Performance Evaluation of Computer Systems

Prof. Krishna Moorthy Sivalingam
Department of Computer Science and Engineering
Indian Institute of Technology, Madras

Lecture No. # 09
Stochastic Process

So, now we come to the first part of the one of the main tools of performance evaluation and that is our Markov processes, chain, Markov chains we have heard those terms. So, what is the Markov process from what you know, what is the Markov from your definition, what you remember, and what is it used for? Nobody wants to make the guess?

System can number of states and in time and the so, and given there it is in a state currently the thus the probability that it goes to the next state time (( )).

So, this is to capture the state of the system usually we want to we want to model system that is ultimately performance modeling is what we trying to do. So, we want to model, the behavior or operation of a given system. And we define certain values or parameters to capture the state of a system, these are usually state variables, parameter is not right, but state variables that capture the current state of a system. And then we want to represent that; we are all talk about state transition diagrams; a state transition diagram we know about we use that represent behavior, but that is usually only to represent functional like a flow chart. So, what happens if our event so, if there you are in state 1, and depending on whichever event occurs you go to some other state and so on.

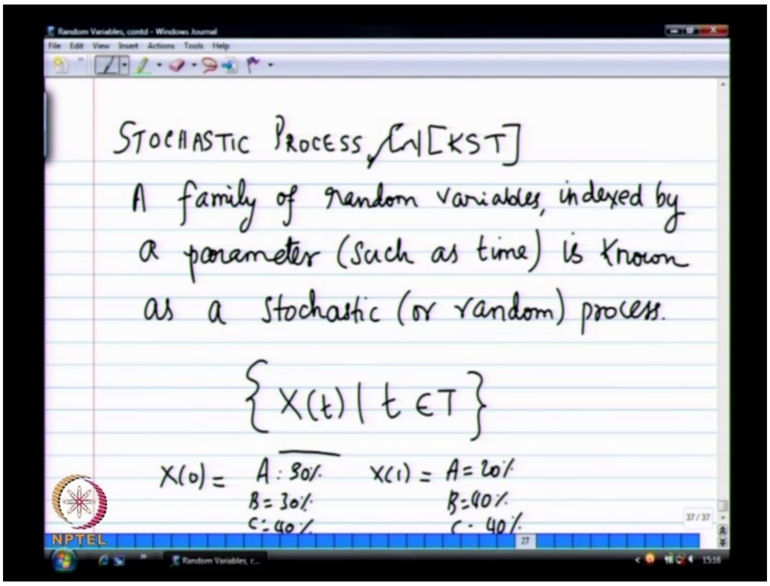
So, that is simply to depict the operation, the behavior of a system itself, but Markov chains are very similar or Markov process general are similar in general stochastic processor, try to capture the state of a system as or in with some variables and it models behavior through state transition. So, you are right now in state X then what is a probability will go to state Y in sometime, how long will you stay in the state X, when will you go to state Y, and what is the probability of going to some other neighboring state, and whether you can go from only one state to neighboring state or one state to some other state and so and so forth. All those things come under this class of stochastic process.

So, a Markov processes are useful many times recommend handy, and some systems are amenable to, some systems are not amenable. We will look at some systems were we can actually use it very effectively. So, the first example we look at is in the cross, in the context of queuing systems. So, queuing systems we can very easily derives several close from solutions, with the help of this Markov chains and Markov process and so on. So, before I get into that, I just want to give some background definition for many of these terms, even I was little bit loosely using processes, variables.

So, now let us try to say. So, random variable is what we have seen now. So, random processes, stochastic processes is where I have repetitions, where each of those let us say in each of those repetitions will have this one some kind of distribution. So, if I say same thing Bernoulli. So, Bernoulli trial I just flip a coin that is one trial, then I keep repeating the trial, then the 10 suppose a Bernoulli process, and then the probability of getting a heads or tails and each of those trials within this process could stay stationary or could be not stationary, because we keep tossing a coin, it is possible that the coin could become unfair or whatever reason it keeps falling on ground therefore, dead sticks some one sticks the coin, and therefore, ends of falling on that particular point. Therefore, it need not be stationary.

So, even if a take a coin in keep tossing it, over time the probability of heads and tails might change. It is a loaded dice or just. So, that is where we look at the random variable which is you know experiment conducted across time. So, the series of random variables over some parameter which we normal is time will be also called stochastic process, and within that some very specific cases are there, which is what we can easily use for analysis. So, that is just go through some of these definitions initially. Others I can simply, here is a Markov process, but let us just know what how we get the Markov process through a series of these special cases.

(Refer Slide Time: 04:14)



So, we are going to look at stochastic processes, and this again is not in detail in the Rajain book. Rajain straight away goes into Markov chain, but this I am using from our KST, it is a usual thrived book. So, this book have his here is the older 82 edition. So, stochastic process so, some little bit of definitions again will be there will be wordier definitions, because are not everything is in equation. So, lots of words to write which is easier do on the board, but anyway let us try to do here. So, the stochastic process is basically a family of random variables, indexed by some parameter.

So, from variable we progress to process. So, series of this variables indexed by sometime. So, for example, student IIT student or elsewhere who never prepares for any exam, and simply flips or prepares that flips in a random basis it flips through some pages, hopes and those questions will appear and then he writes all his exams right way. And you get the GPA every semester. So, GPA is a random variable, at that point is there is random preparation and then we look at all the eighth semester GPA that becomes a stochastic process, because your index is now, but that parameter is your semester.

So, every semester you have a different value which is based may be the same distribution is used, if you randomly flip may be end up with the same you know distribution of great, but somehow that is let us not worry about that. So, that is your example of a random process. The series of GPS where the each GPA, each grade is the random variable that is what that is an example of all stochastic process would be. So, we normally use this notation X of t. X is the random variable, and t is my and t need not be time, t can be any discrete set, but normally we look at what happens over time. (No audio from 07:21 to 07:39)

(( ))

As you have six stochastic processes.

So, the six stochastic total six random variable will make it once stochastic process.

But see I am to define in each for each instant of time I have to capture the state. So, if you are…

(( ))

So, you are taking…

So, only previous semester I am taking only even semesters.

So, you are indexing by the students.

Yes.

You have six students and or may be. So then here there is a see what happens is usually you need with time some changes. Here, there is no notion of time, your discrete parameter is…

(( ))

With the stochastic process, we need to move from one state to another. So, in this case how do we discuss, how do we describe the moving from one student another student is technically unrelated. So, the GPA of the student are unrelated or set of completely or different random variable. So, I would not call that is a given of the student. So, I would rather look at this as a one student over time.

We take one-fourth structures, and they after some do it. So, can we say that the variable then this process and these variables will be moving like one four is this.

That is the moving average of fourth semesters the entire moving average that is also that is also right, then if you look at only every semester completing, but after the fifth semester after the fourth semester then that is also that is also fine, but what happens is you are there is dependences, because now you are in this case these all semester grades are independent would neither we some dependency that is fine then that is why we come and define our Markov where there is no dependence on all the previous state only. For example, in my grade in my fifth semester only depends upon my performance in the fourth semester does not depend upon my performance in other semester within you from prediction.

Viewing other around if I want to predict whether I want take the students as a project associate or not or as a M. Tech or B. Tech student then I usually use the grade as they predict. So, if we look at the four, five semester and then say this sixth semester grade will most likely depend upon the previous five, but that is we are entering at a different space of prediction and so on. That it is something let (( )). (( )) the which is also random the same they may not be same. So, how they dependent on each other.

With time is where I am trying to. So, I am looking at these as the time progression as a series of you know time based event so, I am looking at one student here.

(( ))

So, here there is here there is that is why I am saying that each semester the student is the preparing totally randomly. So, therefore, there is no memory. I can also define a system where there is dependence as you progress from one state to another. So, initially you could be a 7.5 GPA student then, you learn from mistakes and then it came 8.1 you still learn better. So, you could progress that way also that is also a stochastic process that is fine. I am just giving that as a case where there is complete independent, but you cannot dependence between the states two. So, to go from 7.0, let say that I am looking at only physics discrete states a through f these are the only possible states there is no f here.

So, but fine a through f, in each semester give you are you get classified either a or f and so on. So, you start of in a first semester as a say BE student that with the normally issue, we have our 7.0 some so, you are a 7.0 student, then the next semester we look at your GPA and then your GPA will be of course, including a old grades also, but may be whatever you learn from a previous semester you become some other class student. So, there is some sort of time progression there.

And where Markov comes in will see later on this where the state that you are in currently so, after fourth semester you are in a particular state that state probability distribution for the for that state whether you are a whether probability A is 10 percent, B is 20 percent. So, on that depends upon either your previous state or none of the states at all with the completely independent it can be two. The each of the states can be totally independent of each other or there is some dependents on your plastics all of us we will come under your general classification of a stochastic process.

(( ))

So, this X of t so, this X of t that I am showing here, this is a distribution. Your distribution can be stationary with respect to time or it can vary with respect to time. So, for example, at you know let say you just use this example you are only three class. So, you probability of being in of getting in A is that 30 percent, probability of getting a B is 30 percent, C is 40 percent. This is some distribution initially when I start of this system. Then, after may be time instant 1, something has happen whatever things have gone then, let say you are this could be the same as that or it could be changed also, this is also fine.

So, this is so, for every event of time, every instant of time we have distribution for this random variable X, that can be stationary, time stationary or time variable.

(( )) same random variable sir different random variable.

So, now…

Same random variable different time instance.

Different time instances the distribution for this random variable see I am collecting these excess together as wants together as a process.

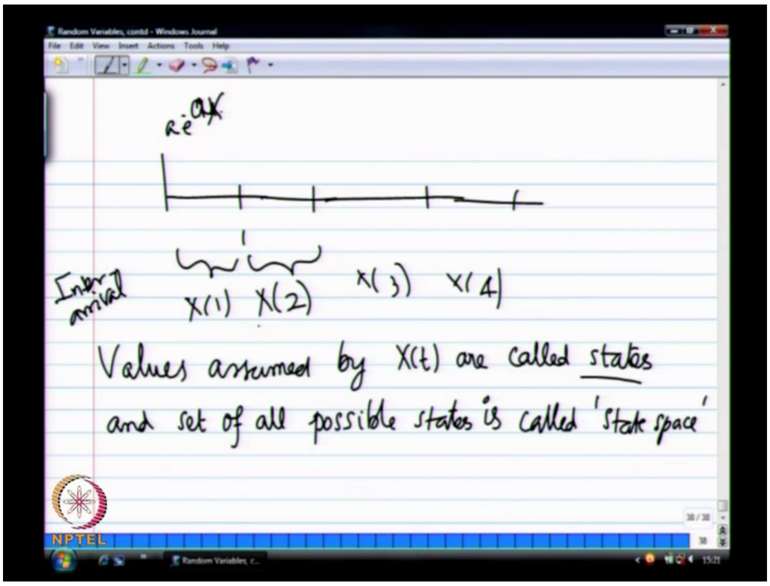
(( )) family of random variable.

Yes. So, my family is X 0, X 1, X 2 and so on.

(( )) one random variable.

So, every instant of time there is one random variable. So, that is this collection of random variables is much too stochastic process. Let us look at this non GPA example. So, let us look at the packet arrival process. So, when the arrival process what a measure is interval arrival time what is the time between two successive arrivals? That is a random variables. So, at the beginning of a system I am saying that, packets will arrive by exponential process with some distribution, and if it is a completely stationary and I say that the interval arrival time is exponential distribution with means say one packet per second or may be mean of 1 second that is my definition of that exponential variable.

(Refer Slide Time: 14: 33)



So, between every successive, see this is the first arrival this is the random variable that, I would say call now X of 1 this has some distribution this is some distribution I am saying that this is ae power minus lambda some parameter a is there. Then I look at the next time to the next arrival what is that? That is also random variable it is also it could be following the same distribution or it could depend upon the fact that the packet is arrival. The this is another random variable and so on.

So, the different inter arrival times the sequence of inter arrival times forms your forms as family of random variables. So, if this is the Poisson process, then the inter arrival times are exponential, and then the parameter does not change, your lambda does not change with respect to here your indexing is based on the packet number. The index is not the time, index is not simply the packet number 1, 2, 3, 4 and so on. So, discrete state not discrete parameter which is indexing. So, it is not necessarily time.

But in this case it is that, I am looking at what is the time between successive arrivals? So, this forms your stochastic process, and are again other example this is arrivals, this is inter arrival time or if you look at a machine, then other life time the machine will be will have falls and get fixed and it runs again and gets fall gets. So, the mean time the failure that is also a random variable. So, the classic case it we know is that between different arrivals that this parameter a it is not change that it is a looks very bad. If the parameter a does not change between the different variables, then that is we will look at it formally as the time stationary process.

(( ))

Yes. So, I am just looking at for a queue packets arrival from one source to multiple sources.

Which you want?

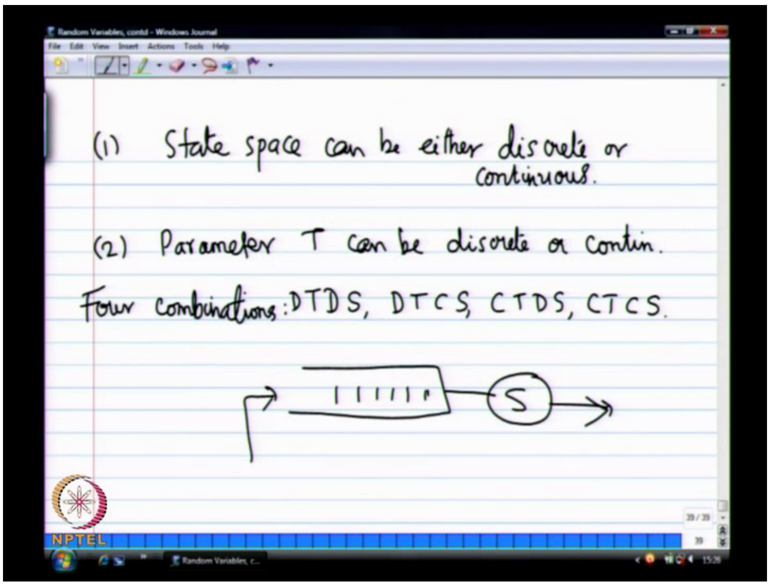
Any other questions, we have been answering? So, you will have to stop here. So, the values assumed by this X of t we will call them states from now on.

Called the stack space…

(No audio from 17:45 to 18:07)

So, there are two things one is, the set of states there are possible, and the other is, a set of parameter values get parameter t, the t is a set that talked about parameter set. So, based on that will have some three or four classifications.

(Refer Slide Time: 18:23)



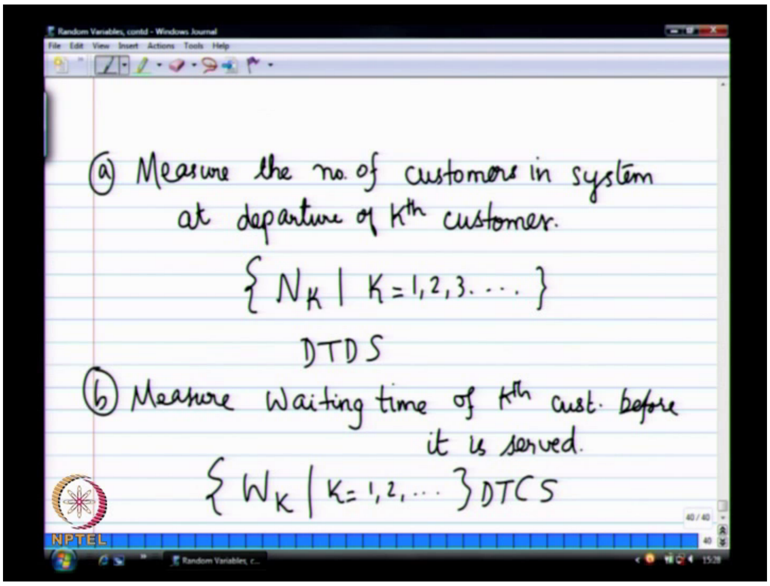
So, the state space can be discrete or continuous. State space for example, if you measuring the number of customers in a system, we can that you count every 10 seconds are. So, that becomes a that is a finite discrete number whereas we are measuring the service requirement that a given point time we look at all the customers waiting in the system in you take the sum of the service times of all the customer that is also state space. That is a thing that represent the state for all other we want to measure that. That is a continuous variable. So, the state space can be either discrete or continuous, and likewise your parameter T, this is the parameter can be discrete or continuous.

So, discrete time would be you just looking at specific instance of time integral values of time, 1, 2, 3, 4 or only looking at arrivals if you only look at the system when the first arrival comes, second arrival comes, third arrival comes and so on. Then, you have a discrete parameter space, but if you are looking at T as general time and we do not care at any point in time I want to measure that average number of customers in the system or the average service time, total weight service time for all the customers and the system then and the come then your parameter space becomes continuous. So, if a parameter space can be discover, this gives rise to four different systems.

We have discrete parameter, continuous state discrete parameter, discrete state and so on. So, we have four combinations, and we will take a look at what those combinations. So, you have discrete state no it is a so, you have discrete usually the whenever its parameter by simply usually referred to us time discrete time discrete state, discrete time continuous state, continuous time discrete state, continuous time continuous state four possible system is there. This is important, because you have when we look at a system each of these as different properties. (No audio from 21:11 to 21:26) So, now we need to come up with some examples of each of these. So, that you understand what each of these you are talking about. So, let us look at a queue so, may not beginning to start thinking about queue. So, this is my representation of a queue. There are there is a buffer, there is a server, customers enter, and customers leave the system.

(No audio from 21:55 to 22:19)

(Refer Slide Time: 22:27)



So, in this system if I define a variable where we measure the number of customers in the system or number of packets in that buffer whatever it is. So, is a mice variable is basically, N K will call at K is 1, 2, 3.

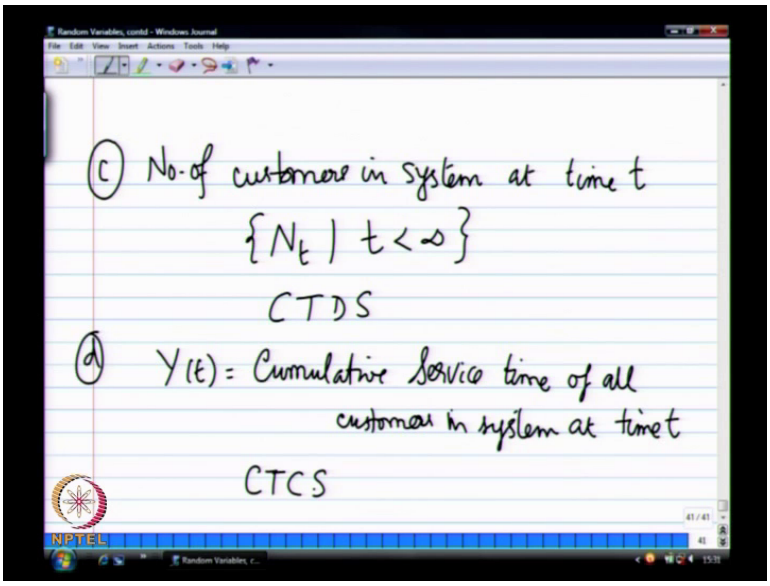
So, what is this, what type of variable or stochastic process (( ))? So, this is your DTDS, then, you measure the waiting time at the K-th customer. (No audio from 23:53 to 24:08) So, you look at every customer, and measure its waiting time before it gets services. So, the amount of time spent in the queue that is what this W K. So, K will be again 1 to infinity. So, this is discrete time continuous state.

As your state is now any where or any non integer value. (No audio from 24:44 to 24:54) So, can you think of a continuous parameter discrete state, because waiting time is it is not an integral, it is not an integer. Waiting time in the queue is the depends on the service. Service times can take your exponential distribution. So, then need not be integer.

(( ))

The number of customers in the queue at any point in time that is, your example of a continuous time discrete state.

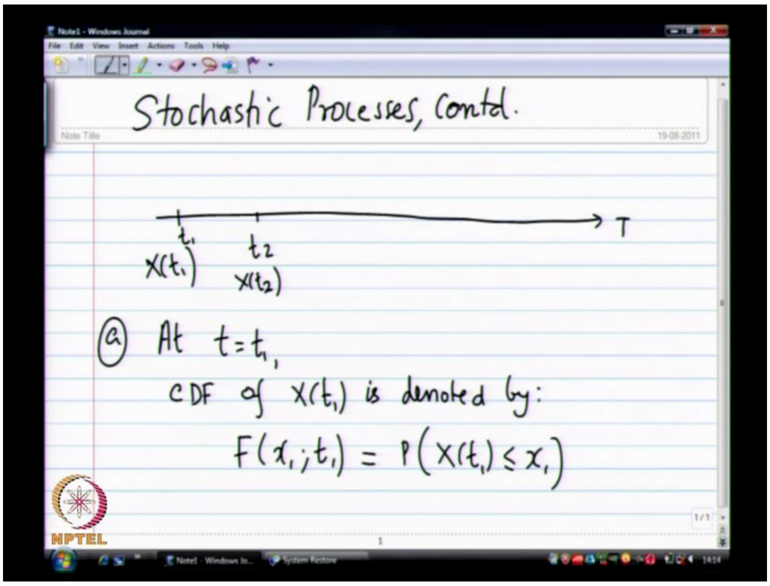
(Refer Slide Time: 25:34)



So, we will say represent this by say N t, we look at in queuing theory in our networks class as I said. So, we actually compute an e of n we only look at the expected value, but you can also look at the distribution. So, t would be, t is possible any value of t that it to infinity. So, that is will continuous. So, this is an example of continuous time discrete state. Then, for continuous time continuous state total waiting time, total service time something the discrete. So, if I use say at Y of t is a cumulative…

(No audio from 26:54 to 27:23)

(Refer Slide Time: 27:27)



So, let us revisit our stochastic processes. So, from random variables when to a collection of random variables or a family of random variables that is what this one represents. So, from this I want to go to Markov, I might skip some definitions along the way. So, essentially what we have is, if we look at some parameter which we call as T so, there is a random variable so, whatever is the state of the system is capture by X of this is some time t 1. So, X of t 1 is the state of the system at time t 1, and that is described by some random variable X (t 1). So, and then time t 2 you have another X (t 2). So, these are essentially potentially different random variable. So, they may be dependent, they may be independent but that is all different classifications their independent then you forms renewal process things like that.

So, all we have is now a series of this random variables that this capturing, the state of the system, and the evolution of the state of the system with respect to some discrete parameter some parameter T could be discrete or continuous.

(( ))

Yes, you have multiple variables also. So, mostly look at single in just as state as sequence of 0, 1, 2 and so on so, but that is suppose to some of encode what are the different values such you have will later on we will see if we good in Markov chain, when we look at sequence of queues in random.

(( ))

So, we saw this queue would be other day, we saw set of examples, set of variable. So, for example, the number of customers in the system at the K-th departure, that is the state of the system. So, N K you look at the state of the system at those discrete instances, when customers are leaving the system. When a given there is only one server customer is finishing service in his leaving the system, but at that time we look back and see how many people are still in the system, that is your N K. So, that is an example of the state of the system, and then what we would use that use that as model to find out how many average customers in there in the system.

And then from that, we derive what it delay last time, delay throughput things like that or it could be simply at every time instant simply that is only when a customer K is leaving or at all times t so, as a continuous time on a continuous time scale look at the number of customers in the system. So, it is like almost monitoring every nanosecond. So, and look at how many customers are there how many customers are there, and then use that represent system or look at every 1 second then, say at every 1 second interval you measure the state of the system, where the state of the system in this case as the number of customers in the system including those an service. So, whatever be defined as state you will look at the transition of the state, over time with respect to this time or some other metric that is can that is used characterize.

So, just a couple of definition then I get straight into Markov some of these things are enough, too much distraction, too many distractions. So, at so, time t equals t 1. So, we can look at the CDF of this random variable you can represent that by so, just want to get to get to the fact that, the system state is captured over a series of time instance t 1, t 2 and so on. So, this is a simple that is x 1…

That is interval which we are taking whether these are timing or something (( )) continuous.

No, it need not.

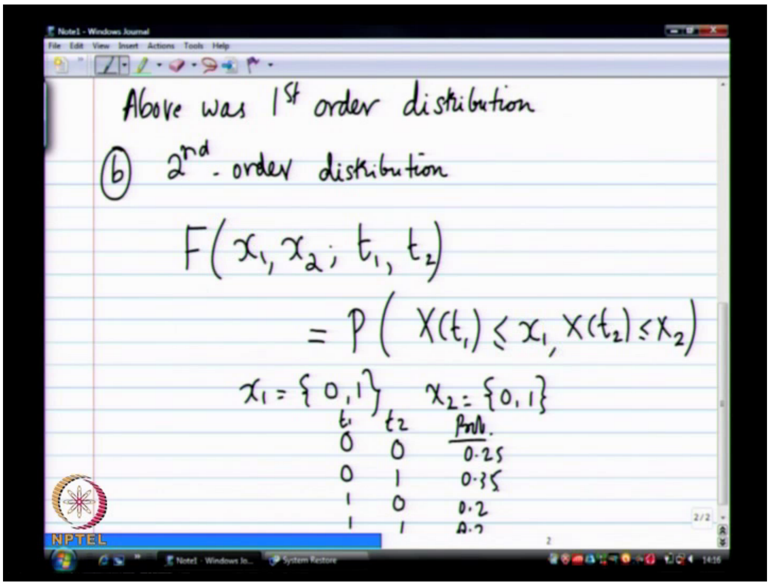
You know.

No, it need not that is why we looked at continuous time, discrete time and so on. So, for example, in the case of customer leaving a system there those are specific time instance. I am only interested in the departure. So, K is for example, first customer K goes from the index that parameter set T is discrete K 1, 2, 3, and so on. But there I am only looking at when the

K-th customer is leaving the system. So, we do not care what time it is, we only caring about what is the K, and for K is simply an integer that is indexing the different customers dealing or if you look at time then, it is even they are.

Other example you look continuous time looking at the average waiting service or the number of customers in the system at every instant of time, and that is there your time is continuous. So, that is why we have this combination of discrete time, continuous time. So, this is our standard this is our default definition at t 1 what is that CDF of X of t 1? This is of called the first order.

(Refer Slide Time: 32:29)



So, now let us say that the system is progressed to some time t 2. So, the above was 1-st order… So, to describe a stochastic process I have to what happens in the next time instant, and third time instant and so on. So, it is get to be fairly complex if I want to exactly capture the behavior of the system. So, 2-nd order distribution is given by so, x 1, x 2 variables t 1, t 2 so, this is the CDF of the of this x 1, x 2 combine together. So, that is… (No audio from 33:15 to 33:26) So, what is the CDF or what is the probability that, the system will have less than or equal to the state of the system will be less than or equal to x 1 at time t 1, and x 2 at time t 2 that is the 2-nd order distribution of the system itself.

So, ideally we should have some description of this which is not possible in all cases, but this is what is describing the system behavior as a time evolves. So, I need. So, the system is defined by this distribution the CDF function that the. So, if it is a discrete variable will have a set of PMS that says, what is let say there are know two possible.

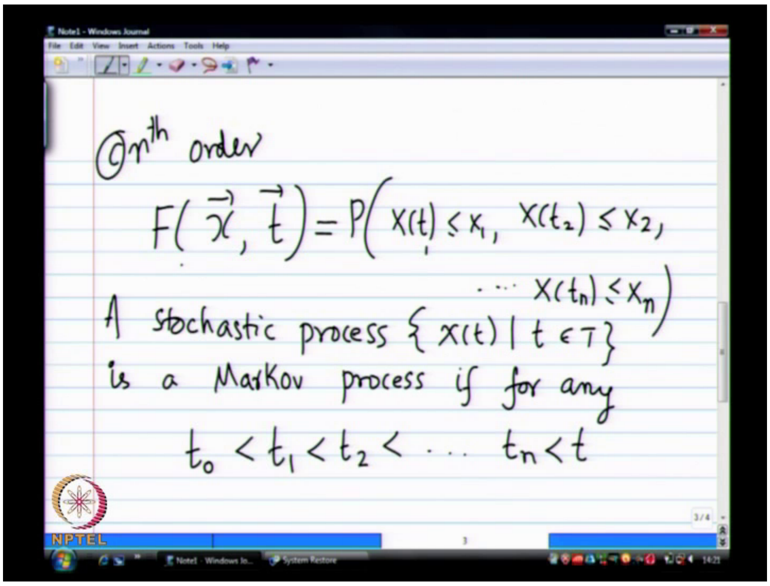
So, if x 1 is 0 and 1, and x 2 is also 0 and 1 then I have a joint PMF is describes what is the probability that x 1 is at time t is t 1. So, if we look at t 1, t 2 at time t 1 what is the probability in the system of the state 0 and at time t 2 what is the probability system was a time in state 0 or (0,1), (1,0), (1,1). So, this would be my cumulative, the CDF (( )) mass function and I will say the probability. System being in this could be whatever they think of. So, 0.25 and 0.35 that integrate of 0.2.

So, this is the characterization of the system behavior now, how do we get it this? This depends upon understanding how what is the probability of going to one state in other state for example, in channel being transmitted initially may be in a good state, so, let say start of with being a channel in good state then, you measure the channel in some other time in the future, and you find the channel is entered noisy mistake and therefore, the next state is noisy. So, 0 is good state, 1 is noisy state. So, what is the probability of being in a good state before and good state now? And, likewise so, this is determine by the thus the channel characteristics has to what is the develop noise in the system, what is the kind of modulation scheme and so on. So, all those piece will enable us to calculate the certificate.

(( ))

Which one? This is and is not might this is simply and conditional where seen this is simply describing the behavior of the system where what is the probability for each of these combinations happen that is all.

(Refer Slide Time: 35:52)

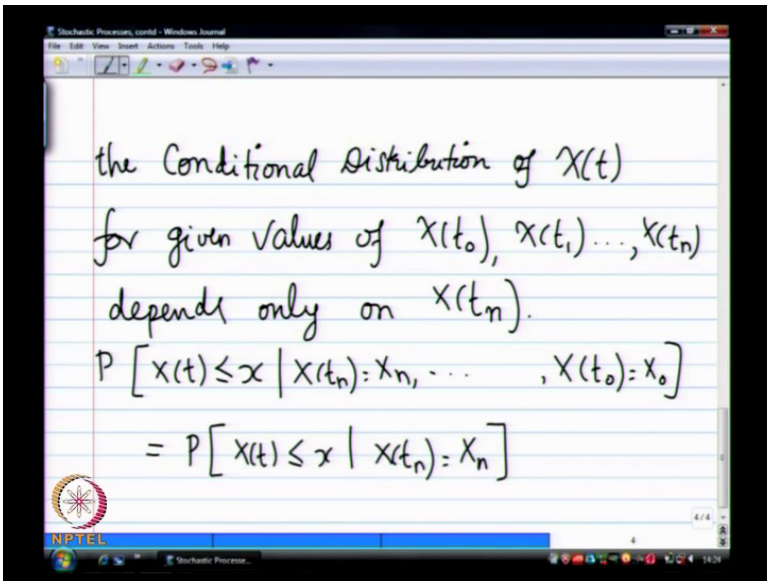


So, then I can extend this further to the so called n-th order, where I will use this vector x to denote all the possible x values, and t to denote my time values. So, this is this massive what is the probability that x. So, t 1 is less than… (No audio from 36:13 to 36:31) So, if I can come up with the distribution like this either you know analytically or simply measured that is what describe the state of the system, with this will have to do some other manipulations if know this I can do this some other operation on the system to get to some.

So, this is what is called n-th order distribution of the system as it is evolving with respect to this discrete time instance. So, what will find is that as a system evolves are long term this my converge to some stationary probability distribution, some stationary distribution where regardless of whether it is 1000-th interval or 1000 of first interval the probability of all these CDF of this individual of this combine set of variables is going to be station does not change respect to time initially it may be changing, but over long period of time it may simply be stationary. So, that I am not define specifically, but is there in the book just assume that long run in this distribution become stable.

Where will be use? This will be useful for computing your stationary probabilities will come to that later on. Right now just assume that, I can describe this way what are the distribution is? Let see we leave it we leave it at there for. So, now we come to your question about what is conditional about this. So, I write this and explain that. So, stochastic process… So, we have a stochastic processes t like before it is defined as a Markov process, again this Markov is memory less. So, we looked at the geometric the memory less, and exponential the memory less. So, that in general memory less property is called a Markovian property. So, this process is called a Markov process if it has this so, called memory less property, it is not completely memory less, but. So, I have a set of t values of n time instance.

(Refer Slide Time: 39:15)



Then the conditional probability that the conditional distribution of X of t.

(No audio from 39:32 to 39:55)

Given the values of, so given that at a time t 0, X took this value time t 1 X took some value and so on. So, even though I have this past history of I have looked at the n plus one time instance t 0, t 1, t 2 and so on up to t n, now looking at the system state at time t the system state at time t, only depends upon the system state at time n, t n then that is considered as Markov process. So, that is right now definition and later on will see how they actually can be lead into exponential time for Markov holding time is exponential and so on, depends only on X of t n. In other words, the probability if we want to formally state this. So, t is the current instant the time looking at. So, what is the distribution for X of t given that (No audio from 41:08 to 41:23) I am sorry for. So, this is the formal property for stochastic process to be called a Markov process and mostly we look at this requirement.

So, this need not be the true all the time. So, let us say that, you are trying to you are ask to measure so, called mess quotient you are satisfaction with the mess on a daily basis. Every day you are ask to your given scale of 0 through 10 and 10 is very high and 0 is very low, and each day you are told each day get X of t as on it daily basis and X is basically your report of satisfaction with mess.

Now, in a real system what will happen on a daily basis depending upon a if you to you ask you today, the food may be good today, but you may might also thing that, last 3 look at the last few days of history there is some short term memory that always share like that the food was bad on 3, 4 days ago there. Therefore, the act will always the modulate the way that you express your mess quotient. So, in that case you are trying to hold on the history whereas if you completely well this still there is some memory only depending on the current state of the system will detect the will dictate the feature state of the system.

So, if I have to where to have a system there you are on mess quotient simply depends upon what you quotient was yesterday and then what are you having today there is some distribution is so much having today, then the rest of the events, the rest of the state of the system is completely this is not needed. So, what it means that your current system state has capture these is the summary of all the past states of the system, that is the particular property if it has then we can prove some nice property use this to essentially try to solve the sum the sum get equations and things like that. So, that is this is one simplification, but this is a if it have this then it is easy to do some of otherwise, you do not deal with this madly complex distribution that the p of know combine n-th distribution you get simplify you look at something like this.

So, this is essentially what we mean by a Markov process. Markov process is by definition memory less, this future state of the system depends only on the current state of the system that it is. All the other past history is simply (( )) in the current state. So, that is what we mean this; regardless of this is, what is the distribution of X of t, where t is greater than T; and even though all this were the previous states it simply depends upon the act that what is given that then last state was x 1 that only dictates my feature state of the system.