Introduction to Automata, Languages and Computation
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Lecture – 07
NFA (Nondeterministic Finite Automata)

So, we are talking about Finite Automata. So, so far we have discussed the DFA, the deterministic finite automata. So, today we will talk about the NFA non-deterministic finite automata.

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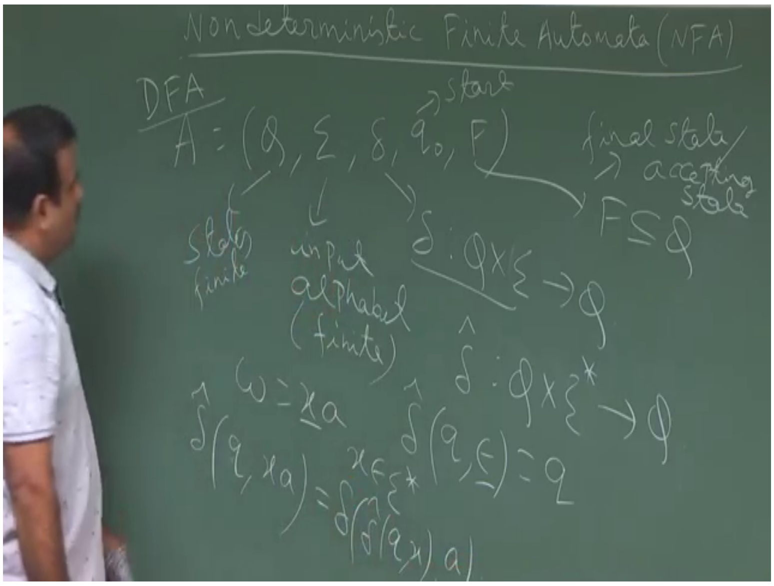

So, for DFA just to recap, for DFA we have 5 tuple, this is the set of all states, this is the state set which is finite and this is the input alphabet, input alphabet this is also finite set. From binary alphabet it is 0 and 1, and we have a transition rules which is a functions form which is taking a state and the alphabet and it is receiving here another states. And this one is the start state or initial state starting state and F, F is a subset of Q which is some of the states which are called accepting state or the final states or accepting state, ok.

So, this delta is, so this is the functional form we are taking us we are at a state Q we take it, so delta of q comma a this is a it is going to fixed state, so deterministic way. But in the non-deterministic NFA there is no fixed at for it there is a set of state where it can go. So, that will define. And also in the DFA. So, this is DFA. Also in the DFA what we define the extended delta hat which is taking a string Q cross to Q. So, delta hat how you are defining delta hat? It is a recursive way the base case is delta hat of q comma epsilon is q, epsilon is the low input. So, empty input or no input.

And if you take w to be x a this x is a string, then delta hat of q comma x a is defined as delta of delta hat of q comma x then the finally, we did a. So that means, meaning of this is we are at q at state going to read this.

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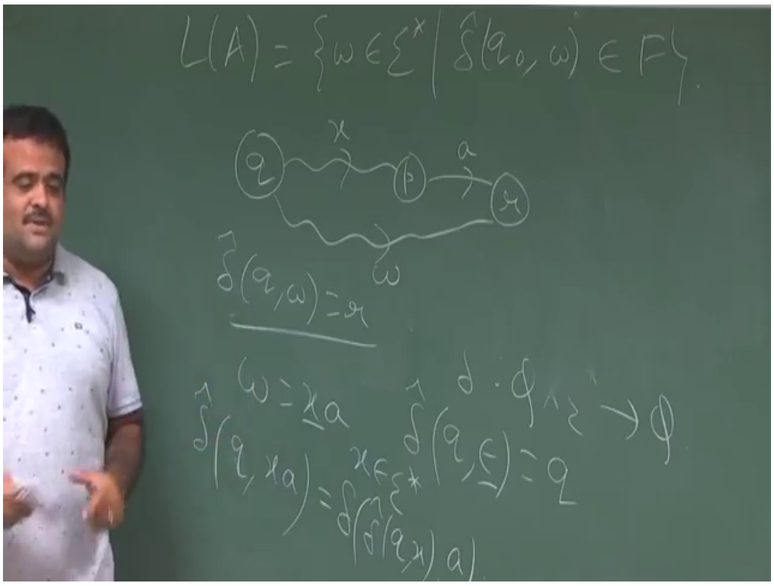

So, we first read x and suppose we are going to a p x state and then from here if it is going to r a state by a. So, this is nothing, but this is nothing but w. So, this is the delta hat of w q, sorry the q w is this is the recursive way.

Now, then we define the language of a, language of a DFA that is basically a L of A which is nothing but the set of all strings such that it is at the end it is reaching to the final state so that that means, delta hat of q 0 w this belongs to F. So, we start with q and then we read the string and we change the state each time and if we can end with a final state. So, then we say that string is accepted by that DFA. And collection of all such acceptance string is called language of the DFA. And we have seen a regular language a language if it is, if there is a DFA which is accepting that language is called regular language, ok.

So, these are the DFA deterministic finite automata. Now, we will define what is called NFA non-deterministic finite automata, then we talk about their language accepted by NFA.

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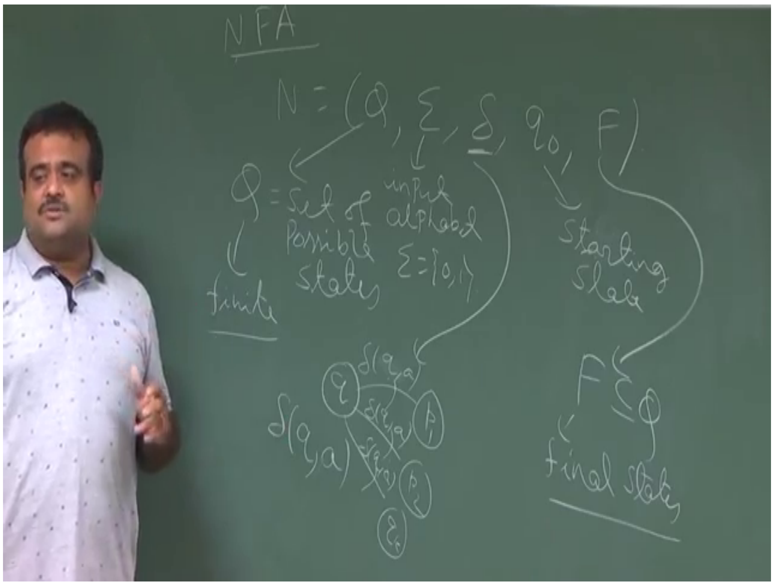

On-deterministic finite automata; so, it is also a 5 tuple Q sigma delta q 0 F. Now, Q is same as t q is the states the set of all possible state which are finite, set of possible states. This is a finite set and this is in to the alphabet which is also finite if it is binary then it is 0 1, but in general it could be a any finite set this we will define. So, this is a starting state or the initial state, and this is a again subset of Q which is the set of all final states. There could be one or more final states. Now, let us define this delta.

Now, earlier delta was a function form it is taking a state q. So, delta of q and a it is taking a input a. Earlier it was going to a fixed state p say, so delta q comma a. But here it is not unique it is not deterministic here we have many options like it can go to state p 1, it can go to state p 2, this is also a move this can go to state dot dot dot p k, this is also move delta q comma a. So, it is not a deterministic going. So, it is an all that is why it is called non-deterministic, ok. It can go to any of these states. So, this is a subset of this.

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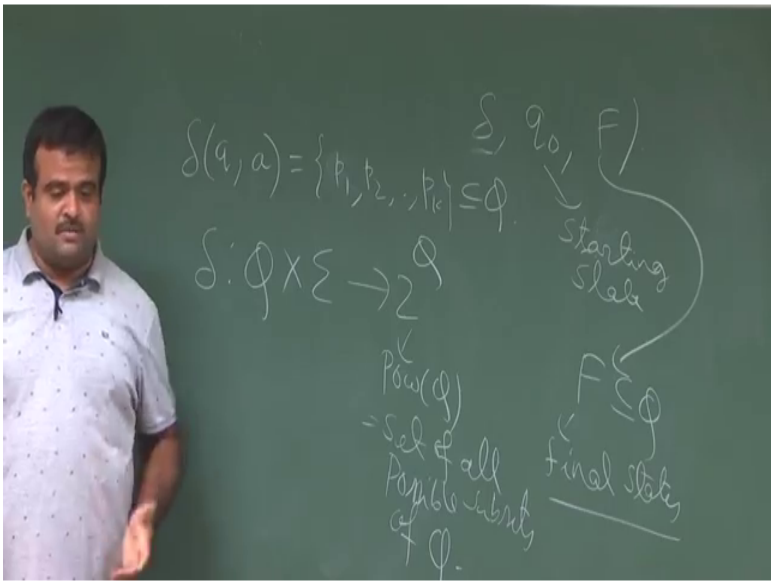

So, basically delta of q comma a; here if it is this subset p 1, p 2, p q; so this is a subset of Q. So, it can go to any subset of Q. So, the basically delta is here a function non-deterministic function delta is a function which is taking at a state, state and a alphabet and it is going to a subset of Q, this is called power set. This we can denote at power Q also. This is basically a set of all possible subset of, set of all possible subsets of Q, that is called power set. So, it is a non-deterministic function. So, we can go to any one of these states having this among this subset I mean, ok.

So, this is the difference between the DFA and NFA. The DFA move is deterministic, but the NFA move this non-deterministic. So, among this I mean it is a it is a it is a collection of it is a subset where we can move. Now, we can move to any one of these states from this subset. So, that is why it is called non-deterministic. So, we can take an example of NFA. So, let us take an example of NFA, ok.

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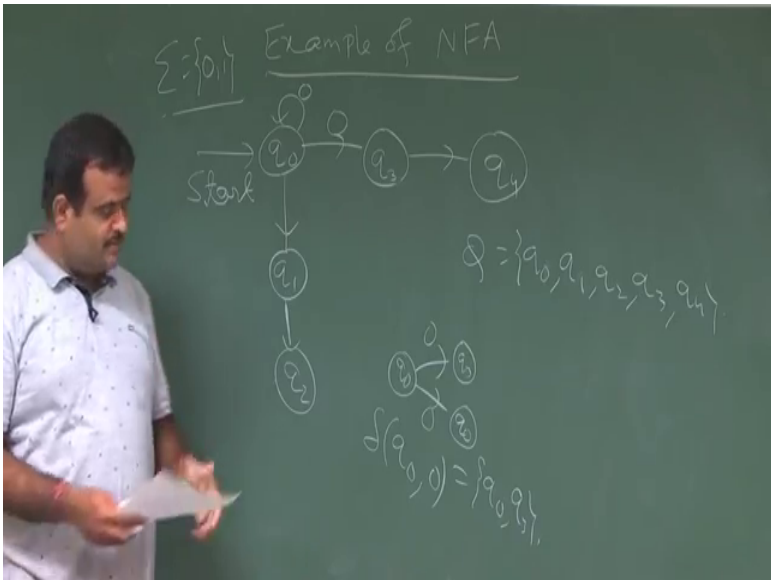

So, suppose we have a starting state, this is you have to start this is same as earlier. Now, from here we can have two inputs. So, suppose sigma x binary 0 1, ok. Now, from here we can have two input. So, if we have an input 0 it is going to if we have input say q 1, q 2. See if we have input 0 and here I can follow this. So, q 2 then let me write this first, q 2 and then we are telling this q 3, then q 4. These are the states. So, Q is nothing but q 1, q 2, sorry q 0, q 1, q 2, q 3, q 4. So, there are 5 states in this automata, ok. Now, let us do the moves.

So, if we see a 0 we can go here or we can go here. Because this now the move is the transition function it is a subset. So, this subset with from q 1; so from q 0 we can go either to q 3 with the 0 move or we can go to q 1 again ok, q 0 again with the 0 move. So that means, delta of q 0 0 uses this subset q 0 q 3. So, that is why it is non-deterministic. So, you can go either to we can either add be q 0 or we can move to q 3, ok. So, let us just complete it.

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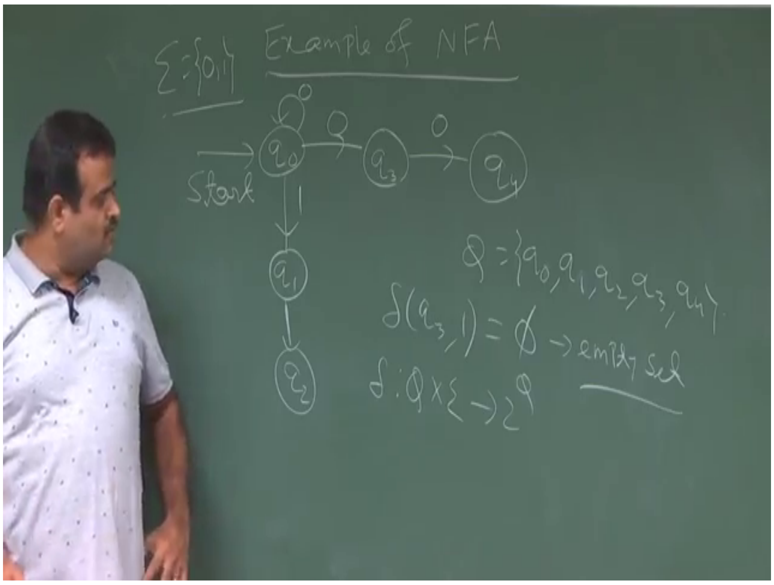

And from one we can go to here this is a singleton set it could be again can go to here. I mean there any kind of this is just an example. From q 3 we have a move for 0 and there is no move for 1. So that means, for from delta of q 3 if we have a 1 this is empty because there is no move empty set this is the empty set it is not going anywhere. There is no move from q 3 with the input 1 now, it is possible because delta is a function form Q cross sigma 2 2 to the power Q. And phi is also a member of 2 to the power Q, phi is the empty set, 2 to the power Q is the power set of all possible subset phi is also a subset. So, it is possible to have no move. Let me complete this, yeah.

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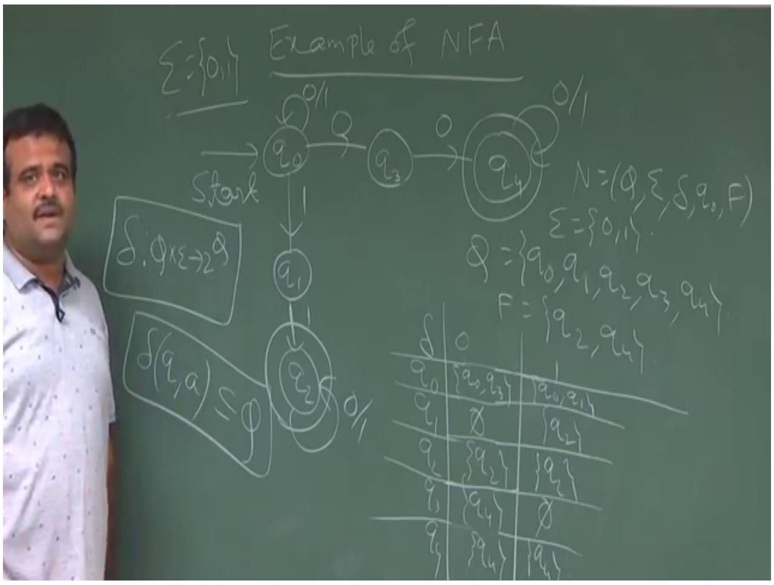

Now, this is 1 and this is if we have 0 1 and then here also if you have 0 1, and they can define some states as a final states. So, these are this is an one example where these two are.

So, what is F? F is, so this is our NFA in NFA is Q sigma delta q 0 F. So, this is Q and then sigma is 0 1, delta we have to define q j is the starting state this one and F is these two, q 3 like q 2 and q 4.

Now, we have to define delta. So, delta function is like this. So, these are the state q 0, q 1, q 2, q 3, q 4 and these are the input 0 and the 1, ok. So, from 0 if you take input 0 from Q, so you can just need some more space over here then you can put over here, yeah. So, now, suppose we are at this state q 0 now if we have a input 0 then we can go to q 3 and q 1. So, this is a subset sorry q 0 and q 3. And now if you see a 1, it is only going to this, if it is 1 then it is going to if we can make it 0 1 again I am following this example yeah, so then it is going to q 0 and q 2 sorry q 1, this is q 1, q 0 and q 1 if it is 1.

And now again from q 1, if it is 0 it is empty because there is no move for 0 and if it is 1 it is going to q 2. And from q 2 if it is 0 or 1 both it is just going to q 2. From q 2 if it is 0 1 what, it is going to q 2. And from q 3 we have a 0 move it is going to q 4, but we have no move for 1 so it can be it is empty. From q 3 yeah, we have no move for 1. And from q 4 q 4 everything is q 4 again q 4, ok.

So, this is one example of a NFA, non-deterministic. Because here what is the difference between DFA and NFA? Differences in the DFA we have a deterministic move. If we have q 0 if we have a input 0 there should be deterministic way that to tell the output why we should go, but here we have two options you can either go to q 3 or we can be remain at q 0. So, that is the way it is that is the sense it is called non-deterministic, ok.

So, this is this is that is why it is here delta is a function form delta is a function from delta is a function from q cross sigma to power set of this. So, it is it is taking, so it is a, so q comma a this is a subset of Q that is the difference and the earlier it was a single state, but here it could be a many state subsets of Q. Now, our option is remove more we can go to any one of these states, ok.

Now, we will talk about the how we can extend how we can this is for single input like for alpha, for a input alphabet. Now, we see how we can extend this for a string how a

NFA can execute over a string. So, that we are going to extend this delta hat that we will define. We will come back to this example. So, we are going to extend this delta hat.

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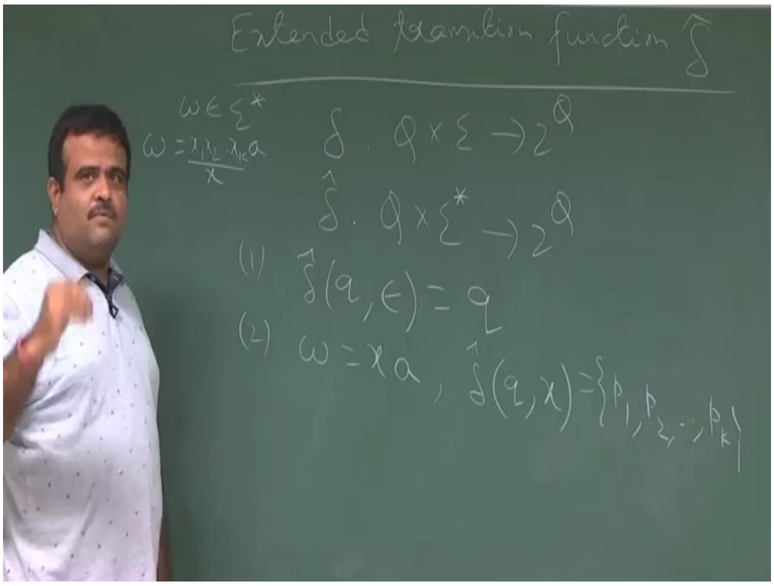

So, extend, take the extended transition function that is called delta hat for NFA. We know how we could extend this for the DFA, but here we are talking about NFA, ok. So, for NFA delta is a function from Q cross sigma to the power set. Now, we need to have we want to have a function delta hat which is taking a string instead of a single alphabet it could it could be single also yeah it could be empty also empty string then it should move to this, ok. So, we are going to do that.

So, this is the similar way we did it for the DFA, but here for DFA we had the single state, but here we have multi state. So, this is the base case. So, we define that delta hat off q comma epsilon is q that means, if we are at q and if you have no move no input will be remain at q that is the idea. This is means no move. If we are at q will be remain at q, I mean with the epsilon move we can say epsilon move. Means we are not reading any string, we are not reading there is no input alphabet in the tip, so there will be no move. But we will after this NFA we will talk about epsilon and NFA there we are allowed to take the epsilon move of him. No input still we can move to some states. Anyway that will come later stage, but for the time being, this is the convention.

And the second one suppose this is x a. So, we take a string w, w is a some x 1, x 2, x k a. So, this is we have to say x a, x. So, w is coming from sigma star. So, it is either this or it has some alphabets. So, now how we define this on this?

So, first we execute this is a inductive process I mean. So, first we execute delta hat on q on x. So, this is an earlier for DFA does a single singleton state, but it is a now this could be a many state. Suppose this is p 1, p 2, p k, and then from there we take a move we take a input a, and it can go to again multi states from that state. So, that is our w of this. So, let us just complete this.

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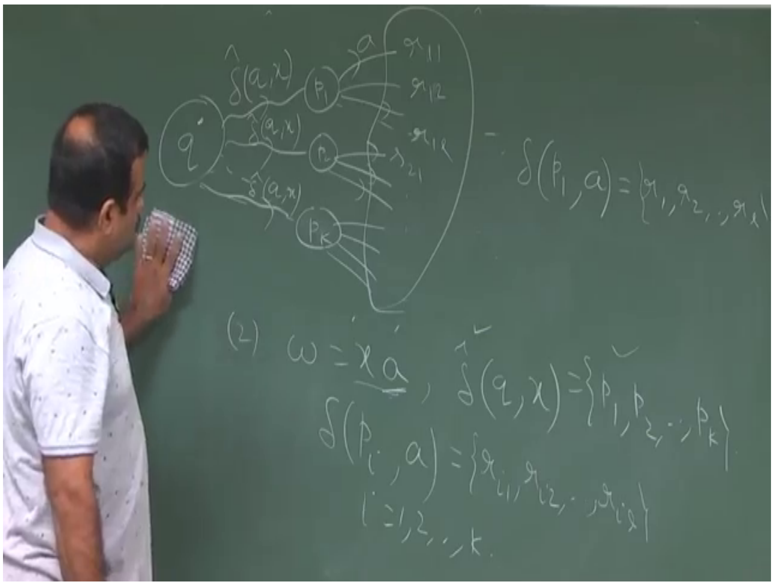

And suppose from suppose we know this delta of p i comma a, so a is the last symbol. Suppose this is some r i, so r i 1, r i 2, r i l. So, this is called or r i n. So, this we denote by this.

Now, so this is for i is equal to 1 to k. So, for each of these states we will go we are able to go to many other states, so like this. So, this is our q. Now, we want to read this. So, we first read x. So, for by x it can go to many state like p 1, p 2, dot dot dot p k, I am sorry p k. So, these are all delta of delta hat of q comma x, delta hat of q comma x, delta hat of q comma x. So, this is the possible state after reading the string x.

Now, from each of these we are going to read a now. So, now, again delta hat of p 1 a. So, this is NFA, so this is again a subset. So, if that subset is r 1, r 2, r l say, so from here we can go to this r 1, r 2, like this r 11, r 12, like this r 1 l, ok. And again from p 2 we can go to some other states say s 1, s 2, s 22 like this. So, these are the sets.

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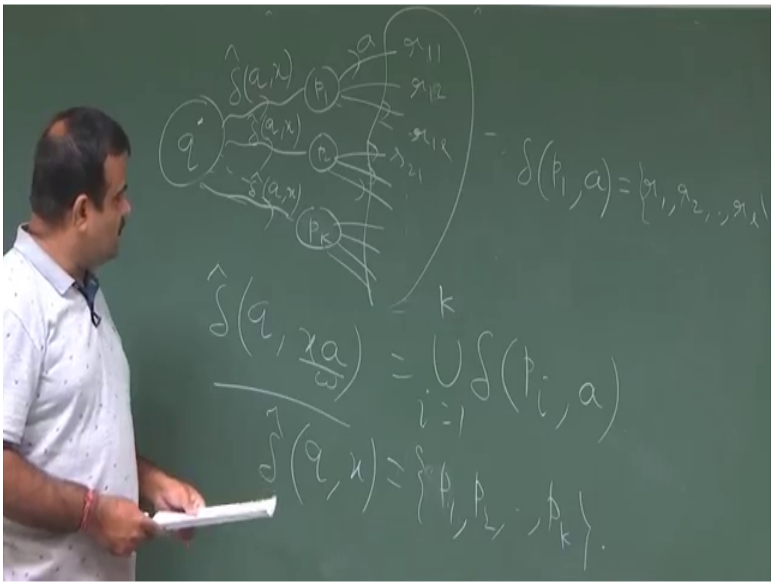

From again p k we go to many other states. So, these are the sets which is basically delta hat of q comma x a. So, it is nothing but the union of, it is nothing but the union of delta of p i comma a, where i is from 1 to k. And what is p i? p i, I mean delta hat of q comma x is nothing about this p i's p 1, p 2, p q. So, this is the p 1, p 2, p k, ok. So, this is the way how we define this.

So, to compute delta hat of x a we first compute delta hat of q x and then from those state we just directly take the input a and we will go to many other states, so that collection that will be the delta hat of q comma q comma w. So, w is our, this is our w. So, this is the way the inductive way recursive way we defined this, ok. So, now if we can take one example on this so that we will discuss in the next class.

Thank you.