Performance Evaluation of Computer Systems
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Lecture No. # 40
Petrinets-II

So, will look at other properties of Petrinets, and look at how we can actually solve some solution to get performance matrix, the performance matrix on these petri nets.

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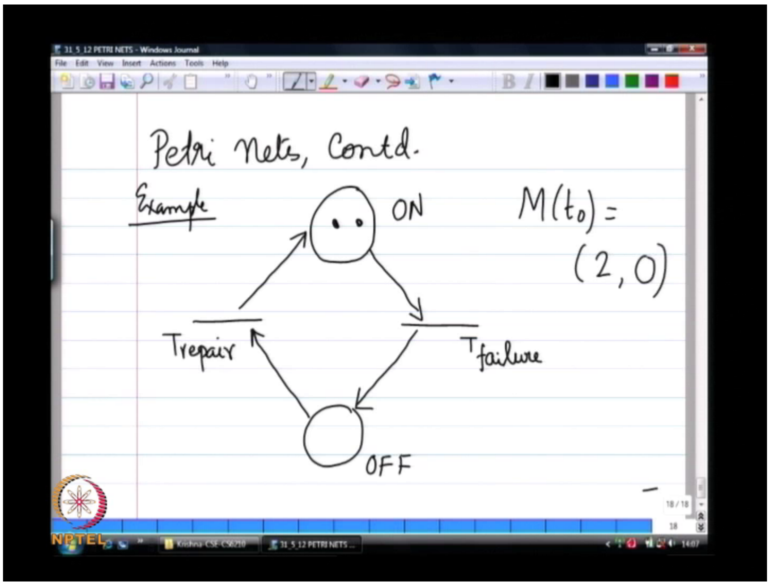

So, we look at some more examples, likewise such in earlier. So, let us consider system where, there are two machines, that are in operation right and the default state for them is on state, and the which some probability of are, some mean time to failure, these system will fail and then they would be repaired in then brought back to the normal working condition.

So, therefore, this is the name of the one of the places right. So, will have a two place petri net, one is labeled on and the other is labeled off. And then we have two transitions,so transition 1 and transition 2. So, transition 1, we will call this is T failure and then the other transition 2 which is T repair. So, this T failure, T repair could be exponential, could be instantaneous, we are not right now talking about that. We have two places, two transitions, and from the on state right, it is possible that will go to this failure state and then likewise some. So, when you fail, then the particular machine is in repair has to be fixed, therefore it is in the off state. And then whenever there is atoken in the off place, then the repair process will start for the particular system, it will get repaired and after finishing the repair, it goes back to on state.

So, this is the general, this is the set of transitions and places right. So, on place, off place and two transitions, and this T failure would be the mean timed failure that is what it means. So, initially you will have some number of tokens in this system and every T failure time units, you will end of essentially failing a process failing one of those two units and then you go to the off state. And then T repair is again mean time to repair that particular unit after finishing the repair, it goes back to operational state.

So, the number of tokens in this system could be anything, right I could have 10 machines, 20 machines. So, in this examples, we are simplifying it basing there are only two machines in the system, therefore the initial state and both of them start in the on state right. Therefore, what is the initial state, what is initial state? 2 gamma 0 right, there are two machines, both are of the on state.Therefore that is this to 2 corresponds that and neither of them are is in the off state.

So, that is an initial, so you will have, so that is the two tokens, it have currently in the system, both are in the on state. So, this is now the system description, so if you look at the system state place right, how do the system now move? So, we have what is the possible, what are the possible states of the system. So, the state of the system here is defined as the number of units plus number of tokens present in each of those two states, two places, on or off right.

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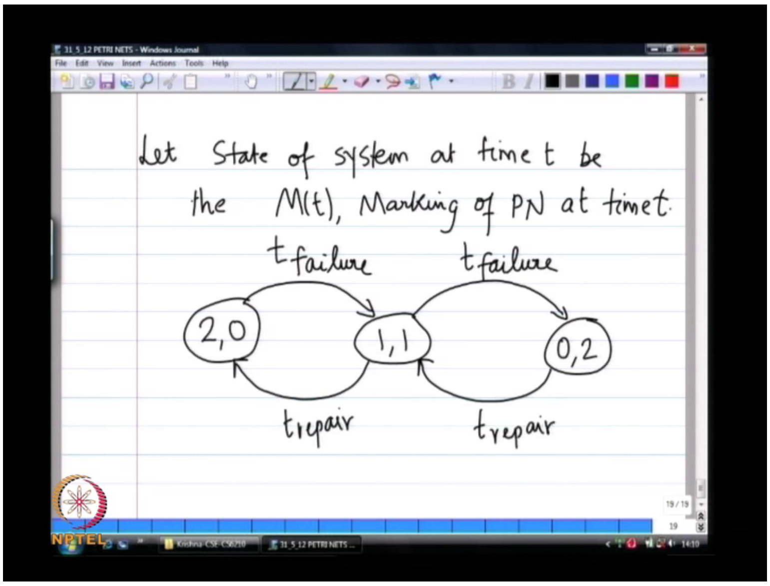

So, what are the possible, so we will let state right are marking of the system right, let the state of the system at time t, it simply the marking right be the M of t right marking of the system at time t are marking of the petri net at time t right. So, what are the possible system states now? There is 2, 0 2 comma 0 represents one state, and then we have in other state, which is 1 1, where 1 is off and 1 is operational are the third possibilities both are in off states, Therefore, right both of them, so sorry 1 is on, 1 is off. And therefore, the one of them is operating, one of them is, one of those as under repair.

So, then the, if we look at this is continuous mark out chain, we look at this state transition probability matrix right. So, we will look at the rates, I think you will have to go back and recollect your continues time and discrete time mark out chains. So, we represent the rate of transition from one state to other state. So, the rate of transition would be assuming that this is exponential,would be simply the rate of which will go from the, this state to this state is with the rate t failure and likewise, the rate of transition from 1, 1 to 0, 2 is also t failure, and likewise from 0, 2 which means that both the systems are currently not working, one of them will getting repaired.

So, when the repair gets finished for one of them, you will go back to one operational, one under repair and the rate of transition for that is again t repair, and here one is operational, one is under repair. And therefore, we go back to after the repair is completed, you will go back to 2 gamma 0 right. So, there this is called, this is here the rate of transition from 1, 1 to

2, 0 is t repaired again. So, this is this is how we have represented the marking right. So, now, you have a graph right we looking at a graph will defined this (( )) we have a graph that connects the different marking of the system and the rate at which you are going to move between this, a rate of probability of movement between those different markings, any questions on this part? So, now, let us, so we talked about the petri net having an initial marking right.

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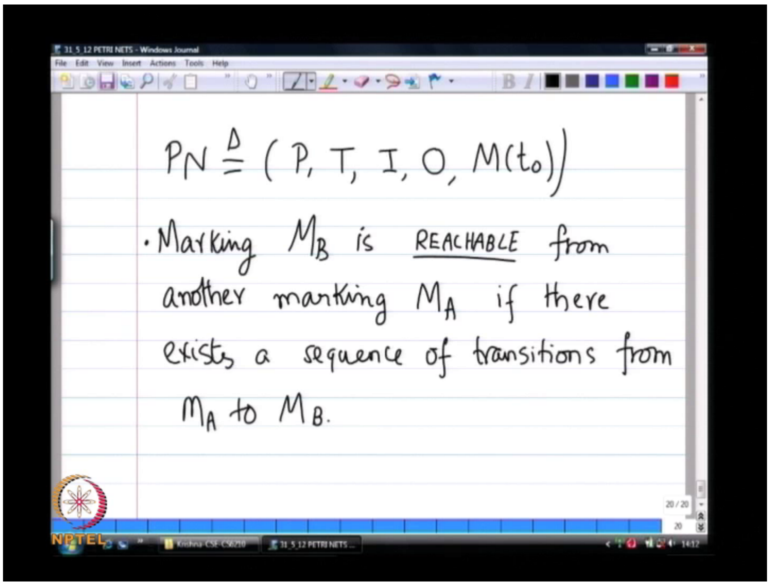

So, the Petri net is defined, some petri net is defined as consisting of this set of places, set of transitions, the input information for all the transitions, output for all the transitions and we also specified this initial state of initial marking of the system right, so that is what defines. So, now, we will serve some more definitions. So, some marking B rightwe will called as some marking say M B right, it is called reachable is defined to be reachable from another marking (( )) marking is just the values of all thetokens in the various places. So, this some other marking defined you know to do us M A. So, M B reachable from M A if there exists a sequence of transitions.

So, there if right one it there is some combination of transition in a particular sequence, there exists the sequence of transition from M A to M B, so that is. So, from M not M of T not, I can go to some other state after two transitions, and then could bedifferent possible combinations right. So, many transitions this system one or more of those can fired in whatever of its T 1 can fired, then T 3 can fired or T 1 can fired, then T 2 can fired if it is three state of three transition system and so on right. So, that is the definition of how of how you go from one marking to another marking, that is a reachability definition.

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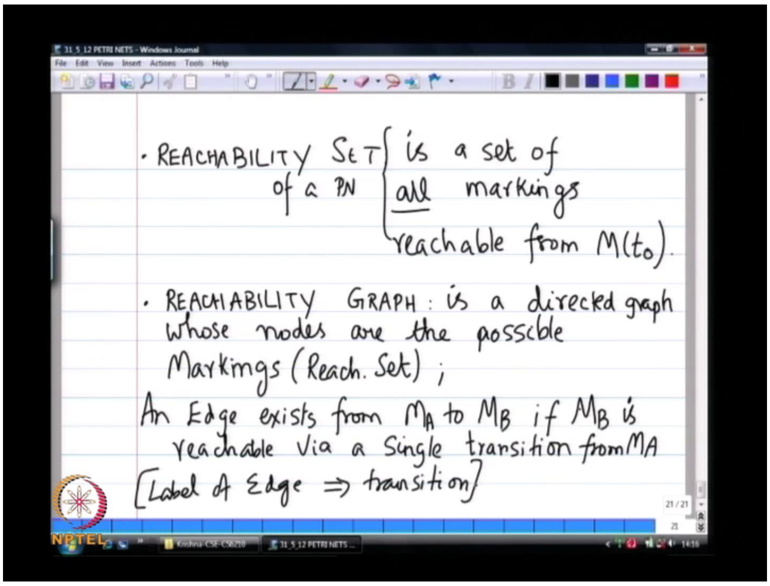

So, now, we can extent this to define what is called as reachability set. So, reachability set right is a set of all markings reachable from some particular starting point right. So, itis a set of, this is reach ability set of a petri net as specified earlier, is a set of all markings that is reachable from M of tnot. So, this is an enumerative process, how do you find reach ability set, great actually enumerate right. So, that is we talked about in the last class where you can look at all possible combinations. So, falls right, all these sequences have to be laid out and then you keep on adding right. So, that is an enumerative process is exhaustive but that is one way of finding out the reachability set, but that is the definition of this reachability set. So, this is the set of all markings, it can reach from initial state M of t not. So, then we define a reachability graph which is what you had you know drawn earlier.

So, a reachability graph has a name defined and this is a directed graph. So, reachability graph for petri net consists is defined as a directed graph where the nodes are, whose nodes are the, so what are the nodes for the petri nets for this reachability graph, all the markings right. Set of all the markings that are possible in the particular system, all the possible markings otherwise reachability set, yeah reachability set from whatever M of T not right, so this is the reach ability set. So, those of the set of nodes and edge exists from from one marking M A to another marking M B, if there is the if M B are if there is a transition from, this not reachable, this is just immediate single transition, reachability reachable means it sequence of transition, here it is only a single transition right. So, from M A, I can go to M B, if there is 1 transition that will take me from one to the other right.

So, if M B is reachable via a single transition from M A and we label that edge with transition name. So, what did we do before, you can look at the ctmc only for a special case, but so the label of the edges the, you simply label it with the transition, that is your basic reachability graph,is it clear? So, if you go back and the previous diagram. So, here this is I talked about this being rates, for time being justrates, but this simply says this this is marking 1 2 3 is a three possible marking and the label on this edges nothing but the transition that is used right, one or the two transition is T failure or T repair is what is being used going from one to the other (( )) define the rate we will do that in just a second. So, (( )) graph is just that set of markings from M of T not and the label is simply the name of the transition that is used to go from one marking to another marking. So, this now this graph if you look at assembles like a mark out chain right, it could be discrete time, continues time, we will get to those in just a second. So, that is the basic notion of water reachability graph is, (( )).

So, given any P N, I can come of this, only thing is you have to sometimes use a tool for automatically generating which you can do manually, but sometimes automation helps right. So, there are some tools are there, which have not had time to try at, I downloaded something call pipe which is can be, which can be used for with (( )) actually draw this diagram all the labels and transitions and so on. Then it will take care of giving some of the performance results from there right, it will construct the underline, if its stochastic petri net will construct the underline mark out chain and then solve that for particular performance result. So, we leave that an exercise for you to figure.

So, now this is the basic notion of what the petri net is, we have seen several examples of those. Now, there is some extensions right, so there are lots of different types of arcs and lots of different types of Petri nets also.

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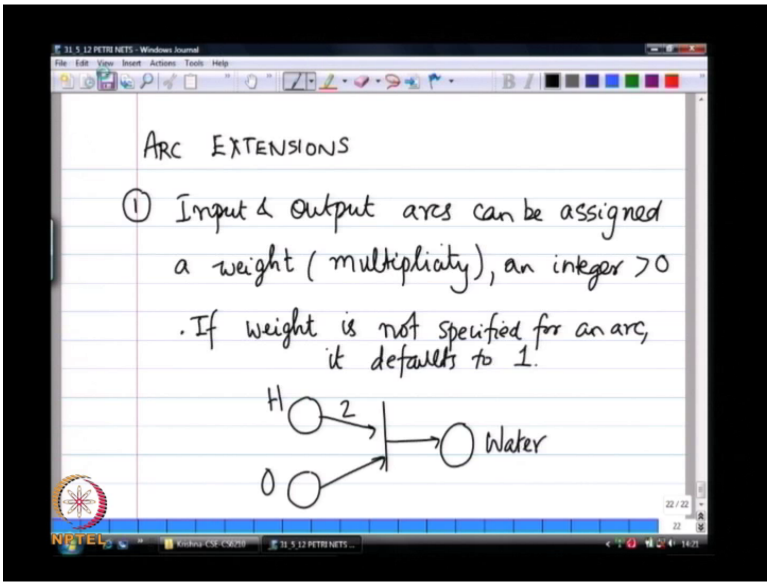

So, we look at some basic extensions right and see what those imply. So, the first extension is, we look we talked about arcs having whenever saying that whenever whenever transition is fired, right one token is removed from the corresponding input place and one is kept in the output place that is what. We can generalize by saying that, in star having one necessary from each of the input places, you can make it some arbitrary integer. So, as long as there are at least k elements in a particular place, only the corresponding transition will take place right.

So, we can add no weights to each of those arcs one in the input side, it says if I say inputs side, I need put the number 3, it means at least three units or three tokens should be there in the corresponding input place, you could 5 on the output place, it means there five units are going to, five tokens are going to added to the corresponding output place, that is one particular extension and some cases that is actually useful say will see how that is right. So, the input and output arcs can be assigned some weight, we are used link weight, so we will call this the weight, but technically they also call this multiplicity.

You have done UML(( )), in UML also you have specified, we are creating an object right. One object is composed are composed of other object, you can say how many multiplicity you can specify on those arcs, we can back look at UML diagram anyway. So, that was that was this is weight can be specified which is an integer, non negative integer right which is to be greater than 0. And if we the multiply, if its weight is not specified, so what we is do is right P N, but we do not specify the weight, just leave that as it is, it means that default is 1 right, if weight is not specified for an arc, it is default to 1. In fact, all our diagrams before have been implying that right, I simply said only one token can go, even though there two token in this on state by the transition, I say there only one will go to the off state through the corresponding transition right, so that is the first.

So, if you want to represent this, so let say this is H,this is O,think this example some of you might have seen right. So, if I have two attempts of H and one of O, you get a quarter right. So, that is your H to a combination, that is have you can specify, we can of course generalize this to other examples that we know of.

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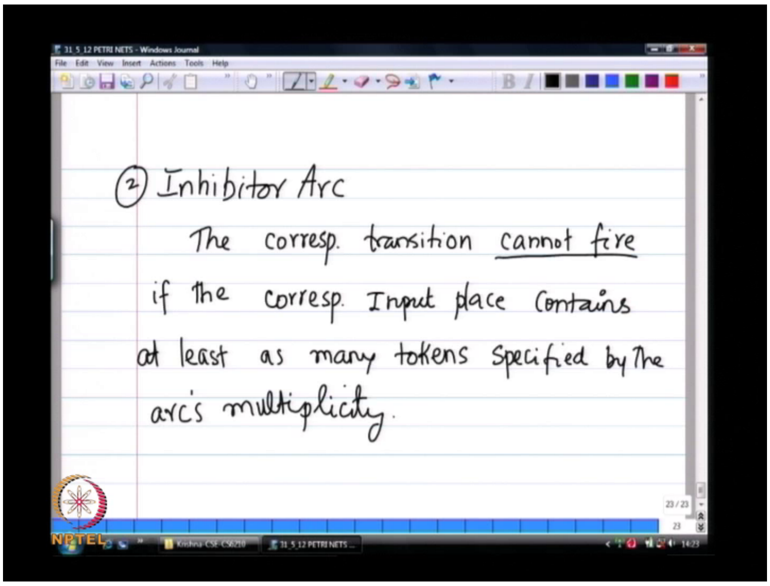

Then there is something else,call the inhibitor arc, sometimes inhibiter arc is used to prevent the transition from firing in certain cases. So, this means that if there is in inhibiter arc, then the corresponding transition, so inhibiter arc will end on a transition, this input your transition, the corresponding transition cannot fire cannot fired, if the corresponding input place contains at least as many tokens as specified by the multiplicity of that arc. So, this means if I put 5 as the arcs value, so then as long as number of tokens is less than 5, I will fire the transition. At least one should be there, but if it is anything 5 or more, it cannot fired, right; therefore that is that is that is again you know, we will at look at some examples where that can be actually useful could be even 0 right.

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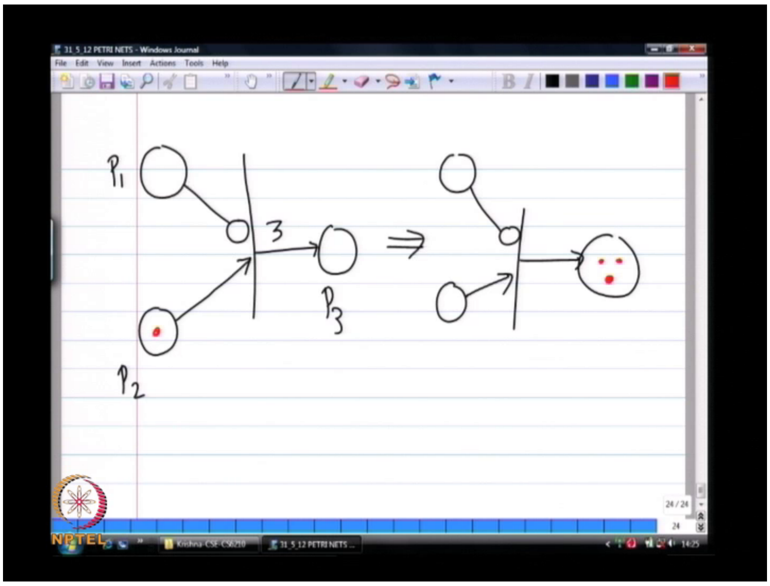

Let us look at an example for this same say if I define, so this circle at end of the arc in star of the air over represents in inhibitor arc that is all. So this is p 1, this is p 2, so this is the current state of the system and there is in other place p 3. So, what will be the behavior of this system? So, the definition is, if I do not specify the arcs weight, it is equal to one and by definition inhibiter arc will fire if the number of tokens is less than are lest if we less than the weight of that arc right. So, at this stage will this transition fire, will it transition fire? No, it cannot fire, because there is no p 1 is ok, p 1 actually meets the constraint, because it is empty. If it has got at least one, I cannot do anything I cannot fireright. So, I cannot fire at this stage, because p 1 is satisfying the condition, but p 2 is not, we need at least one, because I have not specified token value it weight is on each of those tooling.

So, therefore, at this in this stage cannot fire, if I add another token to p 2 right, then what can happen? Thenthen it will fire. So, therefore, now the conditions are satisfied, if I no token here and 1 token here, then you can certainly fire. So, what will be the output or the result, the resultant of this particular system,of course this stages the same, if I look at it pictorially. So, there will be no token, here as usual the single token here is now gone, right and how many tokens will be have in p 3. So, that is our, so that is what this extension of this arcs. So, we will see when necessary we will use this and so can we try to use this inhibitor arc to model our M M 1 B system.

Morning, we saw MM1 B, those extra buffer can we try to model that with inhibitorarc, how will you do that? So, where will we place that inhibitor, I do not have, I do not remember the exact solution, so wewill have to guess this.

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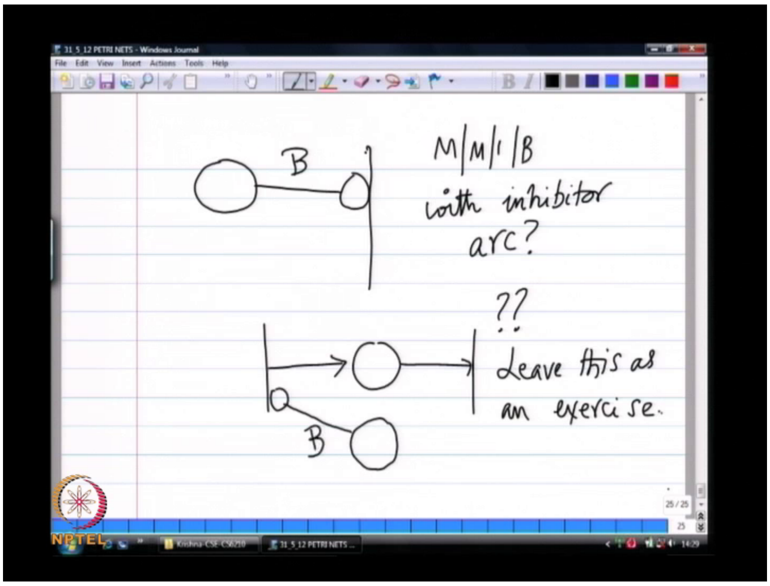

So, somehow I have to, there is some place here right which is holding packets and I have to inhibit this from sending packet if this is right. So, if it is greater than B, I cannot send it somehow, right this is wanted to somehow capture in my system. I have to feed this to some transition. If there is less than or less than B packets I can forward it for servicing. If there is more than B packets, I cannot forward this for servicing, how will you capture that, so how do you model M M 1 B with the inhibitor arc?Can we somehow capture that, you need a transition before. So, let us a let us try to this piece means, so I have they arrival process.

So, we needed have an input before here or no, we do not have, so packets are getting generated here. So, then where does it go? It goes to the queue it goes to the queue here, then (( )) machine come? I have to somehow prevent this from firing, if it is got more than B elements in the queue. So, it is the packet is come and I want this to be added to thisqueue only if some condition is satisfied, right what could that condition B. So, let us say that I have another entry here, another place here and I have that also as an input this particular system. So, this will fire and I specify this to be, say B if it of and I am trying to the figure this out, this is probably not completely correct.

So, what will happen is, this transition will fire as long as the number of items held in the place is less than B. It is less than B, then what will happen? We still have to somehow process these things right. So, what I am putting this in the temporary buffer, so where how will you the processing part of it? So, this has to go to some other transition, this has to go to some transition and will this is be the server transition or it will be the some other transition. So, initially there is nothing here, packets comes in, it gets it goes here and let us say it gets service and then therefore, it simply leave the system.

Now, the second packet comes in and I will put those two packets here, again put two packets there, it is going to get service. Then third packet comes in, where will I put those to? Somehow I will keep track of that token counter, how will you do that with with this inhibition, somehow I will have to store those other packets, we saw a different way of doing buffers in the morning. So, here how can we do that, we will leave this as an exercise. So, keep thinking before the class ends, if you find the, think of the answer then you will include that. So, these are some other cup in basic arc extensions, now there are other extensions also.

Sofar, we have talked about this notion of timing present in any of this, in the petri net itself. We said there are states, places we said there are transitions, but there is a need to also bring in the concept of timing into the system for certain kind of models. So, there are different places in which time can be included in a petri net. So, we will go through those and we will finally stick to just one of those.

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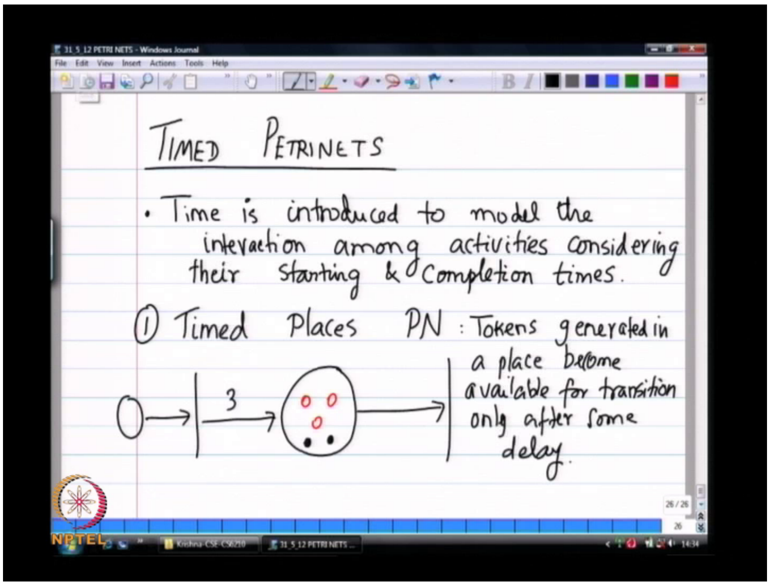

So, we saw the extension; now let us look at a different kind of extension called the timed petri net extension. So, the basic ideas to introduce time, the notion of time. So, time is introduced to model the interaction among activities, considering the starting and completion times, because time is very crucial for lot of the modeling that we tried to do. It is not a static system, there is like every every activity take some amount of time.For example, processing, get data packet, getting data from input, output. All these things are getting out from from the I O bus, all these things take time.

So, how are we going to bring the time notion into the picture? I gave a brief glimpse of M M 1 earlier in the morning right, even though we did not talk about timing yet implicittime there. So, let us look at some things, we can introduce time in four different places. So, one is called timed place petri net. So, we are we can introduce notion of time in a place which means it. So, what happens? When a tokens get added to a place, it will takes some that, that is like cooking, that is like getting puttingthe water, but token is not ready for consumption until some period of time is elapsed.

So, may be take set of few milliseconds before the token can be considered as available for consideration for the corresponding next transition to fire. So, what we say is, I could have a state sorry place which is feeding into a transition and this itself in turn is fed by some other transition. So, there is some place, there is feeding this into this particular transition, and when after an token is added to a place, it is not instantly available for consumption by the next transition. Even noodles take two minutes to cook, so that is what this is not really instant noodles; it takes some amount of time.

So, there are two kinds of packets or two kinds of tokens. These red ones can be considered as not available, not ready and the black ones can be considered as available, ready. So, that is the time factor, so the time for the token generated input place becomes available only after sometime. So, this is tokens, so when will you generator when of this transition fires tokens are getting generated. So, let say that this is 3, the multiplicity of that is, so the tokens generated in a place become available for transition only after some delay that is called a timed place petri net, this is only definition for now.

We do not really we are going to really use this, but if you look at it logically, I can add time inmany places and this is this is one of those place right. So, there is some time for this to become available, it is like you know it is like trying to introduce some processing delay in a place itself. We normally talked our processing delay in a activity, but this is now saying that even within the place, that is going to be some time delay. But it is not that much relevant to what we are going to talk about, so that is one place.

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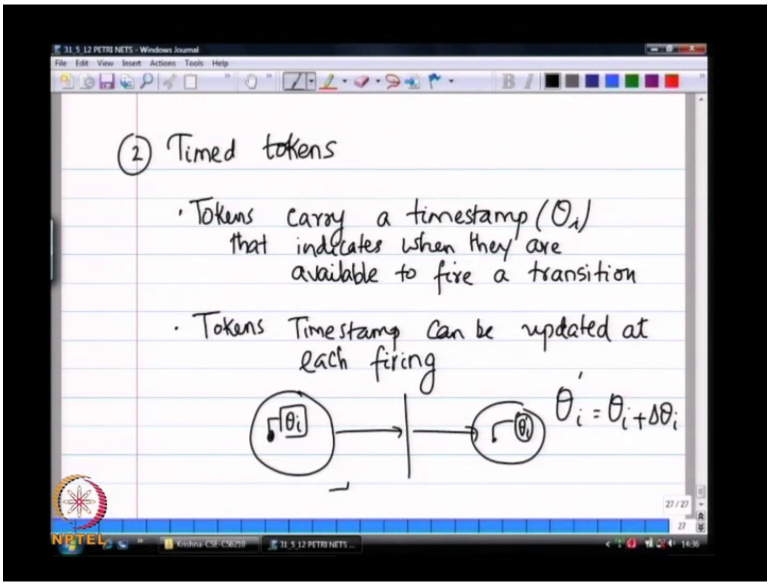

So, where else can we add time?We can add time in the transitions and in the tokens, so we will come to transitions. But before that, we will come; we can also add time in the arcs. So, four places, I can addin the place transition in that token itself, I can add and then I can add delays in the arc itself. So, let us look at timed token. So, now, tokens will carry a time stamp. So, you created token, then you carry a put a time stamp like we do with packets. We do put t t l on that, tt l says it has to be dropped by this time. This is little bit different; it says that it is available only after this time is elapsed. So, token can travel, but it is got time stamp implicitly built in it, similar to the place business. But now, the token itself can be, you can update the token on a time, on the time on a token as it goes through difference places. So, tokens carry a time stamp, say theta i that indicates when they are available to fire a transition.And as the tokens, that tokens time can be updated, time stamp can be updated as set progresses in this system at each firing that ten counters.

So, again pictorially we can again, we candraw this things by hand, but some of there has to be a system that understands how to attach times stamp token. So, here is this thing right, and then it goes to the next thing and then it will have you know updated token on time stamp on that. So, this for example, this time stamp, the new token would be the old time stamp plus some delta i,it could be a fixed variable, random variable, all this things are theory possible right How it is going to be useful? That depends, that is we are not talking about that, but I can add notion of a time stamp to a token, is it clear?

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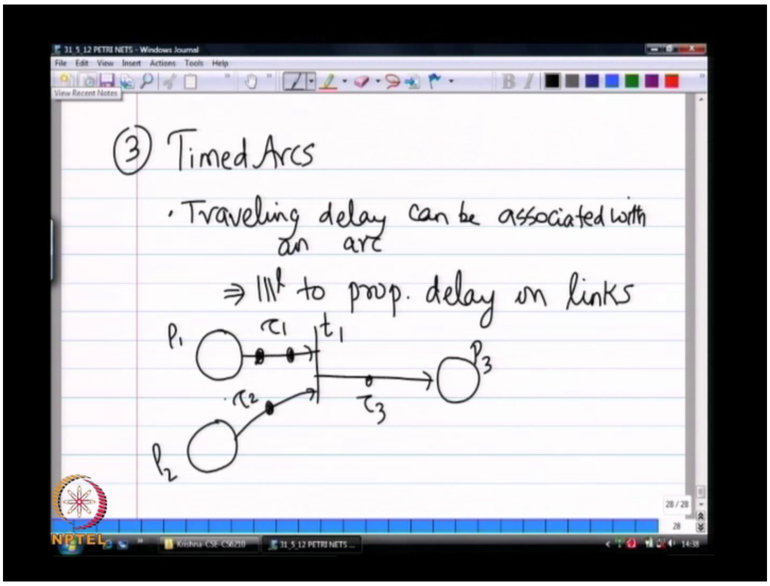

Now, you can add times to arcs, it is like propagation delay, there is a link on this and it take some finite amount of time. Once a token leaves a place, it takes a finite amount of time for it reach the destination transition. So, there is a delay that is we also incurred that right. So, therefore, the travelling delay can be associated with an arc right, this is similar to propagation delay on links.

So, it is now possible if you look at transition, it could have several packets travelling on the transition. So, let us say tau 1 is the delay besides the weight that we normally specify one packet could be here, one packet could be here, one token.Therefore, along at via different tokens can be travelling and different stages and likewise, I can define something like this. So, this is tau 2 and then it finally, you know goes to tau 3 and p 3. So, I can also time the arcs, this could be useful for some cases and that will make some if here looking at system like 5 planning.

For example, our assembly where it takes some time to move this part from one other part of the factory it is some other part of the factory and that is non trivial time. It could take 5 minutes, 2 minutes of ship and that is also has to be considered in the total delay of this system, right it is not just the processing time, this travelling delivery time is also crucial, so now, that is timed arcs.

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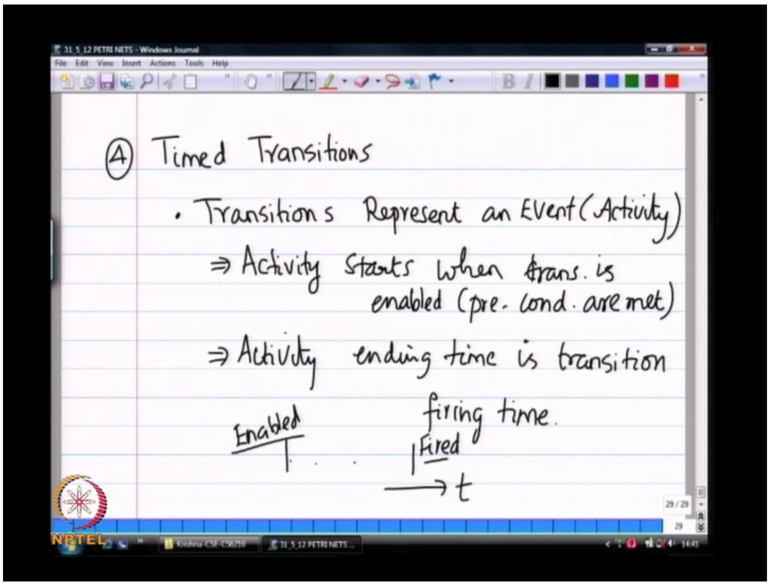

So, now we will come to our timed transitions, and a timed transition is something that we can quickly all relate. So, transitions represent and even are an activity right its some processing there is going on event or an activity, that was the transition represents. So, then we define the, so then activity right start. So, there is a notion of a transition getting enabled, which means that is get started, but then theactual firing of the transition will take place some tau units later on right. So, transition enabling means the conditions are met right starts when the right transition is enabled.

So, all the preconditions are met right and we do not worry about what happen is conditions change right in between the, before this transition complete. We assume that the, those conditions still hold and the activity ending time, basically when the processing whatever is complete is when technically the transition firing time.

So, there is a gap between enabling the transition and firing the transition thatis what this is. So, there is you know, so this is this is time, this is the enabling of the transition,transition enabled and then transition is now fired, so that has to communicate function. So, this is so called transition time, retransition. There are different types of firing, we just there something will atomic firing which is what this used, we will for now just not worry about the detail, because is that default thing that we will talk about.

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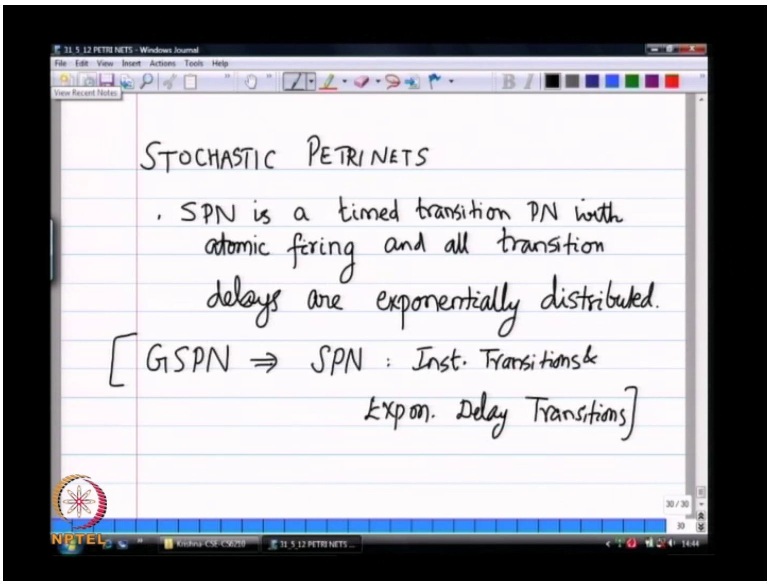

So, now, we come to the special class of time petri net called stochastic petri nets. So, stochastic petri net or S P Nis a timed transition petri net, where with atomic firing and all the transitions delays, all the delays associated with all the activities are distributed with some well- known random variable distribution and what that might be, exponential right or convenient. So, all the transition delays are or negative exponentially distributed or exponentially distributed. So, it is a special class of petri nets. So, it includes delays for transitions and the delays are all distributed with some exponential distribution.

There is also another class are called G S P N or generalized stochastic petri net which is basically stochastic petri net with two kinds of transition delays; one is instantaneous firing and one is exponential time firing, that is what is called as the generalized stochastic petri net. So, this is the stochastic petri net with instantaneous right firing, instantaneous transitions and exponential delay transitions. Because sometimes in a system, you will find the there are some activity that are just instantaneous, there is no need to delay them at all, it is like you know some things are, just take almost negligible time. So, you can consider that this is immediate transitions. So, questions, no questions.

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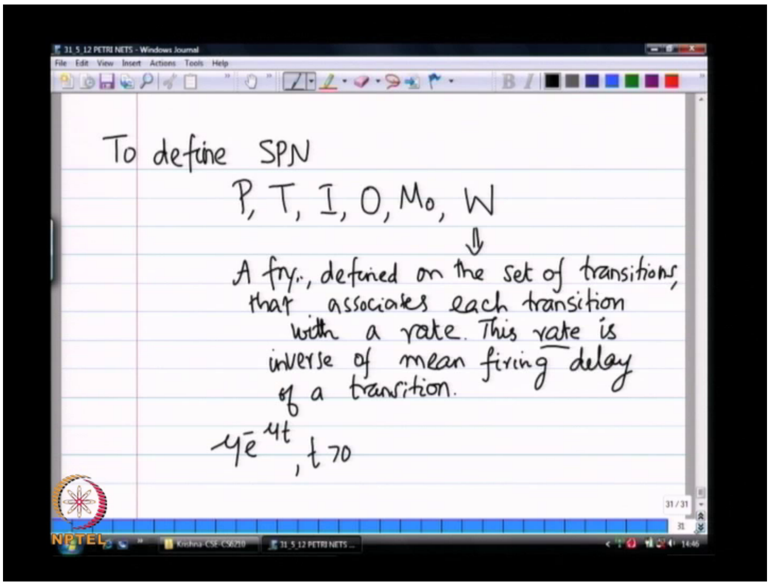

So, now when I define the S P N, so it define the S P N, we need this same things as before, the set of places, set of transition, your I your O, and then your M not is also needed. In addition, you need your W, soW is the set of this is simply right, W is the function of, this is basically the set of all transition delay, say transition rates as we callthat. So, because when we talk about exponential, we talk about new parameter rate exponentially (( )) mu x where mu is a rate we called at, that is rate parameter for the corresponding distribution.

So, there W will specify, this is the function defined on the set of transitions that associates to the function, defined are set of transitions that associates each transition with the with a rate and this rate is nothing but the inverse of the mean firing time. Ifthe delay is exponential distributed, it is got the mean value is 1 over mu right for when youdefined as mu e power is a mean firing delay of a transition. So, if I define right mu e power minus mu te power minus mu t, this is our definition of the transition time t greater than 0.

So, this is the probability of, this is our probability function and we know that mu represent the rate and 1 over mu represents the mean time that is spend in a given transition, mean time to fire a transition, questions on this? no.

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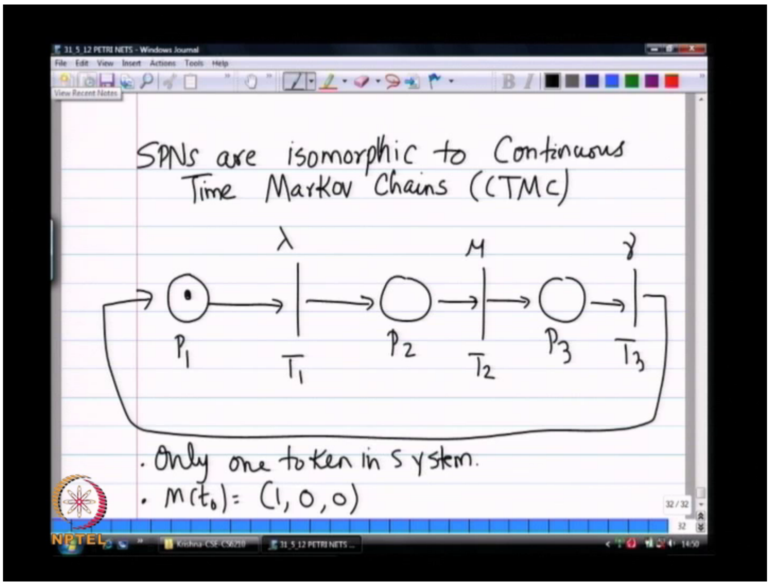

So, why are this stochastic petri net special? Because the reach ability graph of stochastic petri nets or isomorphic to the state transition diagrams of continuous time mark out chains, also called C T M C. So, I can take a petri net in a spine, derive the reachability graph and from the reachability graph, I can derive the corresponding C T M C.Once I get the mark out chain, I can simply solve that by all the techniques of seeing before and then once I solve that, I can find out the probability of given state system being in the given marking and then use there for getting through put delay in all those various possible calculation (No audio from 43:07 to 43:23).

So, let us look at some sample values, see how we can do this. So, here is in example of stochastic petri net, there is a place P 1 and there is a transition that it feeds into T 1, the rate of this transition is lambda. Then there is another place P 2, which goes to in other transition transition that goes to in other place, rate of this transition is mu, this goes to one more transition call T 3. I will call the rate of that is that, and then this a feedback loop here. So, this is our set of classical right close queuing network, we will impose the condition on this that there is only one token in this system, there is only one token, no token generation is possible.

So, I start the system by placing one token in anywhere P 1 or P 2 or P 3, there is only one this again this system right specification only one token, I can make this multiple tokens it is not a problem. But for now, we will say there is only one token in system where that easy at set that state space, otherwise the 2 minutes space state state space combination it. We saw this when we did mean value analysis on the extension, two different techniques itself, only one token in system and the M of t is 0, it is simply 1 in P 1 and none in the other tool.

So, now what happens? There is a token available, these two transition is will not fire, this transition will fire and the mean time to firing is 1 over lambda right, this is exponential distributed. So, after that time the token will get transferred from here to here, and then it will spend some time here, will get transferred and so on. It is like going through sequence of 3 servers, 3 M M 1 server where each server's service time is given by this lambda mu and gamma parameters right. So, this is the S P N specification, now how do it translate this to the C T M C? So, this state of the system is defined as a (( )) because C T M C require states right. So, how do we what of the states?

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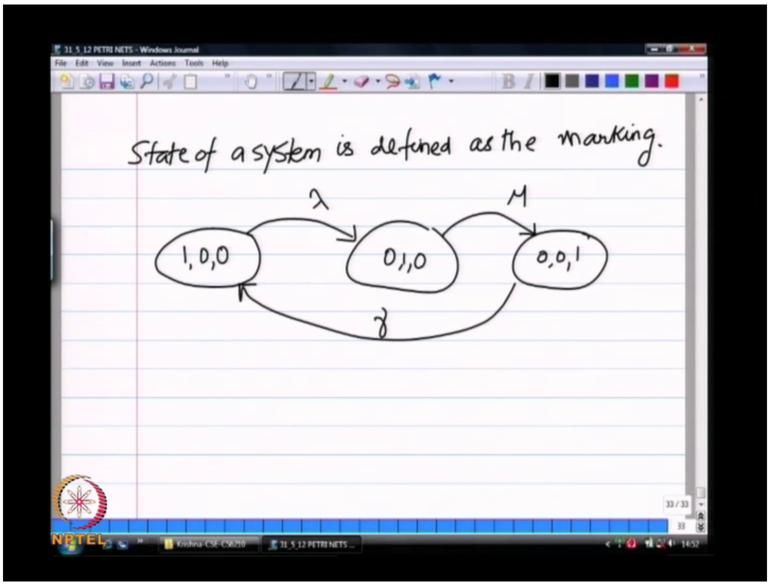

So, state of the system is basically the marking right. So, if you look at the as the marking value right. So, what of the possible marking values? The marking values are 1 0 0, then there is another one which is 0 1 0, and then there is 0 0 1 right, because there is only one customer one token in the system where any point in time, this is the only three possible state, three markings. And the rate of transition from here to here is lambda, rate of transition from here to here is mu, and the rate of transition from here to here is gamma,this is my C T M C right. And then I can solve this whichever way want to quite happening, I can solve and find out what is the time that is spent and right each of this state.

What is the probability of spending time in this marking, in this marking, this marking and then you can do some set of delay analysis in this system. How long this take before a packet that are token that is start here comes back to this right. Basically when we come back, that means that all the three, all the three servers have been red dealt with, you have been service by all the three servers, you will come back to starting point, that does not defines the through put of the particular system right. So, probability of being in this state times this lambda which is the rate of the particular thing. So, questions? Clear enough.

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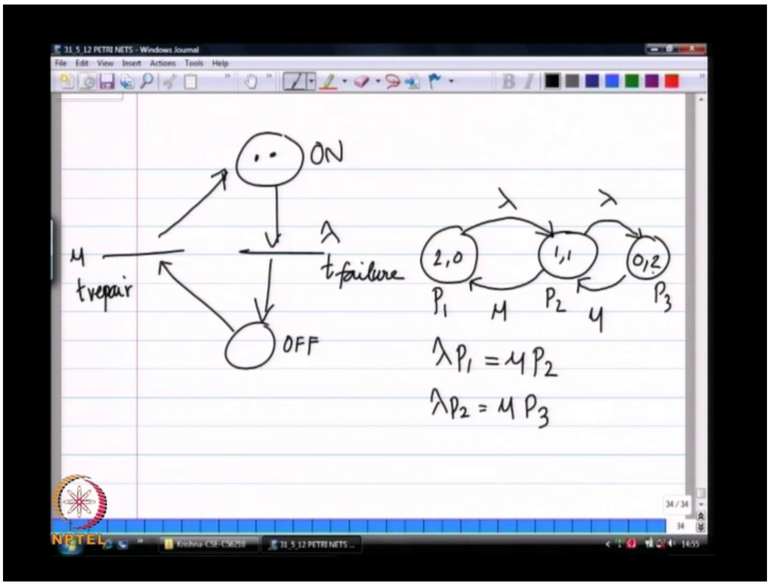

So, if we can again apply this to the other model that we saw before right, the model with failure that on, off situationwhere I can I can see the same thing as before, I have youknow this is on state and there are two tokens start with and then this will be right, this will trigger. And so again if I say that the failure it is lambda, this is the transition failure right, so t failure transition. So, then we go to the off state, then the mean time to repair is mu, this is other transition, I get simply goes like this. So, this is the, for for this particular system, what will be the corresponding C T M C? What do the What do the possible states for this system? We saw that before three states are possible. So, it is given to 2 comma 0 1 comma 1 and then 0 comma 2. So, if you want to find out what is the fraction of time both the systems, both this machines are operational, I simply need to find out the steady state probability of being in this state.Now, what is the fraction of time that both are them are down, that will be the steady state probability being in this state right.

So, what are these? So, from here to here, what will be the rate of moving lambda and then from here to here it is again lambda, from here to here it is mu, from here to here it is mu that is here corresponding C T M C, and this if you remember, it is like a M M 1 B system, M M 1 this is like M M 1 2 where there are almost two packets in the system. We can very easily solve for this correspondingtransition probability, we will do one example at the end. In fact, even we can even, here we can try to solve the corresponding transition probability right, how do we solve the transition probability?

I want to find out the probability of being in the state 2 0. Let us you know call that, since we call this is right. So, this is P 1, these are 3 states,solet P 1 be the steady state probability of being in state 1, P 2 of being in this state, P 3 is the probability of being in that state. So, can we write the corresponding balance equations, so what will have be? So, lambda P 1 equals mu P 2, and then lambda P 2 equals mu P 3, because this is the probability of being in P 1. So, the rate of leaving is lambda into P 1, the rate of entering in the state is from P 2, I will enter the state, and rate is again mu probability of being in the rate is P 2.Therefore, this is lambda P 1 equals mu P 2, likewise lambda P 2 equals mu P 3, and these are the two equations that we have, any other equation possible? We have only two equations right. So, can we solve for P 1, P 2, P 3? How will you do that? So, let us express everything in terms of P 1.

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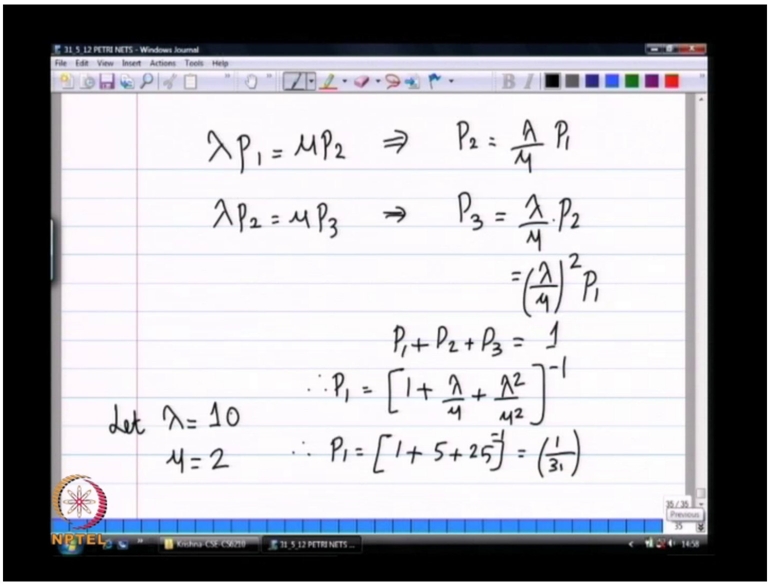

So, what is P 1 equal to? So, lambda mu 1 equals sorry, solambda into P 1 equals mu into P 2. So, if I express P 2 in terms of P 1, what will that be? P 2 equals lambda by mu into P 1 and then we said lambda P 2 equals mu P 3. So, what will that give me, P 3 equals lambda by mu into P 2, which is again lambda by mu squared into P 1, so I have expressed P 1, P 2, P 3.

Now can I solve how will I solve and get the value for P 1,P 2 and P 3? I still need one more equation, I have three unknown variables and what is the third equation?(( )) one right I said this is the probabilities. So, P 1 plus P 2 plus P 3 should equal1. So, now we know that, so therefore, P 1 is going to equal to 1 plus lambda by mu 1 plus lambda squire by mu square to the power minus 1 that is myP 1. So, therefore, the probability of being in steady state in the system is just that 1 plus lambda by mu.And if you want plug in values, let us say lambda is mean timed failure, let us say 10 that is we want lambda to be very large, we want mean timed failure to be fairly large.

So, let us say that equals 10 and I want repair time to be fairly small, so let us say that mu equals 2.Therefore, what will be P 1? 1 plus 5 plus 25 to the power minus 1, so 1 by 31 to the power minus 1, is that right? No, 1 by 31, they are right is in to be right. So, probability of being in the system where both the servers are operational which is actually 1 by 31 is that high or low, that is very low only 3 percent right, one y is our because our lambda value is so large right. So, what should happen here? So, I want my lambda, if my lambda is even 1000, then what will happen?(( )) that is M C M , I want lambda to be very large. So, the mean timed failure is very large, then what will happen what should happen, the system should never even go to 1 1 state right sorry, this is the, lambda equals 10 means that the mean time to failure is actually point 1. So, lambda should be very small right, so that meansthat every point of 1 units of time, one average I am going to fail and that is not the particularly good scenario.

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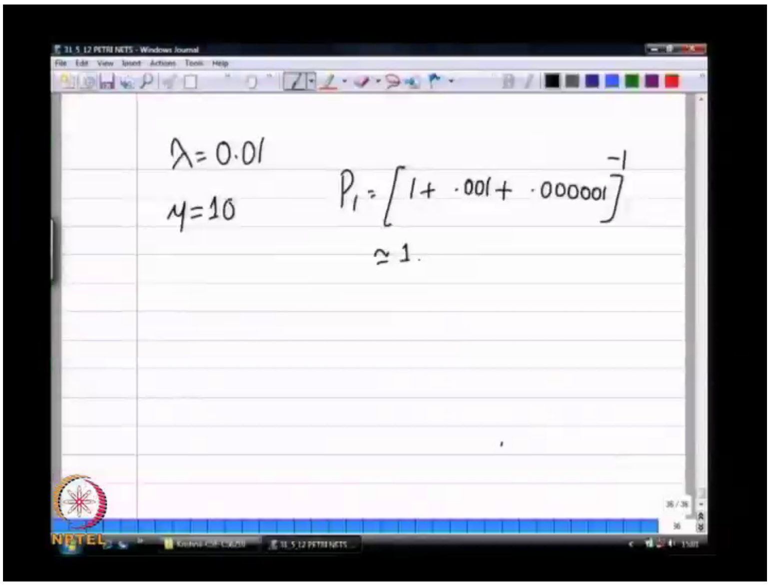

So, we will go to back in revert this, so I will pick some small value for lambda. So, which means that the time to fail right is the mean to time fail is 1 over that which is 100, and I can pick again mean time to repair also should be small right. So, if I want the mean time to repair to be say 10 units of time. So, every 100 units of time I can fail, but I want if repair to happen and average with say 0.1. So, mean time to repair should be 0.1,therefore mu should be say 10 that case, then you will find out that lambda be mu is fairly low. So, row is low right we remember this M M 1 B what happens?

We want row to be row is very small, what will happen to row M M 1 B what stated going to being P not where the system is mostly ideal, a same interpretation,, but here it is not ideal, it is simply means that both the servers are essentially operational. So, if you want be in P 1 in this particular case to the extent possible, soP 1 is now 1 plus 0.001 plus 0.000001 inverse right. So, therefore, that is a very very small addition to 1,therefore this kind of going to be hopely closed one, some small value. So, that is serve the representation from the petri net side converted to C T M C side and you can get the corresponding. You might ask, why should I go to petri net actually do that could on the straighten with the mark out chain, you could have, but that is ok. We want to show an example of how you go from one to the other, but sometimes systems can be very complex, especially concurrency is not directly capture in a mark out chain right.

So, you will have to capture concurrency with the petri net and then translate this state into what will do here. And so that is where the potential benefits of the particular system are. So, questions any questions?

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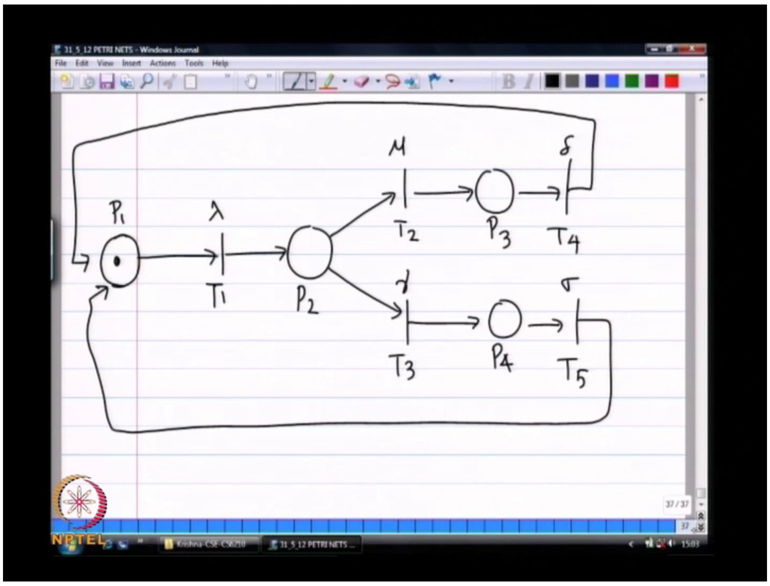

So, now let us look at another similar example, only thing is this, we will come to this will again look at notion of conflicts. So, there is one state P 1 which is getting fired by transition or connected to the transition, then from P 2. So, transitions T 2 and T 3 are again defined by the corresponding expansion, this parameters mu and gamma, this goes to in other state P 3, we will say this goes to in other transition T 4, and this is with some parameter delta.

After that, I come back here and from here I go to P 4, from P 4 I get connected to transition T 5, and say sigma is the parameter, and I come back here, and I can start this system, I can start this system 1 token, 5 token does not matter, any numbers of tokens I can have. So, now, what do we see here, this is I can also represent this. Now there are, let us assume that there is only 1 token in the system as such, only 1 token and it moves among this process, in that n equals 1 in the M V analysis that we done earlier. So, what are the possible markings of the systems now? How many process able markings are there?There is only 1 packet right, so it is either P 1 or P 2 or P 3 or P 4, so only four combinations.

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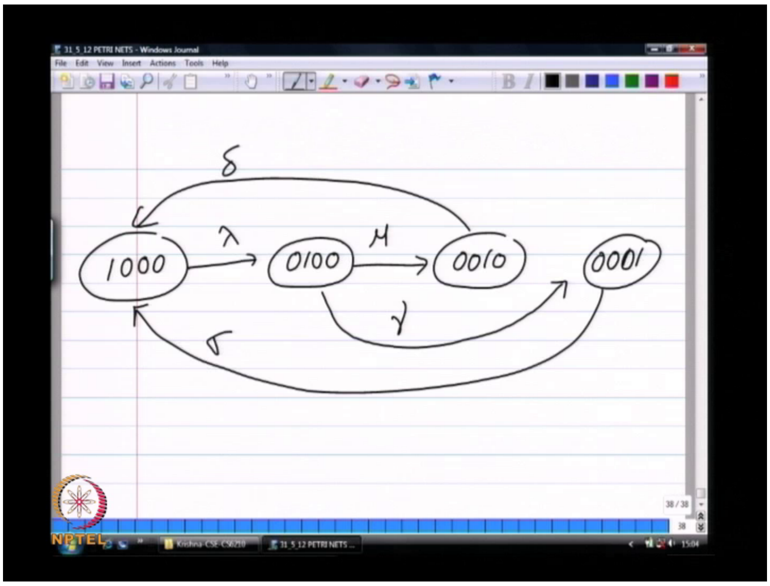

So, just like before, so this just productive of what you done before. So, 1 0 0 0 represents the state where the token is P 1, 0 1 0 0 where it is in P 2 and then and so on, P 3 and P 4. And here is lambda, there is transition from here to here, right this is with rate mu, this is with rate gamma from here we come back, and from here we come back with a rate sigma. So, here that is only one catch what is the catch? Did anybody get that?

I can go to either T 2 or T 3when there is only 1 token in P 2, which of these 2 transitions will get fired? You did not know, there has to be some other information, is it random or some priority based information has to be provided, this is remembered what the system this is look like,does this look like any system we have seen before? You seen this lots of times and class, a CPUto disk, this is one CPU and two disks. So, what happens? The process get processed in P 1, this is the CPU is represented by T 1, after processing it can go to either disk, this disk or this disk after processing in that disk, it simply comes back to the CPU, this is one CPU, two disk representation and the notion of the probability has to be now brought in.

So, this state transition matrix will now also include the rate as well as the probability of that rate. So, we can this is something that we can you know if you are going to, so what is the probability of going from P 2 to either P 3 or P 4? When the service times are defined this way, to know what the probability of what will be the rate going from this to that, it will be mu by mu plus gamma and then gamma by gamma plus mu. But that is something that we can derive, but anyway that is one thing that we wanted to talk about.