Performance Evaluation of Computer Systems

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Lecture No. # 11

Slotted Aloha protocol model and discrete-time birth death process

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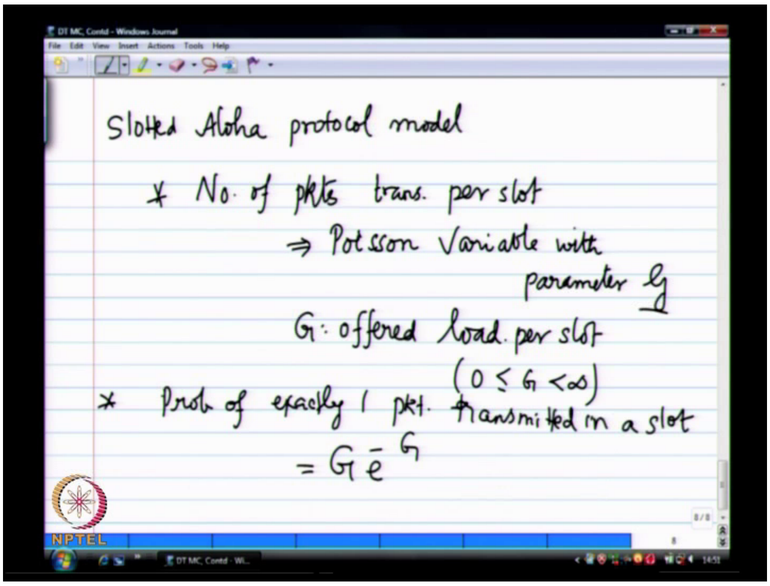

Now, another example of a discrete time Marko chain; so, this is our famous… So, all of us know this slotted aloha protocol, you know how it works. The performance question is what is the average throughput utilization of this aloha channel slotted aloha channel. Now, what is the fraction of time or what is the probability of success in each slot, because there is several users competing. So, what is the probability of success in a given slot?

That is what you would like to know and what is that, what is the probability of success in a slot what is maximum throughput in a slotted aloha 0.3 6 so, how did we derive that the Poisson. So, that is one way of doing that so one can that is the very it is there are lot of approximations is there, with the Poisson model. We look at the Poisson model first to give us where this thing. At the derive that already before G m minus G, it is the base the simple derivation which is a so have a gross derivation says that the number of packets, transmitted per slot so at each slot I am trying to calculate, I am trying to estimate the number of packets transmitted and the given slot. Now, if you look at a real system let say there are ten user out of that 4 user might out try to send a packet in the previous slots and.

They need up having collisions. So, these are so called back logged user to already have a packet to send and because of collision, they have a going to retry in this given slot. So, that is one kind of user other kind of user is who did not send a packet earlier, but certainly as a new packet in this given slot. So, in a given slot I have so back logged users and un back logged users or non back logged users and the both of them will try to send their packets together. And I need to I will be successful only if exactly one of them sends it because of this probability, there is some back of probability then and if it is a new user you always send a probability one. So, in a given slot if I can have some weight to modal the distribution for the number of packets transmitted in a slot, then I can do some computations based on that so the number of packets depend upon depends upon these two users.

The respect to probabilities of know retransmitting in a given slot and so on. So, we just know the early assumptions that is a in the others used is that, we simply preceding that this is represented by a Poisson process that, their number of transmissions per slot is in actually a Poisson variable in a given slot it is a variable, it is a Poisson variable with some parameter G this is so, G is basically the offered load in one slot in packets per second. It can be from 0 up to infinity slightly. So, G is basically the offered load first slot infinity. Now, tell me you what is the probability of success in a given slot which is basically, utilization so that means that exactly one packet has to be generated in this slot and may simply using my slot length is one.

So, whatever the slot length is so that's why the slot fixed slot length. So, the number of Probability so probability of exactly one packet transmitted in a slot is 1 t into 1 minus t into t into 1 minus. T into 1 minus t to the power that is geometric this is the Poisson variable. So, for this Poisson variable what is the probability exactly one packet getting in generated remember it is e power minus lambda into lambda to the power I by I factorial. In this Case I equals one therefore, this is simply my lambda is know G, G e power minus G that solve this is a Poisson variable is not you are not looking at the number of atoms or you can also think about it as binomial variable to, but binomial in the long in the when n an n p is very small, then it becomes a Poisson variable, the product of number of trials.

So, each user is trying with some probability p and n is very large and p is very small then a binomial becomes essentially of Poisson. So, the probability of one packet therefore, this is simply your throughput. So, the throughput of slotted aloha is simply G into e power minus G.

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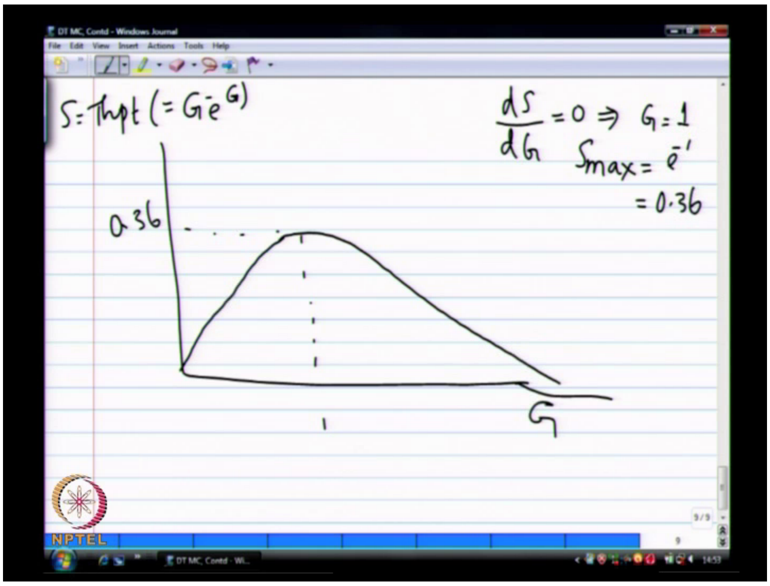

So, if we have to plot this you have to look like so, this is G and this is my through put which is nothing, but G e power. So, when G was to 0 throughput is 0 it has to be because no offered load therefore, there is no so as it increase. So, this will increase G e power minus G, if you plot it will increase and what point will it maximize one how do you determine that, how do you mathematically show that will their maximize differentiate with respect to so, if I call this S. So, if I said ds by d0 dG equals 0. So, this will be achieved at G equals one therefore, S max is so as G increases m so, so and actually it goes to 0 so at G.

Equals one we achieved. So, this is one way of getting a asymptotic understanding of behavior of the system and ideally, it should never operate close to one because system will become unstable lot of approximations in this modal is it does not really capture. The retransmission probability back of probability retransmission and then getting's captured in this modal, it gives you very, very high level view of what could be the maximum. We actually run this when a simulation, will find that point you some time get 0.42 even as I guess 0.42. So, this 0.36 is simply because of the fact that returning that its a Poisson variable, but its not so, we want to say refine this modal. Let us be little more realistic and let us try to look at an so that is what this next modals. So, when we got go to slightly more realistic modal it is more complex, but you can start adding more parameters.

So, in this case there is no other parameter at all I cannot specify, the rate of arrival for particular process and back log users the probability of transmission of retransmission of a back log user, none of those are specify assuming I want to have final modal then that is where our next approach comes in. So, questions on this basic modal, transmission in a spot, but I do not the probability the thing property change are slow already do it using this distribution.

If you want to call that you simply the channel is idle. So, what we want to know is in what fraction of time or what fraction of slots was there is successful transmission. So, if we want to calculate it will be simply a probability of x equals 0 is what e power minus G because your lambda to the power 0 goes to one I factorial goes to 0 factorial goes to one simply G. I am sorry, whatever be the offered load.

Even see you, if your parameter is lambda or G. G is the average number of packets or having at a given time slot. So, there is a probability of no packets arriving in a given side time slot to the simply e power minus G. Now, for a little bit more complex modal with Markov chains now, that we have a seen Markov chains everything is a Markov chain. yeah Distribution in is where in this binomial is essentially, what they you are will converge to Poisson see you are say, if you are saying that I have n users and each of them as a problem generating a packet p.

So, the average number of packet generated is n p. so, you would say the probability of n p is so, much you will do that binomial expression yes, but that in the very in we look at that example next how binomial is going to be actually useful. Here n is very large the reduction make sense to do that it essentially, converges to a Poisson process. And the proof for that is in Thrivedi's book by the way, we want to see why it converges to Poisson lets there. Now, our Markov chain based slotted aloha modal.

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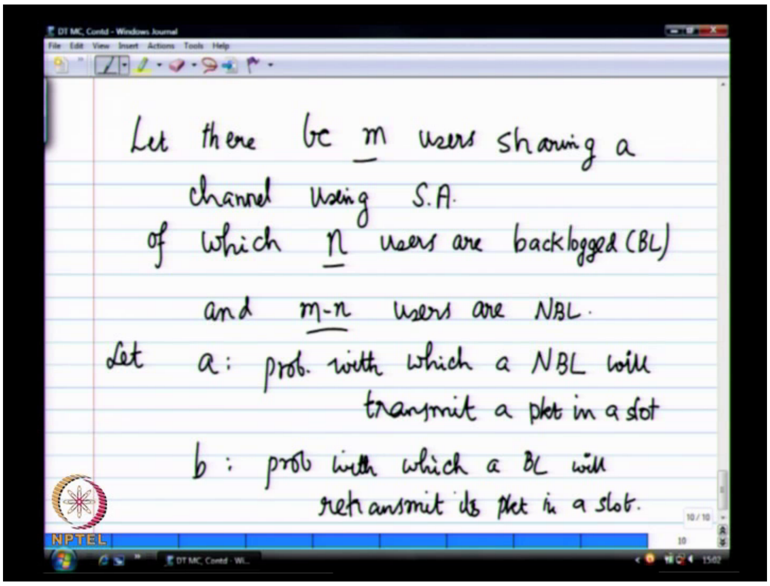

So, let there be m users sharing a channel slotted aloha, this is say m is a input in a couple of other probably is also given input. So, we are trying to modal the system behavior so, one way to modal a system behavior is look at the number of back logged user. So, user is back logged. If the user has a packet to send in the next slot that is the definition of back logged user; and un back logged user is someone that does not have a packet to send it all. So, the queue is empty that is a board just look at one queue, one packets told in every users queue. If the queue is empty you are not back log, if the queue is not empty you are back logged so, not back logged user has some probability.

You know, we call that a or b yeah a. So, non back logged user as a probability a of getting a new packet in the next slot that is like your geometric distribution. You are tossing a coin sorry not geometric itself Bernoulli trailed error you are with probability a you are going to generate a packet in the given slot in the next slot 1 minus G you want to generate a packet, this is true for all the users in the system then the back logged user who has a packet to send if the way in slotted aloha.

Works as the first time you get a packet you simply send it you do not really wait for any thing you do not wait for accessing other, but once you get collusion you go into a back of mode. So, in the back of mode with some probability b you will try to retransmit the packet in every subsequent slot only than aloha will work otherwise. What happens if two users try to use in the same channel? Then both of them colloid the subsequent channel both of them cannot try to send again.

They have to do some, they have to compute some probability and depends some probability b with which try to sample you b is very small. What happens then? Geometric comes it by the time you try sending a packet you will keep on wait. So, if the one hour be could be the average number of slots, which end of waiting before we send the packet we keep b very small the waiting time becomes large, if you keep b a very large then. What happens if keep b as 0.75, then the collusion probability will become very high? So, there is in there is a trade of so, this will help analyze a trade on. So, given the probability if this is what will define so, there are m user sharing of which n users are back log and m minus n are not back log. So, than let a be the probability with which a non back logged user will transmit in a given slot.

So, this a effectively captures the packets generation rate, if a is very large which means that in every slot which go into generate a packet in which case, aloha will simply collapse. So, a is usually small so, this is probability of any node generating a packet new packet in a given slot () coming to it so, that is a than b is the probability in which a back logged user itself. So, back logged user it is not generation of a packet it is retransmission of a packet that's the difference.

We cannot go this we cannot, but we would like to vary that because b is different from a, but a is quickly depend up on the load of the system where as b is controllable parameter users are getting back log. How can we vary the packet generation rate? You are a is let a fixed you start the system specifying m and a. B is the parameter that lets you control the operation of the system, if I like a said b is very large then your retransmission rate in.

Probability so, high will end up creating more and more collusions nobody level get through. So, b is something would like to find the optimal value for a is something you cannot control your system is generating you have put hundred user. A is whatever the users rate at which that generate traffic that is not really control your parameter it is a variable parameter, but not a controllable one. Whereas b is controllable you choose b, which gives you the best performance controllable. So, that's why b is a defined with the different probability. So, given a, b and m. What is the prob, what is the through put of the system so, b is the probability () yes that will make it even more complex.

Is b always less than the a I need not. Could not, Need not, it is a load is light actually b can be dynamic. If the load is light you can more aggressive policy with respect to that and yes, you can make it b independent of every one that is also possible because if you look at really, if you look at Ethernet for example, if your if a binary back of where as if I absorb more collusions, I reduce my I will keep on reducing b equals b by 2 and so on. So, that is your exponential back of that you try to do.

And yes it will be different for different users which is little bit that is where we go to simulation. When I want relay do that that can of intricate this one be able I can, but it gets more complex. So, here for know your simply assuming there all of them have the same retransmission probability b and then does not change.

In every every will one no a can be anything a can be 0.1, 0.2 that is load on the system. If I am saying that I will generate with probability 0.2 and I will generate a packet in every slot then we can load. No non block queue the back logged back logged the that an if I just simulated he start for with the. No user will let say user will user will go between these two states you will be back logged your packet goes through then a become non back log.

So, we can be back or of course, we are not in looking at the queuing implications here. So, if you look at really a queue than what happens so, the user successful in the next log you will continue to be a back logged user, I am not going there here let us assume thus only one packet per queue for every user and once you finish it you have to wait for to, for to next packet stream there is no queuing as such here so, this is where some of the you know. This is the is we as you want add more more reality, the modal gets more and more complex your are making is lot of assumptions here, I only have space to store one packet at a user that solve I generate a packet and I am try to send it yeah I am not generating any more packet.

So, may be that it hell it higher layer just pretend that ip layer is hold in its buffer there is no in your Mac. If your Mac called only has buffer for one packet that solve it cannot send any more packets its helps somewhere, else of this basic definitions clear. Now, to represent the state of the system I simply use the value and the number of back logged users and then based on this, we will try to find out the probability of successful transmission and so on.

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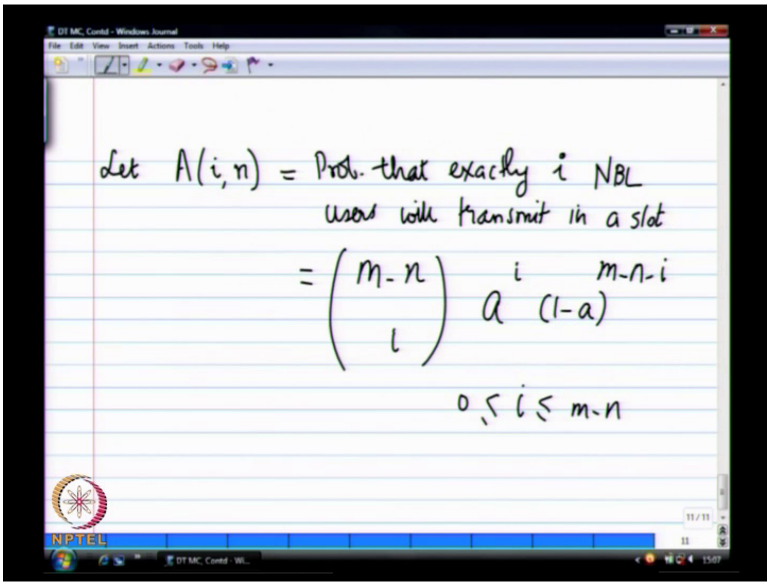

So, the book as use this know a, b little bit without any mnemonics associated. So, let a so A i, n. Now, your binomial is here that you want it. So, this is the probability that exactly i non back logged users start so, given the state system or state is n given that there are n backlogged users, how to the m minus n back logged users i out of them or going to transmit a packet so. What is that probability? I am sorry. We should also modal n.

M is what is giving m is an input total number of users in a input out of which, n is the current set of back logged users where, n can go from 0 to m only n is so, only n is getting modal. So, the status of the system is simply n the number of back logged users. That dependence on whether the packet was transmitted correctly or not. Yes so a back log user can stay in the back logged user in the next class to so, here I am only looking at the non back logged users, who do not have any packets to send. They are generating a new packet with this probability a that I mention only that solve that can, So, if you so this is Simply m minus n, choose I will do a so the time varying is only the states state is the number of back logged users that that is one the changes from time to time from between time slots so, that is why n is the state of the system. So, this is a A i, n everybody get that go to the next slide.

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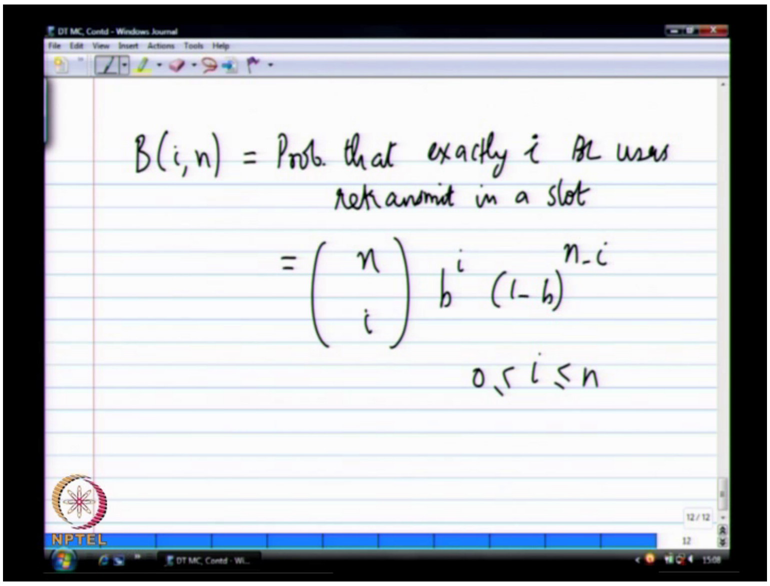

Then now, lets look at the back logged user. So, the same philosophy so this is the probability that exactly I back logged users retransmit minus log and that is again n choose I it where it defined b as the probability of retransmission. So, 1 minus b i goes from in a (no audio from 2:00 to 2: 32) this are still no definitions.

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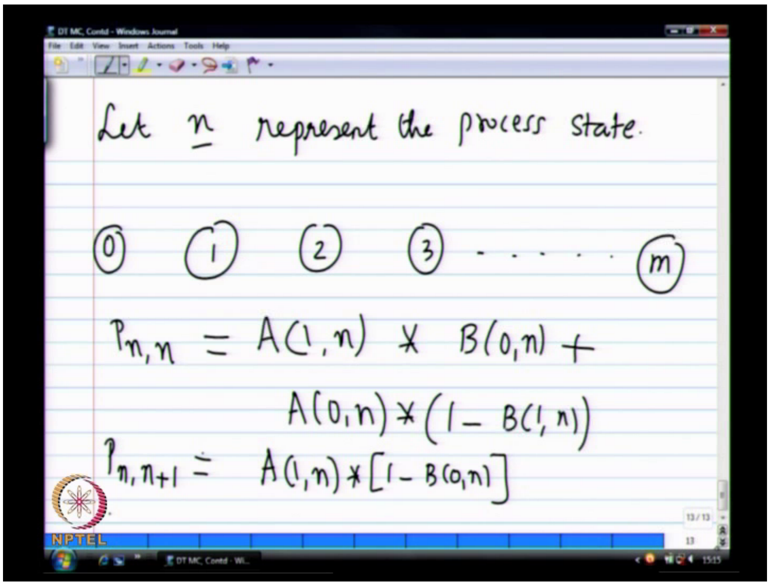

Now, let us so let n represent the process state. So, I will start of with the system starts. How many back log users will be there? What is value of n 0? Nobody is back logged. So, 0 is a starting state initial state and then I can go up to m back logged user. So, my system states are simply, this 1, 2 and so on. Now, I have to define my transition probability. What is the probability? That given in some state n I will go to state n plus 1 n plus 2 n minus 1 n minus 2 and so on. Now, what is the probability that given I am some state n, I will remain in the same state n in the next slot. So, beginning of the slot I had n back log end of the slot I still have n back log. So, what is the probability for that. one non back log is A 1 back log user or one non back logs A 1 back log users transmit in an goes to n minus 1.

So, all the n back logged user did not send and only one non back log is send a packet so. What is the probability for that? We know in terms of A and B this is so, this is A 1 n this is the probability that exactly assume that everybody is independent. So, this is without independence in say do all this.

So, assuming that all users independent is where that binomial also comes under. So, this is exactly one non back logged user sending and I do not want any back logged user sorry. Now, I am reading the wrong expression sorry this is to go from sorry this is B. Why I am looking for an eraser is here? Now, I am starting in state n I do not want any other back logged users to have send and I want to end up in state to remain an state n, I have only one un backlogged high, who was successfully send the packet.

It should a of 1, n minus 1 into B of 0, n. Now I represented a of one as if you remember the definition, n is captured in the formula there it is m minus n A i, n is defined is in a I is choose it is not A i, m minus n plus nobody sends a packet. Go you user generate a packet and no back log user as try to send. If no you know user generates a packet and nobody, use back logged is also try to send or sorry no as no more than one user is send. So, there are there are two cases so, let us look at this one so first is no non back logged user as sent to packet into if exactly one back logged user as try to send state would I gone to n minus 1 where as if nobody.

As tried are if more than one person is tried. What happens? Collusion if nobody is tried nothing as happen, if more than one back log user tries, what you have collusion. So, I simply want to this is multiplied by 1 minus B, one. So, only one back logged user as tried or sorry opposite the head it is not the case that only one back log user as tried therefore, the back log a stay put and there.

Is now new generation from any of the un back log is so, there is the probability of staying in state hand this are the two cases, if you not able to get this away if you go back in spent 5 minutes on it you should be able to then, what is the probability of going p n, n plus 1. So, to go from n users to n plus 1 back logged users what is the probability 1 minus k 1 n, 0.2 minus A 1 n that means nobody as try to send or more than one the same probability of trend.

More than one no you go to state n plus 1. 1 if there is a new user to get the back logged user you have the you have to one unblock log high trying to send the packet exactly one. So, that is basically A 1 n and at least one back log guy as try to send because you have collusion. If the collusion any two guess to send one is unblock guy one is a back log guy. So, back log guy is captured by 1 minus at least one back log user has to send so, more than 0 and there as to be at least one new generated from the un back log so that is your p n, n plus 1.

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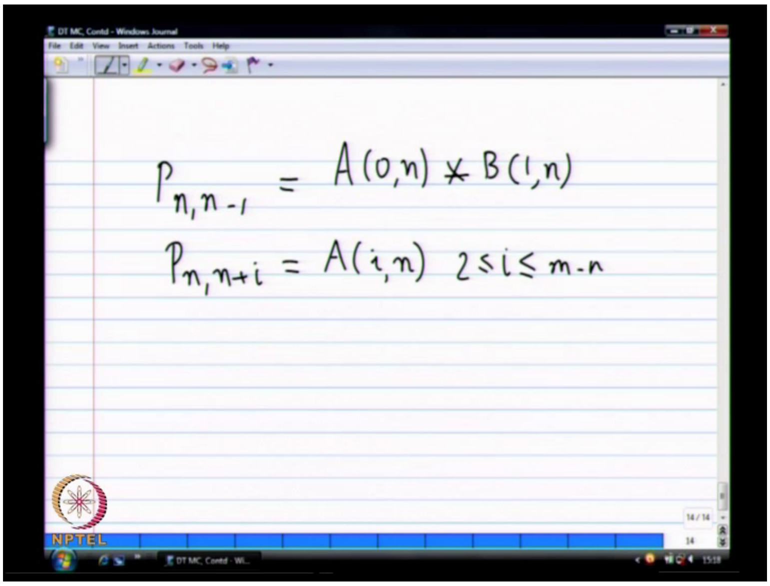

And then for p to go from state n to state n minus 1; so, how will have they should be they should be no new non back log transmission in exactly one back log transmission therefore, it is a 0 and which means nobody from then un back Log group as tried and exactly one back logged users tried. So, that is so this are the 3 special case n from plus 1 minus 1 and staying in an, but you can also go from n to n plus i.

Sir

Yeah. n to n plus I will they not be one more case, where more than one non block user tries to transmit at the same time.

Yeah that is what is capture by 1 minus B one.

Sir, non back log users.

Answer say. Non back log, if you talking of two back log user two. Say any two n; two, two non back log. So, in that case of so, you go to n plus 2 then, first you will not stay in n plus 1 you good n plus 2, that the next two that if you trying to do to go to n plus 1 you need exactly one back log user, if I go to n plus 2 that means does not matter whether, they un back log they back logged guys tried on or not as they as have two non block guys are trying then, they all become back log so to go from here to here it is simply a of i, n does not matter whether, the back log guys send it or not all other simply add of to 1the back log guys could have send could not have sent.

I have collision any of two way of sending where your i is greater than 2 that is an equal at 2 as long as any two un back log guys, try the rest of what the back logged guys do is a relevant to our probability. So, simply A i of n so, there I un back log guys trying to send they all end up with colliding and therefore end up with collision and now, this pretty much gives us the now, you define this set of transition probability questions on this. So, for so if I give you n and I give you a and b simply you will write this large.

Metrics write a program to do that in we do that in mat lab or whatever it is to generate the values all this probability you will get your p than you can simply solve that v equals v, p and then if you, if the study state exist then you will get the value for v then you can get the probability that the system will be in a study, when the study state what's the probability will be in state with 0 users 1, users 2, users 3, users and so on. And then will some all that things will some up to some up to one. So, the reason where computing all this p is to fill up our probability matrix our transition probability matrix v, use that solve for v once I solve for v then I can compute the throughput of the system. Which we have intent talked about before that, let us go back to this diagram here so, from 0 or this 0 is a special case state 0 will there be a transition from state 0 to state one can I go from 0 back log to one back log because in that case transmission will happen this only one person.

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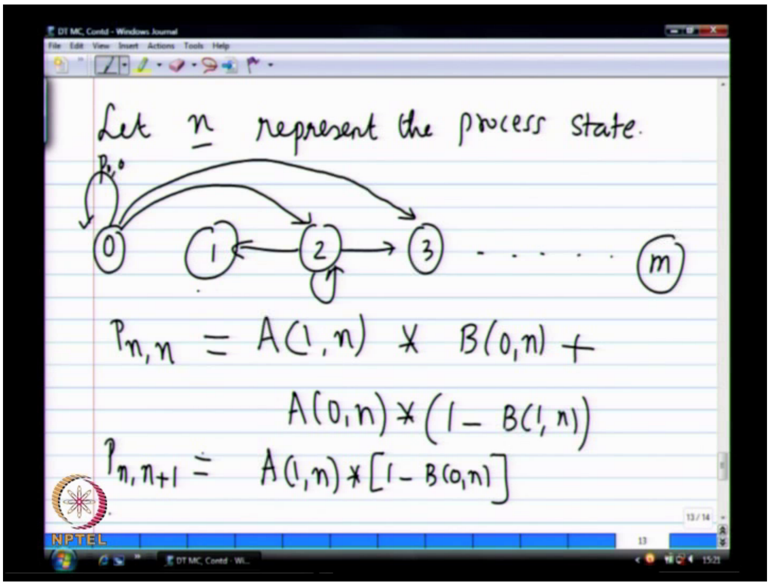

I will have to one back I can go from 0 back log to 0 back log diagram can go this is my p, 0, 0 which is captured by this one there is only one user everybody is un back logged. So, there is only one guy who tries to send that guy is successful therefore, is stay in this state if more than one back log un back log user tries to send than I will simply jump to state 2 state 3 and so on.

But we never go to so that’s the only special case p 0 1 is basically 0 you can also try that here when p 0 so this is A 1, 0 which will be yeah you will get figure it out you end up going to 0 is B, 0, 0 is going to be 0. So, 1 minus z 0 so, whatever is this so 4 p 1 0, 1 0 you do not have really think about it we simply code this mindlessly you will get all the probabilities such one and from one. What are the states I can go to for one back logged user?

I can come to one back logged user will never A 1 back logged user you can because from two I can come to one as two back log one of them succeed. So, I come back to one back logged user I am from 2 I can go to 3, 4 and so on. So, these self loops all there so, this is after that everything is kind of you know pairly routing and again from one I can go to three. So, that is all of the square if looks pre early, but one I can go to two, I can go to three whatever, it is can go to and become one can I come back to one from one back log I come back to one back log. Yes, yes, yes.

Either this back log try did not try at all and the new and that is about it this only back log is I never to try are there is guide did not try and there was a new user, they try to send anywhere. So, I may tend of staying here either there is no new packet generation or there is packet generation and they get succeeded if both of these happen, then I will end up pull state to with this back log guide transmitted and the un back log guide also transmitted, I will end up will state going state to because there is a collision therefore, this is self loop is and so on. Now, we have we can compute our steady state probability so, being each of those states so the question is so what. What? We do that steady state probability ignore, how can I compute the through put we did the simple analysis for throughput with the G equals minus G.

So, here how do I define how do I calculate the throughput so, we have to one step further and use this. So, in every system you have to figure out how to compute the relevant performance metrics using this steady state probability. So, Markov chain will stop at this point you have the generic frame work, but doing this but then will have to interpret how do, how do you represent the metric that you want in terms of this probabilities that is where extra thinking is needed the now, in this system how will I define my throughput of the system. Number of is the here the throughput is simply the fraction of slots, which are successful is if I have n users, what is the probability that if I have n users, I will have a successful transmission. So, this is a notion of a reward so for every state we have established a reward.

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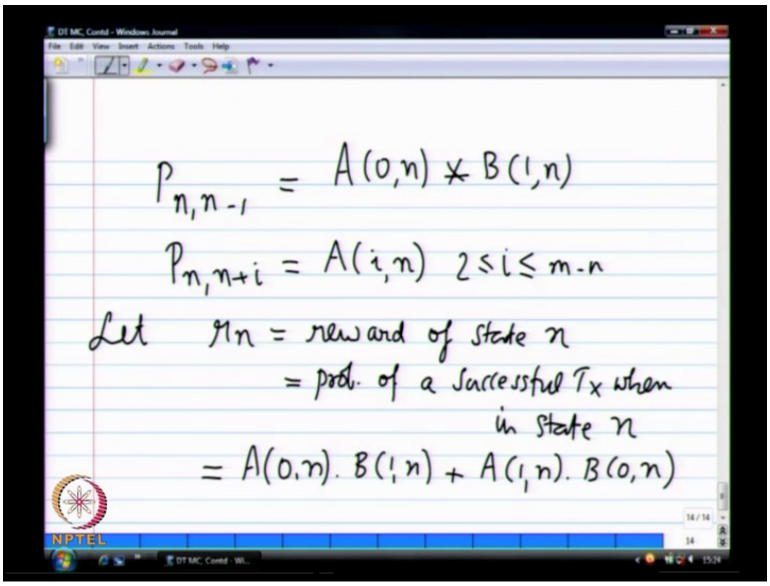

So, in general this are in a some reward and we define the reward in terms of probability of a successful transmission, when system is in state n so, what is that probability? What is the probability of successful transmission when your state n? Either there is no new transmission from the un back log side and there is exactly one transmission from the back log side or so you define r n to be this is the, this is the probability of one successful transmission. When you are in system k, n this is the reward with you. So, this is like computing f of x you know the distribution of being in the different states that is your x variable then, we compute f of x so this is basically r, n.

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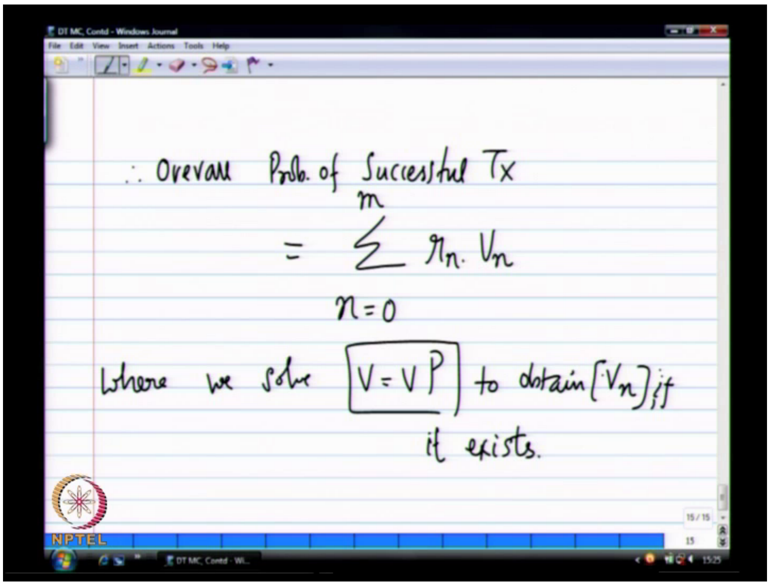

So, you are an throughput of the system is therefore, over all probability of successful transmission is summation over all the states going from 0 to m or n into V n. Where we have to solve probability it depends the values of a and b, if a is very large then you will be always stuck at the other side of system into probability of successful transmission will be very easy, will be you know you might never leave that state m at all if a is 1 b is one everybody is going to send and you under having collisions always. Nobody they will transmit. No steady state is you will slot the system. So, there are hundred users everybody is generating packets initially, there will be some set link down period then beyond the time, when a none of the packet rates change and your the retransmit probabilities is also fixed.

Then you land up with the system, where there is always somebody trying to set some body somebody, succeeding somebody not succeeding and so on. So, your system will enter as your so called steady state, where the there is no long term change in behavior of the system as such. So, if we have to calculate we are in this case, how you will calculate that you know the metrics p, the metrics p is what all those the p of n that we computed alpha. So, you have that metrics p that a simply solve V equals V p to the set of linear equation that we saw before. So, with simply you have take that we thus this is what this is what want we talked about earlier solve for this when you solve, if the solution exist you will get the values for the probability of being in state with know 2 users, 3 users, 4 users and so on.

From that if you define a, r, n recording the you will get your average value of the system. Steady state after this you cannot a more users that this maximum throughput or something like their achieved, the system is has a fixed number of users and so this is what if you, if you keep your a to be very small. Then your throughput also be proportionately roughly m into a, where m is the number users and probability generation is m into a is what you will, when the system load is fairly like and the system load is very large then you lend up your throughput going to this other. If you plot this with respect to a, a is if know fixed parameter b is let say fix to 0.0, 01 or 0.05 are something like that. So, if you really want to understand the behavior of the system so your m and a are fixed.

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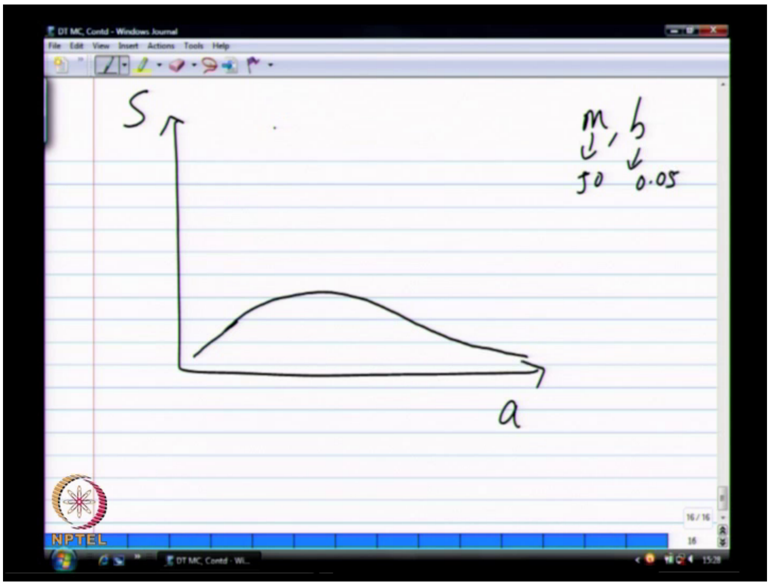

So, given m and sorry a, b let say I am fixing this to be 50 users fixing, this to the point you know as 0.05 something like that some numbers that I am choosing, then I vary a is the load of the load of the system a is the fraction, where probability with which I will generate a packet in every slot and then I am computing my a, S the throughput of the system. So, with varying a with for given m you will see something like this where. Throughput increases and then starts decreases only difference this is the for more accurate representation of the system behavior compare to what the Poisson arrival try to sure. So, you might find that for it because it depends on these value of b also b is hidden in that particular case m is also not take on solution, you will get a similar curve like this. So, what steady means is if I look at the certain probabilistic behavior of the system at time say one million units.

And look at time two million units the probabilities will not change of being in any of these different states that will still essentially, converge to that value if such a thing is as independent; for a given set of users, if we depend on the value of a. So, if I say 50 users are there then a should be small enough so, that the collective will a load is a into m that is what G is calculate representing that so, the a will dictate the steady state region I mean in terms of the throughput as situate the value of S will depend upon a, but as to whether we have what time will get steady state, what you have seen about.

linear steady state is independent of the number of users is it so you have for a given set of users I am looking at and then after sometimes, you have been these two steady state and you have additional of the user addition of the users, our number of user is derived in the system for a given set of users for a given packet generation rate, you are you are witching steady state; if you change the number of users in a different system an altogether; yeah if I change my I will come back and recomputed my because my matrix is it m plus 1 by m plus 1 therefore, that will change. So, if it exist that is why that condition that we said before one it is it should be able to go from this irreducible condition and so on so.

If you are able to solve for that in you get those values, when you have you know that it is existing in this particular case dip. See if your values of a and b will dictate whether, your end up completely staying in one side of the system, if your a is very small than b is does not matter to the system at all you will be mostly in an back log state, if you a is a point ten to the power minus 5 most of the times you will have no packet the throughput also be very low, you will be simply a into m that is solve. So, that is so in that case in theory you will be able to go to the other states, but you never end up going there which does not really matter.

Now, the case is where we have a in theory you can go from one state to the other this particular scenario, yeah if way approaches way equals one then what will happen you will end up with being in state m all the time, but you can all is the just the probability of coming back to that is the slightly non 0 probability of coming back to the lower state, but you mostly end up being in state term.

So, your steady state will be simply 0 a, 0, 0, 0 and than one for being in the state to m in which case the probability successful transmission m users is again negligibly small you will never being able to succeed. So, you will throughput always be end up being close to 0 you are reward further, will be pretty much to close to 0 so, your effect to throughput will be 0. So, guess this is no this is not complete everybody wants to know.

What is the throughput? I well that is your home work. So, like one home work I will put it formally on the where, which will take a small like ten user system set this various of n the set a its small ten by ten matrix solve it, find out what the steady state throughput probabilities are and then from that you can club compute, the effect to throughput of the system others no big deal.

So, we can either solve it yourself manually by hand or let to get one this software that gets you that does it for you tried on mat lab. So, these things are set of or incidental that expect that you will. It go does not take that much time to figure out to in a system of equations in the system in look for there is a come out our interfaces you considered as an then you have a Markov chain. Yes, that is what is coming next so the this is I am still building up the base case for Markov.

We were queuing, I will be getting into queuing after this so, that is other example of how you can use this Markov chain representation for a two modular system behavior. So, then there is a special case of these Markov chains of the discrete in general Markov chains. What is called as your birth death process?

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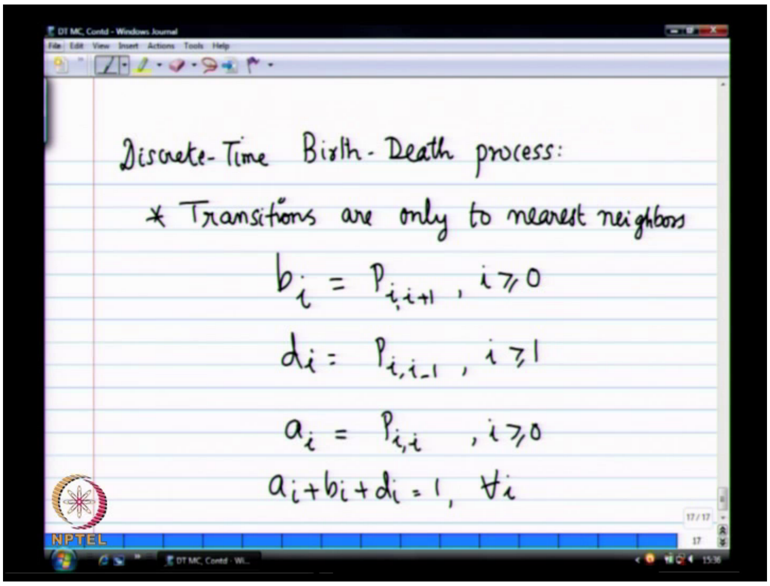

So, we are still in discrete time. So, in the previous example aloha we saw where from state, I can go to n plus 1, n plus 2 and so on. That is fairly complicated but, there are some special systems where the transitions or only to your nearest neighbors. So, it is effectively birth death. So, if you are in state i then if there is a code and code birth then a go to state i plus 1 and there is only probability of one such birth happening in a given state more than, it is simply not allowed as to not definition.

So, from there is a probability so, b i represents, the probability of a birth occurring in state i. So, and again if I am in state i, then the probability of a death occurring is represented as d i. When the state will move from I system will move from state i to state i minus 1. So, b i, d i and then this will be i greater than or equal to 1 or there is also a i, which is as is so you can also continue discrete time in the stay in this same state. So, for every state I we define this probability.

And of course, all these 3 should some up to 1 there is therefore, there is no other transition possible. The state i will go to either i plus 1 or i minus 1 or stay in the same state. So, this can be used to modal, the queuing system in a discrete in a discrete time in a, we are also use continuous state continuous time representation also is possible. So, in this case yes, if I keep my delta t to be very small; so the number of packets arriving in a more than one packet at arriving in this very small time interval is negligible that is what this could represented.

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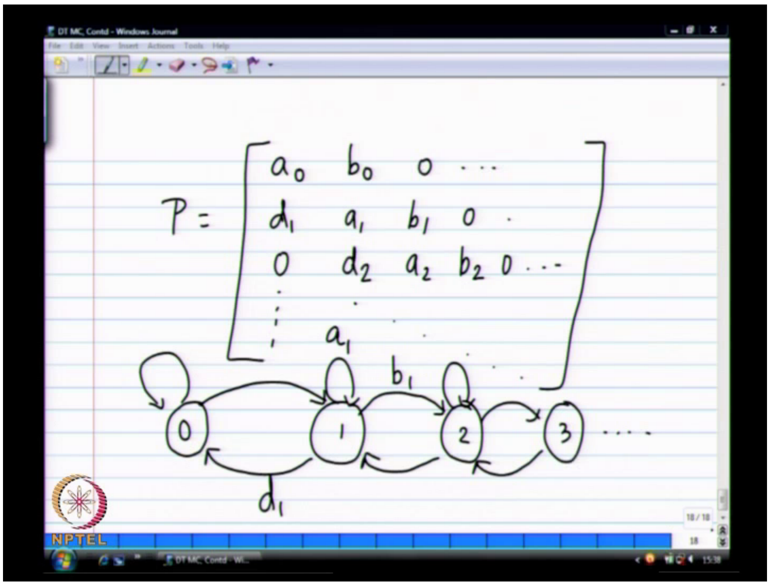

So, my transition matrix will look something like this and so on. So, that is of the metrics looks like and pictorially, if this looks like this. So, this is birth 1 this is probability so, the death 1 this is stay in this same state.

So, this is the special case and why this is special is there close form solution that you can simply apply, you do not have to go and try to solve the equations from scratch. Whereas, for a general process if we saw aloha case, we have to i do not feed in this value is run it in the system that will give me the solution for the set of equations here, it is easy to find close form solutions. So, if no other that the particular process is birth death, you simply go and apply the set of results from this particular thing.

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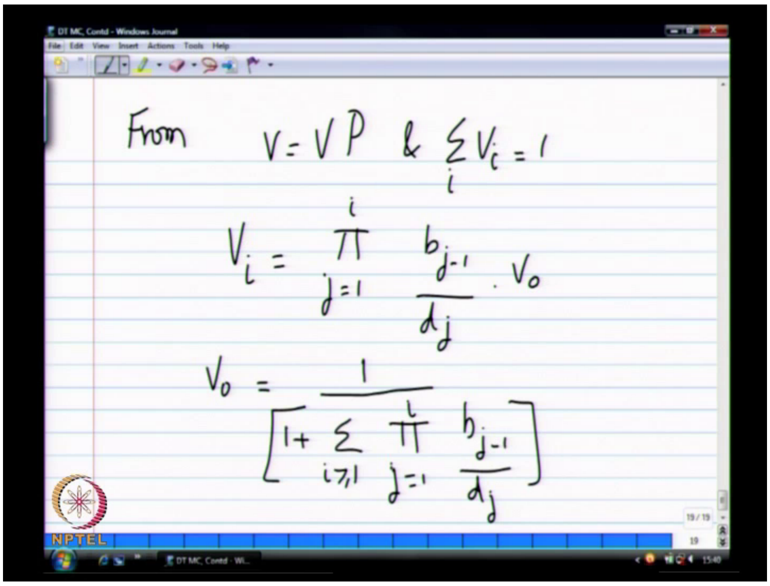

So, from here you can derive this. So, from I am not going to go through the derivation, because there is fairly in a straight forward derivation (no audio from 53:29 to 54: 22) no not V i, this V i has definition for V i, we cannot and third del this lazy and skip the definition that no. So, that is expression, we will work out in example. In total session on Friday to see what that better, but in general all needs to know here is the system, if here it I know that this is birth death, then we have to know that solutions close form solutions for that exists.

So, I can use it as it is very conveniently to more minutes in V j minus 1 divided by V j, we can check that the way we can use that. So, the last two minutes on this topic, before we go to continuous time Markov chain. So, what is the average time spend in a given state i in a discrete time Markov chain?

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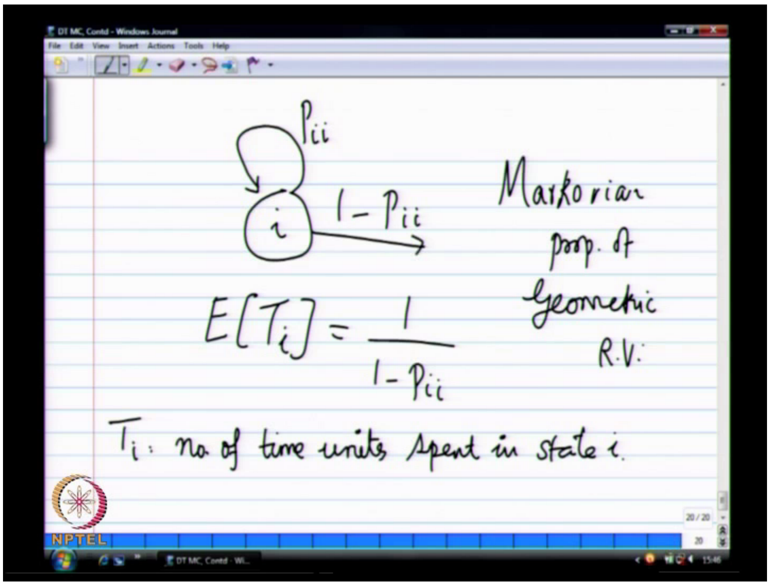

So, I am in a state i, I will leave the state and go to some other state I will stay in the same state probability P ii, this is every time instant. So, what is the number of time instance I will spend in state i in a given discrete time state, yeah I know steady state probability if saying in state i is known, but what is the time that you will spend, if you where to imagine this as a simulation that is running. So, you are I have the code for that I have probably pull on the web; I do not have to displayed on this one or may be in the laptop session.

Already I show that you can actually very easily write to simple Markov chain simulator, it takes about less than ten pages of could same they could 2 by 2 examples that, I showed is very simple to do all. We have as you have basically, what will happen is in every you start with the system in some state 0 or 1 where you flip of a coin P, then you simply loop for i equals whatever one million steps, ten million steps in each step you have.

Depending of the state if state equals i, then you know the probability of transition, you know your single step transition probability simply if flip a coin and again this side whether the go to state 0 or state, one based on the probability that is define and than just repeat this forever, than you will get you will see the sequence of state.

Changes as the system are going ever. So, if you look at it you will be in state 0, then will be is state 0, state 0, state 0 and finally, go to state 1 and 1, 1 state 1 like 0, 0 and ones is what you would see. So, what is the length of that 0 of being in a particular state number of steps that you spend in a given state. yeah. On the whole process.

Continuous number of process at a given instant at for even is for once you start in a given state 0, how long will you stay in the state 0 before we go to state 1. () So, it will be sigma i P of incoming into P of staying there i minus 1 times and P of outgoing. So, would so, that the geometric variable so, this is geometric variable if I look once I enter over state i, then that the time spent in the state is the geometric variable and finally, at the end of that I will go to the next state any other state I will go to some other state.

So, that is what I have here so, therefore the expected time spent in T i, this is my probability of success. I am simply tossing a coin than every unit, time unit with probability P ii stay which is failure 1 minus P i, I leave the state which is success therefore, one over the probability of success which is 1 by. So, this is the geometric variable.

So, that is why this Markovian I said earlier on geometric variable is Markovian. So, this is why our the time spent is basically, Markovian this is the Markovian property of so the number of time units spent is a geometric variable, geometric random variable when we go to continuous time, the time spent in each state is Markovian, because time spent in exponential. So, that is the parallel between the discrete in the continuous time so, this is I have to define T i.

So, T i is the number of time unit spent in state, with discrete time birth death process. We will see example, but when we go to continuous time where mm 1 is automatically a continuous time Markov, we can model mm 1 is both discrete time as well as continuous time, if we look at the term text book values mm 1 as a continuous time Markov chain, if we look at this one the old book by burst case their they deliberately modal a queue, as a discrete time Markov chain and then, then try to get the same set of result, but the derivation is slightly different so, will come back on Friday and then go through that questions before we closed out. Questions?

Sir, here a is i, it also include the situation that you are entering a state one. Once you entered this one

Yeah that time, Yeah that from that’s from that, we from the time we entered the state how long it is before you need, I am separate at the same exactly, the same as geometric the geometric, we do not care what as happen say see in every state start. Now, how many flips will be say, how many have happen after this yeah number of failures yeah. Here T i equal to I means that you enter it then you spend I and then come out something like that.

If once you enter the state, you stay in the state with probability pii and leave the state probably 1 minus pii that is what the geometric nature. So, since I am doing this every time instant during the becomes the time spend becomes the geometric, the number of time unit spend is a geometric variable.

Sir, could you explain the steady state metrics for then birth death process? This one so, this is the transition. The transition are so, How do we?

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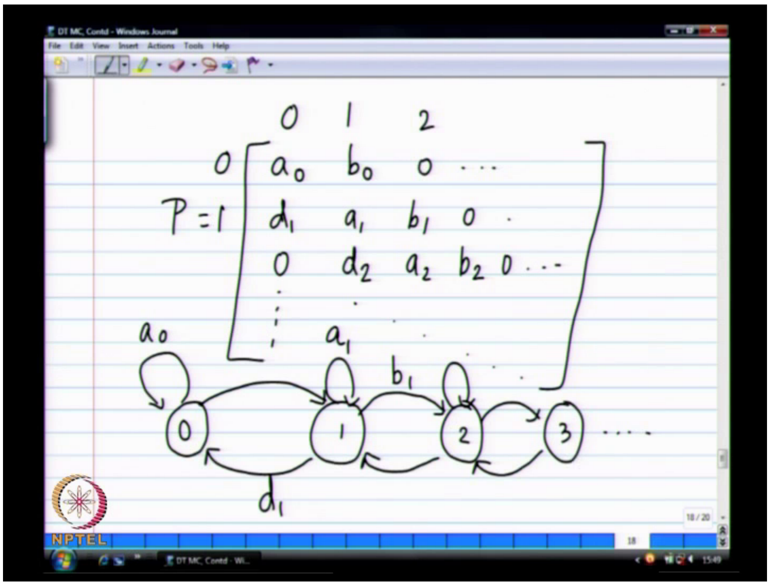

So, from state 0, I will go to state 0 with probability a not I go to state 1, state 2 and from state 1, I will go to state 0 with probability d 1. Let us be the dou y column wise. Now, and then I will stay in the same state with probability A 1 and I go there is a new process or new birth than I go to state. So, this is your basic and then thus repeated for all the state. So, each row as only 3 entries and their up to one and than all the other entries are 0, never go to any other state only your immediate neighboring states are your self is a only one that you go to a steady state.

So, this is the one step consistent probability so, at any point in time we can go from only in the state to the next state. So, then we go back to if we can solve so, if we solve this the V equals V p with that you will end up with this setup equation. So, this is your this steady state will be this one. we will go through of one example of this in the tutorial class because I want go to c t m c's and then start looking at queuing modals arrival that will no further questions then will see you on Friday.