Discrete Mathematical Structures
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Lecture # 26
Functions (contd…)

We were considering about functions. We were also considering when a function is said to be injective, surjective and bijective. If f is a bijective function from A to B then f inverse is the inverse function which is defined from B to A such that f of f inverse f composition f inverse is a identity function and f inverse, f is a identity function on A, this is the two sided inverse. But you may have one sided inverses for a function that is what we considered towards the end left inverse and the right inverse.

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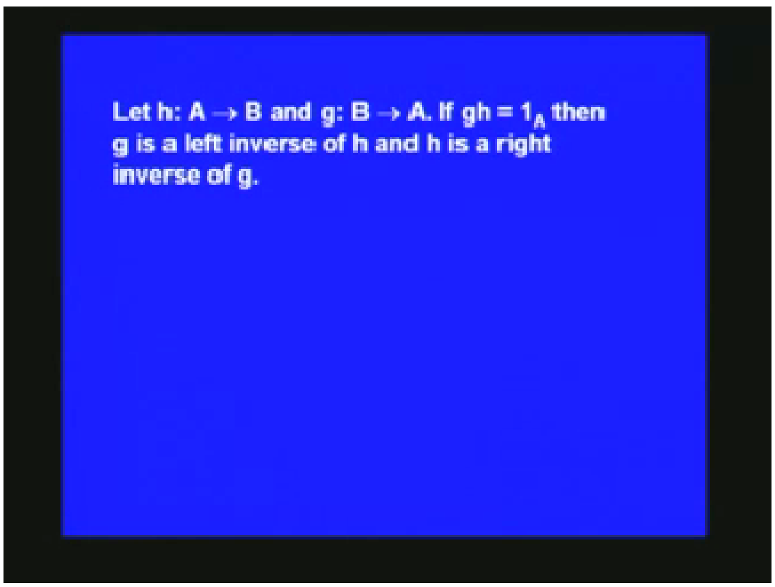

Consider two functions h from A to B and g from B to A. And if g h g composition h is equal to 1_A that is the identity function on A then g is called the left inverse of h and h is called the right inverse of g, these are called one sided inverses. Let us study some results about one sided inverses.

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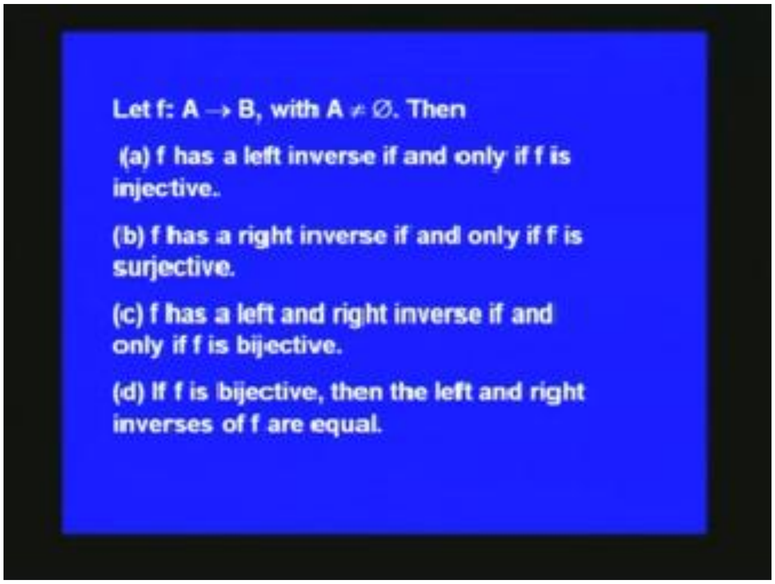

Let f be a function from A to B with nonempty domain A, then

(a) f has a left inverse if and only if f is injective.

(b) f has a right inverse if and only if it is surjective.

(c) f has a left and right inverse if and only if f is bijective.

(d) If f is bijective then the left and the right inverses of f are equal. We want to see all these results.

Let us prove one by one.

The first portion is, f is a function from A to B, f has a left inverse if and only if f is injective. Now, there are two portions, we have to prove f has a left inverse. If f has a left inverse this implies f is injective. This is one portion we have to prove. The other portion is, if f is injective then f has a left inverse. So let us prove the first part now, f has a left inverse, left inverse of f(g) is the left inverse. So g will be from B to A and g dot f is equal to 1_A.

Now we have to show that f is injective. Suppose f is not injective then there exist a a dash belonging to A, a not equal to a dash such that f(a) is equal to f(a) dash because two different elements are mapped onto the same elements. That is, it is violating the injective property so we are taking it as suppose f is not injective what happens? We will show that if f(a) is equal to f(a) dash then you will show that a is equal to a dash.

So, consider this. What can you say about a in this case? a is 1_A of a the identity function on a and what is 1_A? It is g dot f(a) that is equal to g dot g(f(a)) and f(a) is f(a) dash so g(f(a)) dash and that is g(f(a)) dash that is 1_A of dash gh is 1_A is equal to a dash. So if you have f(a) is equal to a dash your assumption a is not equal to a dash is not correct. You will show that a is equal to a dash. That means this is not correct f is not injective f is injective.

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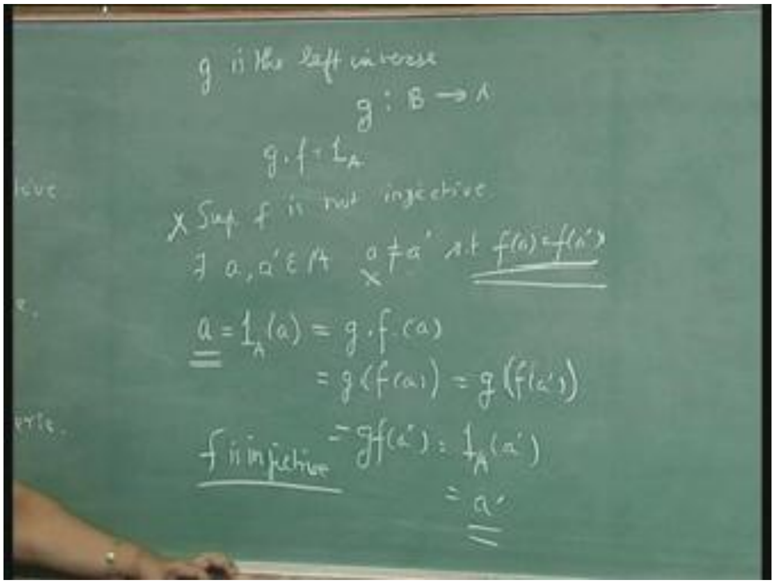

This is one way we have proved. That is we have to prove two parts f has a left inverse then f is injective. The second part is if f is injective then f has a left inverse, this is what we have to prove now. Let us see how to prove that. f is injective. Then define the left inverse g as follows: g will be from B to A, g(b) is equal to a if f(a) is equal to if you have an element a such that b is the image of a then define g(b) is equal to a. There may be elements in b which are not the images.

For example, the function can be something like this: it is an injective function so different elements would be mapped onto different elements. So in that case if this is a, this is b then f(a) is equal to b you define g(b) is equal to a like that. But there may be other elements like this which are not the images of anything. For them choose an arbitrary element some element a dash and map, you may have one or two like or some of them like that and you map everything onto that element. So if there is a such that f(a) is equal to b define g(b) is equal to a. If there is no a such that f(a) is equal to b then choose an arbitrary element a dash and then define g(b) is equal to a dash otherwise, that is in this case. If there is an element a such that f(a) is equal to b define g(b) is equal to a. if it is not the case that is b is not the image of any element choose an arbitrary element a dash and then define g(b) is equal to a dash. Now this g satisfies all the conditions of the left inverse.

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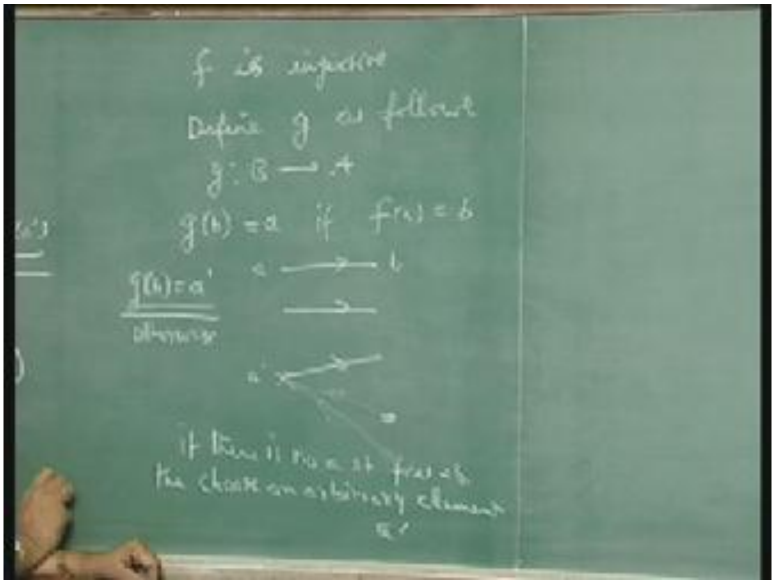

If you have g of f of an element a will be obviously g(b) f(a) is b will be a so that condition will be satisfied and so g is a left inverse. But you must note that I could have chosen this as the arbitrary element, I could have chosen this as the arbitrary element and so on there are several possibilities. This element could have been chosen in several different ways. So the left inverse of a function is not unique.

What we have proved is f is injective would imply f has a left inverse. It may have several left inverses it depends on how you define that. So, by this we have proved the first part, f has a left inverse if and only if it is injective.

Let us now prove the second part, f has a right inverse if and only if f is surjective. So here again there are two parts f has a right inverse then f is surjective, if f is surjective then f has a right inverse. So we have to prove it in two parts. First is if f has a right inverse then f is surjective. Let h be the right inverse of f then h will be from B to A. So you have f dot h is equal to 1_B. In this case you have to show that f is surjective.

Now, if you take an element b, take some element b belonging to B, f is from A to B so if f is surjective then b should be the image of some element of a under f. Take some b belonging to B. Now what can you say about f dot h(b)? This is 1_B of b is equal to b so f(h(b)) is equal to b.

Let h(b) be is equal to a then you have f(a) is b. so there is an element a belonging to A such that f(a) is equal to b because h is a function h is properly defined. So h(b) is definitely some element of a and that is a. Then you have f(a) is equal to b. So for every b belonging to B you can find an a in the set A such that f(a) is equal to b. So every element of B is an image of some element of A under f. That means f is surjective. Therefore f is surjective.

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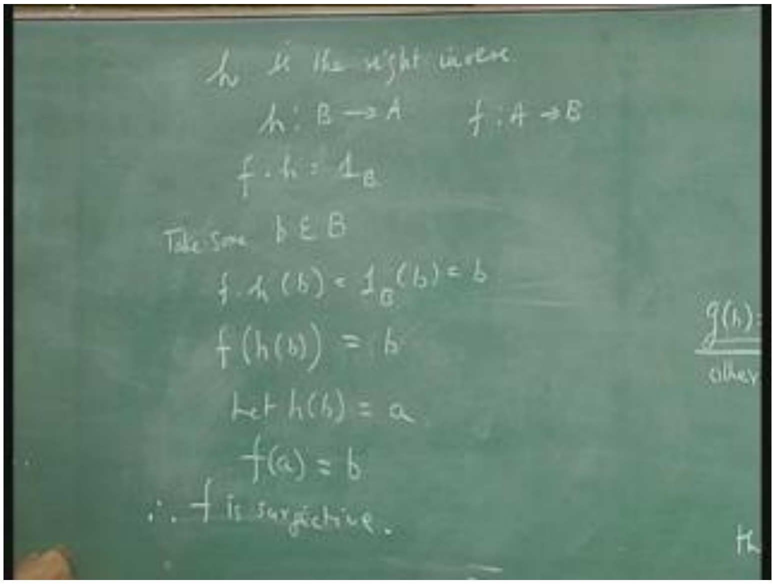

So we have proved this part. That is if f has a right inverse then f is surjective. The other way round you have to prove that if f is surjective then f has a right inverse. So f is surjective. It will be something like this. This is mapped onto this, this is mapped onto this like this, f is a function which is surjective. It is from A to B A has say 1 2 3 4 5 and this has got elements b_1 b_2 b_3. So 1 and 4 are mapped onto b_3, 2 and 3 are mapped onto b_1, 5 is mapped onto b_2 this is just an example I am saying. So in this case define h as follows: f is from A to B we know this, we have to define h from B to A.

Define h from B to A as follows: it is like this h(b) is equal to a if there is a unique a such that f(a) is equal to b. If there is only one element a such that f(a) is equal to b then define h(b) is equal to a. For example, here 5 is the only element which is mapped onto b_2. So when you define in the reverse order you can define $h(b_2)$ is equal to 5 like that you can define.

Now, in the other cases like here; 2 and 3 are mapped onto b_1. When you want to define in the reverse way the inverse you can take either 2 or 3 $h(b_1)$ will be either 2 or 3 any one you can take. You can choose any one of them, so h(b) is equal to a_1 if there exists a_1, a_2 some of them such that $f(a_1)$ is equal to $f(a_2)$ etc is equal to b. Many elements are mapped onto b in that case you can choose one of them like this. If you define like this what will be the inverse here?

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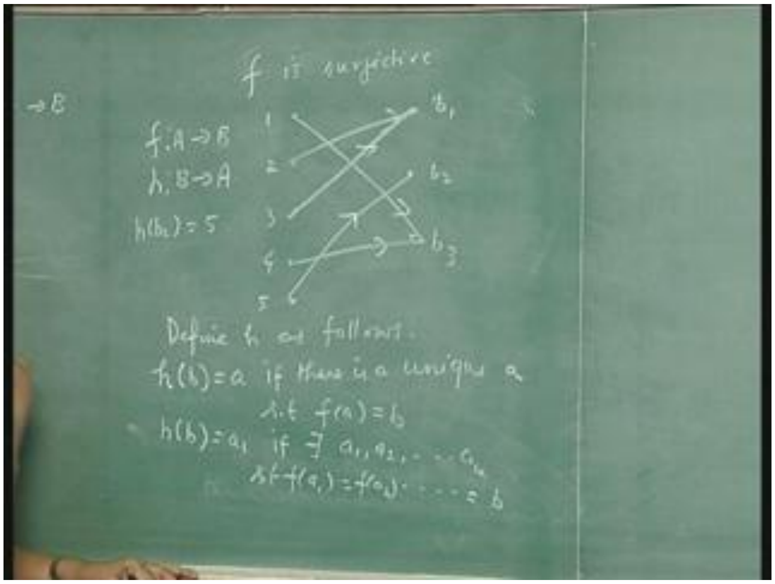

In this example the inverse will be like this:
b_1 b_2 b_3 first b_2 will be mapped onto 5, b_1 to any one of them, b_3 again to one of them like this. Again I could have chosen 2 or 3 for b_1 or I could have chosen 1 or 4 for b_3 so the right inverse need not be unique, that you have to see. Now just to satisfy the condition of 1_B what can you say about f? We know that f cross h is 1_B so take some b belonging to B then what can you say about f(h(b))? This will be equal to f(h(b)) but h(b) will be one of the a_1 which are mapped onto b so it will be some f(a) such that a is mapped onto b by f, so this will be b. So this condition f(h) is equal to 1_B will be satisfied so h will be a right inverse. As I mentioned to you earlier in this example you could have mapped b_1 to 2 or 3 you could have mapped b_3 to 4 or 1 several possibilities exist, so the right inverse is not unique.

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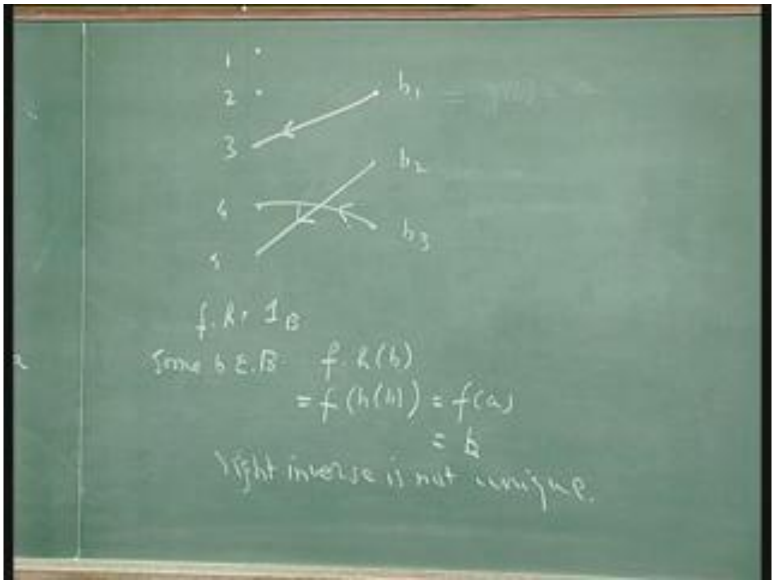

So we have proved the second portion. What we have just now proved is f has a right inverse if and only if it is surjective. Now the third part says that f has a left and a right inverse if and only if it is bijective. The third part c follows from a and b because if f is a bijective function it is injective as well as surjective so because it is injective it will have a left inverse and because it is surjective it will have a right inverse so it will have both left and right inverses.

The other way round is, if f has a left inverse and the right inverse because it has a left inverse it will be injective and because it has a right inverse it will be subjective and so it will be both injective and subjective and hence bijective. So the third part will follow from the parts a and b. Now the last portion says that if f is a bijective function and then you have the left and the right inverse they are equal. So in the case of bijection it is very easy it will be like this as we considered f inverse earlier if f is bijective then the mapping will be like this, something like this, different elements will be mapped onto different elements and everything will be the image of something and to get the inverse you have just reverse the arc that is all this will be both the left inverse and the right inverse. This is what we want to prove now.

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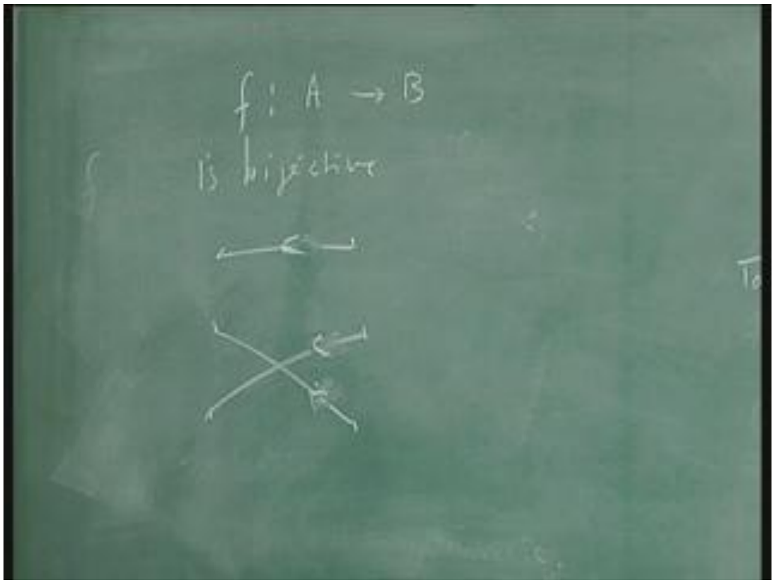

We can prove it in an easier manner like this:
Let g be the left inverse of f and h be the right inverse. Then both g and h are from B to A. they are functions from B to A. And what do you have? You have g into f is equal to 1_A and f into h is equal to 1_B this is what you have. Now we have to show that g and h are the same, g is equal to h how do we show that? Take g, g is from B to A so you can write g as this is from B to B and this is from B to A. So g is equal to g(g dot 1_B) but what is 1_B? Here, 1_B is f dot h so this I can write as f dot h. And because composition of function is associative you can write this as g dot f dot h and what is g dot f? g dot f is 1_A. So you can write it as 1_A dot h, h is from B to A and this is from A to A so this is properly defined. And what is 1_A? h identity function when you use in composition it is the same as h so this is the same as h. So you get g is equal to h and this proves the last part.

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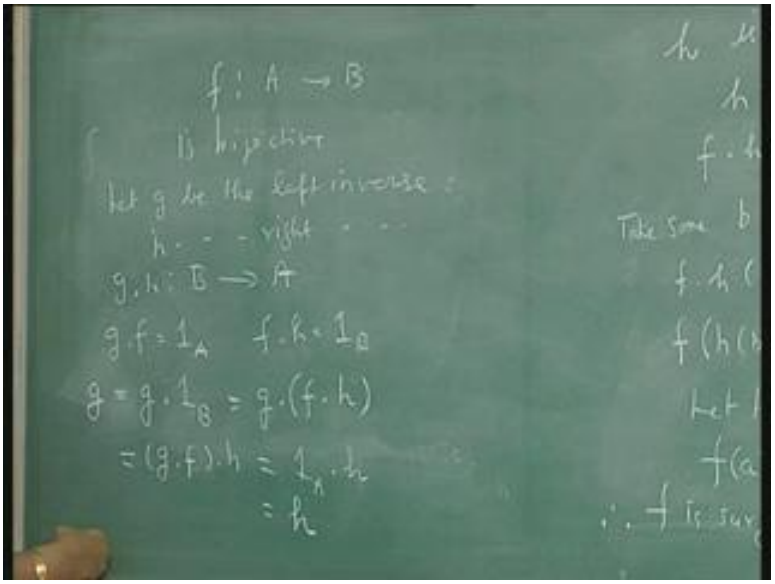

So we have this theorem connecting the properties of a function being surjective, injective, bijective and the property that it has a left inverse, right inverse and both the inverses. So, we have considered some properties of functions and some facts about the functions and some definitions also. Now let us consider some problems, how to tackle some problems using functions and also in relations.

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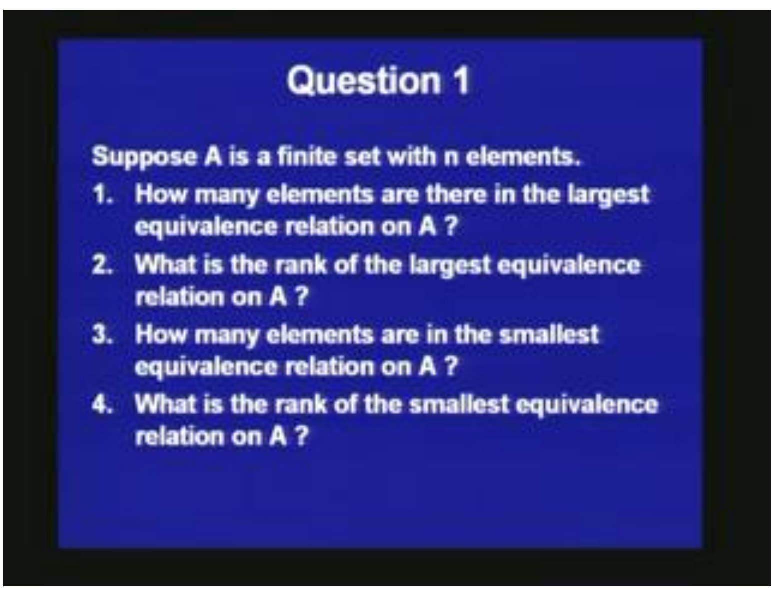

Consider this question:

Suppose A is a finite set with n elements how many elements are there in the largest equivalence relation on A? What is the rank of the largest equivalence relation on A, how many elements are there in the smallest equivalence relation and what is the rank of the smallest equivalence relation.

Now, if you remember, if you have a set with n elements then if it is reflexive if it is an equivalence relation it will be reflexive and symmetric means when there is an arc between this node and this node there will be arc in the other direction also and transitive would mean if there is a arc like this and like this you will also have an arc like this. Now, the largest equivalence relation means it should contain the largest number of ordered pairs. That is possible only when every two nodes are connected then it will satisfy the symmetric property and it will satisfy the transitive property also. So the largest equivalence class having these n elements would be a complete diagraph where there is an arc between every pair of nodes.

So how many ordered pairs can you have in A cross A?
This has got n elements, this has got n elements so n squared elements you can have. And all these n square ordered pairs are present in this relation it is a largest equivalence relation. So how many elements are there in the largest equivalence relation means there will be n squared elements in the largest equivalence relation on A. This is the answer for the first one.

In this case every pair of element is related. So if you have a_1 a_2 a_3 a_n then a_1 and a_2 will be in the same equivalence class because everything is related, a_2 and a_3 will be in the same equivalence class and so on. Like that all of a_1 a_2 a_3 a_n will be in the same equivalence class. So what is the rank or the index of the equivalence relation? It is the number of equivalence classes. And in this case the number is just 1 you have only one equivalence class so that is the second portion. The third portion is how many elements are there in the smallest equivalence relation on A.

What will be the smallest equivalence relation?
It will be like this, you will have only the self loops at the nodes. There will not be any arc between the nodes. In this case the symmetric property is satisfied, the transitivity is also satisfied and this is the equality relation on the set A. This is the minimum number of ordered pairs you can have in an equivalence relation on the set A because it should always satisfy the reflexive property that means you must have the self loops. So in this case the only ordered pairs are of the form; a_1 a_1 a_2 a_2 a_3 a_3 and so on. So the minimum equivalence relation or the smallest equivalence relation on A will have n ordered pairs of the form a_I, a_i. In this case each element is in one equivalence class, a1 is in one equivalence class, a_2 is in one equivalence class, a_3 is in one equivalence class and so on. So the rank or the number of equivalence classes is also n in this case. So, for 3 and 4 the answer is n.

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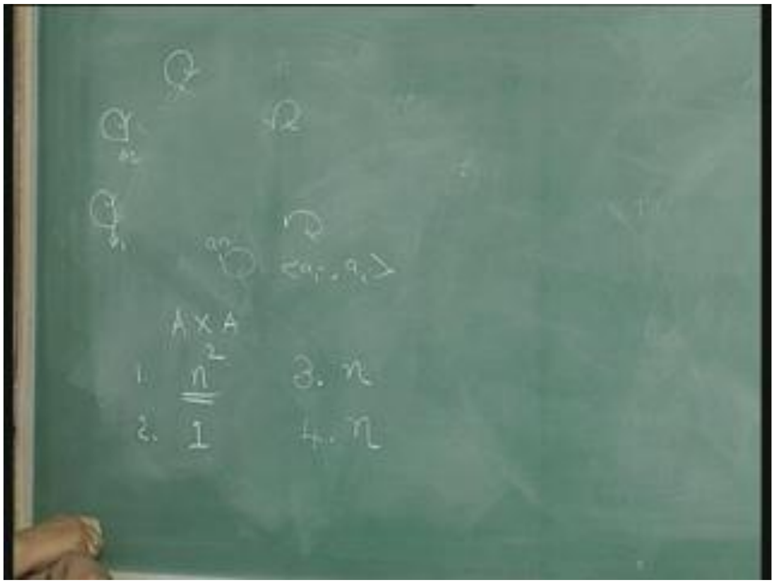

Now let us consider some more relation.

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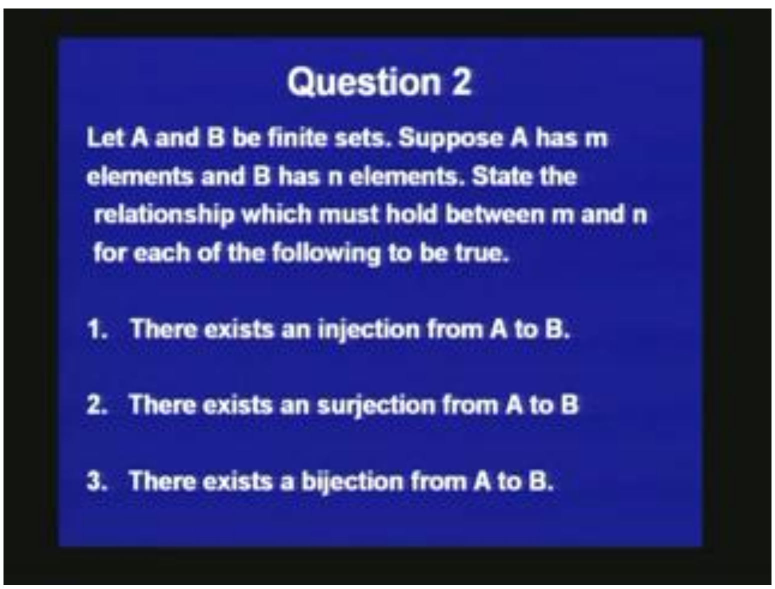

Let A and B be finite sets.

Suppose A has m elements and B has n elements state the relationship which must hold between m and n for each of the following to be true. There exists an injection from A to B. A has m elements and B has n elements. You have a set A and a set B this has m elements this has n elements A has m elements.

Now, you want to have an injection from A to B. That is different elements should be mapped onto different elements. You may have some more elements left out also. So what is the relation between m and n? n should be greater than or is equal to m. If A has m elements they should be mapped onto different elements in B so B should have at least m elements so n should be greater than or is equal to 1 this is the answer for the first one.

Now, there is a surjection from A to B. That is, every element is the image of some element here, you may have something like this also, if we have more elements here then there will be some elements left out so the number of elements in B should be less than or equal to the number of elements in A, that is n should be less than or equal to m in this case. If you want to have a surjection from A to B A having m elements and B having n elements n should be less than or is equal to m. And if it is a bijection it is both surjective and injective that is you will have something like this and like this.

Different elements will be mapped onto different elements and at the same time every element will be the image of some elements. So if it is bijective n has to be is equal to m. Or in other words it has to satisfy this condition and this condition so it has to be n is equal to m. So if there is a bijection from A to B the relationship between n and m is n is equal to m.

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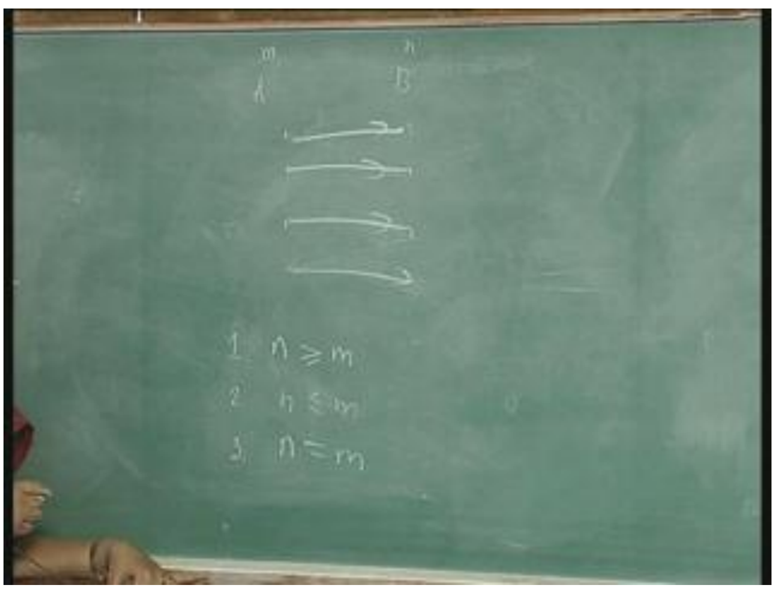

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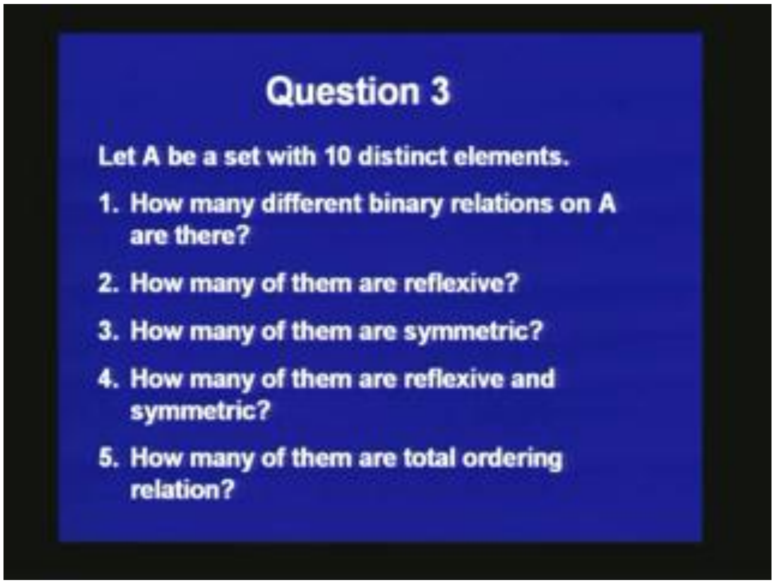

Consider another problem:
This is on relations, let A be a set with 10 distinct elements how many different binary relations on A are there?
The relation is on A cross A, A has 10 elements so how many different distinct relations can you have on A. A cross A will have 10 squared ordered pairs or 100 ordered pairs. Now in a relation each pair may be present or not present. So you may have a pair present or you may not have a pair present. So there will be totally 2 power 10 squared possibilities. So with 10 elements the answer to question one is this, question one is how many different binary relations on A are there? The answer for that will be 2 power 10 squared or 2 power 100.

The second is how many of them are reflexive so among these 2 power 10 squared relations how many of them are reflexive?
If you have n elements if it is reflexive there will be self loops this has to be present at every node. So among the 10 ordered pairs there are 10 elements these 10 should be definitely present and leaving out the 10 there are 90 other ordered pairs from one to another that is 10 cross 9 is equal to 90 between the nodes we have 90 arcs. Now each one of these arcs may be present or may not be present in a relation. So if it is reflexive these self loops are present but the other arcs may be present or may not be present. So, if the relation is reflexive the number of distinct relation you can have is 2 power 90. Out of this 2 power 100 distinct relation 2 power 90 of them will be reflexive.

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The third question is how many of them are symmetric?

What is the condition for symmetric?

Either if you have nodes like this, either you must have both of them present or you will not have any of the arcs present. So you can look at it this way. You are having 10 nodes and the self loops may be present or may not be present. Between any two nodes either two arcs are present or no arcs are present. Therefore, of the 90 ordered pairs like this from this to this connecting the nodes you can group them as 45 because either this should be present if it is present the other should also be present like that. So you can look at them as 25 sets of ordered pairs. Either a set is present or not present in a symmetric relation. But the self loops also may be present or may not be present so add that 10.

Hence, out of these 55 things something may be present or may not be present in a relation. So the total number of possibilities will be 2 power 55. The number of relations which are symmetric is 2 power 55, so we have tackled the third question.

Now, what is the fourth question?
How many of them are reflexive and symmetric?
We have seen that there are 2 power 90 which are reflexive out of which how many of them are symmetric also. That means again as I mentioned to you either the arc is present in both ways or not present. That is you can group the 90 ordered pairs as 45 sets.

Therefore, each one of the 45 pairs may be present or may not be present but the other 10 should be present if it is reflexive. So if the relation is both reflexive and symmetric the number of possibilities will be 2 power 45 in this case. So out of the 2 power 100 relations 2 power 90 of them are reflexive, 2 power 50 of them are symmetric and 2 power 45 of them are both reflexive and symmetric.

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The last question is how many of them are total ordering relation?
A total ordering relation is a linear order and it should be represented as a chain. The elements will be represented as a chain like a_1 a_2 like that how many such relations can you have?
Now there are 10 elements so the first element you can choose in 10 possible ways. After choosing the first element you will be left out with 9 elements, the second element can be chosen in 9 possible ways. After having chosen the first and the second element you can choose the third element in 8 possible different ways, the fourth element in 7 possible different ways and so on. So the number of such relations you can have is 10 factorial or 10 if you represent like this. So the answer to the fifth question is 10 factorial.

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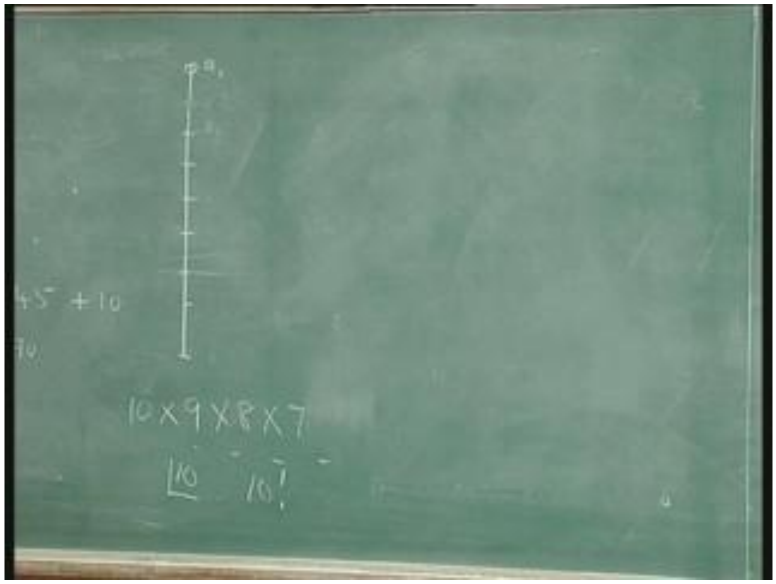

Now let us consider another thing.
You are having a set A with n elements. A is a set with n elements, n_R represents the number of number of distinct binary relations on A, that is A cross A. Now what can you say about n_R? What is the value of n_R? You know that n_R is 2 power n squared we have seen this just now we have seen for the case of 10 elements. With n elements you can have n squared ordered pairs. In a relation an ordered pair may be present or may not be present so there will be 2 power n squared possibilities. Let n_F denote the number of distinct functions from A to A.

Now what can you say about n_F?
Now you are having n elements like this a_1 a_2 a_n. again in the codomain or the range also you are having a_1 a_2 a_n. Now what can you say about the number of distinct functions you can have? Now a_1 can be mapped onto any one of the n elements so there will be n possibilities like that, a_2 can be again mapped onto any one of the n elements so there will be n more possibilities like that, a_3 can be mapped onto any one of the n elements that will be another n and so on. So, each one of the n elements can be mapped onto any one of the n elements in the codomain. So the total number of possibilities will be n cross n cross n cross n cross n cross n cross like that n times which will be n power n. So the number of different distinct functions you can have is n power n.

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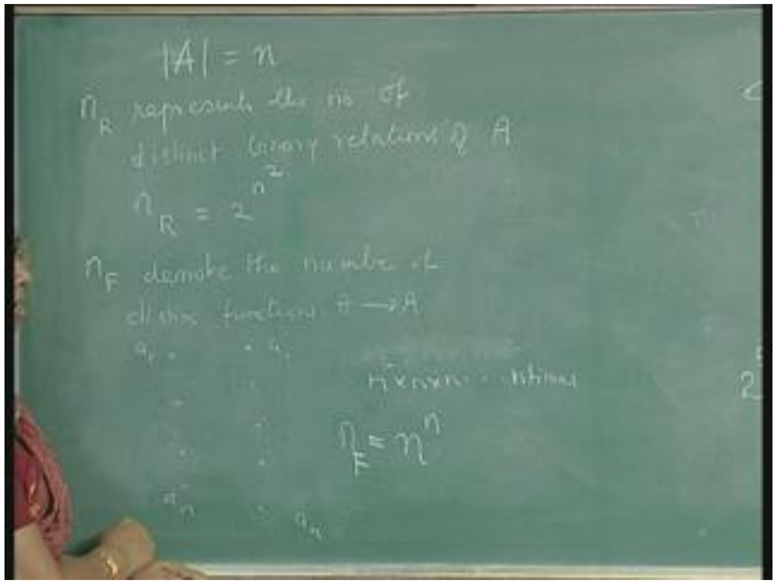

Now which do you think is greater?
What is the connection between n_R and n_F which is greater?
Of course, while defining the function we have seen that function is a restricted version of a relation. So intuitively you must realize that n_R should be more than n_F. So a relation, you have n elements then in a relation there will be n squared pairs in that. But in a function if you represent as a relation there will be only n elements and n arcs there. But those n arcs can be different that is why you are having n power n. Anyway a function is a restricted version of a relation so n_R should be greater than n_F.

You can see this with some elements, take n is equal to 2 if you take n is equal to 2 say the set having a_1 a_2 how many ordered pairs it can have? It can have four ordered pairs and so how many distinct relations you can have? You can have 2 power 4 distinct relations, that is in this n R will be is equal to 2 power 4 is equal to 16. Now, how many different functions can you have from n? a_1 can be mapped onto a_1 a_2 the different functions you can have is like this a_1 a_2. One is a_1 can be mapped onto a_2 and a_2 can be also mapped onto a1 that is one possibility.

Then another possibility is both of them can be mapped onto this, this is second possibility. Third possibility is a_1 mapped onto a_1 and a_2 mapped onto a_2. Fourth possibility will be a_2 mapped onto a_1. Or in other words $f(a_1)$ you have two possibilities a_1 it can be a_1 or a_2 $f(a_2)$ you can have again a_1 or a_2 two possibilities and two possibilities so totally there are four possibilities so n_F is equal to 4. With increasing in the difference would be more and more. So n_R will always be greater than n_F.

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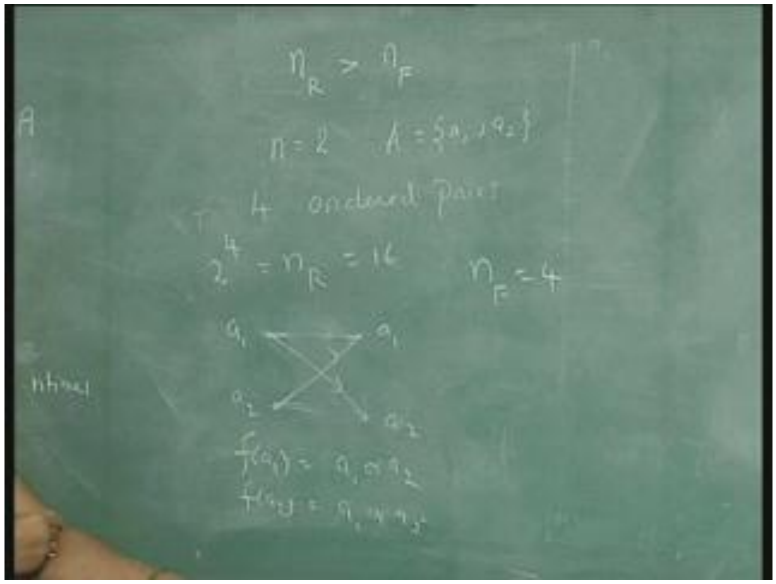

So like this we can consider some problems. You can also see that an equivalence relation induces a partition and a partition induces an equivalence relation. Let us consider some problems about partitions.

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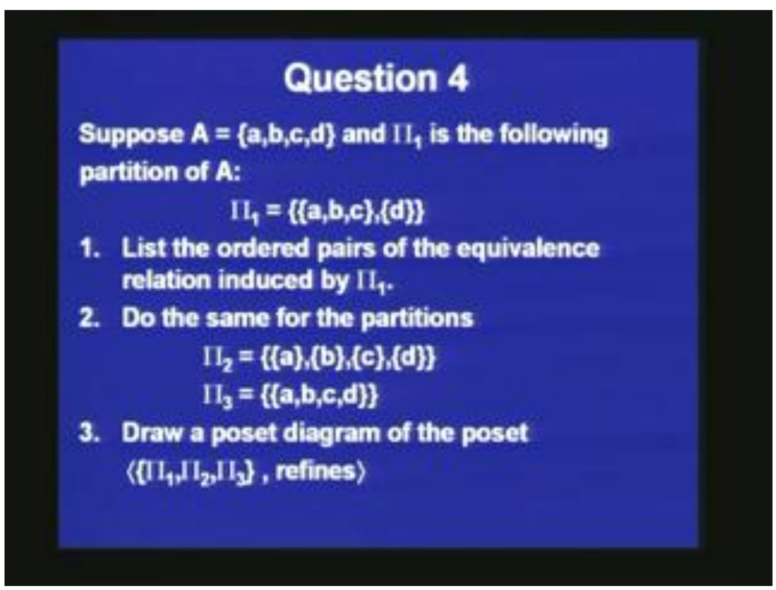

Take this problem; suppose A is equal to a, b, c, d and pi_1 is the following partition of A. pi_1 consist of two blocks a, b, c and another block containing d. List the ordered pairs of the equivalence relation induced by pi_1. Let us consider the ordered pairs; pi_1 consists of two blocks a, b, c in one block and d in one block. That means the equivalence relation induced by pi_1 a, b, c are equivalent they belong one class that is the equivalence class d belongs to one class. So the graph will be like this a b c which is a complete diagraph and d. So what are the ordered pairs? The ordered pairs will be a a, b b, c c, a b, b a, a c, c a, b c, c b and d d. These are the ordered pairs which belong to the equivalence relation induced by pi_1.

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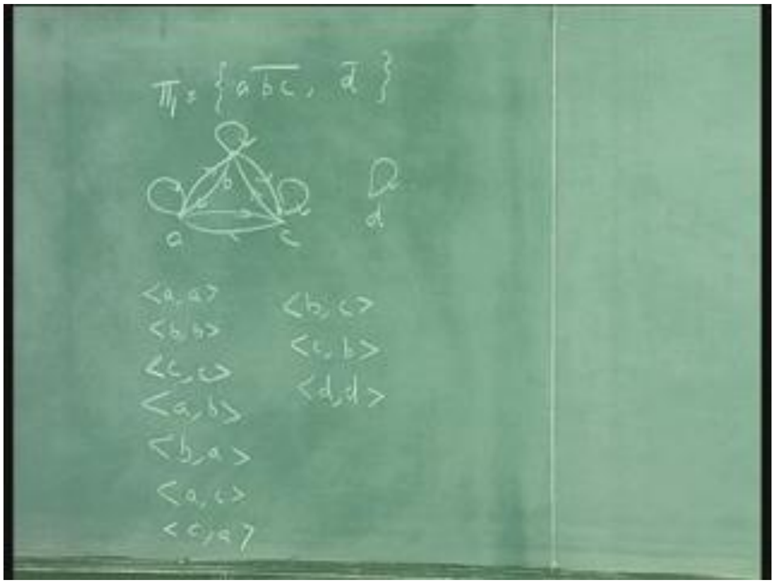

And consider some other partitions pi_2 has four blocks a is in one block, b is in one block, c is in one block and d is in one block. Pi_3 has only one block all four elements are there. So, similar to this one you can write down the ordered pairs for these. This will have only four ordered pairs a a, b b, c c, d d and here the graph will have a complete diagraph on four vertices.

Draw the poset diagram of the poset pi_1 pi_2 pi_3 refines. Now pi_1 is this, abc, d and pi_2 is a in one block, b in one block, c in one block and d in one block. Pi_3 is all of them in one block. So which one refines this? You can very easily see that this refines this and this refines this as well and this also refines this. So the poset diagram is like this; pi_2 refines pi_1 and pi_1 refines pi_3 pi_2 also refines pi_3. So the poset diagram will be just like this. Let us take one more problem which introduces a new concept called the substitution property.

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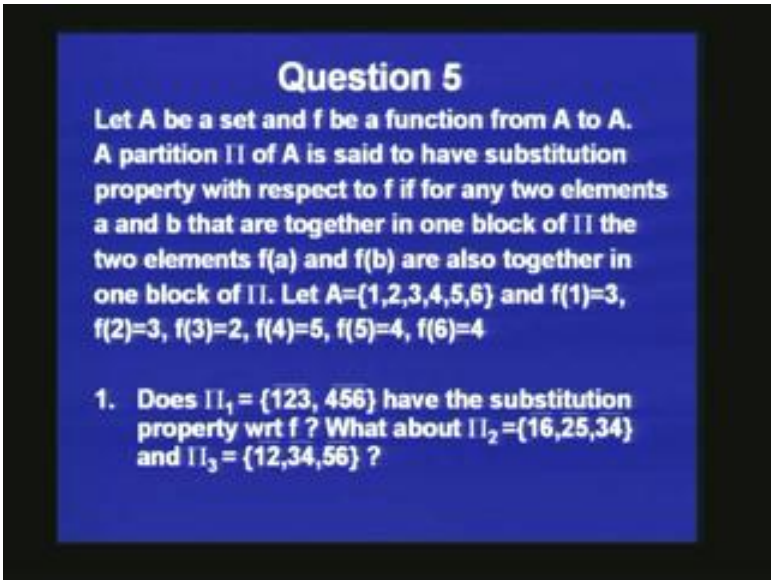

Let A be a set and f be a function from A to A. A partition pi(A) is said to have the substitution property with respect to f if every two elements if for any two elements a and b such that they are in one block the two elements f(a) and f(b) are also together in one block of pi.

And an example is given:
Let A is equal to 1, 2, 3, 4, 5, 6 there are six elements in A and the function is defined like this f(1) is 3, f(2) is 3, f(3) is 2, f(4) is 5, f(5) is 4 and f(6) is 4. Does pi_1 which has got two blocks 123, 456 have the substitution property with respect to f? What about pi_2 what about pi_3?

A substitution property means two elements are in one block they are a and b, a is mapped on to f(a) and (b) is mapped onto f(b) and f(a) and f(b) should be in the same block with respect to that partition. In that case you say that that partition has the substitution property.

In this example A has six elements and you have f(1), I will write down like this; f(1) is 3, f(1) is 3, f(2) f(3) f(4) f(5) f(6).

What is f(2)?
F(2) is 3, f(3) is 2, f(4) is 5 and this is 4, this is 4. Now the partition is like this; pi_1 is 1, 2, 3, 4, 5, 6 where 123 are in one block 456 are in one block. Look at this; 123 are mapped onto 3 and 3 but 3 and 2 are in the same block 456 are mapped onto 4 and 5 and 5 and 4 are in the same block. So, when elements from the block are mapped onto elements they are mapped onto the same elements in one block. Here similarly 456 are mapped onto elements from the same block so pi_1 has substitution property with respect to f.

Now what about pi$_2$ what is pi$_2$? pi$_2$ is given to be 16, 25, that is 1 and 6 are in one block 2 and 5 are in one block, 3 and 4 are in another block. Now 1 and 6 are mapped onto 3 and 4 and 3 and 4 are in the same block 2 and 5 are mapped onto 3 and 4. Again 3 and 4 are in the same block 3 and 4 are mapped onto 2 and 5 and 2 and 5 are in the same block so again this also has the substitution property.

Now take pi$_3$, what is pi$_3$?

Pi$_3$ is 1, 2, 3, 4, 5, 6 that is 1 and 2 are in one block, 3 and 4 are in one block, 5 and 6 are in one block, 1 and 2 are mapped onto 3 no problem, 3 and 4 are in one block they are mapped onto 2 and 5 but 2 is in one block 5 is in another block so they are not in the same block so this does not have substitution property with respect to f. Pi$_1$ and pi$_2$ have substitution property with respect to f. Pi$_3$ does not have substitution property with respect to f.

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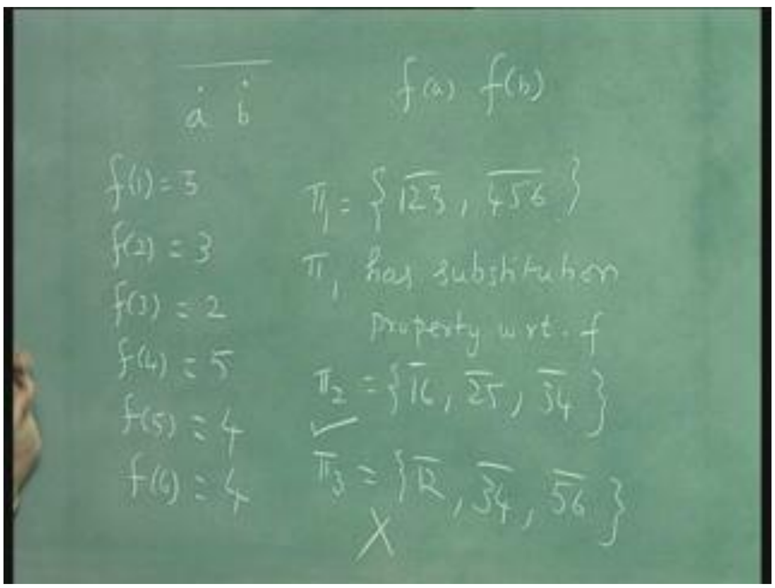

Let us continue with the problem.

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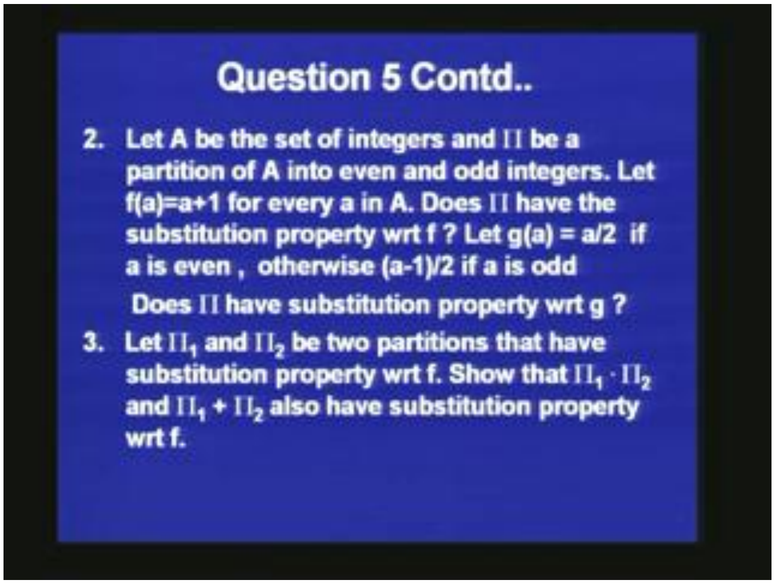

Let A be a set of integers and pi be a partition of A into even and odd integers. Let f(a) is equal to a plus 1 for every a in A. Does pi have the substitution property with respect to f? And let g(a) be a by 2 if a is even otherwise a minus 1 by 2 if a is odd. Does pi have substitution property with respect to g?

So let us consider this. We are considering the set of integers and splitting it into even and odd integers. One block consists of even integers and another block consists of odd integers. And the function f is defined as f(a) is equal to a plus 1. That is, any even integer will be mapped onto an odd integer and any odd integer will be mapped onto to an even integer. So all even integers are mapped onto odd integers and all odd integers are mapped onto even integers. So whenever you take two even integers they will be mapped onto odd integers and whenever you take two odd integers they will be mapped onto even integers. That is one block is mapped onto another block, this block is mapped onto this, this block is mapped onto this. So this particular partition this partition if you call it as pi then pi has substitution property with respect to f where f is defined this way:

There is another function g which is defined like this; g(a) is a by 2 if a is even and g(a) is a minus 1 by 2 if a is odd. Does pi have substitution property with respect to g?

Suppose I take a to be 6 then g(6) will be 6 by 2 that is 3 and if I take a to be 8 g(8) will be 4. So 6 and 8 are even integers they are in the same block here but one is mapped onto 3 and another is mapped onto 4 they are not mapped onto elements from the same block. Here it is mapped onto an odd integer and here it is mapped onto an even integer. So they are not mapped onto elements in the same block. So pi does not have substitution property.

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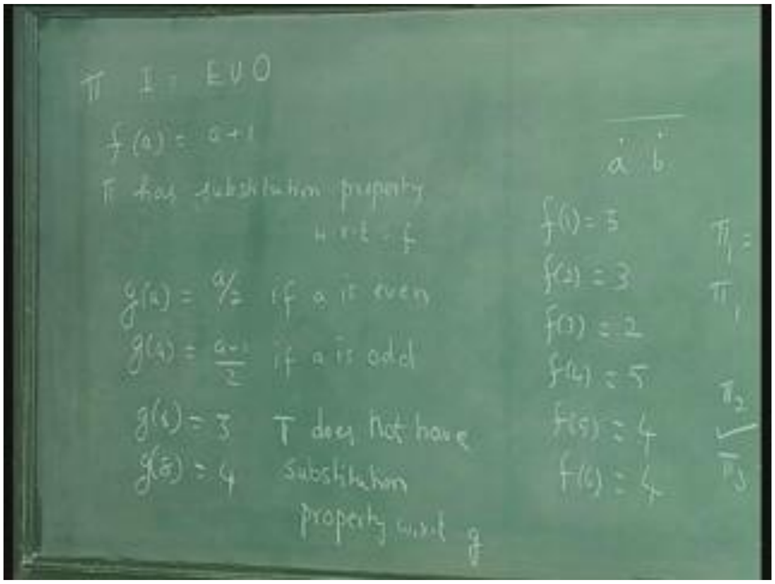

In a similar manner you can also show that, the third problem that is we have already seen what is meant by the product of partitions and the sum of partitions. So if pi$_1$ and pi$_2$ are two partitions that have substitution property with respect to f. There are two partitions which have substitution property with respect to f. Then if you take the product of pi$_1$ and pi$_2$ then that will also have substitution property with respect to f. If you take the sum of pi$_1$ and pi$_2$ it will also have the substitution property with respect to f. This is very easy to prove, the reason is if you take two elements which belong to the block in pi$_1$ and which belong to the block in pi$_2$ then those two elements will belong to a block in the product partition and also in sum partition.

Similarly in pi$_1$ because of the substitution property f(a) and fb) will be in one block in pi$_2$ also f(a) and f(2) will be in one block. And in pi$_1$. pi$_2$ f(a) and fb) will be in one of the blocks which is obtained by the intersection of a block in pi$_1$ and pi$_2$. A similar reasoning can be given for the sum of partitions pi$_1$ plus pi$_2$. So these are some problems with respect to partitions and the whole concept of order relations, equivalence relations, partitions everything can be very clearly understood by working out more problems.

In the beginning of the course I have mentioned several books. There are several exercises given at the end of each chapter in those books. By trying out many problems the concept of all partial order, equivalence relation, partitions will become very clear to you.