Performance Evaluation of Computer Systems
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Lecture No. # 29
Convolution algorithm-II

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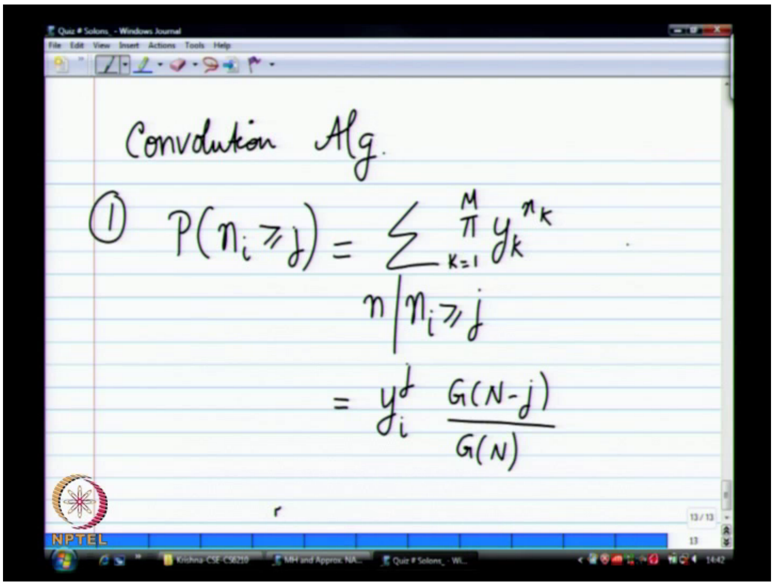

So, we saw exact MVA - approximate MVA,and this is a convolution gives you even more detailed results compared to the mean value results that you get from MVA, and soon. So, last time we saw how to compute that G of N, M right, that is again(( )) oneexample for the G of we did. We did we compute G values for particular system right; G G of 3, 2, we did computed right, so we have the table, that we computed last time. So, now, to look at. So, now that we compute these; this table right, and then given n. So, we know how many combinations are there, and soon right. So, now let us look at how to extract some performance metrics with help of this convolution algorithm.

So, last time every (( )) gave you one metric, now let us look at ourthismetric. So, probability. So far some particular queue right, what is the (( )) at least j customers in some queue I. So, this you can simply enumerate right. List all possible combinations right, we have this entiretable available right. So, just you can simply manually go through the table, and just routinely counted, but we can also do(( )). So, basically this is summation over all the possible combinations of rightn, such that n iis greater than or equal to j.

And we know this is our standard form right, this is our y i orand this which we can actually figureout, may be (( )).

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So, if you know G of Nthat remember thatwe compute the table, they will away looked at last time right, you also compute all the other values in the table right. Your G of N minus j by comma m is also computed in running that algorithm sequentially. So, you can simply store all the G values from one to n right, and then you would get your probability of that will be. So, greater than or equal to j customers in a given queue is given by this value.

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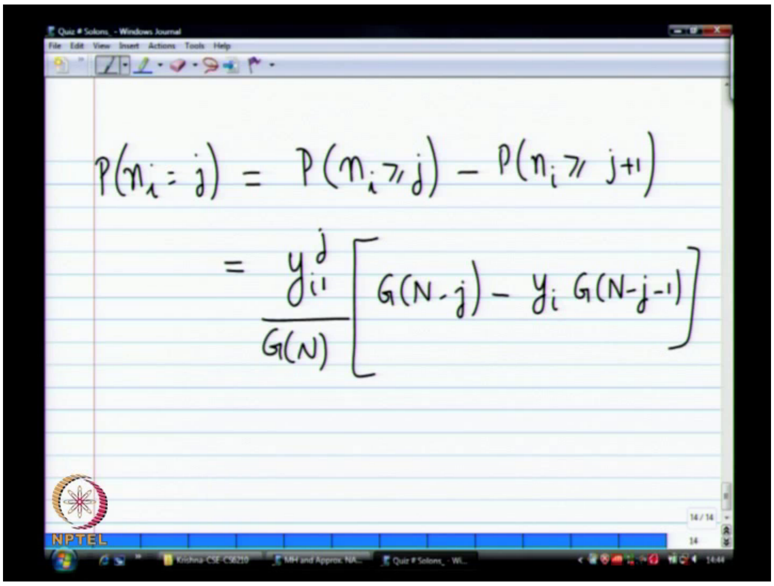

So, let us say one,if you want to know exactly how many customers are there; if there are what is the probability of exactly j customers in a queue i. So, that is simply probability that there aregreater than equal to jminusgreater than equal to j plus one. So, it corrects. So, that give you the probability that queue customers exactly equals j.This is second Ican to do this. So, this is y i to the power j ignore that something there at is…

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So, with just the demand value,you scale demand values, and the G table that we are computing, we compute this.

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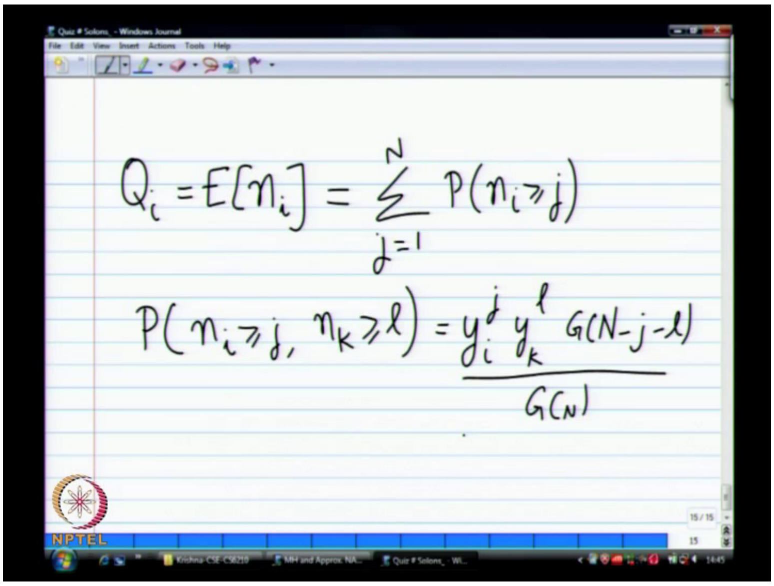

So, now, Qi,remember Q i you areyou are doing this mean value a right, we are write last time. So, this is the expected number of customers in Q iisgiven that the n possible customers, this is simply the probability that,n i is greater than equal to j. This is another defining defining expectation right. So, summation of all the CDF’s going from one to n is also the expected value of that.

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Then, soif you want to know the probability that there aresay, j customers in the i eth queue, and some k customers are sorry l customers in the k eth queue; that is almost similar to what we saw y i to the power j y k to the power l G of N minus j minus l. So, that is(( )) actual queue lengths right.

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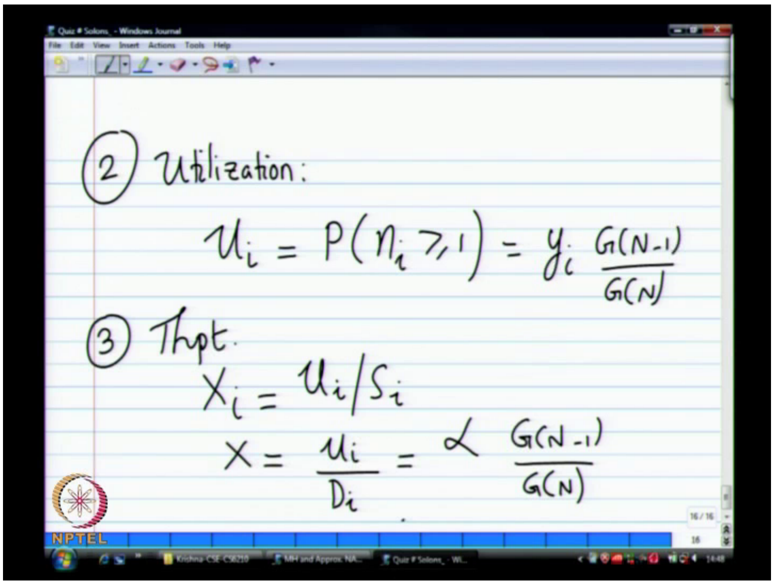

So, we now need the utilization, the delays and all that stuff right. So, what is the probability that a queue given Q i is not idle. Based on what we know sofor how do you define that probability.What is a probability is soutilization is basically the probability, this is theu i remember -u i is the probability that the queue is not empty. So, what is that? n i is greater than or equal toone right. So, if we already know that. So, that is simplyy i to the power one. So, this is G (N minus 1) by G(N). So, utilization is now deriving from this. So, then what is the throughput of a device x i.

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So, how do we get a throughput of device, remember u equals X, S;that is our first law right,utilization equals throughput into service time. So, throughput equalsS i is usually known service time right. The average service time is known, that is any parameter - input parameter.

So, we know u i you can compute X i.Then once you know x i you can always computeSi,and Vi,you can compute x right, but x can also be expressed differently right. So, x is also equal to u i by D i right; remember u i equals x into D i. So, u i you have computed here as a y i right it G (N minus one) by G of N.And what isD i equal to remember, we did y i is actually alpha into D i right. So, y i is the alpha D i divided by D i. So, it is simply, that is it?

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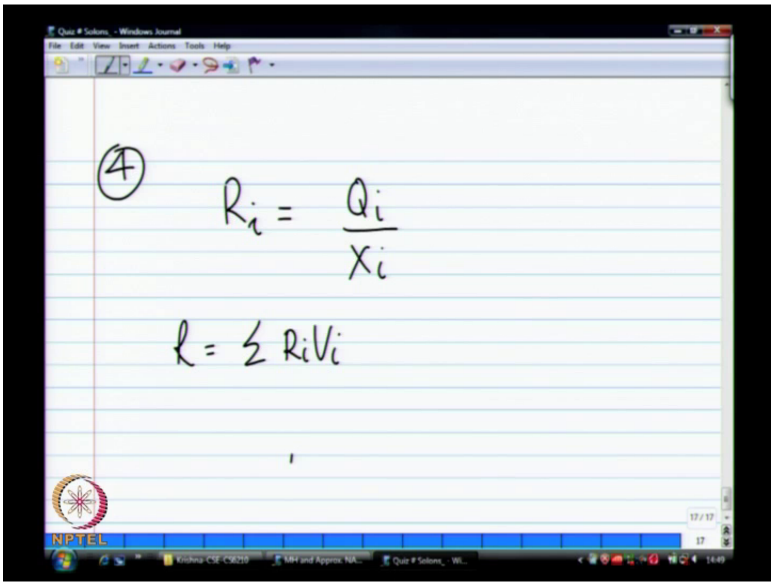

So, now, do we have everything to proceed to calculate R? So, we need R i. So, what is R i?R i is simplyQ i by right; Q equals X, R.We have computed Q i or expression before we have computed X i also right. So, therefore, R i that then after that we have. So, then you compute R,then you can compute the overall(( )); we know X, we know Rthat is what we now will try to compute.

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Questions. So, once you compute that table in everything is a just routine calculation.

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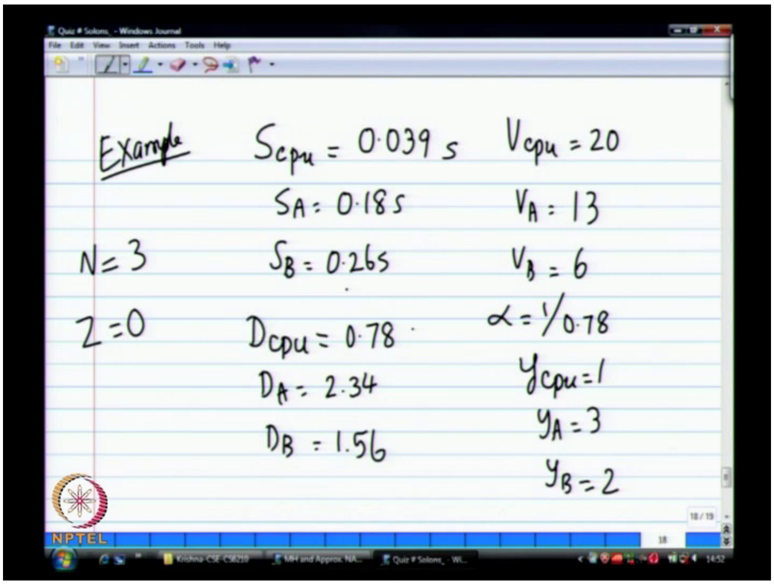

So, let us look at the same example that we saw before same cpu, two disks example. So, in that example we had computed S cpu to be or it is given to us that this 0.039. Then V c,VAis given,V B is given. So, thatis right. This is what we had last time, and then we computed D cpu is 0.78.

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And for alpha equals 1 by 0.78, we had computed that y cpu was one, y A was 3, and y B was 2. So, this is what has been given to us right. So, with just this,we can now proceed to get all the queue values(( )). So, with the mean value analysis with this information would be have, because N has to be given to right. So, N is given;N is 3,N is 3, and yournumber of devices also 3 right. And yeah z actually in this case we have taken z to be 0. So, there is now, there is a slight modification for delay centers. In this particular derivation, we are not considering delay centers; delay centers this is small extension that is all.

Number of devices…

Number of devices is 3, M is 3, then only 3 queues; cpu, disk A, disk B.

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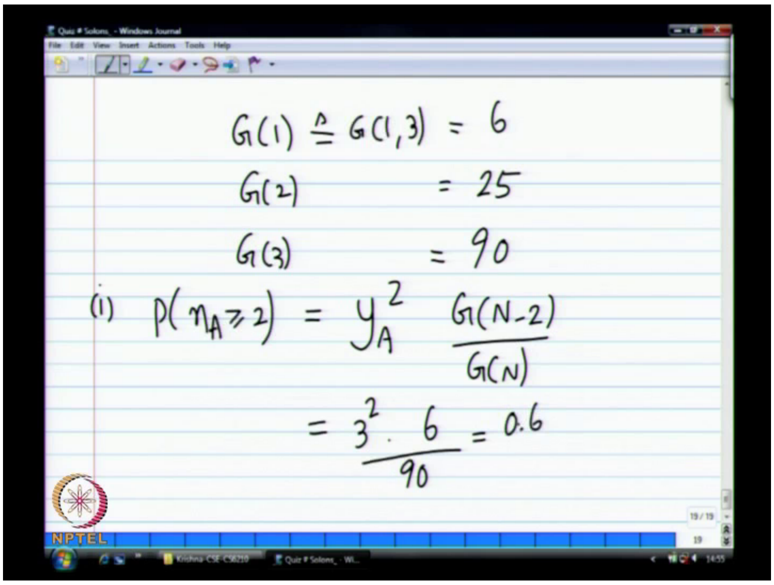

So then,we were interested in (1,3) right, we set G of N, M right. So, if G of N is basically nothing but G of1 comma 3in this example right. So, from the table what is G of N? What is G of 2, and soon for G of 2. This is basically a last column, columnMin the table.

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25, 90. So, we can compute a whole bunch ofthings(( )). So, what is the probability that there at least two jobs in disk- in a disk A. So, tell me just compute into the answer, then I write it down, 0.6. So, this is y A squareG of N minus 2G of N. So, this is 3 square n is three; therefore, this is 6byand 6. So, that is how we get the probabilitiesof more than 2 jobs in queue A. If you refer to the first table at we compute, ever we computed longer table tall at ten possible states right, where three jobs could be intend differentstates.From that table,can you find out what p of n a greater than or equal to 2 is.

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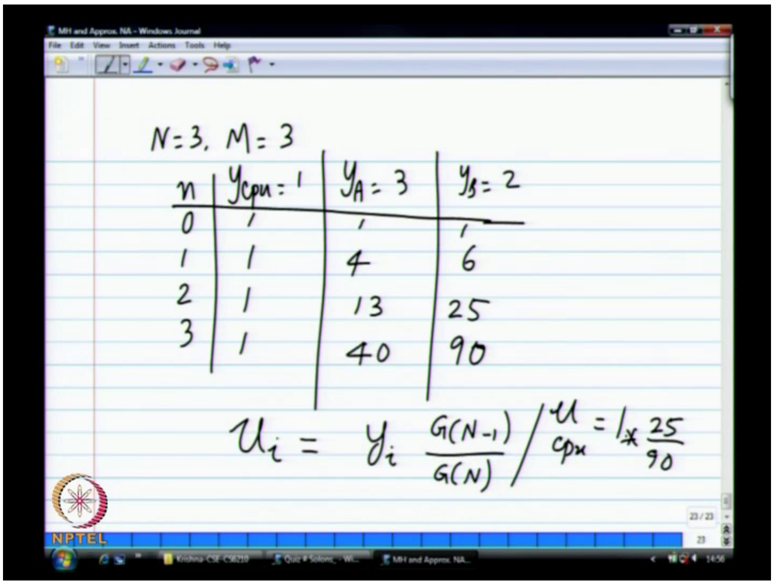

So, that table isbeing computed,here we do nothave to compute that table,because we add anywhere the table at a fairly substantial hash table right. So, if you do not want to do that, then Isimply we just the G value is right, Ican compute this. Is that is the order N, M; that was at one to enumerate all the items in the table is N to the power M minus 1.

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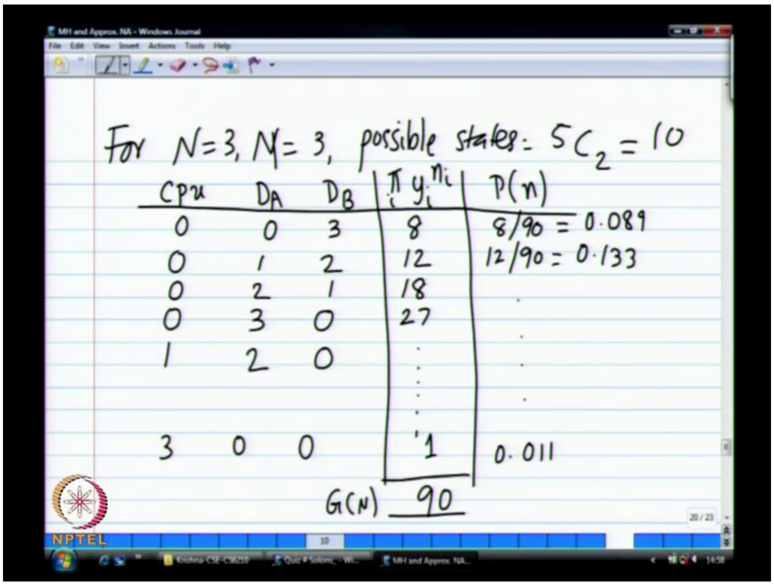

So, there are basically what are the states there.

12 states are there.

Out of that how many states will correspond what will looking for y A greater than or equal to 2;(( )) as this that table completely, but there will be3 states.3 states which will have this property. So, this is one state, this is another state then will be one more state well Iguess, if you are write it down that is not(( )) here, but you can work it out.

So, that isanother verifying that what we are computing is actually correct.This table also gives you the same thing right, if I compute this table I do not to go through all the other calculations. I can simply run my right, just run this as all excel formula in just add about our things that Iwant, but that is what it tries to… Given N and M this table that we did…

Sir, why have you last.

We have only three possible states, what are the other states in…

We can have 1 incpu,1 in D A, 1 in disk b; if I have one here; can I have 1, 2,and then 0. Any other any other combination, I cannot have any other combination; there only three jobs right. So, the only othercombination valid combination is this1 to 0; these are the only three cases where number of jobs in queue A is more than or equal to...This y a greater than or equal to 2.

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In fact that as you get the table here. So, this is a 0.2, 0.3, and then 0.1. So, that is all; proceed can.

(( ))

Is already there.

(( ))

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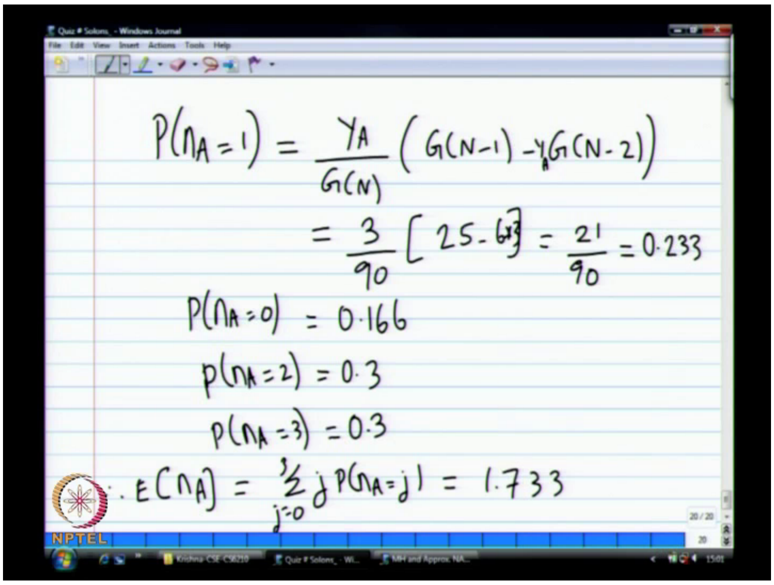

So, then if you want. So, what is a probability that there exactly one customer, there is exactly one customer on this queue. So, there is y A by G(N)G of minus 2 which is 3 by 90; this is 25, this is 6 sorry, there is a Y Ahere.

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So, therefore, exactly one customer is that, and then we can right (( )) do this (( )) for n A equal 0 which we want to compute now.

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So, therefore, Ican compute E of n A is simply.

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So, now, I have the mean, but how do I get the variance of the number of jobs in a particular queue, I have the probability distribution right.

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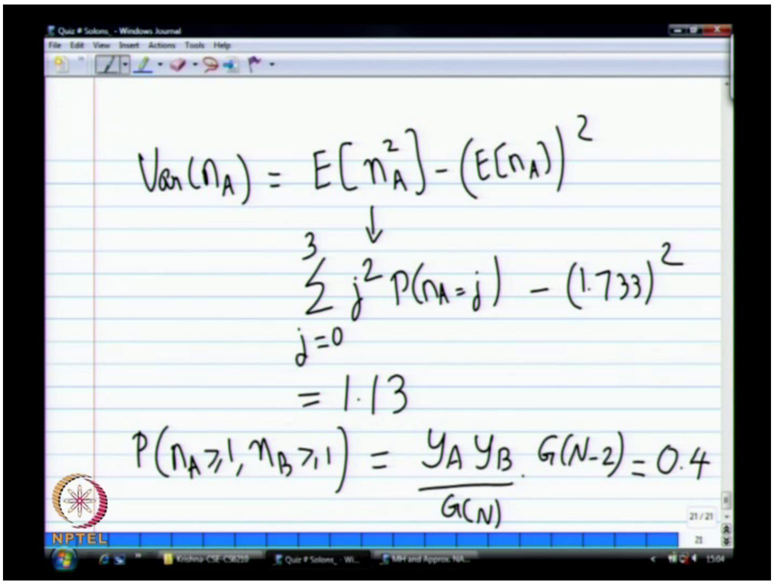

So, variance Ican compute simply, but E f j square, Ican compute E of n A square, and then right. So, we can, so if I want to know the variance of the… And E of n A square is a simple Iknow the distributions. So, this is simply j square (No audio from 20:44 to 20:56) and so on. So, this will work out to 1.13.

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So, what is the probability that, there is at least one customer in this queue, and one customer in this queue.

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So, at least one in n n Q A, and at least 1 in Q B.So, what is that? So, what is the formula? Y A into Y B this is divided by G of N up yeah into G N minus 2. (No audio from 22:35to 22:49) 4.The second again go back to the table and verify right, look at all the cases where these two these this condition is met; simply enumerate that right.And you can get the value that you want. So, we can now get all these probabilities. Now,what is the average queue length right. So, yeahwhatis the average queue length.

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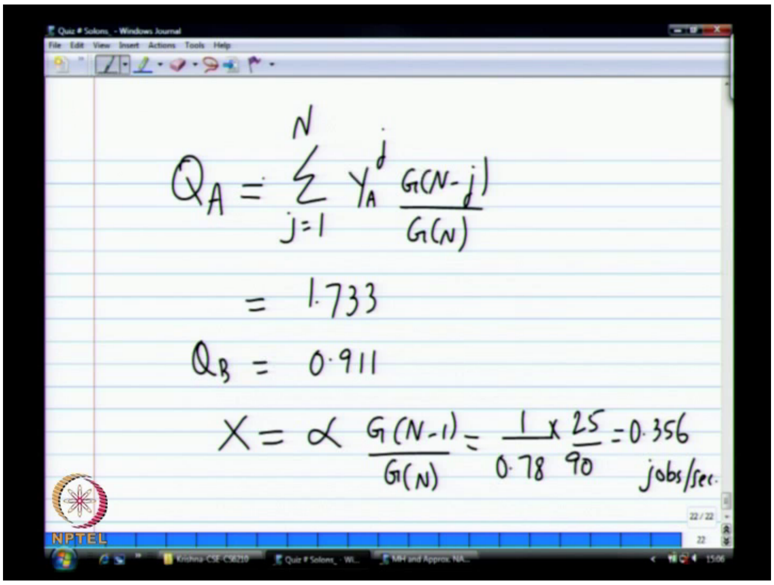

So, Q i is equal to probability that N A is greater than or equal to j, and that we know is. So, this is basically j equals1 to N. So, if you do this is best to write a program, but yeah you can,this we have already computed by the way which is right, you can also do it this way.You can compute Q B also the same way in that transfer to be 0.911right. So, we computed Q of A are basically of E of N Apreviously with N A equals 0, N A equals 1, 2, 3 and soon. But you can also get the same thing rightin a different way. So, after that this is fairly straight forward, togive a system,and Itell you compute this exact values onenot a big deal at all.So, what is X now?Alpha intoalpha was 0.78.

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Its correct. Yeah sometimessuspect a books answers. So, sothat is all you can just run through this, and then compute everything as that we want. So, that is the idea.Question on this part, sothis is the more accurate way of getting what we want.

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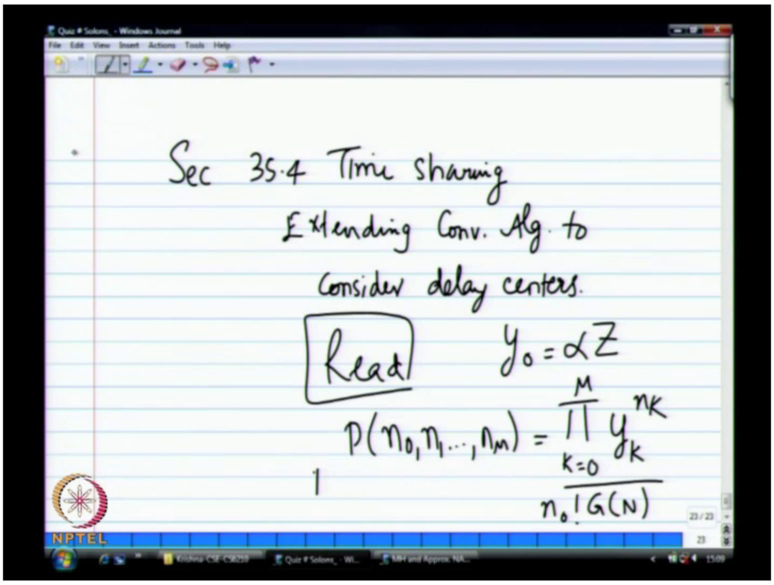

So, the next section which is a basically extend in this to include delay centers also, section 35.4 on time sharingright. So, extendingis convolution algorithm(No audio from 26:10 to 26:20) right,it is a fairly straight forward extension for this aread. So, only thing is the delay center is considered as another device y naught right, with with y naught is the parameter right as the demand for that. So, you simply scaleZ by the sameat alpha that becomes a y naught,then everything elseis the same. So, then you will have pn naught right. So, now, we have some customers in thethinking state which is n naught,and then the device queue is n 1 through and one1 through m. So, this is as same as beforeexcept.

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So, that is only term.If you have this n naught factorial in your denominator, which we did not have right. So, (( )) n naught equal 0 automatically became one. So, that is the only extension; that is the end of chapter 35.

Sir, if youZ value Z value should be 0.

(( ))just ignored than in the previous case that. So, if Zequals 0, you are then thestudent work, then y naught could have been 0; and then 0 to the powerknowthat case Iam just assuming.

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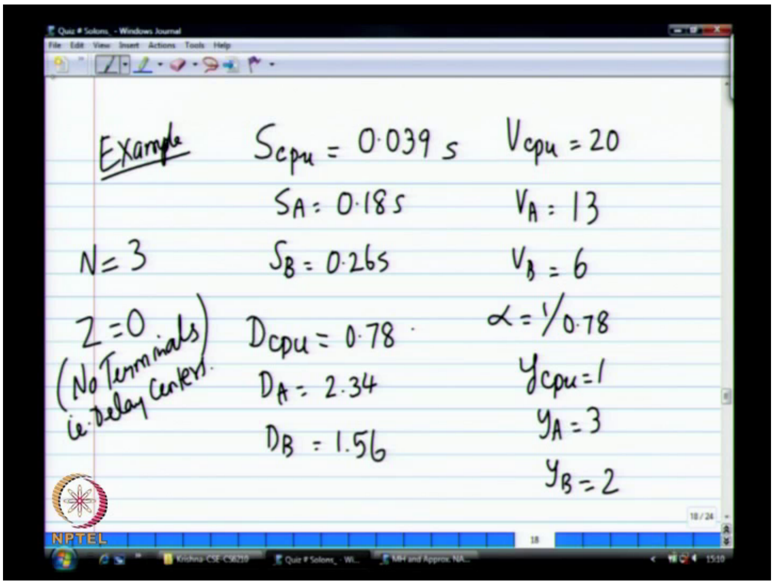

So, what Isaid here is, and Z equals 0 is basically right. So, this is there is no terminalsin the system, if you have terminals withwhere there is no this is the delay center in the system right. So, there is no. So, without delay center is what we aredone.

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So, two big topics still remain, the other ones are… After this third chapter 36, there is no other chapter in the bookbeyond that. So, we will go back to some(( ))chapters which are not as heavylittle bit easier to deal with. So, the remaining topics thislow dependent service center, that is one addition. Then this hierarchical decomposition, and I will finish those balance job bounds; Isaidcome back to that later. We are the simpler bounds first. So, those are the three big topics left to be done.