Performance Evaluation of Computer Systems

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Lecture No. # 06

Probability distributions-I

Let us come back to random variables. I think, last class (( )) probably lot of equation with not much explanations it become fairly monotonous.

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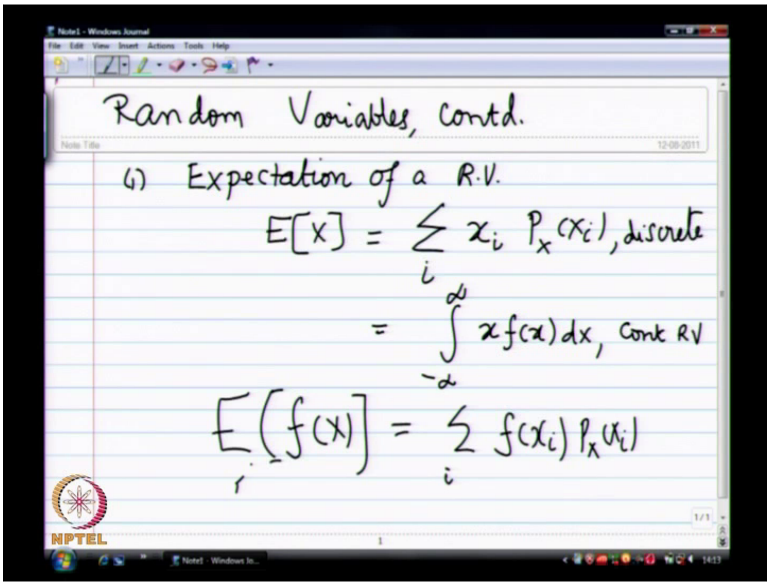

So, let me see, if had some explanations. We have been through a bunch of the sort of typical variables and are there are (( )) so, few definitions which that I should come back to and then let see. So, first of all expectation of a random variable; so that is standard definition; these all practice, I have to you get used to infrastructure; some particular angle that is have to this. So, if it is a discrete, three times for I get that. So, if x i is the value of the variable and P of X x i. So, i is your, x i are the set of all possible values. So this you know, this is your mean basically and if it is a … We can also represent that it was as if it is a continuous random variable, it is simply a summation of all possible values x into … My infinities are looking like alpha, but you know that its infinity, then this in always close so this is your continuous random variable of i.

And there is another little button here which a press something funny happens. We will figure all this out, I will figure all of this; this is continuous; this is discrete. And in generally, I can extend this expectation to any function of the random variable f of x so, that is E of f of x. It is again the same; it is simply f of apply to x i and then the probability. So, this P X x i, thus mean that what is the probability; that random variable X takes the value x i, we in this before and this is i. That is again definition; sometimes will use this later on, thus needed. Then we have the special E of X square.

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Expectation of function, a function applies to the variable x.

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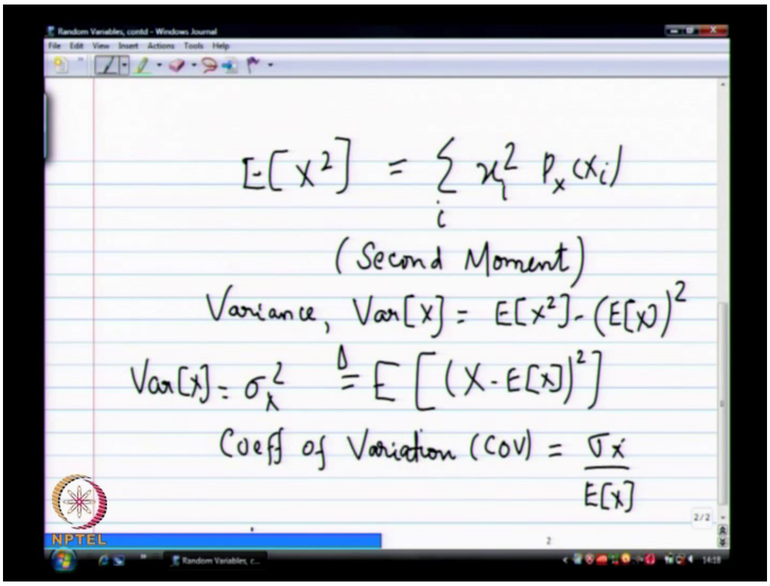

So, in this case X square is for example a function. So, if f of x is X square than E of X square is defined. So, this is I show you, x square, x i square… We can always go back, if you want. It is not there is permanently where once it is whatever it is for each variable we are applying the function f and then the multiplying there with the corresponding probability. So, X square is the simple example; this is X square for overall thing. And what is special about E of X square? It is other name for that.

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Variance, no

Moment, second moment

Second moment; E of X is the first moment; this is your second moment. So, later on we talk about moment generating function is too. So, this is your second moment. And then your variance is actually defined as… So, what we normally write that as variance of X that is E of X square minus E of X whole square; that is the variance. And then standard deviation is simply square root of here, variance of here. This is following us from form and came to the back. There is another definition for variance two; thus we will other definition for variance X minus E of X. So, it is expected value of X minus E of X whole square. That is also a definition, but both are equivalent.

We can derive this either now or tutorial session, ask why this is true. So, that is something that is I want to and then and that is here. So, variance of X is also a standard deviation square; sigma X square. So, then this sigma or sigma X with it. And that is one more parameter or one more metric that one can measure for the coefficient of variation, for a particular random variable; Sometimes we call it is COV.

This thing is not well.

Then what is the coefficient of variation? ((No audio from 06:23 to 06:36)) Sigma X for at here. All clear, I think that is about the basics that you want to be first.

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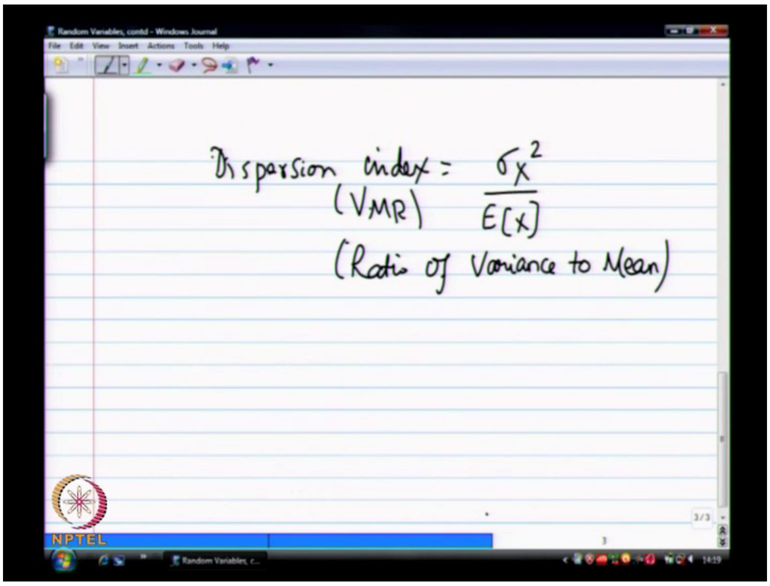

And that is one more metric, all the dispersion index or index of dispersion or VMR. Sometimes we also just use this index of dispersion. So, I am trying to avoid touching this screen where apparently that is not feasible, where it is simply ratio of variance to the mean. So, it is called VMR- Variance to Mean Ratio. So, this also is called… So, VMR is something special. So, this, it is either the one or less than one, greater than one; these are three possible cases. If it is equal to one, then what variable is it? There is only one variable for which VMR equal to one.

Poisson distribution

Poisson distribution so, the poisson variance equals mean. When we look at poisson again we will discuss this. But that usually as a test, if you want to see whether you are measure data. So, you normally what we assume is, we assume that distribution is Poisson distribution do all our assumptions. But see other way around all the time, when you do your experiments and you are trying to take some measurements for says of which times in many things. Then you want to retrieve of that you some of known distribution, because you want to take that and use that to generate traffic later on. So, sometimes you do some test measurement, traces and then you want to use that as a way to generate new traffic models.

Because sometimes when you want to do something mathematically, you should know what the part of will distribution type is. So, anyway VMR is on only in the case of poisson. So, these are some of the essential definitions that you want to cover. So, next we will get into, again this is still we lost in the probability of the random variable (( )). So, you will hear about the moment generation functions.

Now only see, two heads shaking, how many of you heard about moment MGF? MGF is only two, three four, five. So, I will go through that about probability generating function is PGF; it is try to MGF it is fine. So, you hear a Laplace transform, some of those things you have. So, now let me just again this is for background some case for which we are deriving the mean and variance and so on those giving you the values. But I would like you try those that also. So, we will do that the tutorial session; I will just go through some of the highlights of that today.

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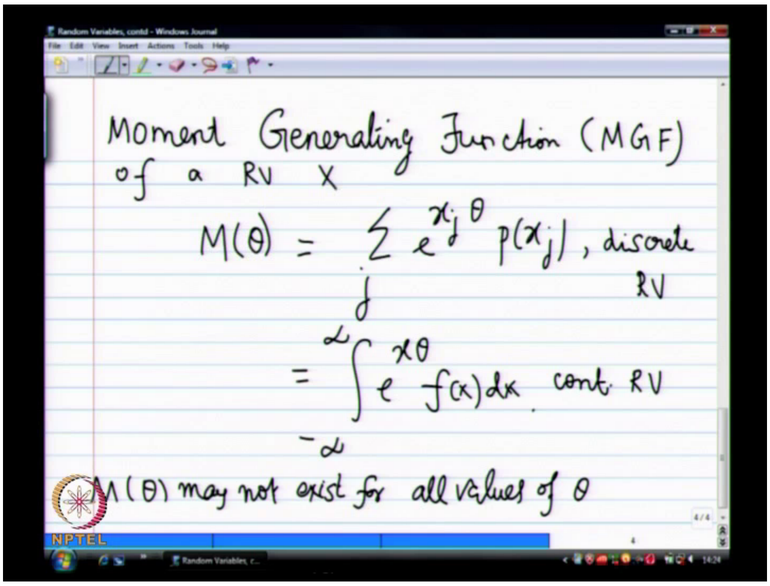

So, we are now in page three; you go to page four. So, Moment Generating Function MGF of a random variable X is simply defined as for as … or just use the multi source definition. So, that is summation or all the possible values that this variable X can take. So, here is your f of x, f of x in this case is simply e power x j theta, when theta some whatever will see, what this theta can be integer, can be integer also. And in the case of continuous random variable or correspondingly your summation will go for minus infinity to plus infinity of x theta f of x d x so, continuous random variable. So, this M of theta may not exist for all values of …, but for most of the cases that we look at we can derive this MGF in these cases. Questions?

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So, for definitions only so, now this so, instead of theta if we have some other terms like Z or minus S and so on end of the different transforms; I look at only two of them. This can also be a Fourier transform, but since we do not use Fourier transform and the derivations are simply leave that.

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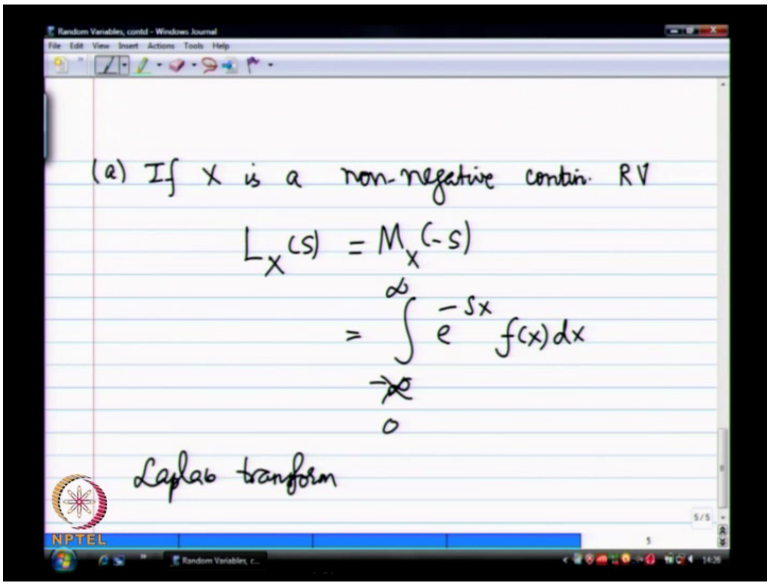

So, there are two sub cases or special cases of this MGF. So one is, if X is a non- negative a continuous random variable. So, say if X is the non-negative and continuous random variable then our Laplace transform comes again; it is a… So, if I define L X of S Laplace transform can be applied in any function between these cases, we talking about.

This is the same as applying my instead of theta, I just a prior use minus S, and because it is continuous going to be using by non- negative so, technically this can be zero. So, zero to infinity is… this is defined as your Laplace transform of this particular random variable and why this is useful? You can help as easily determine your mean and variance some things like that without having to go through this. If we use the f of x directly, it sometimes hard to get to the mean and variance, but using this we can sometimes get the mean and variance quite easily.

So, that your Laplace transform got it. Question, we look at one example. So, that this gets clear. So, that was for continuous case, in the case if X being non- negative discrete random variable.

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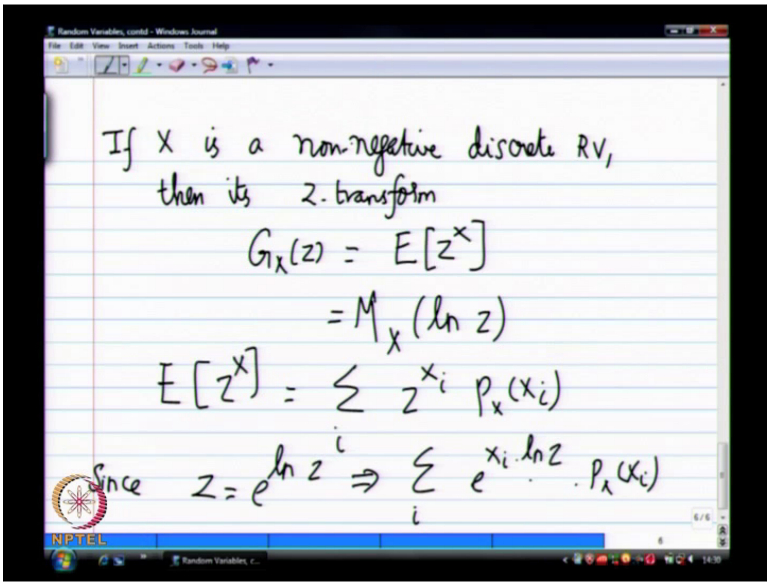

Then we have it is then the zee transform, Z transform. So, Z transforms which is usually using this G x of Z by simply. Now this is by definition, your probability generating function so, you would have this as Z to the power X. So, there are some variables Z to which you are rising your random variable. And this is the same as, I did something to it since x mean (( )) Are you got that? So, E of Z base to the power X is summation of all i. So, this is Z to the power X i. And then the probability of this is my definition that I defined E of f X, f X is in this case in the some variable Z to which you rising at the power X. And depending on the value of Z, you get different things; Z equals to one get something done to some terms and so on.

So, it simply for every possible value I simply rise into the power, rise Z to the power that. Now this I can write in terms of my exponent; so, it is Z in terms of e, natural log of. So, my summation becomes, e power X i into natural log Z.

So, then why the non negativity condition there?

Why is the non negative condition there? I am going back and check that.

Laplace transform it is should be non negative, but for seen, but for this also why?

Okay. That is why or not. That is the definition as it. So, this now looks familiar; this is e to the theta into X i and this is your theta basically.

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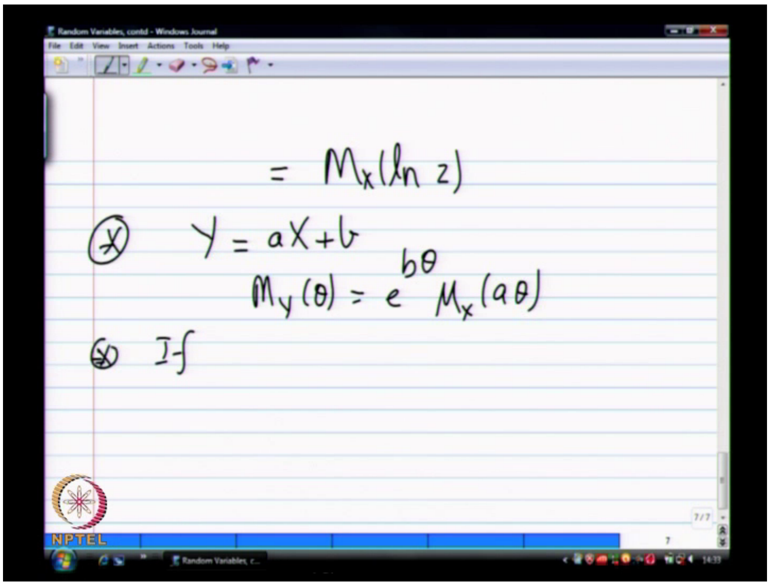

So, therefore, I can write this. Therefore, this is nothing but M X to the power ... So, theta becomes an ln c. So, again rather this is still definitions you come, boring a two clock in the afternoon which is there. So, then some few properties, let we will see where needed. So, if y is a variable which is defined as dependent on X, with is this y is equals to a plus b, then the moment generation function for y is basically that... so, a theta so, a X plus b. Again this you can go back derived from basic principles. This is the just a property that if needed I try to use that.

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Why I have doing, because when I have I am talking about the random variables, I need to do some of the basic properties of the R Vs, before I go into the exponential and so on. So, when I say the exponential is equal to lambda or one over lambda and so on. So, then at least you should know why it is, how it is the best way. So, this is the definition just (( )) it for next twenty minutes and so on.

And because many times you may want to actually derive when we will doing later performance analysis for example, when you have a several independent traffic sources coming to a multiplexer by that is, then you know the independent distribution of all those which you want to know the collective distribution is of that variable itself. In those cases, then you it is, if you know the distribution for the traffic comment to the single queue mathematical analytical, then you can use those analytical results. So, that why is the sum of so, for example if you know the each of these next one is there. If I have said X or n different traffic stream is coming to this queue and now you are not looking at simulation I could last time.

Now you want to know, what is the actual distribution of this particular composite traffic sources? In that case, I can use some of these things to derive the corresponding distributions.

(( )) you have to just to spend in the (( ))

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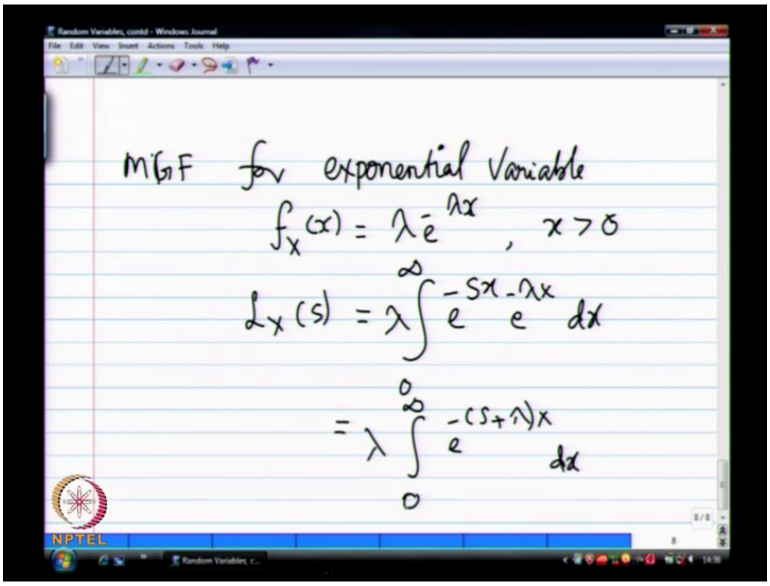

So, here I for that I will just come to that example with exponential if you want, let me stop these properties for just a second. And I will go to your … Let us look at this MGF for exponential variable. So, what is the… I used A e power minus a x which is your case which is some is lambda e power minus lambda. The text book actually use as this one hour, A e power minus, one hour a which is sometimes confusing. There is you have to think that one hour a, one hour a X 2 is the mean, a is have to inverse of the mean.

So, I will just use for now. I will just lambda which is what we used in the case of the Poisson traffic arrivals. So, this is my f of x. So, lambda is the rate parameter so, it is called rate. So, when this lambda specifies, this is so called rate and one hour lambda is the mean of this variable. And always if we look at Raj Jains book (( )) x greater than or equal to zero and when Kishore is (( )) very particular for an x greater than zero only. So, it is the lambda yesterday. We will try to plot x equal to zero for exponential. And this according this book, we want to look at x equal to zero only, x greater than zero which in sometimes if an x terms you may look an inter packet arrival times, your perceiving the two packets.

So, it is not going to be… it is at least zero plus some none some epsilon values is there, before the next packet it can arrival. So anyway, equal to zero really does not make a difference, but we just use and other book definition. So, this is my function. So, know what is the Laplace transform for this one? We simply replace theta in our equation with minus S so, all the double if of probably, as seen thus the thousand times. I even at that we did this in second year Math; second year if a college Math or first year college Math, we have done we have derive a Laplace transform to left and right for also some complicated for functions without knowing why with it is used for. So, here is one case may be sequentially used.

So, what is it now? It is e power minus S or e power theta x is, what theta is minus as therefore, it is e power minus x. And then of course lambda in one, I take the lambda out and so, because this is positively adjust.

Minus to the power minus

Minus so, then it is lambda is this basic… ((No audio from 25:20 to 25:38))

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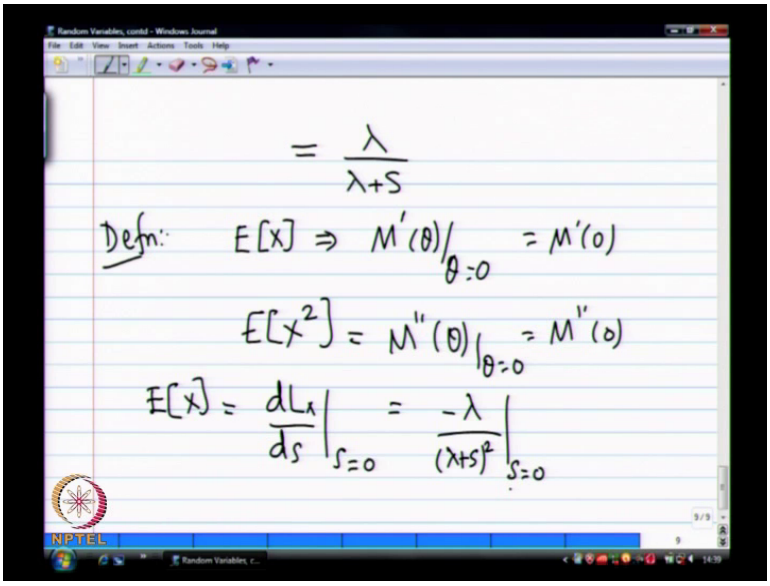

So, what is the value?

Lambda by x square (( ))

Lambda,

Lambda over s plus,

Lambda over s plus lambda or lambda plus s over here so, that is you are your MGF. So, with this we can generate all the moment that you want. Now if you want to find out the mean, I have to, I want to how to get the mean sir. So, E of x can be obtain by simply differentiating with respect to theta and setting theta equals zero or M dash of zero or whatever you have the you want. So, you can generate the moments by simply differentiating the function left and right or continuously E of x square is M and so on. So, this is the definition of moments that is why it is called a normal generator.

So, now when this particular case E of X equals d of your Laplace transform divide by d S; setting S equal to zero and that is nothing but, minus lambda by lambda plus S whole square S equal to zero.

Minus S of, L of minus S,

Yes.

So, it is a like lambda by lambda minus S that is what the MGF.

Lambda by lambda,

Minus S will be the MGF.

If you could be are the…

If you are compute the Laplace transforms so, L of S.

So, it will be minus S equal to zero.

M of S should be lambda by lambda minus S.

So, we computed of minus S.

No. Laplace S (( ))

In our definition, at least we used minus S as the… So, that why it is lambda by lambda plus S, if I use S to be lambda by lambda minus S when using, you seeing it should be S instead of minus S.

(( )) computed the Laplace transform, we gave a differentiating the same thing with respect to S.

With respect to S, we are only respect to S only I am differentiating.

. But answer is Laplace not, if we have done only Laplace not MGF; MGF should be…

The MGF is your generalization of the Laplace. So, where MGF was e power theta X. So, in that theta, instead of theta might use their minus S.

So, with respect to, with respect to minus S, not respect to S

Theta which is minus,

If it respect to S, what I will get? Then you… let me go back, go through the computation again if the CPU is actually if I do. Let us check the (( )). So, with this if I get if minus one of lambda then I may travel, I do get that.

(( )).

With respect to M minus so, here it will be, and I will be getting minus.

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So, this will be minus. So, we lot to refer, just put it plus differentiating with respect to theta. So, theta is minus S or will just a minus of this. So, only this term will change. If we lambda by lambda M plus S are your Laplace, you differentiate with this than I get this negative term of runaway.

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Catch that. So, this is a lambda by lambda square.

(( )) you put their minus.

(No audio from 30:35 to 30:58) you actually, I miss that minus one to the power K. There is actually I will go back and write that formula of as…

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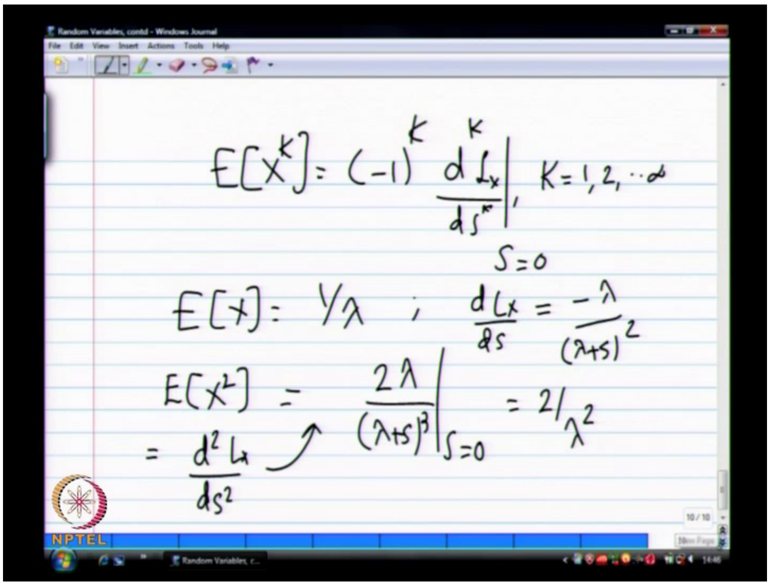

So, if you have the Laplace then your E of X K is minus one to the power K d of L X divided by d S, all the terms, because when we go to the second, I cannot do the minus. So, i is that of … at S equals zero. (No audio from 31:50 to 32:13) So, let us one way of train to compute the mean first moment. I could also the computed the mean in another way which simply using the definition itself which will do in the tutorial; this is one way. So, if I want to find out the… So, we saw that E of X is now is, now one hour lambda. So, what is E of X squared?

One by lambda square,

One by lambda square because everything is know is one by lambda square

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E of X square is not two way. So, we have lambda by lambda plus S whole cube.

What is the differential of that… See they are figure out that the d L by d S was lambda minus lambda by whole square. This is just your d square and what is that two by lambda square? So, that is the purpose of using in the sense of minus.

(( ))

Because minus is now absorbed here so, this is your… this is the definition. So, the minus one to the power K, when I differentiate lambda by lambda plus S, you will get this one minus lambda by lambda plus S square. And because of the fact that is the negative one and K was equal to one that negative got that. But I when go to the second differential, when I get the string so, second derivative. I am sorry.

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So, this is for the K the derivative. I forgot the K here slashes and K here does not look like a K is that.

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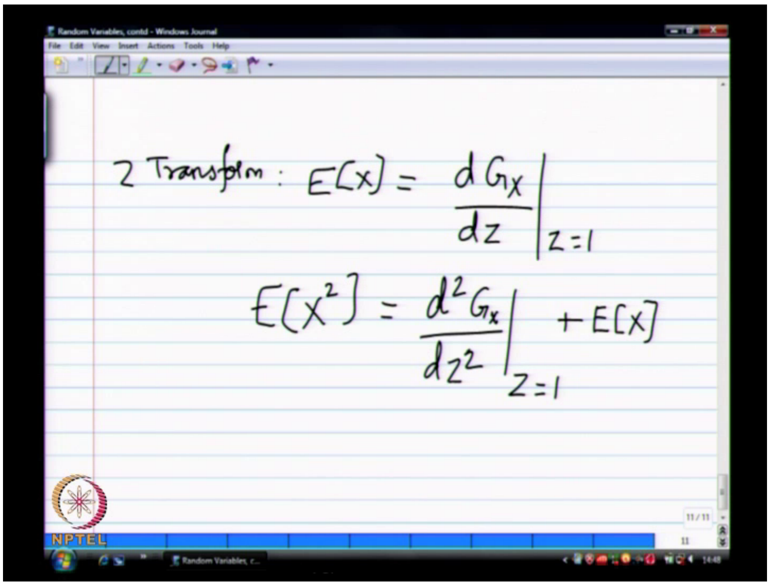

That is in anywhere here; and in the case of your Z transform, remembers that it is E of X. So, I use the term G, Generator probability generating function. So, this is basically d remember that X in terms of Z. So, differentiate with respect to Z and set Z equals one, zero, one. In fact with these we can also not get not only get all the probabilities P of X say is also can be obtained. But that we are not look at now and this only two E of X and E of X square is all that is to define and that all like the other one this can be obtained. So, simply differentiate with respect to Z and once, twice that the sets equals to one.