Performance Evaluation of Computer Systems

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Lecture No. # 07

Probability Distributions-II

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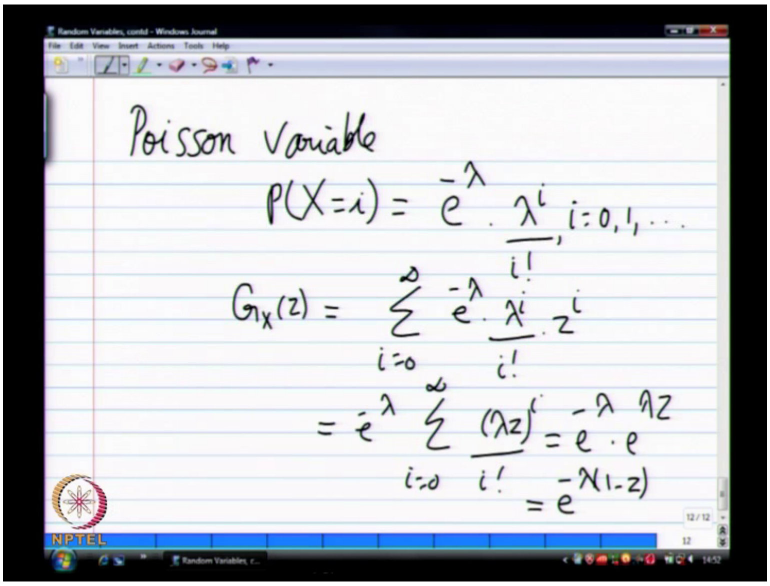

So, variable, so probability that this variable takes some values, so we will use X equals i its we will again use lambda here as the parameter. So this, so that is, e power minus lambda, that is our Poisson definition. So now, I need find the expectation of X, e of X harder way tutorial, easier way I do not know, will try this out the Z transform for this, (( )) if I have them.

Now, the generator for the Z transforms. What is the definition for that, summation 0 to infinity e power minus lambda? Lambda i divided by factorial and this is simply Z to the i that is all to the power ln, so simply Z i. So, we can extract e power minus lambda out and then what do I have lambda Z to the i divided by i factorial i equal 0 to. So, that is simple enough. So, what is this that of the term equal to e power lambda Z. So, therefore, that is e power minus lambda Z, which equals we can we can I did either i, but I am just writing it like this, I can make it lambda or into Z minus 1 or minus lambda into 1 minus i. So, I am writing it as e power minus lambda that for whatever is (( )) so, that is my generation. Questions?

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Say, so now, what is E of X? Do you want to derive easier from the lambda X is to we derive. So, it is d G divided by d Z. I am just dropping the X there. So, set at set Z equals to one.

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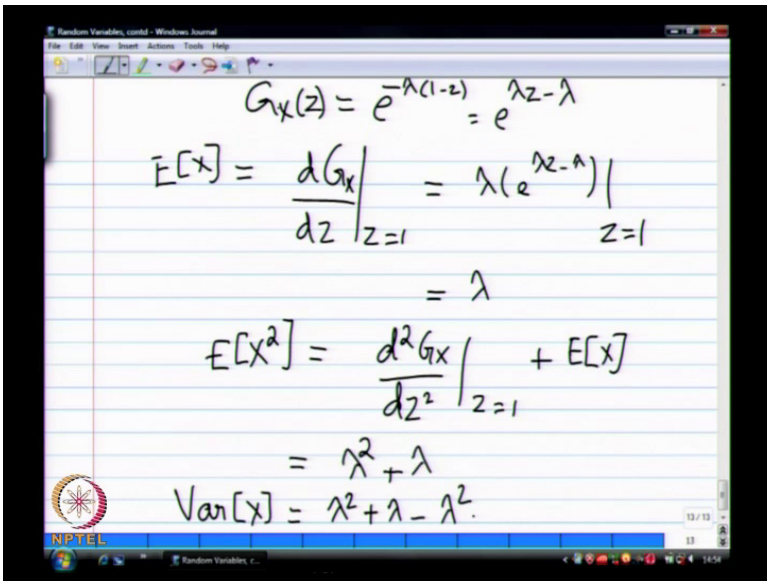

Again for reference, we will see minus lambda it is easier to differentiate when it is lambda Z. So, what is this? Lambda into, parentheses is meaningless Z equals 1. So that for lambda, is again we set this d square G X, then if you remember my variance is minus lambda square is its remember variance is, e of X square minus e of X whole square. Therefore, your so variance and mean are one in the same and its same for derive that. Let us, what about that a little bit some more properties that some point later on we may have, sometimes when you look at papers we make this assumption, and is just background for reading those papers. So, questions on this what? So, we have seen two MGF, two different MGFs and how they are used. So, then so many times like I said, you have several independent random variables coming in a there is a composite random variable that is summation of all those in the case of Poisson arrivals and so on.

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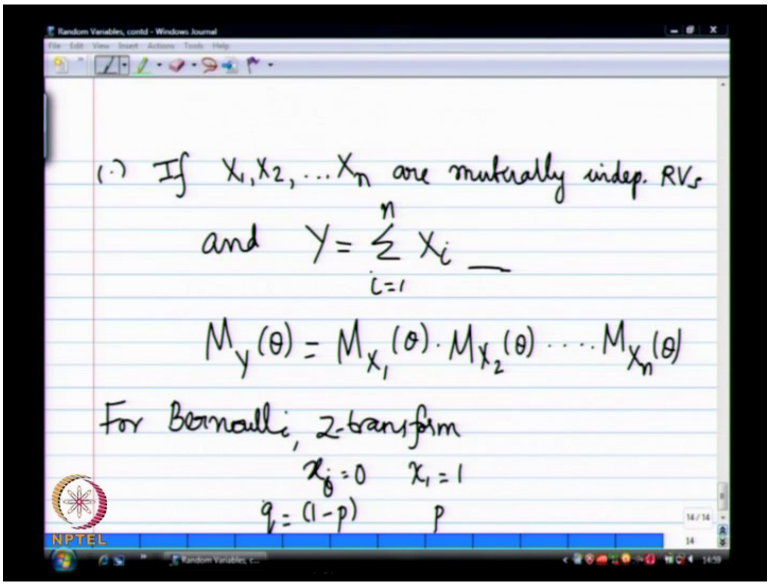

So, if X 1, X 2, X n are mutually independent there it is. So, there are n mutual independent random variables. So, think about packets coming to a router from several on several links to be same cube, they are mutually independent and they have their own distribution in terms of number of packets per second. And I define the variable Y which is aggregation of all those streams. So, what is the sometimes we need, what is this distribution of this, why become a distributions of X, sometimes which is figure out, sometimes which is not figure out. Occasionally, it is convenient to use this fact that, this is your convolution theorem to moment generation function of M of y is simply, we can this spell it out, so simply the product of all those.

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It can any think of an example where, we can actually use this from the variables that we saw before. Is any variable that we saw first class or last class, somewhere we can use this what variables we saw some discrete variables last class; first one was Bernoulli and Bernoulli is related to use anything else? Geometric is also related to Bernoulli, but that is counting the number of trials until the first success, that is geometric and binomial is counting a total number of success. So, let us see, if you can use this in that context. So, what is the for Bernoulli, what is the corresponding Z transform?

So, for Bernoulli remember that there are two values X i or X 0 equals 0 and X 1 equals 1, only two possible values and the probability of one is p and this is one minus p, sometimes we also use the term q thus for convenient.

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Agreed? So, what is the generator of the polynomial? Sorry, not the generator the generator Z transform can did the polynomial is something else. So, it is simply Z to the i in to probability of that.

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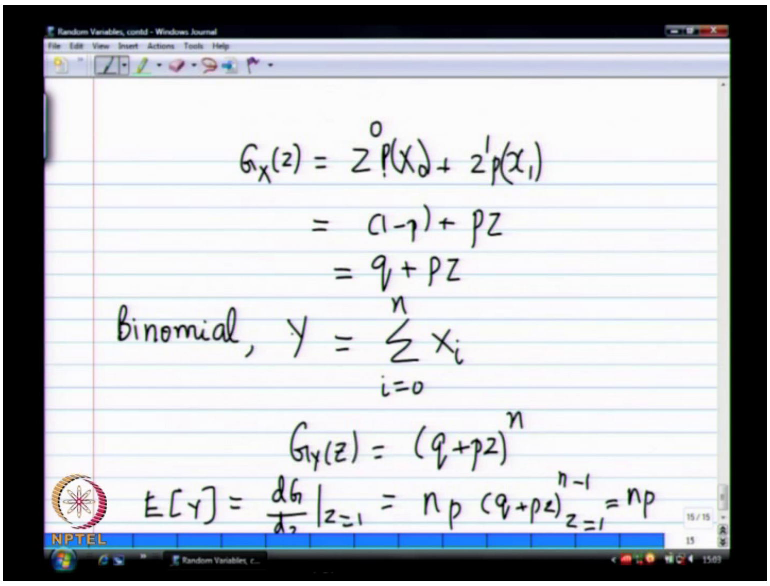

So, for the two values only; so that is to the power 0 into X 0 or probability of X 0. So, probability of X 0 is 1 minus p therefore, this is 1 minus p and this is Z looks like 2, but these Z right or else sometimes, we also write that as q plus, we write p is that more, because where use to X where Z is your variance, p is the constant; so we normally, since the write as A X plus b n and so on. We can p that plus q that 2, but we put coefficient of the variable, for the put p for there is on. So, this is a generator for Bernoulli. Now, binomial is simply enter, else and I am counting the number of successes is. So, therefore, if I simply, so now, binomial is, if I represent the binomial as y and each of those X is where independent Bernoulli with its experiments. So, therefore, this is simply where X i equal one or 0 right X i takes value one or 0 and I am simply adding a term in case of Bernoulli values 0 , 0 or one the probability p or q or q or p. So therefore the…

Now, if I want derive the Z transform for this binomial what is it? I can derive this both ways, I can go to the basic definition of binomial and then start with there and then come back to this or it is, remember that my convolution theorem function. Now, my y is summation of all those independent variables n and therefore, the MGF is simply the product of the corresponding MGF in which our transform. So, this is simply and now I can compute E of X.

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So, what is the differential what is the derivative. I can erase and very clearly.

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So, what is this can be, if you want to go through the, you know it is n p If we want to go through the full derivation. So, p plus q equals 1. So, therefore, when z equals 1 everything equals 1 form therefore, it is 1 and so forth. That is what the convolution theorem has begun.

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I think I cover pretty much always, now will again go through some more variables we stop last time, in the exponential and trying to see why exponential memory less, let we will get to just couple of slides.

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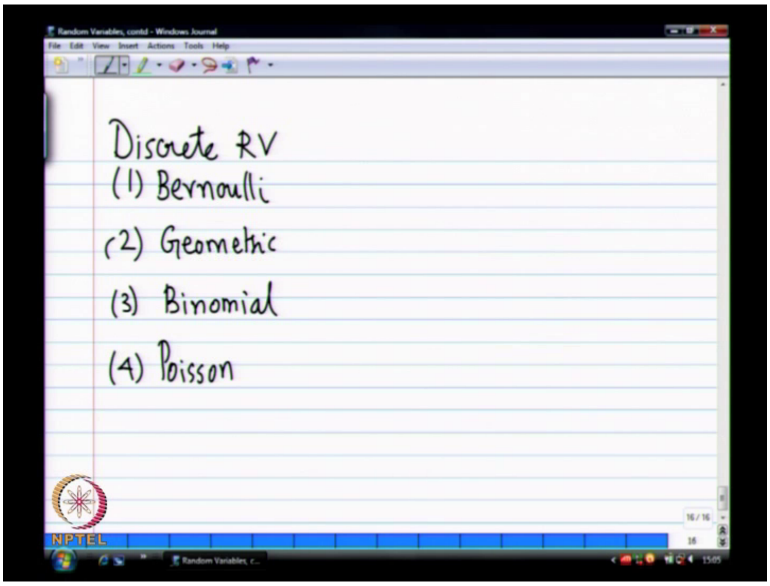

So, is the round up of all the discrete random variables that you have seen so for, these are Bernoulli, we saw Geometric, saw Poison this is what you have seen so for. Just couple of more to fill our knowledge, some other distributions and then we could move on. So, the next one is try to the binomial one sometimes. So, in the binomial what we do this we run in n trials and count the number of success. And the distribution with the occasionally, we need to know the other way around us to how many trials; should I run before a get n number of success, that in other way right if I want R successes, how many trial should be run with the given the probability of success p for each trial that is called a negative binomial distribution.

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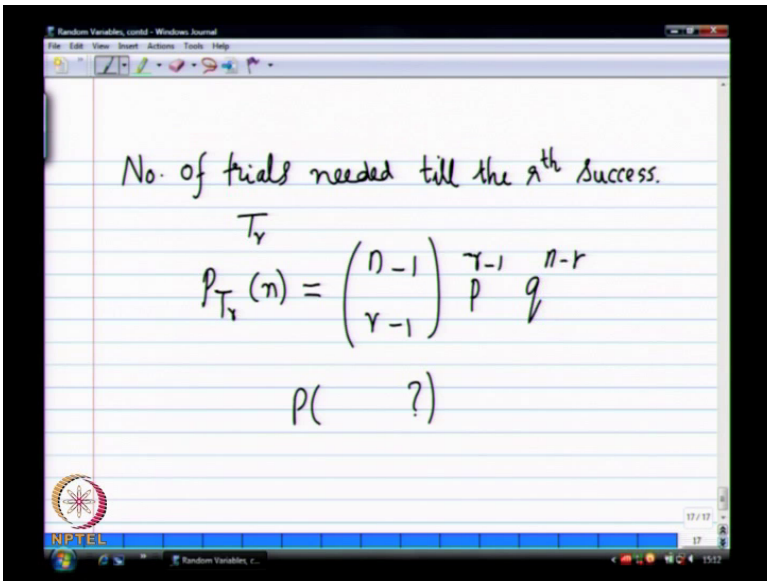

So, that is basically the variable or the random variable representing the number of trials needed till r eth success. So, I keep repeating the trials, until I get r success. So, what will be the… So, how to be derive that there, which is enough geometric is only to the first success. Yeah its correct yeah. So, r equals 1 would be geometric that sense negative binomial, but in the sense stay to the variable is in the binomial properties. So, let us call this variable T r then the probability that this variable T r right.

So, what is the probability of needing n trials? What is the probability of needing n trials to get r eth success? See that it at from memory or derived on the spot derived. So, anybody else wants to try. So, I have r successes. So, if n exactly n trials need r successes then what should I have, the last one is the success. So, I should have r minus 1 success before this one. So, therefore, it is n minus 1, choose r minus 1. So, in the n minus one trials preceded, this one I have r minus 1 successes and so, out of which r minus 1 on successful, this only on for the n minus trials q n minus r. The last that should be one more p or c, I looked at once, that why is in the… I look that book why is there p is not there, but that is because we are assuming that the n trial is the success.

So, this is conditioning on the fact that, you until the success. I could not, say what we are trying to do this is the, for having r minus 1 successes in n minus 1 trials, given that the r eth trial is the success. So, that will be into the probability that r eth success which is p than divided by the fact that is the success. I think your p and p will cancel it condition for probabilities. Will work out the derivation later, it is the connection probability that you are r eth trial or n eth trial is a success. So, n the trial is success given that you have r.

Arun what is it? To for yes.

(( )) Defined as.

Yeah.

Then it should be then it should.

Conditional there.

We have the extra one p is go to your countering. I looked at Jains book, Jains book defined it is, try to differently where it is that is, modified to same thing, sorry in his a definition is looking at the total number of failures X, before the M eth success same thing, total of M success M plus X trials is what is there, there the term is correct p to the M is one minus p to the X right in this particular case. This because you are very looking at those this is the combination of having r minus 1 failures or r minus 1 successes and n minus 1 trials and that is what we are reporting in this particular case.

Only.

That the n eth.

N the trial is the success.

Trail is the success. So, that n eth trial success is already given.

So, that is why you only looking at this is number of base in which you can have say, this is what is this is number of base which you can have because you consumed a pointing as number of ways, in which you can have r minus successes and n minus 1 trials and therefore, in there we which is the n eth trial is effectively a success. Still not convinced? Fine, we will go back then check to the basics for the derivation right. So, that missing p will figure out, that is where that how that the p.

Having…

No r success.

R successes in n trials.

In trails yes.

Which is the probability of getting n minus 1 successes which r minus 1 success in n minus trials?

And…

The probability of getting success in the final trial.

Final trial.

Given that we have got R minus 1 success in n minus 1 trials, but we are assuming this an independent events.

Yes.

So, probability of success on the final thing given this is equals.

This is the...

Probability and success on final thing which is p.

P.

So, that there it will be one trial will be a p.

But then, when you do the conditional probability.

But…

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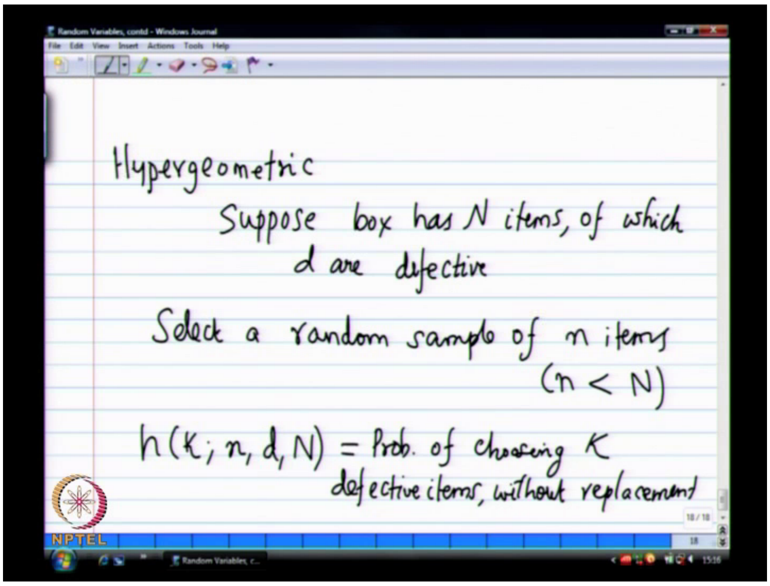

We will derive; we will come back in the try to say. No way. So, I missing p, that is what you are asking, will you that then there another is hypergeometric is there. We may wonder why are we looking of these thing, some of these things it may never encounter you never no, sometimes you will need it, if you look at one of the conjunction control algorithm derivation the actually use something like this. This probability to find out in case how many trials are needed and so on. I will this is in volt watts book derivation is there or actually trying to find out their performance of throughput of a window conjunction control system, window based mechanism in which sometimes we end of having to use that. So, any way this is again this is for that, for now just look likes a generic knowledge you store it away.

And in case, you are encounter this in a real system, then we will this will come in handling, but this also a standard this is useful and why are we twelve. See, some of the analysis is so for, performance sometimes is also from reliability perspective. So, we are only looking at, we are talked about three types of performance metrics. So, one is that speed, so the second one is reliability and the third one was try to that availability and so on. So, reliability measures require such can be distribution. So, we will say look at this case where, I am we have a box of light 1000 toys whatever toys and you want to find out you know that probability of the ten percent of these toys are always quality, there is that is your measure of your, so you know that is a definite probability that there some d out this thousand toys are faulty. And then, your job is to pick has somebody randomly pick toys. See, have to some instead of you can pick all thousand is whether the faulty or not.

So, you have to picks an only 10 or 100 test them and then put it back and then. So, you take it out of the box. So, you take one it face a is without replacement. So, if it is defective, you take it out or any you take it out the defective through outside and you want to do hope that within hundred of your testing that you are basically found majority of the defective items. That is one way of looking at. You no need to test everything is again, we have a test in, go you are sampling you are doing as a sampling a test and see and you want to maximize the probability. You want to get hundred percent all the defective item. So, some defect when will go through will give QA whatever it is, but you want to maximize the probability of detecting as many items as pass. That is, where this hyper geometric will come again. So, suppose there is a box with N say, items d are defective and then d is some either known or its is simply guessed based on passed information you know that some fixed fraction of n is been defective. Therefore, may be you know ten fifteen percent. So, the d is some fraction of this N.

Then you select a random sample of n items where, n is definitely should be less than if equal to n; that means, you are testing everybody, but you want to do that, this is your sample testing, which will be and M should be radius much smaller than n right, but for now a we know at least less than l so in the probability, so h, of getting k finding out what is the probability of having detecting k faulty components, faulty items given that your than n samples d is the total number of defective items n is the total number of items in the box right. So, this is the probability of choosing without replacement.

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So, and it is you sample it outside, you known put it back the box we want derive this today something which we will the expression.

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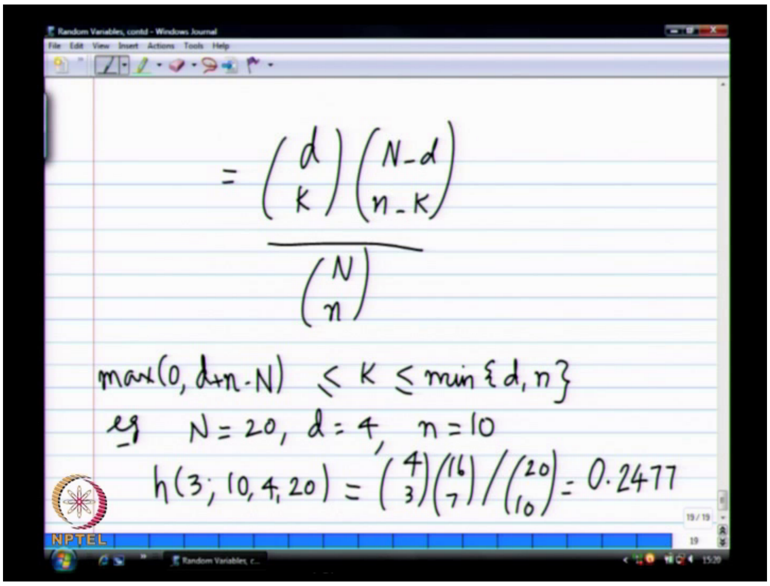

So, that is given. So, we will just do one quick example to see. So, if I have N equal to 20 and you know there are 4 faulty items in there, in a choose an items in the box. I do not put the back and I want to find out the probability that I found three defective ones because my goal is to get the defective ones. n minus k should be seventeen. I met a cup first we n minus small n minus k.

So, what is that tell you at it only 10 samples right 20 difficult to get event 25 percent of the time you want to be able to find even three defective items forget about all the four. So, your systems will have fairly large number of defective it cap that you going, but that you deal with in warranty face. Then, what about the next step quality control fix. Questions? This again just for storing in memory for now, but this is without replacement; so I take an item defective or whatever set it a site or if throw it back in the box and I keep sampling than what is it become? It becomes a binomial that is with replacement, because your probability of finding a defective is item always the same there always d by n. So, every time we look for something always d by n and then finding out with they are here your success based finding a defective component.

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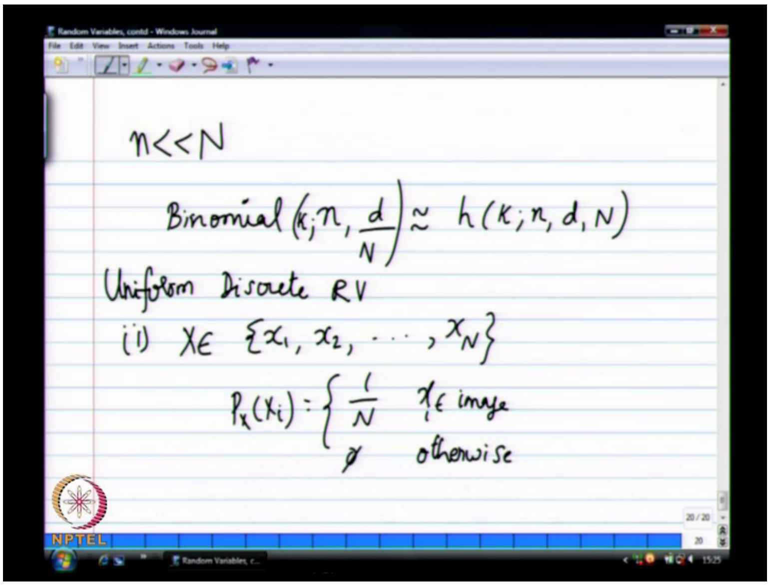

So, therefore, for large when your n is substantially smaller, right or n is very large then this basically. So, k n trials; so, getting k successes in n trials, the probability of the success which d by n will simply approximate your. So, you can use a binomial as a short form, shortcut instead of (( ))

Which one?

The range that when given maths.

That is basically the second,

The second is once again part. So, why do we need again, this one is evident. So, the lowest possible value for k right; so, you have in and your n minus k should be less than or equal to n minus 1. So, from that you get your and pointing the, see here, it comes from these condition that n minus k that be less than that is from the formula point of view, but from the actual physical point of view, lets the what should the number of defective elements can should be at least as large as your sample size plus the total number of defective elements. The number of non defective item should be as, that is the this is the like low bound of the non defective items that you can choose.

The number of number of non defective items is n minus G.

n minus G.

You cannot choose more than n minus T.

I do not know this minus is a lower bound n minus l, that is non defective.

So, there it is.

Non defective items are choosing the greater non defective item on the set.

So, in this particular example, if you carried this fourteen minus twenty minus six, there we see cannot have the as therefore, it cannot be negative. So, where will it be case will we have if everything is defective right? So, it is n is defect twenty, these are the twenty then if the same like twenty out of twenty samples fifteen are defective as. Say, I am choosing ten from it, you can only five max forget. You can only, you see the max of the total number of sample that we choose. So, that will be n when we just like this on paper n to figure out. I will come back to you with negative items physical explanation for the last one that will and then will go to. So, this is uniform, uniform discrete random variable thus continuous also.

So, the two ways to look at uniform either we have a set items right X 1 through X n. So, if the all possible values are X 1 through X n, this is the set of possible values for random variable. So, X can and the probability of X i simply 1 by N right, if this is a show called image. Image is a set of right this possible values is this you take then 0 otherwise. So, this is one form of uniform distribution which we all possible values are equally like this, which is normally in the case when you have enumerate it is not the continuous set of numbers discrete in set of integers. So, that is one form of uniform distribution.

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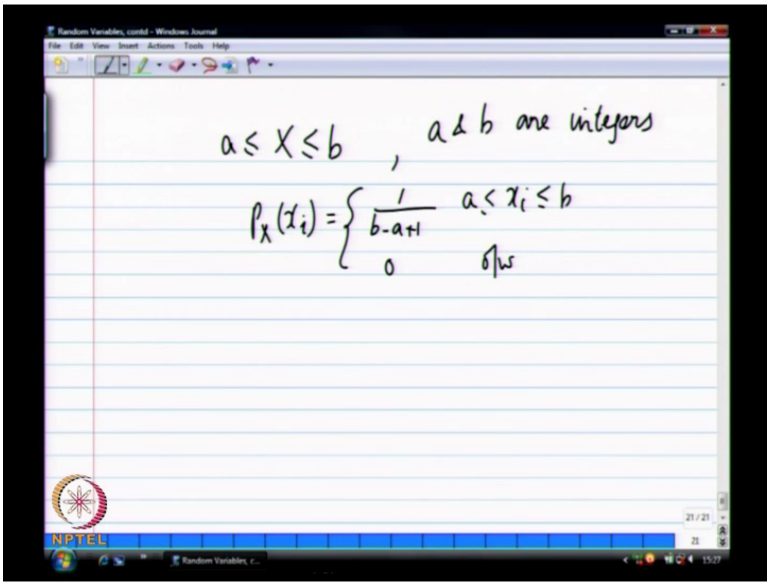

And the other form is where you have the. So, X can basically the between say b and some value k. So, all possible values are so from a to b everything is possible. So, you have a total of how many items, b minus a plus 1. So, the probabilities will b minus a plus 1. This is only integers; this is the discrete random variable. So, the probability of know X i is … And this is also useful again this is very simple variable. That ends the theory part.