Algorithmic Game Theory
Prof. Palash Dey
Department of Computer Science and Engineering
Indian Institute of Technology, Kharagpur

Lecture - 07
Security of Players

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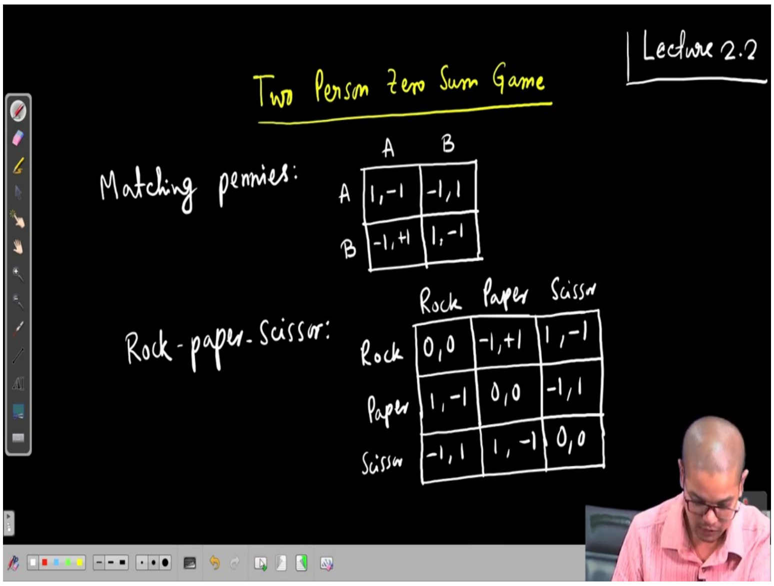

Welcome, so now we will study zero sum game two person zero sum game. So, we have already seen couple of examples of such games, let us recall we have seen matching pennies there are 2 players and 2 strategies A, B. And the row player or player 1 wins, if there is a match and loses otherwise the when row player loses column player wins.

So this is called 2 this is a 2 person game because there are 2 players and 0 sum because some of the utilities in every strategy profile is 0. So, we have also seen another game which is Rock-Paper-Scissor; this is also a 2 person zero sum game rock paper scissor rock paper scissor.

If both person plays the same thing then there is a match and both of them gets a 0 utility. If the row player plays rock and the column player plays paper then the row player loses column player wins and if row player plays rock and column player plays scissor, then row player wins. If row player plays paper and column player plays scissor then the row player loses again and the symmetric thing happen -1, 1, 1, -1.

So, here also you can see that it is a zero sum game two person zero sum game and there are lots of other examples wherever there exist strict competition these sort of games are very useful.

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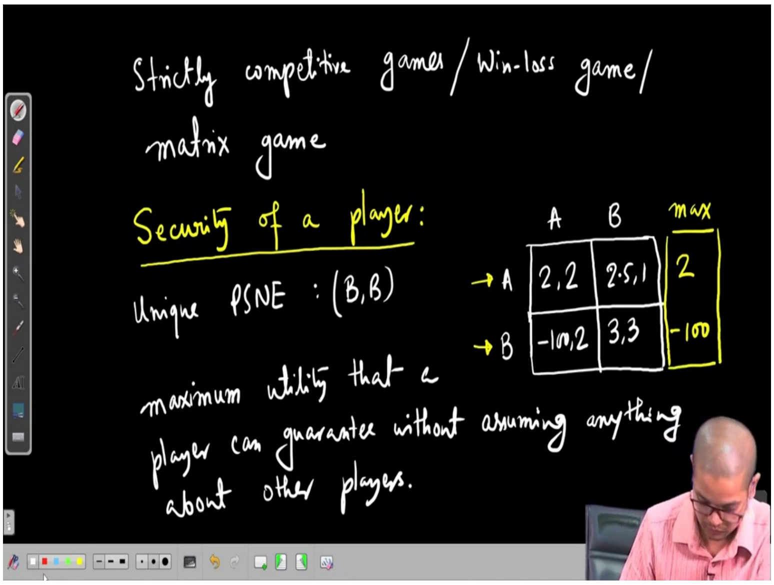

And that is why this these type of games goes by some other names. So, some popular names are strictly competitive games or it is also called win loss game or it is also called matrix game.

Because the utility matrix of one player is minus the utility matrix of another player and that is why to give the utility to represent the game it is enough to give 1 matrix, we do not need to give 2 matrices for utilities of both the players. Only 1 matrix is enough that is why it is also called matrix game.

Now, we will see that this sort of games has very nice structure and very convenient very its very rich in structure. So, let us first define a notion called security Security of a player; this concept is defined for any game not only for matrix games, but for any game it is defined.

What is security? Let us understand this with an example. So, let us take an example of a two player game not 0 sum and suppose the utilities are like this 2 comma 2 say 2.5 comma 1 say 3 comma 3 minus 100 comma 2.

See this game has a unique NASH equilibrium unique which is B comma B. But the question is that will players play according to that, here the security level comes into picture. So, what is the security? Security is the maximum security of a player is the maximum utility that a player can guarantee without assuming anything about other player other players. This notion of security is defined for games with more than 1 player also.

So, what do you mean by that? For example, suppose the row player suppose the row player wants to wants to measure what is the maximum what utility is guaranteed by playing A. Now if row player plays A, then utility of 2 is guaranteed, because if column player play A then the row players utility is 2, if column player plays B the utility of row player is 2.5.

So, the utility is minimum utility guaranteed is 2 by playing A row player can guarantee a utility of 2. How about for playing B? By playing B by playing B if row player play if the column player plays B row player gets a utility of 3, but if the column player plays a then the utility of row player is minus 100.

So, the guarantee or the minimum utility that is guaranteed is minus 100. Now what is the maximum utility that the row player can guarantee without depending on the column player its utility of 2; which is the maximum of these number.

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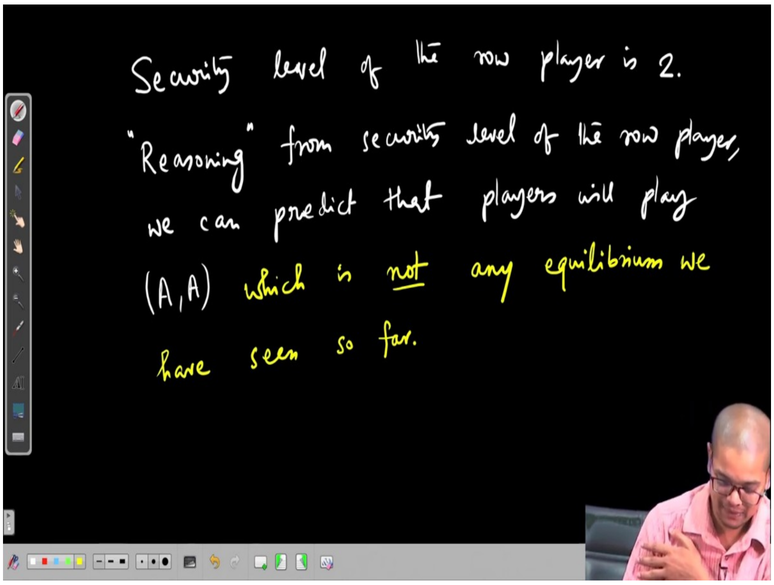

So, this thing we called the security of security level of security level of the row player is 2 and this also I do not know means this also seems very rational, means I want to I want to maximize my worst case utility irrespective of reasoning about how other players play whether they are intelligent at all or they play rationally or not.

You see a few classes ago we discussed important game theoretic assumptions what were they? We assume that every player has a utility players are rational they are intelligent, they have infinite computational power and common knowledge which is quite controversial. So, without assuming anything I can achieve security level by playing my sort of safe strategy which guarantees which ensures my security level.

Now, this sort of reasoning also makes sense and if now by following this sort of reasoning this sort of rational. So, to say if the row player plays A to guarantee its security level column player also will play A and then the predicted outcome will be both players playing A.

So, this is the concept of security level and you see that argument or reasoning from a security level perspective may lead to an answer which is different from any equilibrium A comma A is so ok. So, let me write reasoning from sorry security level of the row player we can predict that players will play A comma A, more importantly which is not any equilibrium we have seen so far.

Now, here is the beauty of zero sum game is that this line of reasoning from security level coincides with NASH equilibrium, it turns out that in we will prove or we will see that for zero sum game every NASH equilibrium every NASH equilibrium profile is guarantees every player their security level which is such an amazing thing. But before that let me define formally what is security of a player in pure strategies?

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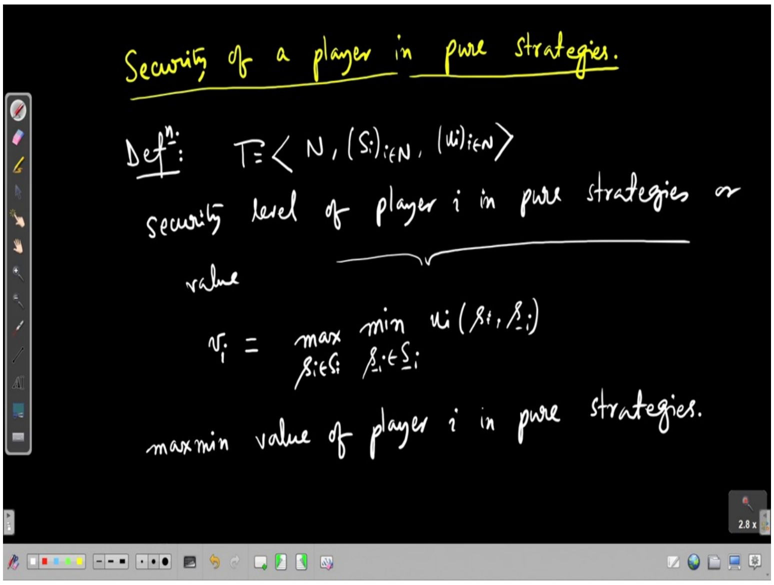

So, security of a player in pure strategies, what is it? Definition so, again as I said this is the this definition make sense for any normal form game not necessarily for zero sum game. Suppose I am given a game in normal form $\langle N, (S_i)_{i\in N}, (u_i)_{i\in N} \rangle$, this is sometimes called security level or security level of player i of player i in pure strategies.

Security level is also sometimes called value of player i in pure strategies denoted by say v_i. So, player i considers each strategy and sees what is it can what is the minimum utility that it will get by playing s_i. So, minimum over strategy profile of other players $u_i(s_i, s_{-i})$ this is the minimum utility that player i will get by playing small s_i and player i wants to maximize it overhead strategies.

It wants to maximize the minimum utility that it gets and because we are taking this max and min over pure strategies that is why it is called security level of player i in pure strategies or value of the player i in pure strategies ok. So, and because of this expression max min this is also called max min value of player i in pure strategies.

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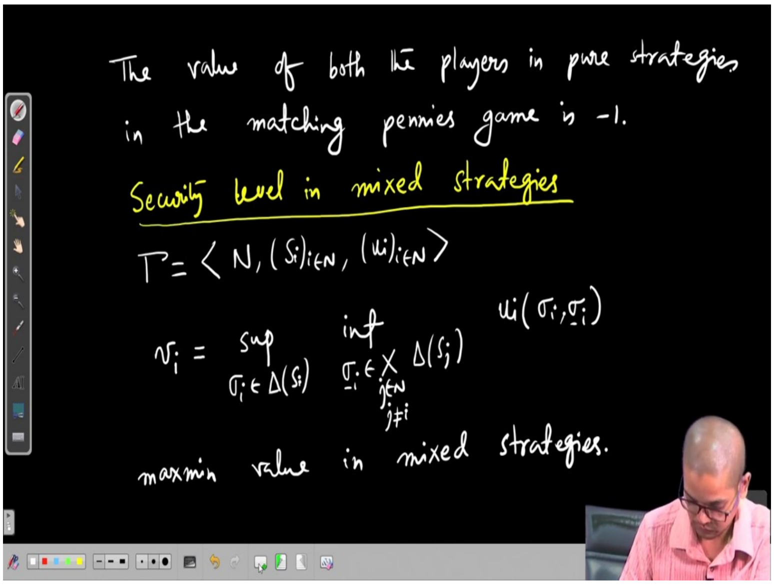

So for example, the value of both the players in pure strategies in the matching pennies game is minus 1 same with rock paper scissor. Now by the very name that the value in pure strategy it indicates that we can also talk about values in mixed strategies and instead of taking max and min over pure strategies you simply take max and min over mixed strategies.

And because it the we are now taking max and min over infinitely in infinitely many elements max and min does not make sense in general and you should use supremum and infimum instead. So, let me write security level in mixed strategies. What is it?

So, again I am given a game gamma in normal form and security value in mixed strategies v_i is instead of taking max you take supremum over all probability distributions overall mixed strategies of player I, infimum of σ_{-i} in this product distribution $u_i(\sigma_i, \sigma_{-i})$ ok. And again this particular value is also called max min value in mixed strategies.

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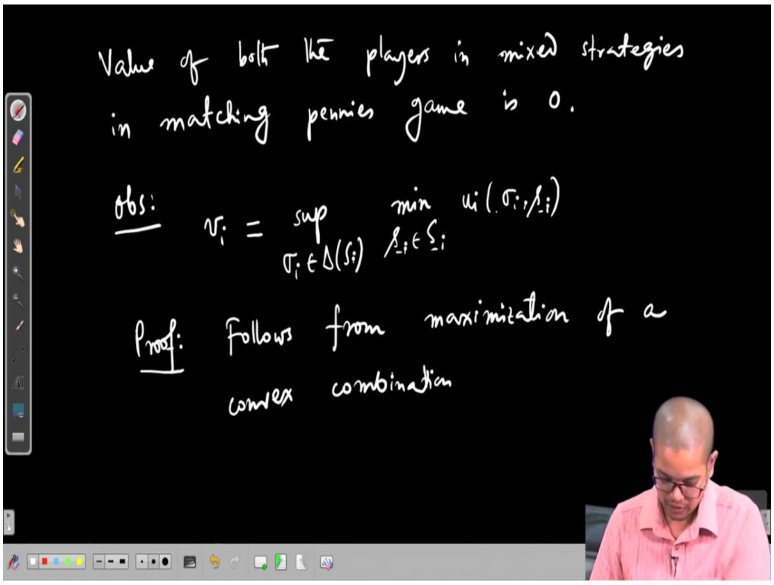

So, as an example I will let you check that the value of both the players in mixed strategies in matching pennies game is 0 and this can be achieved by playing the mixed strategy which puts equal probability on both the strategies.

So, it turns out that this value in mixed strategies is much more useful and we will use means we will use this means whenever we do not mention anything if not mentioned otherwise value means value in mixed strategies. So, only when we need to use value in pure strategies we will explicitly mention value in pure strategies, otherwise we will simply say value and it should be understood that it is value in mixed strategies.

Now, here is one observation this value in mixed strategies v_i can also be written as supremum $\sigma_i \in \Delta(S_i)$ probability distributions or mixed strategies of player i. We can replace in inner infimum with minimum and this follows again from averaging principle or convex combination.

So, this is the utility when other players are play according to this pure strategy profile s_{-i} and player i play mixed strategy. These follows right, follows from follows immediately from maximization of convex combination of a convex combination ok.

Now, let me draw our first connection between mixed strategy NASH equilibrium and these values.

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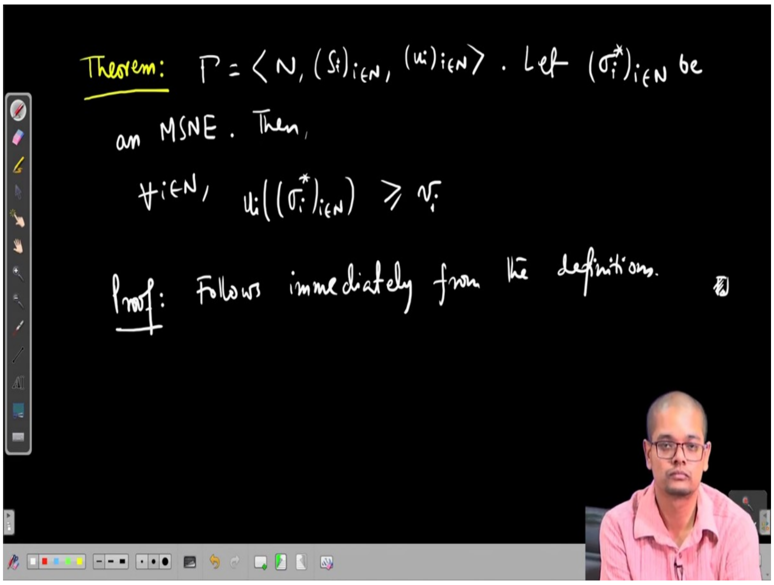

So, let me state a theorem and this this theorem holds for any game not necessarily for zero sum game we have not started anything for zero sum game, all the things we have done till this theorem including this theorem applies equally well to all kind of games.

So, again suppose I am given a game in normal form gamma equal to $\langle N, (S_i)_{i\in N}, (u_i)_{i\in N}\rangle$. Suppose I am given a game and let $(\sigma_i^*)_{i\in N}$ be an MSNE, then we have for all player i in N the utility that player i gets in this mixed strategy profile is at least their values in mixed strategy and it has to be so.

Because if some player is getting utility is which is strictly less than v_i then why follow this thing you just this security level is guaranteed. So, you just play whichever strategy or mixed strategy guarantee security level and then you are better off. So, proof is follows immediately from the definitions from the definitions of security level and definition of mixed strategies straight forward ok. So, we will continue in the next class.