so on.

There are 100 boxes, take a number, if you pick a number i express i as some r cross 2 power k, each number you can express like this r cross 2power k where k will be greater than or equal to 0. For example, if I take 55 that is 55 cross 2 power 0, you can write like this. If I take 112 then 112 is how many times? It is 64 divided by 8, 7 cross 2 power 4, 16 cross 7 is 112 you can write like this.

Now, whenever you have a number like this you write it as r cross 2 power k. So put this number in the box corresponding to r. So if I have selected 55 I will put 55 in the box corresponding to 55. If I had selected 112 I will put it in the box corresponding to 7. So when I select 101 numbers out of 200 numbers and put them in the boxes corresponding to these like this by the Pigeonhole Principle one box will contain at least two elements.

Suppose a particular box x contains two elements then there are two numbers m is equal to x cross 2 power p this box corresponding to x x is an odd number it contains two elements m and n. Then m can be represented as x cross 2 power p and n will be represented as x cross 2 power q. Now obviously p will be less than q or one of them will be less than the other p and q cannot be equal because m and n are different. Suppose m is greater than n then p will be greater than q and m is divided by n and m by n will be is equal to 2 power p minus q. So m is divided by n. There are two numbers such that one divides the other so we proved the result. So you can see how the Pigeonhole Principle is applied here.

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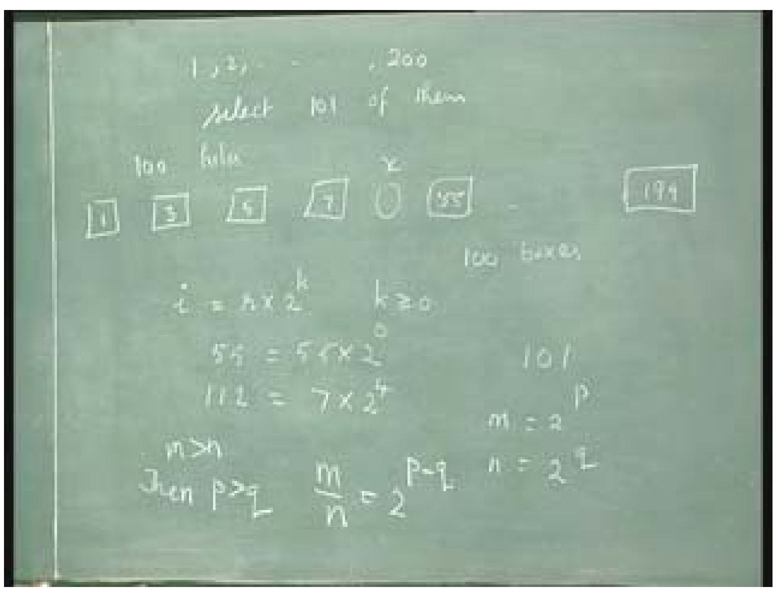

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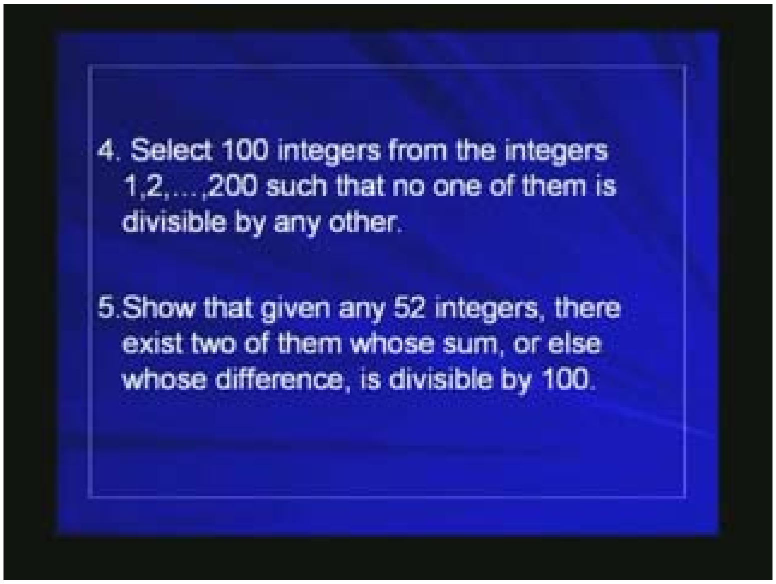

The next is, it is a continuation of this, select 100 integers from the integers 1, 2, to 200 such that no one of them is divisible by any other. So from 1 to 200 you are asked to select 100 of them so that no two of them are such that one divides the other. If you select 101, 102 to 200 you will realize that none of them divides the other. You can see like this, argue out like this, even if you take the least element and multiply by the smallest integer it will be 202 which exceeds this. So this will not divide any of this and similarly any one of them will not divide the other that is one argument. and when we consider the earlier thing try to express each one of them as x cross 2 power some k and try to put in each one in one of the pigeon holes 1 3 5 7 etc these 100 pigeonholes you will realize that these 100 numbers will be put in 100 different boxes each one will contain only one of them. You can try that and check whether it is correct, no two of them will be put in the same box. So you can take it as an exercise and try to put these numbers into those 100 boxes.

The next problem is also somewhat a similar one. Show that given any 52 integers there exist two of them whose sum or else whose difference is divisible by 100. The numbers can range in any way. So let a_1 a_2 a_{52} be the integers, they are some integers. And you are asked to show that there are two of them such that the sum will be divisible by 2 or the difference will be divisible by 2. You can always select two from these 52 integers such that the sum or the difference of will be divisible by 100, how do you prove that? Now any number a_i you can express as 100 k plus b_i you can express a number like this a_i is equal to 100k plus b_i. What is the range for b_i?
It will be from 0 to 99.

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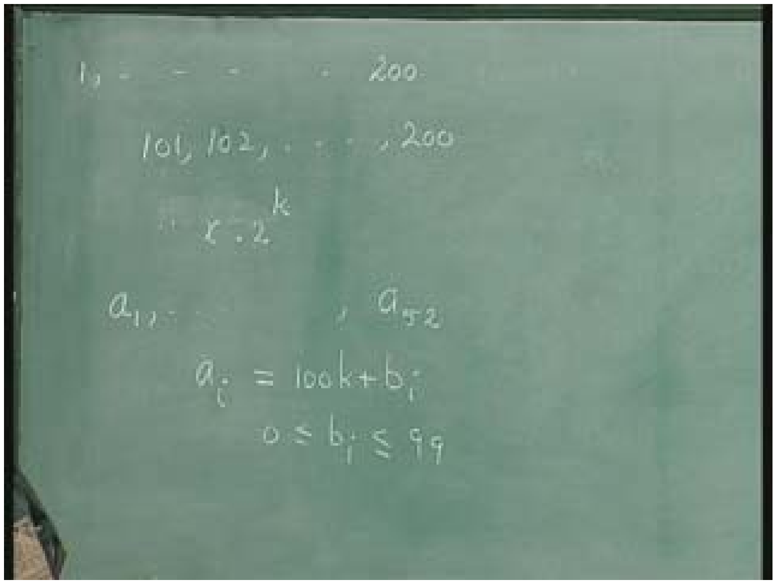

Now, let us consider some boxes like this corresponding to 0 1 2 49 50 boxes. You consider 51 boxes like this. Any number a_i is of the form 100k plus b_i. Now if b_i is 0 you put it here, if bi is 50 you put it here, if bi is equal to 1 or 99 you put it here in the box corresponding to 1. So you put b_i in the box j if b_i is equal to j or 100 minus j. Now, how many boxes are there? There are 51 boxes there. And how many numbers are there? There are 52 numbers. So by the Pigeonhole Principle two of them will be put in the same box. So two numbers say a_m and a_n will be in the same box because of the Pigeonhole Principle. Suppose a_m is equal to some 100 times k plus b_m and an is 100 times l plus b_n they are in the same box and let the box be the jth box. So b_m is j or 100 minus j, b_n is also j or 100 minus j.

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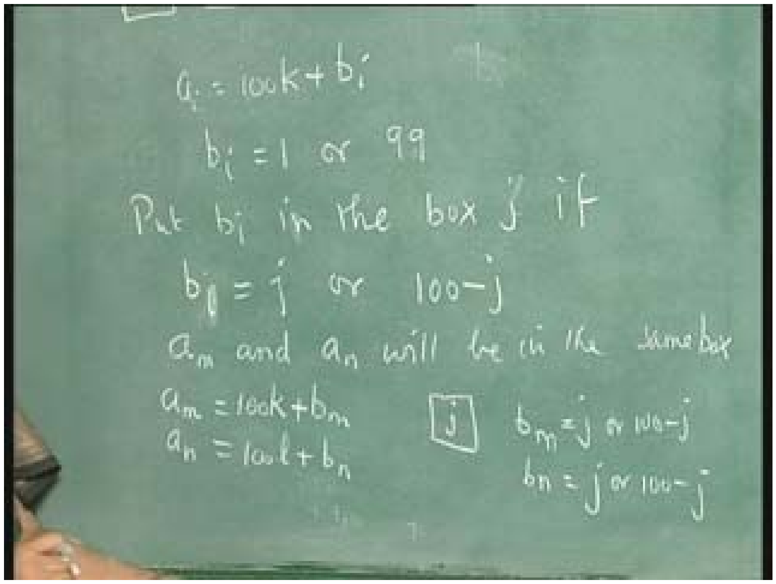

Suppose b_m is the same as b_n then am minus an will be hundred times k minus l so the difference is divisible by 100. Suppose b_m is j and b_n is 100 minus j then am plus an will be 100 cross k plus l plus b_m plus b_n will be 100 plus 100. So the sum am plus an will be divisible by 100 so any number a_i you express as 100k plus b_i where b_i will be from 0 to 99 and you consider 51 boxes like this 0, 1, 2 corresponding to 50 and you put a number in a corresponding box j if b_i is j or 100 minus j. For example, if b_i is 1 you will put here, if b_i is 99 also you will put here, if b_i is 2 you will put here, if b_i is 98 also you put it here, like that you put because there are 52 numbers and 51 boxes at least one of them will contain two numbers by the Pigeonhole Principle.

And suppose a_m and a_n are in the same box j then you can write am in the form 100k plus b_m and a_n you can write as 100 plus b_n where b_m will be j or 100 minus j and b_n also will be j or 100 minus j. If they are the same consider the difference the difference will be divisible by 100. If one is j and another is 100 minus j consider the sum of the two numbers the sum will be divisible by 100. So this way we can prove that given any 52 integers there exist two of them whose sum or else whose difference is divisible by 100. Take another problem. Let us go through the statement of the problem carefully.

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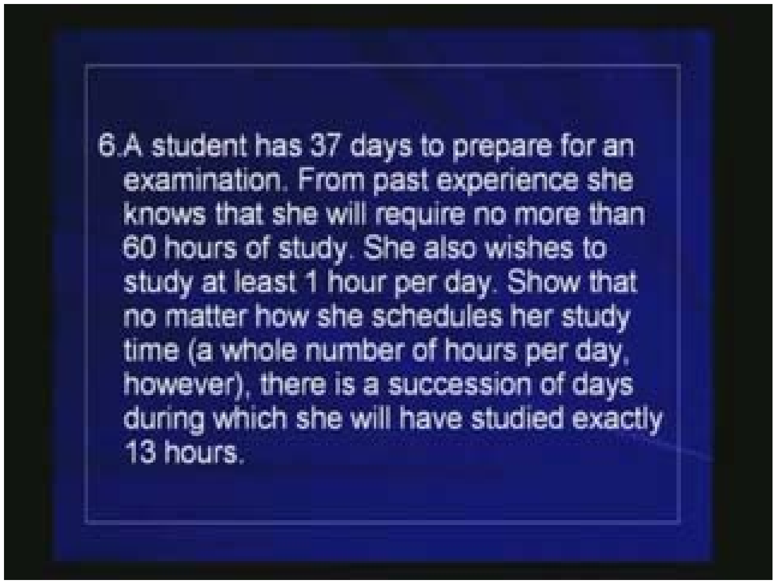

A student has 37 days to prepare for an examination. From past experience she knows that she will require no more than 60 hours of study. She also wishes to study at least 1 hour per day. Show that no matter how she schedules her study time a whole number of hours per day however, there is a succession of days during which she will have studied exactly 13 hours.

A student has 37 days to prepare for an exam and she needs to prepare for 60 hours. A student has 37 days to prepare for an examination. From the past experience she knows that she will require no more than 60 hours of study. She also wishes to study at least 1 hour per day. Show that no matter how she schedules her study time, there is a succession of days during which she will have studied exactly 13 hours.

Suppose on the first day she has studied for a_1. At the end of the second day she has studied for a_2 hours. At the end of the third day she has studied for a_3 hours. That is the first day she studies for a_1 hours. The first and second day put together she has studied for a_2 hours and the third day that is the first three days she has studied a_3 hours and so on. So on the 37th day she has studied for 60 hours. So this is an increasing sequence. So every day a_i is the number of hours she has studied up to ith day. So this particular sequence is an increasing sequence because everyday she studies at least 1 hour. And these numbers are between 1 and 60 because maximum she studies only for 60 hours. And consider this sequence, now what we want to prove is some consecutive days she has exactly studied 13 hours. That is some a_j minus a_i is equal to 13 where j is greater than i this is what we want to prove.

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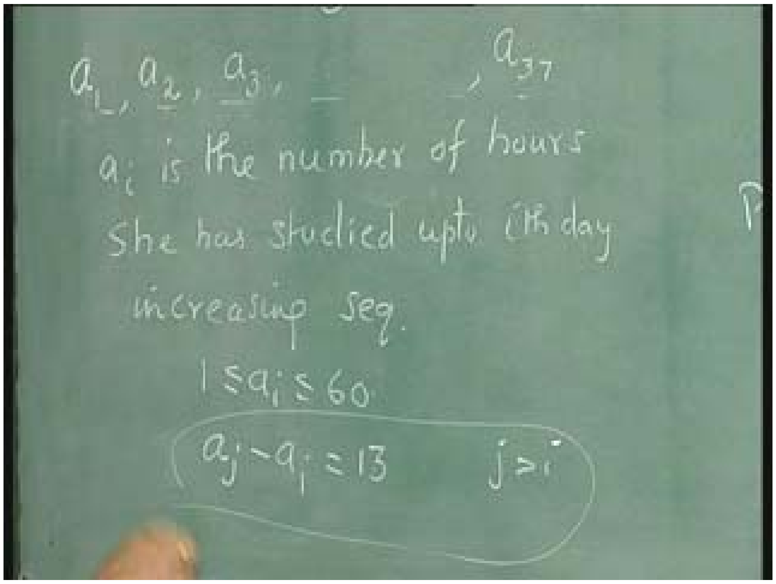

Now consider this sequence a_1 plus 13, a_2 plus 13 etc a_{37} plus 13. a_1 a_2 a_3 a_n is an increasing sequence so we are adding 13 to every number. So a_1 plus 13, a_2 plus 13, a_3 plus 13, a_{37} plus 13 is also an increasing sequence and all numbers are all different. So this is an increasing sequence and the values will range from 13 to 73. A_{37} is 60 so 60 plus 13 is equal to 73. The values will range between these two 13 and 73.

Now consider the numbers a1 a2 a37 a1 plus 13 etc up to a37 plus 13, how many numbers are there here? It is 37 plus 37 is equal to 74 numbers are there. And the value is a_i ranges from 1 to 60 the other sequence the numbers range from 13 to 73. So these numbers the value ranges between 1 and 73, these numbers are between 1 and 73. And there are 74 such numbers so two of them will be equal. Obviously by the Pigeonhole Principle two of them will be equal.

Now you know that none of these numbers are equal because every day she studies at least 1 hour. These numbers are all different and these numbers are all again different because with each number you are adding 13. So the fact that two of these are equal would mean that sum a_j will be equal to sum ai plus 13. Obviously, j has to be greater than i so a_j minus a_i will be 13. That is from the i plus 1th day till the jth day she has exactly studied for 13 hours. That is from the i plus 1th day to jth day she has studied for 13 hours. So, you can see how the Pigeonhole Principle is applied here.

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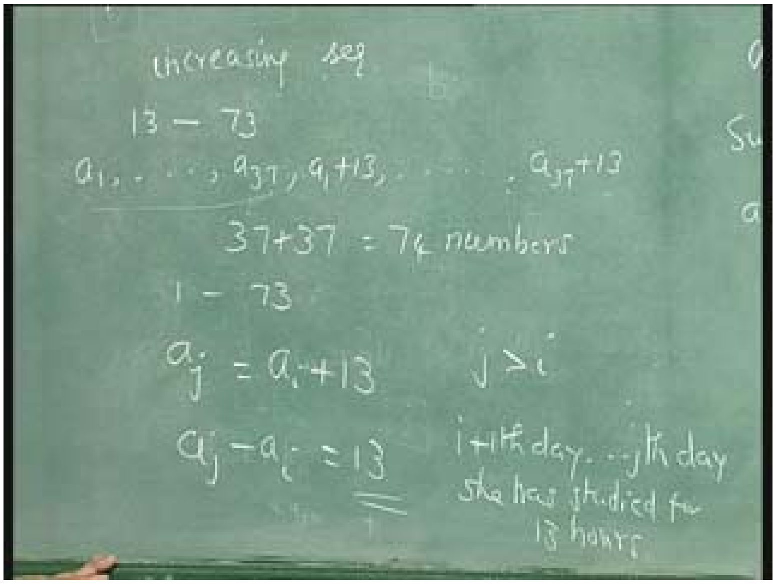

Let us take one more problem.

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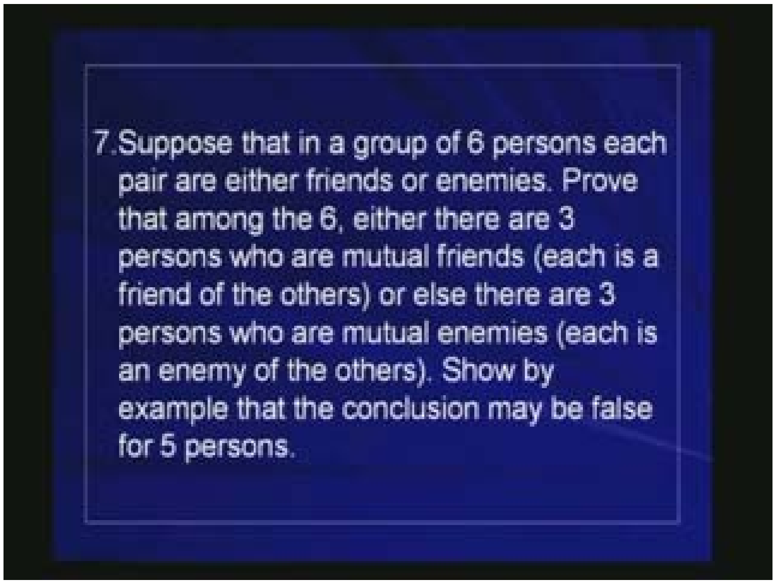

Suppose that in a group of 6 persons each pair are either friends or enemies. Prove that among 6 either there are 3 persons who are mutual friends each is a friend of the other or else there are 3 persons who are mutual enemies each is an enemy of the others. Also show that this is not true for 5 persons. So you can consider it like this. You are having 6 persons a b c d e f. If a and b are friends, I will put a line like this straight line. If a and b are enemies I will put a line like this. So if x and y are friends you consider like this, friends, enemies if x and y are enemies, consider a and consider 3 of his friends. See, there are 5 people and either he will have three friends or three enemies.

If a has three friends you can proceed in a following manner. either among the 5 the possibilities are, apart from a there are 5 people, the possibility is 5 friends and enemies, 5 friends no enemies, 4 friends 1 enemy, three friends 2 enemies, 2 friends three enemies, 1 friend 4 enemies, 0 friend all enemies. So either a will have three friends or he will have three enemies. This is by Pigeonhole Principle. Either a will have three friends or three enemies.

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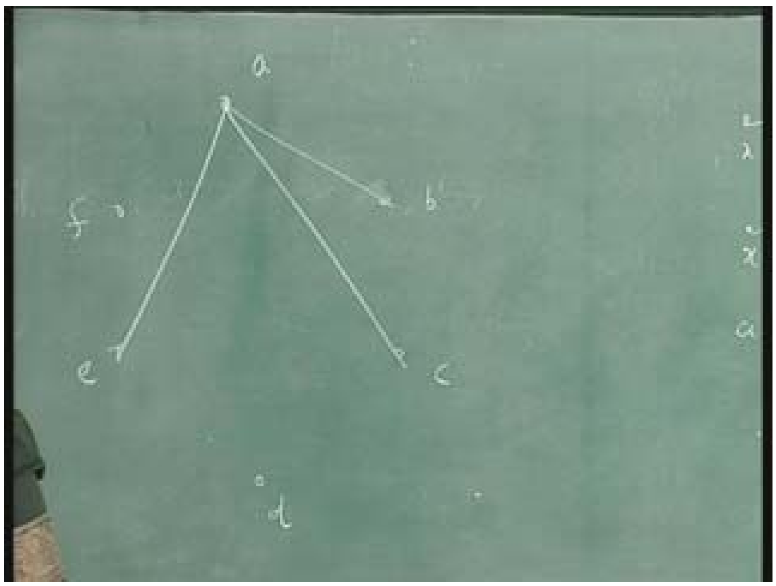

Suppose he has got three friends we will see how we can deal with the case when he has three enemies, it is similar. Suppose he has three friends say suppose b, c and e are his friends then consider b and c or c and e and b and e. Either b and c are friends then you have three mutual friends or if e and c are friends again you are having three mutual friends. If otherwise if b and e are friends again you are having three mutual friends.

On the other hand, none of them are mutual friends suppose b and c are enemies, suppose c and e are also enemies and b and e are also enemies, then you are having three mutual enemies. So in this case you are having three mutual enemies. So either you are having three friends or three mutual enemies. The other way round supposing a has three enemies, 3 or more enemies, a has three enemies like this now consider again b and c if they are enemies you are having three mutual enemies, if c and e are enemies like this you are having three mutual enemies, if b and e are enemies then again you are having three mutual enemies. Otherwise if they are not enemies b and c will be friends, c and e will be friends and b and e will be friends so you are having three mutual friends. So this shows that either you have three mutual friends or three mutual enemies among 6 persons.

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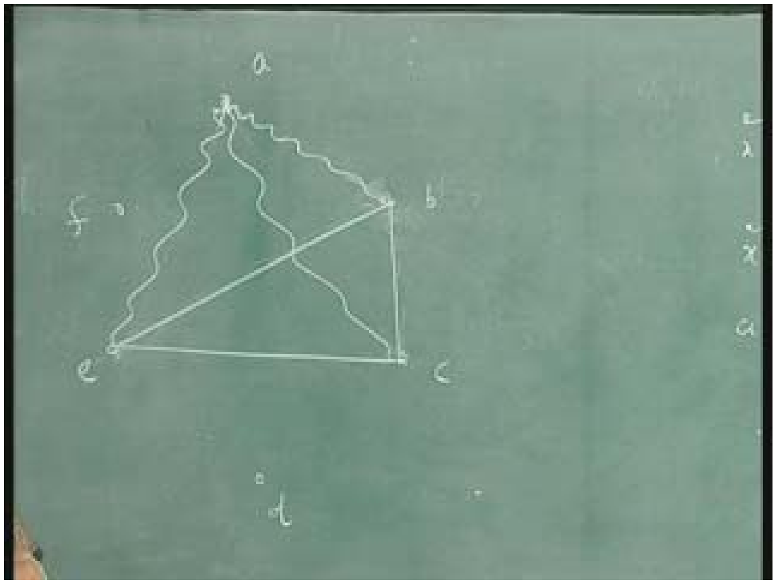

Now, the same is not true for 5 persons a b c d e the reason is, apart from a you are having 5 persons then either three of them will be friends to a or three of them will be enemies. More than 3 will be friends or 3 or more will be enemies this is because you can put like this. Now, apart from a there are only four of them so 2 can be friends and 2 can be enemies. So the fact that a will have three friends or three enemies does not hold anymore. So suppose b is a friend to a and e is a friend to a and c is enemy to a and d is also an enemy to a,

I can design in such a way that there are no three people who are friends or there are no 3people who are enemies. Now a and e are friends, a and b are friends, so suppose it is possible that b and e are enemies and a and d are enemies, a and c are enemies but c and d can be friends, it can happen like that.

Let us draw the other arcs also. Now, in this case you have to draw this arc 1 2 3 and 4 5 6 arcs we have drawn 4 more arcs we have to draw. Suppose b and c are friends then d and c are friends, b and c are friends. It is possible that b and d are enemies, you can have like this. And e and b are enemies b and d are enemies so it is possible that d and e are friends. So in that case you do not have three enemies or three friends. And similarly you are having d and c as friends and d and e as friends. So it is possible that c and e are enemies.

Now, if you take any three of these vertices you will find that no three of them are friends or no three of them are mutual enemies. So it is possible to draw a graph like this with two different types of arcs such that five of them are of one type and five of them are of another type and you have their condition that no three of them are mutual friends or no three of them are mutual enemies. It is possible to draw like this. The reason is, apart from a there are 5 people of which you put friends and enemies in two boxes, friends and enemies, 5 people you try to put them one of them should contain 3 and 3 or more. a will have either three friends or either he will have three enemies. And in that case we argued that how you get three mutual friends or three mutual enemies.

Now, apart from a if you have only 4 people that condition does not hold. If you have two boxes friends and enemies you can put 2 in each of them and then you can construct a graph like this which shows that you do not have three mutual friends or three mutual enemies so the Pigeonhole Principle is again used here. Let us consider one more problem again, similarly. The same idea, it is a very simple idea.

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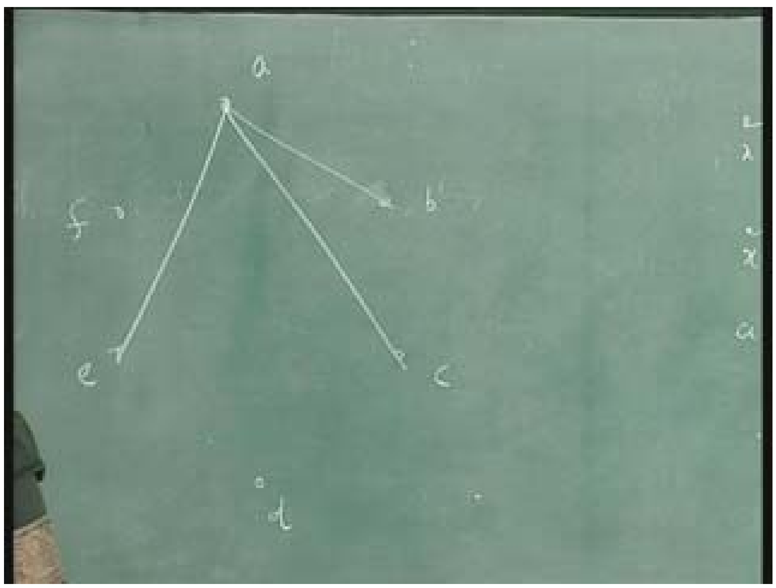

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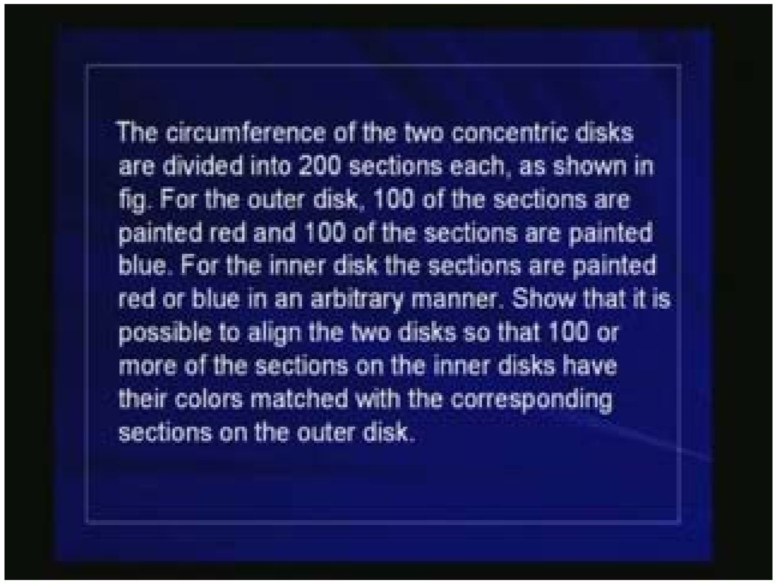

You have two concentric disk. The circumference of the tow concentric disks are divided into 200 sections each as shown in the figure, let us see the figure.

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You have two concentric circles like this and the circumference is divided into 200 sectors like this. For the outer disk 100 of the sections are painted red and 100 of the sections are painted blue. For the inner disk the sections are painted red or blue in an arbitrary manner. Show that it is possible to align the two disks so that 100 or more of the sections on the inner disks have their colors matched with the corresponding sections on the outer disk. The figure is like this; now in one position you are having a disk like this.

Suppose 1 to 200 is there and similarly here also 1 to 200. In how many ways you can align them?
First is, 1 and 1 can be aligned in a manner then you can rotate and 2 can be aligned with 1 then you can again rotate and 3 can come in alignment with 1 so that 2 will come with 200 and 199 1 will come like this. I am rotating in the anticlockwise direction. So, you can rotate in such a way that you get the next alignment. The first alignment of course 1 and 1, I number them as 1 to 200 and 1 to 200 here, 1 and 1 will align 200 and 200 will align and so on. If I rotate through one sector then 2 will align with 1 and so on. So how many possible alignments can you have? You can have possibly 200 alignments and the problem is it is possible to align the two disks so that 100 or more of the sections of the inner disk have their colors matched with the corresponding sections on the outer disk.

Now, in the outer disk 100 of them are colored blue and 100 are painted red and in that disk they are arbitrarily painted in red or blue. So take in one alignment. In one alignment suppose this particular disk is colored red out of the 200 possible alignments in 100 possible alignments it will match with the corresponding sector in the outer disk. So in the 200 alignments the first sector will match in 100 of the 200 alignments, second sector will also match in 100 of the alignments, third sector will also match in 100 of the alignments and so on. So totally the number of matches will be 200 cross 100 is equal to 20000 matches you count the matches.

In one alignment you count the matches, in the second alignment you count, the matches in the third alignment you count the matches, like that in the 200 alignments you count the matches. The total number of matches will be 20000. The reason is, each sector in the inner circle is painted red or blue and the outer sector 100 of them are painted red and 100 of them are painted blue. So if you take this sector which is painted red in 100 of the alignments it will match. If you take the next sector it will be blue or red so in 100 of them it will be matching and so on. So, if you count the number of matches in all the alignments it will add up to 20000. And how many alignments are there? There are 200 alignments, so the average is 100.

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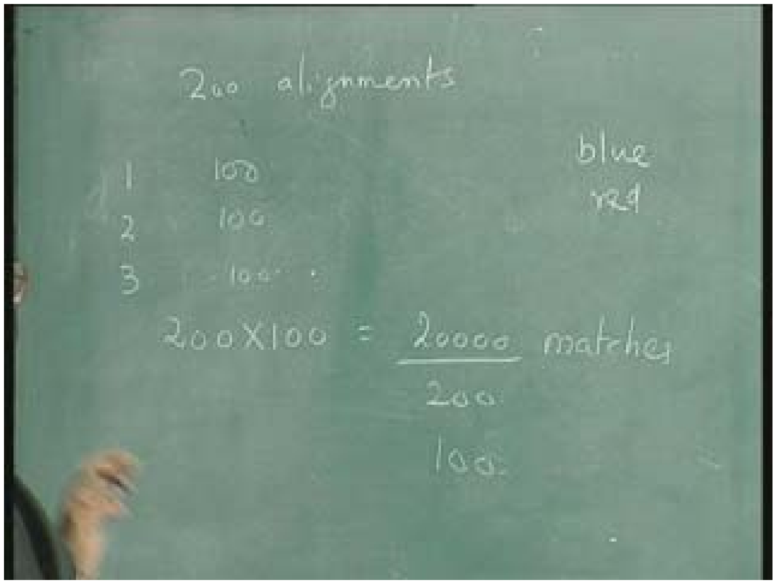

So the average is 100, so at least in one alignment 100 or more of the sectors will match the corresponding colors.

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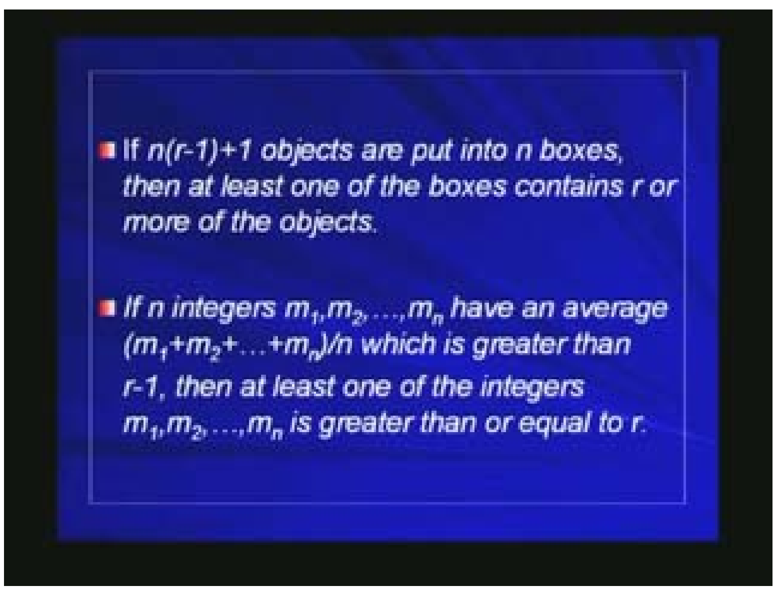

Here we are making use of this; If n integers m_1, m_2, m_n have an average which is greater than r minus 1 then at least one of the integers m_1 plus m_1, m_2, m_n is greater than or is equal to r. So there are 200 alignments. Let us call them as alignment 1, alignment 2 and so on, alignment 200. You call the number of matches as m_1, m_2 and m_{200}. The sum is sigma mi is 20000. So the average sigma m_i by 200 is 100, so it is greater than 99. So here in this example m_1, m_2, m_n represents the number of matches in each alignment and the average is greater than r minus 1 that is 99 it is at least 100 so at least one of them should be odd, in the number of matches at least one of them m_1, m_2, m_n, m_{200} is 100 or more. So we are making use of this form of the Pigeonhole Principle here. So let us recall the Pigeonhole Principle and the different versions again and it can be used to solve problems like this. The idea is very simple but when you tackle each problem you have to think in a different manner for the solution.

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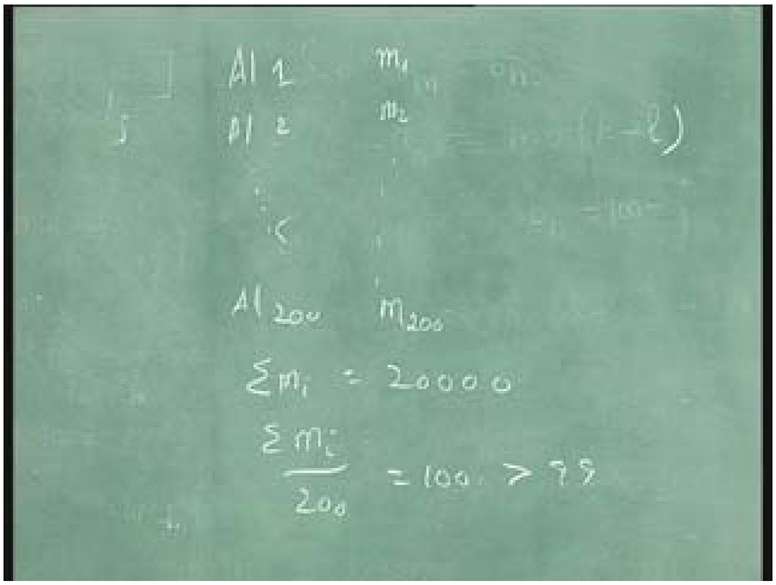

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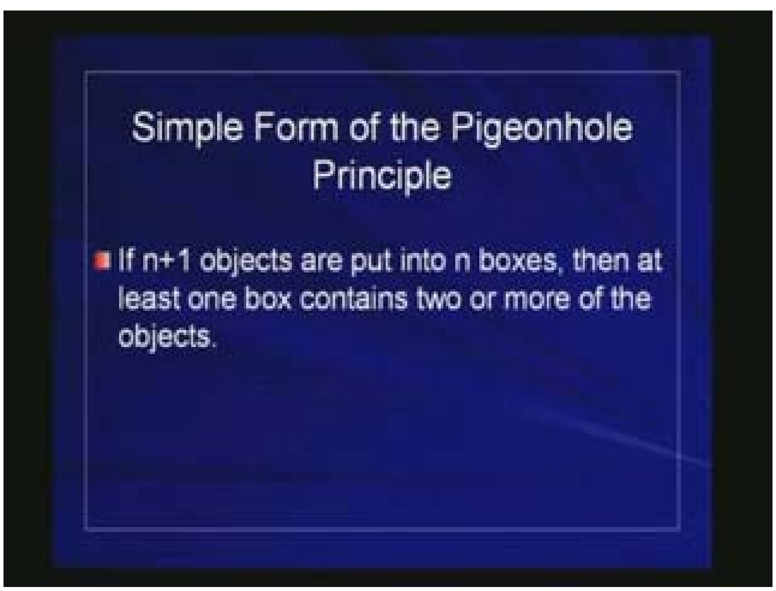

The simple form is this: n plus 1 objects are put into n boxes then at least one box contains two or more of the objects.

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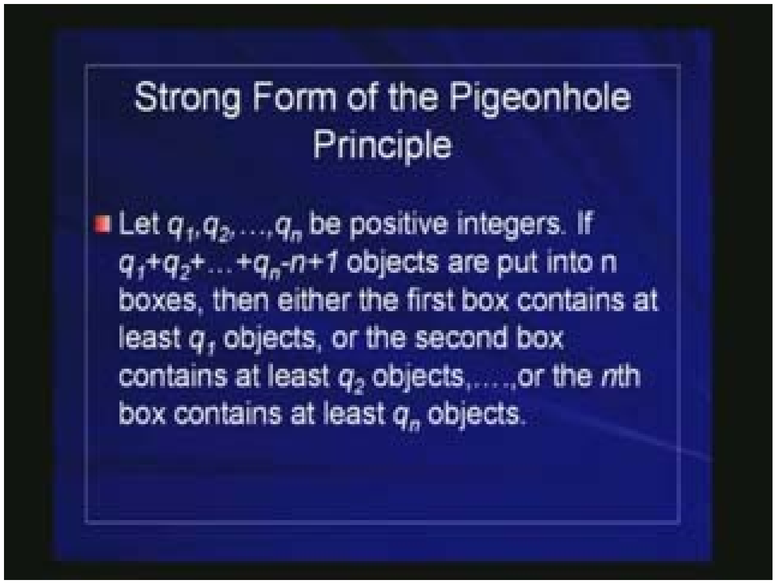

The stronger forms are like this: Let q_1, q_2, q_n be positive integers. Then if q_1 plus q_2 plus q_n minus n plus 1 objects are put into n boxes, then either the first box contains at least q_1 or the second box contains at least q_2 and or the nth box contains at least q_n objects.

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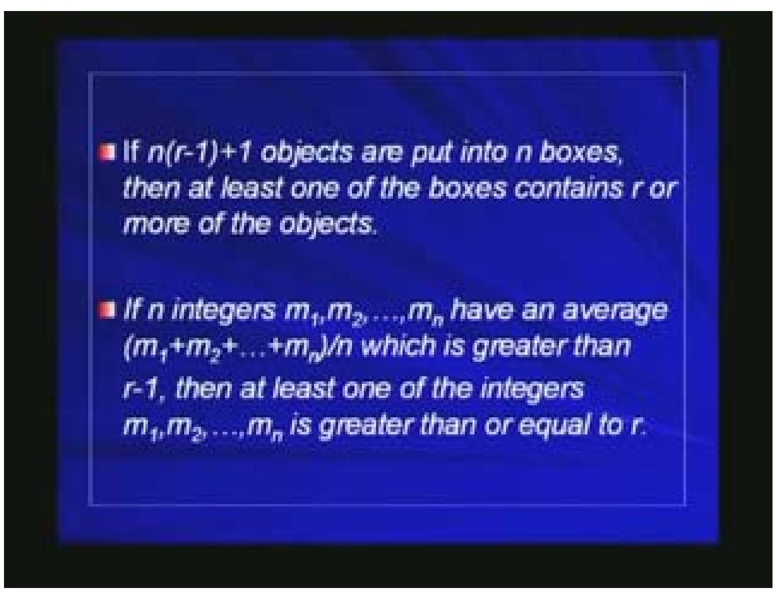

The other two forms are like this: If n cross r minus 1 plus 1 objects are put into n boxes then at least one of the boxes contains r or more of the objects. And another form is, if you take the average of n elements if it is greater than r minus 1 then at least one will be greater than or is equal to r.