Introduction to Automata, Languages and Computation
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Lecture – 26
Substitution

So we are talking about regular language regular set. So, today we will discuss the Substitution; substitution over an alphabet symbol.

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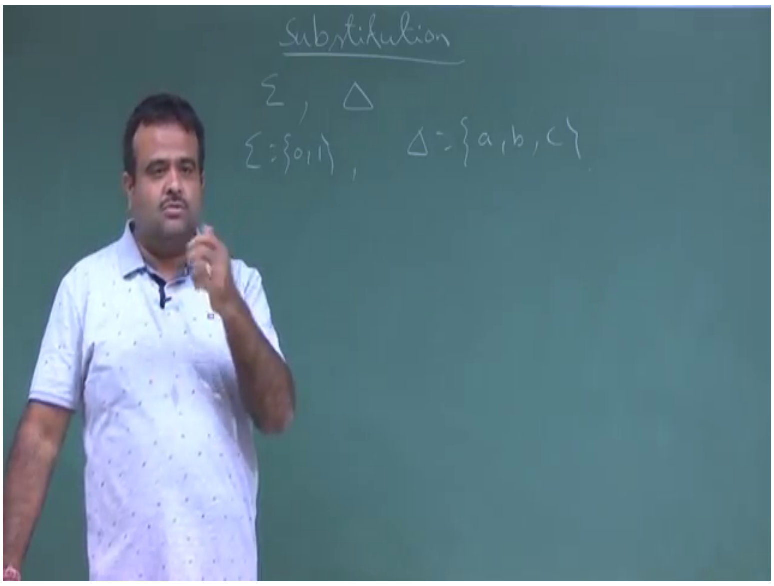

Suppose we have given 2 alphabet, a sigma and delta say sigma could be for example, sigma could be say 0 1 and delta could be a b or abc 0 1 2 anything. So, we have given 2 alphabet, we want to have a mapping from one alphabet to another alphabets string we just want to rename the alphabets.

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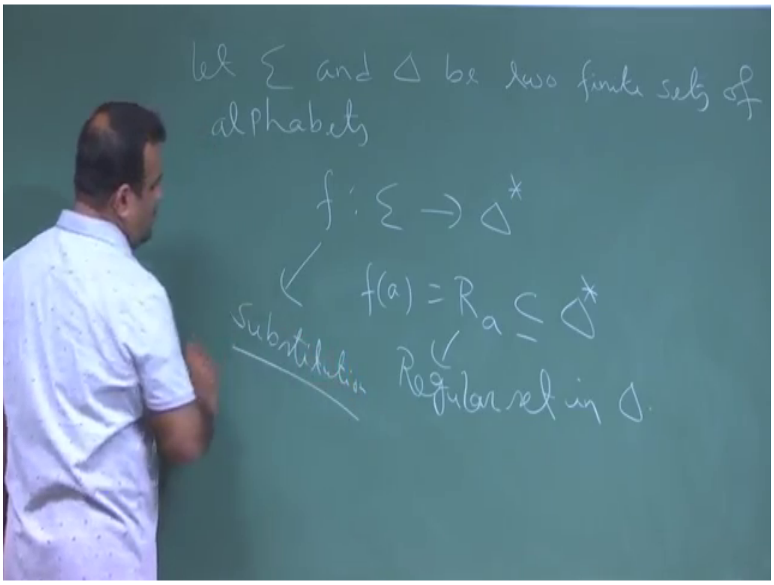

So, let sigma and delta be the finite set of alphabets sets of alphabet; sets of alphabet. Then we define a mapping from f sigma to star. Star means this is we are taking a string, given a from sigma if a belongs to sigma f of a will be a string in this set f of a is a belongs to this star.

So, defined as, so, when you call this is substitution? If f of a we define f of a actually this is a sub set this is sub set of this. So, its going to sigma star and f of a is a subset of f of a is equal to say R a which is a subset of; its a set of all say set of few strings for this delta star ok.

So, now this is regular this is a regular set or regular language, this is a regular set in delta. So, then this is called a substitution we define the substitution by this. So, we take a alphabet from a this set sigma.

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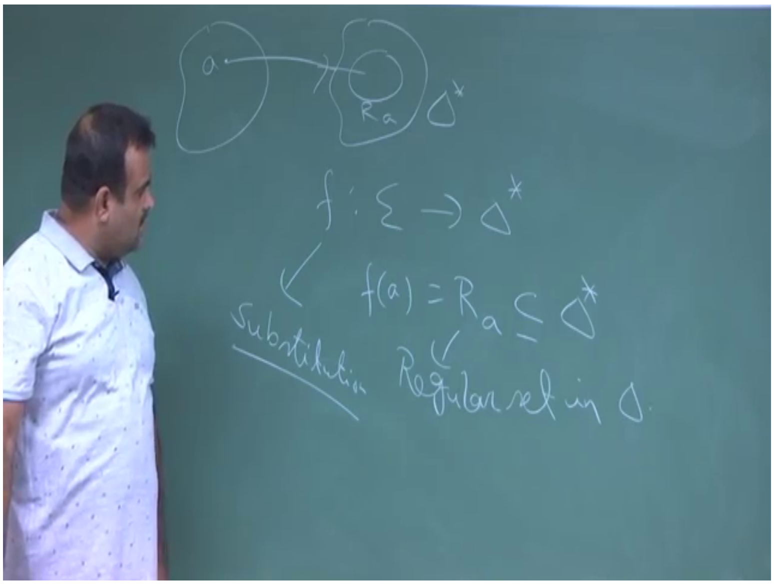

So, we take a alphabet from so, say 0 1 anything. So, a alphabet, this is going to a set over here. So, this is our delta star this is going to R of a; this R of a is a regular language then we call this is a substitution.

So, given a input alphabet in sigma it is going to a regular language or we can represent these by a regular expression. So, such a function or mapping is called substitution, here substituting this a b c d by the new symbol.

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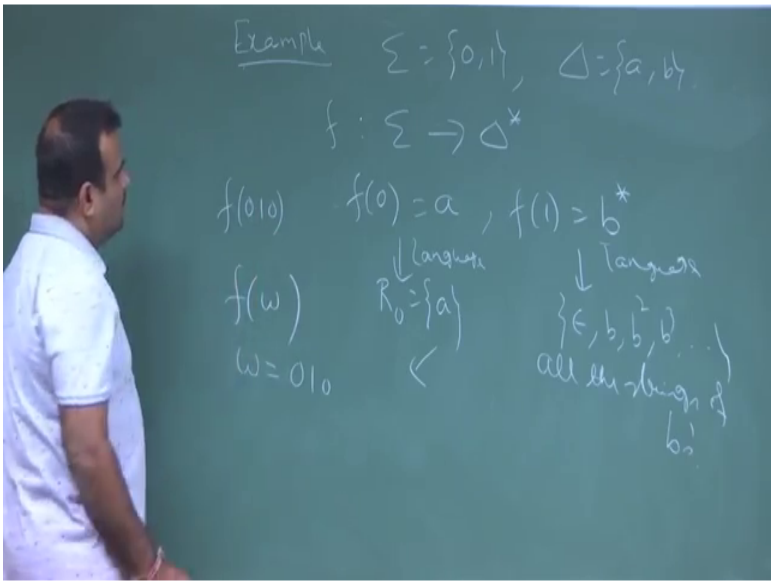

We will take an example. We will take an example suppose sigma is 0 1 and delta is say a b ok. Now, we want to have a mapping from sigma to delta star so; that means, we take a alphabet from here a I mean we take a alphabet from this set, it will give us a regular set in here. So, that we are defining like this f of 0 say is going to a and f of 1 is going to b star. So, what is the language corresponding to this? So, this is R of 0; R of 0 is basically a this is the language.

This is a single term set containing a and here what is the language this is b star the regular expression this is the regular expression a is corresponding to the single transit a and this is the regular expression b star which is corresponding to the b I mean any string of b including the null string.

So, all the string of b, sigma b b square b q, so, this is the language. So, this is all the strings of this ok. Now this is one example now this is a regular set; this is a regular set because we can have a automata which is can accept, this is any input alphabet is a regular and b star is also regular you know b is regular; so, b star is regular that is regular. So, it is going to these 2 alphabet 0 and 1 is going to a regular set. Now we want to extend this for a string this is the definition on the alphabet mapping.

Now, we want to extend this for a string. So, how we can have a say x w that w is a string w is some a 1 a 2 a n something like that here it is 0 1. So, say 0 1 0; so, you want to find f of 0 1 0, so, far you know f of 0 f of 1. So, you want to extend this for string.

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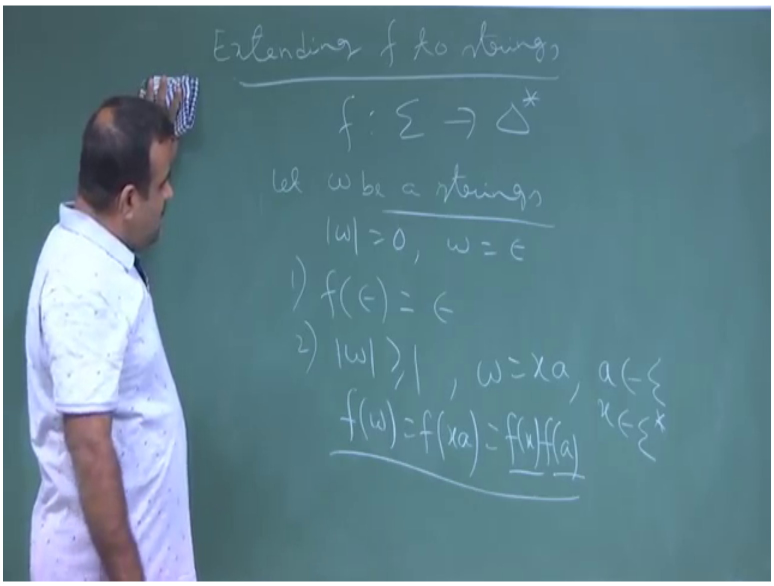

So, this is similar way as we extended the transition rule for a finite automata. So, this is the extension extending f to string. So, f is a function form sigma to delta star. So, given a we will get a regular expression there, I mean we get a language there which will corresponding to a regular expression ok. Now how to extend this for string? Say w is a string, now w could be of any length if w is 0 length of w, then w is basically epsilon.

So, how we define f of epsilon? F of epsilon is simply epsilon, if w length is 0. If w length is more than 1, if the length of w is more than 1; then we can write w is equal to x a where a belongs to sigma and x is a string including the null string if w is length of w is 1 that is alphabet.

So, this will be null and this is this, then we define f of w which is basically f of x a is nothing, but f of x concatenate with f of a. So, concatenate with f of a now this is a set this is a set this will be also set we know the concatenation of 2 sets. So, now, we will we extend. So, this is how we extend this for a string even we can extend this for language. So, this is the way we extend this for string; that means, this is our f sigma star to delta star.

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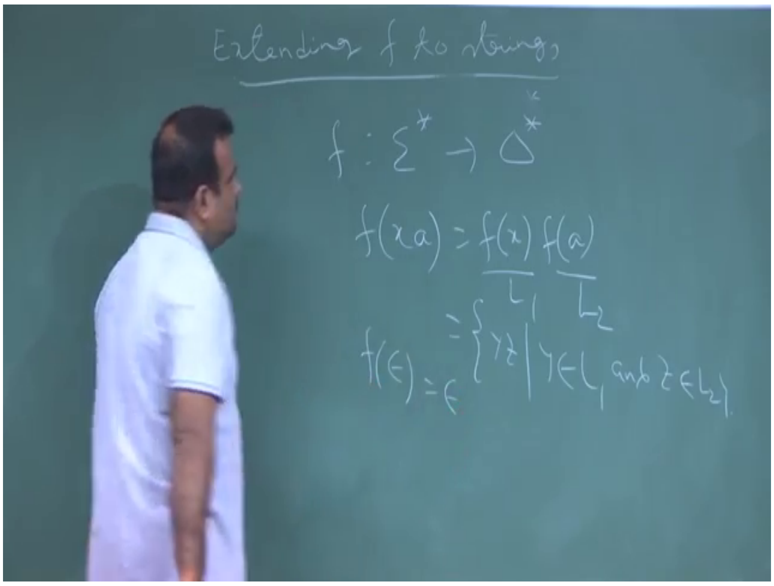

So, f of x a is defined by f of x, f of a, so, this is a language L 1 this is a language L 2. We know the concatenation of 2 language. So, this is basically set of all y z y is coming from L 1 and z is coming from L 2 and obviously, f of epsilon is epsilon like this ok.

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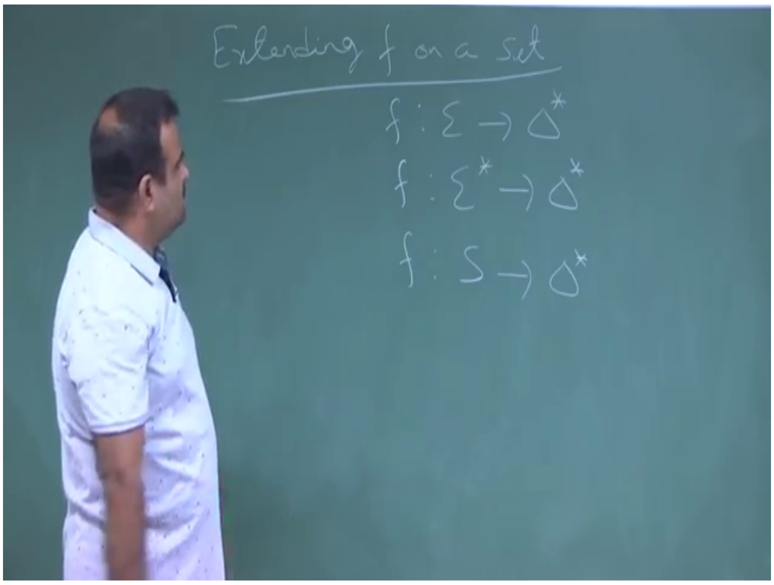

Now, we will we want to extend this for a language. We have given a language in sigma extending f on a set on sets that means, we want to see.

So, f is f now we have a extended these 2 a string. Now we want to further extend to S to this I mean actually this will be subset of that.

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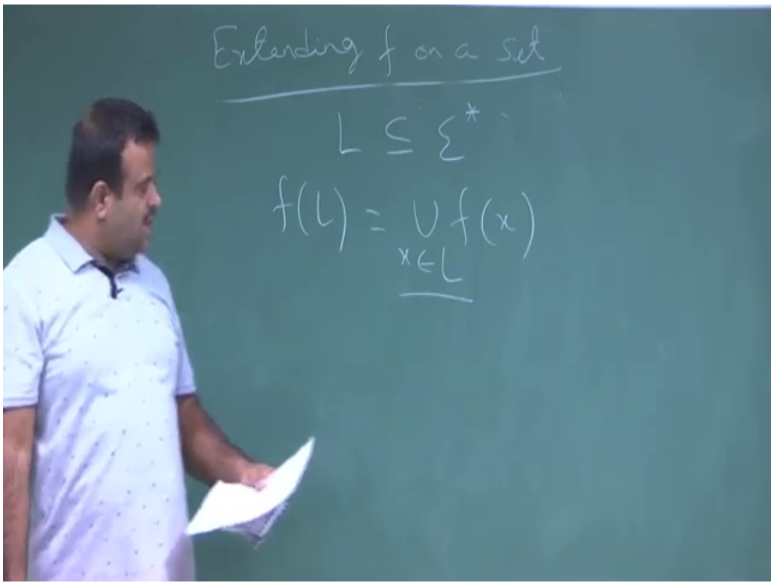

So, anyway, so, how to extend this? Suppose we take a language L from sigma star you have to define f of L. So, f of L will be union of f of x while x belongs to L; L is a string

I mean x is a string ok. So, this is the way we defined f of L this is the union. So, this is the extending this function f on a set or the language ok.

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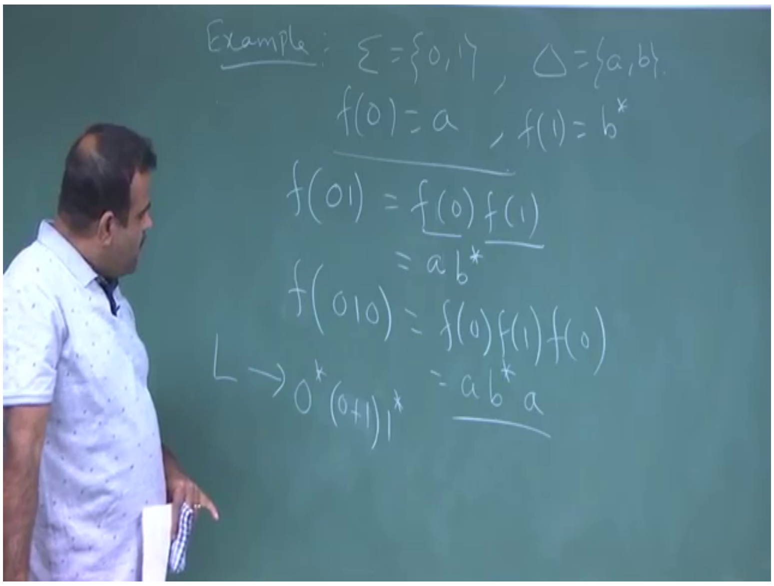

So, we takes the same example our sigma is 0 1 delta is a b and we define a to be a the this is the in the regular expression sense. So, a to be a and we define f of sorry f of 0 to a and f of 1 to b star. This is the definition and then we want to compute this. Say if we want to compute f of 0 1 how to compute f of 0 1? F of 0 1 is nothing, but f of 0 f of 1.

So, f of 0 represent a language, f of 1 represented language it will corresponding to a regular expression. So, what is f of 0? F of 0 is a and f of this is b star. Now if you want to compute 1 sorry 0 1 0 this will be basically f of 0 f of 1 f of 0 again, so, it is nothing, but a b star a this is a language.

Now, if L is a language corresponding to the regular expression, 0 star 1 plus 0 star y sorry 1 plus 0 and 1 star or 0 plus 1 ok. So, this is any number this is the language any number of 0 followed by either 0 or 1, then followed by any number of 1 that is the language ok. So, this language we want to see under this f where it is going under the substitution f.

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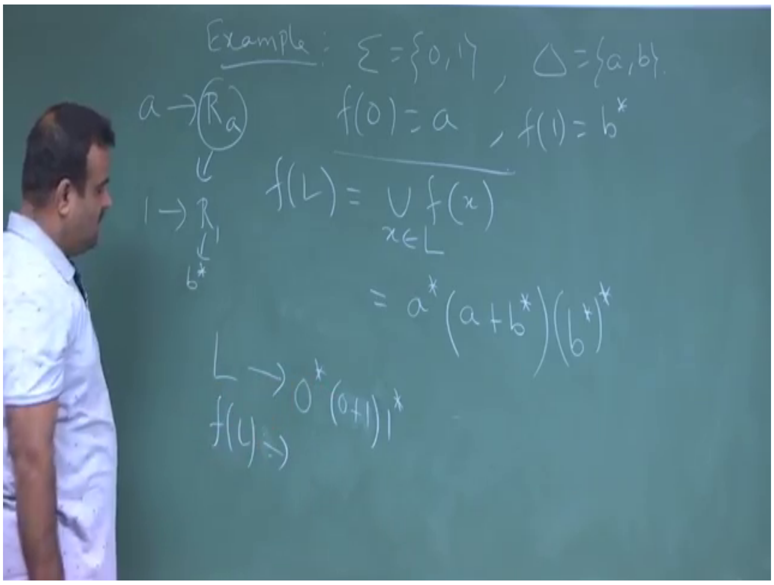

So, if you take; so, f of L by definition, it is nothing, but union of f of x; x belongs to L. So, this is we can show that, this will be actually in the regular expression sense we can just L is this. So, we will just replace 0 by a. So, a star then again 0 by a then 1 by b star, then again 1 by b star ok.

So, this is f; so, f of L is nothing, but this we can prove that, we can I mean. So, we can prove this f of L is nothing, but f of I mean we just replace this symbol by its regular expression I mean where it is going. So, a is going to; a is going to R of a. So, is R of a will corresponding to some regular expression like here it is corresponding to 1 is going to; 1 is going to r of 1, r of 1 is corresponding to b star this is the regular expression ok. So, we will just wherever 1 is there, we just replaced by b star like this.

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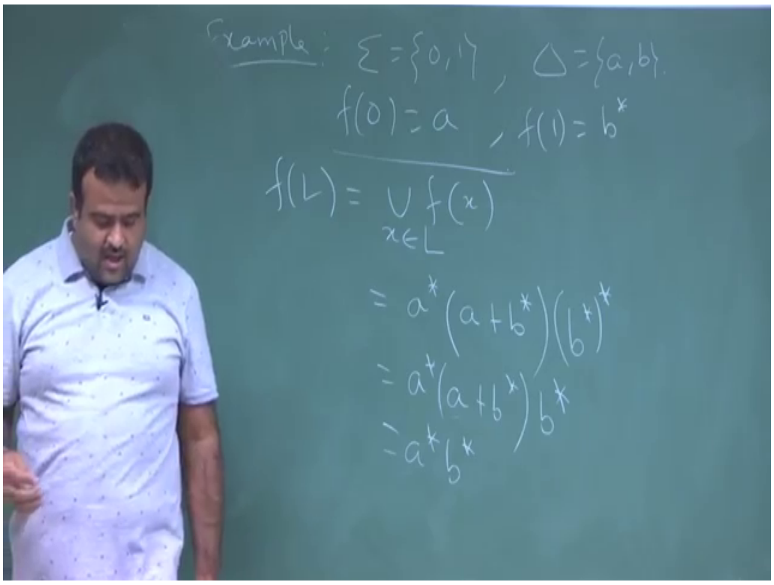

So, then this will be if you simplify this; this will be a star then a plus b star and this is b star. Now if we use the properties of the regular expression, you can prove this is a start b star. We can take this just a a b star then b star b star. So, this is will be and again if we concatenate these from a star you will get a b star.

Now, so, this is how we define the language substitution under the on the language. Now we want to see if the L is regular, then whether f of L will be regular; that means, the is the regular set is close under the substitution. So, that we want to see the class of regular set is close under substitution.

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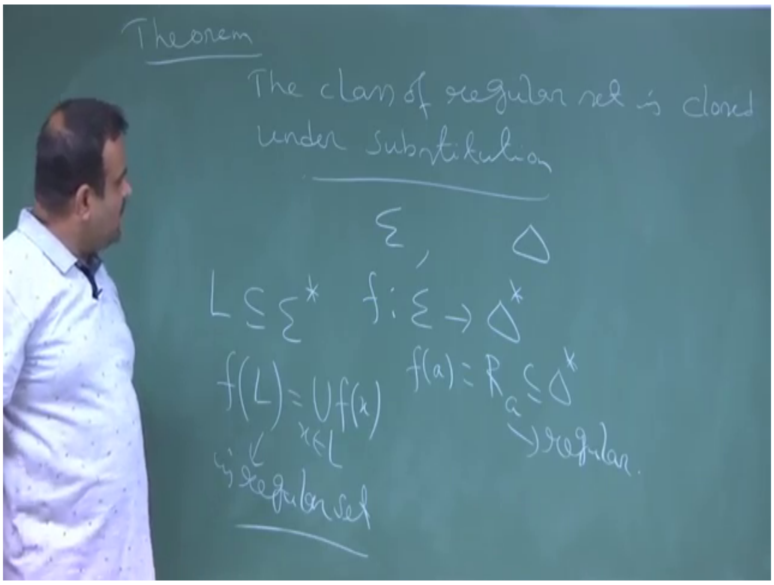

So, this is a theorem the class of regular set is closed under substitution ok. So, what is the meaning of it? Meaning of it is, so, if we have given a sigma and we have given delta 2 alphabets say 2 sets of finite alphabet and if we have a substitution. Substitution means we if we take a from here f of a will be a regular set in delta. So, it is R of a which is a subset of this and this is regular; this is regular this is the definition of substitution.

Now, what we are going to show? We are going to show that we take a R from or we take a language from this and then we know what is a f of L? F of L is nothing, but union of f of x, x belongs to L x is a string. So, in that case we know the how we can extend this f on the string now we are going to show this is regular; this is regular set that is the theorem.

So, we can show this is regular, then we can say the regular set is close under substitution and it is true why? This can be proved like this. So, one way we can see it by in terms of regular expression.

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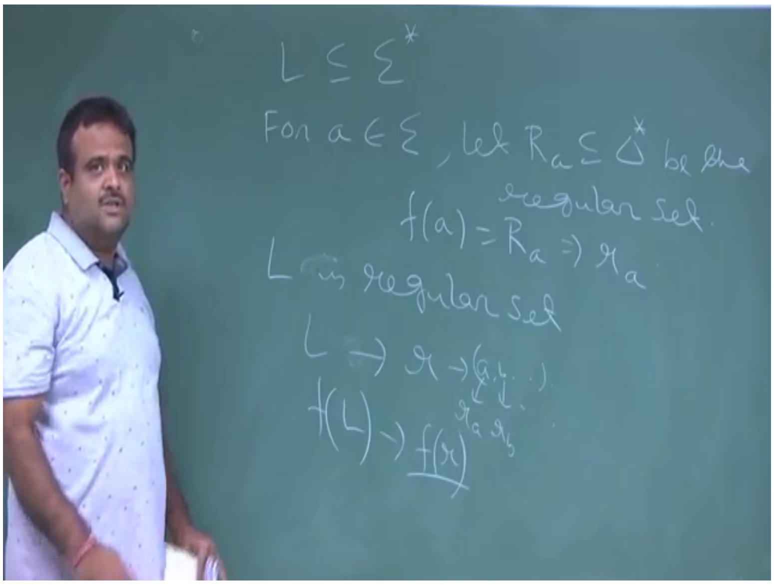

So, what we do? We take a language from here and for each a let R of a which is a subset of this with a regular set, regular set and once this is a regular; that means, f of a is R of a. And once this is a regular set then we can have a regular expression for this r of a we can have a regular expression for this r of a.

Now, L is regular; L is regular set; that means, L can be represent in the regular expression of this L will be represent as a regular expression. So, we have a regular expression for L, so, that is sum R. So, R will be consist of some abc like the some form of this now what we do? We replace a by the regular expression of r of a we replace b by the regular expression of r of b like this. Then we will get a regular expression for f of L and that is the proof I mean since we are getting a regular expression for f of L, we have a regular expression for f of L.

So, what we are doing here? We have a regular expression for r which consist of all the symbols from sigma. Now we just replace that symbol of sigmas by a by the regular expression of r of a b by the regular expression of r of b see like this. So, then we will get a regular expression and that will be the that is the proof. Since we are getting a regular expression, so, we must have a finite automata which is accepting this.

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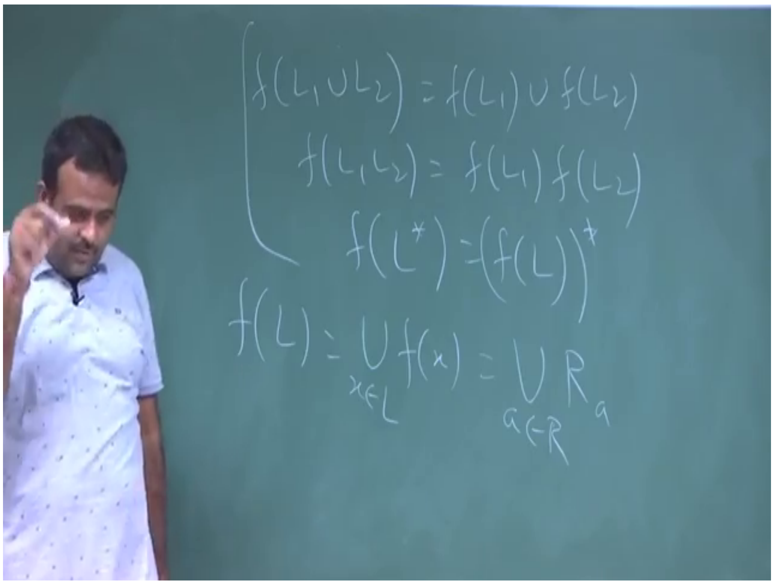

So, this we can even you can justify these using like we have to have to prove like this f of L 1 union L 2, this we can leave as a exercise; f of L 1 union f of L 2, then f of L 1 L 2 is equal to f of L 1 f of L 2 and f of L star is nothing, but f of L star. If you can prove this then f of L which is nothing, but union of f of x.

Now, is to L this we can write as R of a where a belongs to R. So, this we can prove using the induction on the number of operator on the number of operator using the regular expression. So, this proof you can do this exercise. So, regular substitution is closed under a regular set is closed under substitution, now we will take the particular case of substitution which is called homomorphism.

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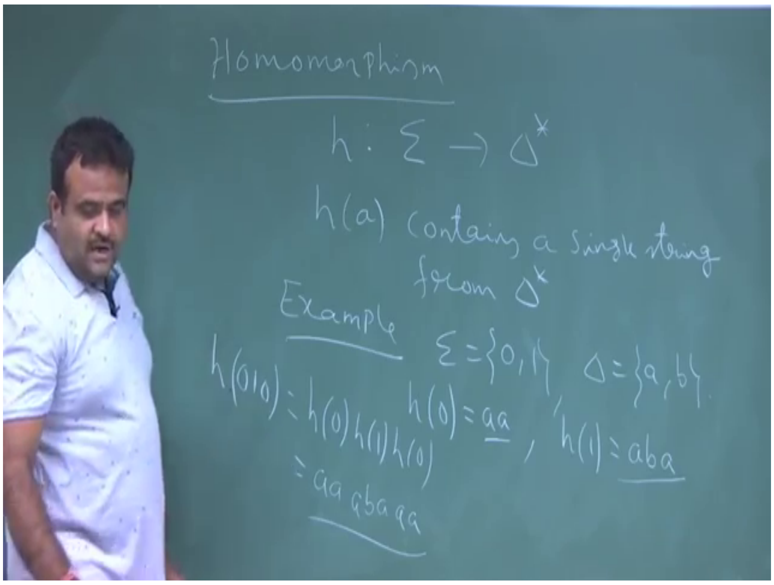

. So, this is also a substitution, but it is a particular case of substitution where a alphabet is going to a fixed string a singleton a single string not a set. So, it is a sigma to delta star, where w a content a single string single string from delta star ok. It just going to a single string its not going to a set its going to a singleton set a single string then it is called a homomorphism they have substitution and that single string is a is a regular set it is a particular case of regular a substitution. So, this homomorphism is a particular case of substitution, but here the mapping is to a single string instead of a set.

So, example is like this. So, if we have say sigma a 0 1 and delta is a b. So, what we can do? So, you can take this h of 0 is going to say h string a b and h of 1 is going to a b a ok. So, this is a single string this is that is a single string. So, still it is substitution because this is a single string.

So, this is a regular set, this is also a regular set single string can be regular set we can have a automata for accepting a single string ok. Then in that case we can again define the this we can extend for a string like h of 0 1 0 is nothing, but h of 0, h of 1, h of 0 which is nothing, but a a, h of 1 is a b a, h of 0 is again a a like this and we can have we can define this over the language.

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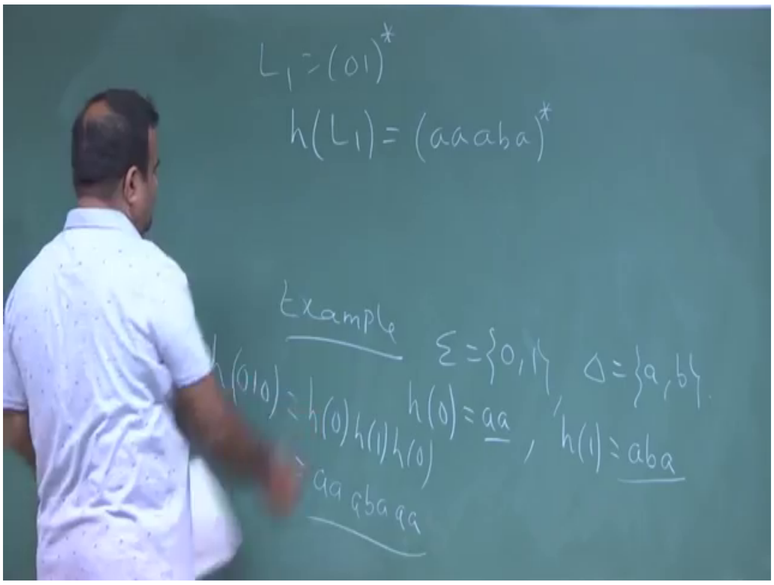

So, this is nothing, but a substitution, but it is going to a single string in set of a set of string ok. So, if L 1 is 0 1 star it is a language, which is having epsilon then 0 1 then 0 1 0 1 and 0 1 0 1 0 1 like this. Then h of L 1 we just replace these by its. So, 0 is going to a b, so, a a then 1 is going to a b a star.

Now, this is also regular. So, this L 1 is regular then h of L 1 is regular. So, regular set is closed under the homomorphism also, because homomorphism particular case of the substitution. Then we can define the inverse homomorphism also over a this language because inverse homomorphism means given this we can come back this.

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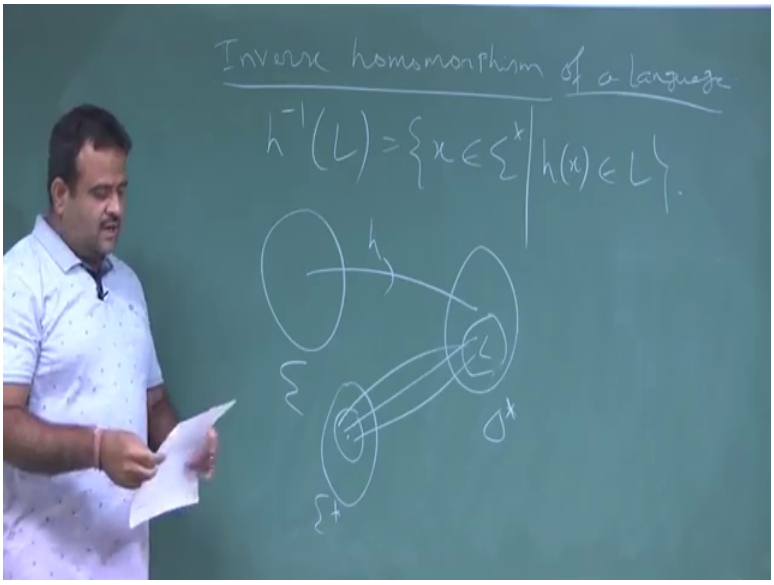

Inverse homomorphism of a language; so, suppose we are given that homomorphism, then inverse homomorphism is defined as like this, this is the set of all x in such that h x belongs to L.

So, we have a this is our sigma and this is our star, then homomorphism is the mapping h this. So, h of L means, so, if you take a subset of this L then. So, we can construct all the strings which is going to here, I mean then it will be sigma star. So, all the string which are mapping here under this h, so, that is called inverse homomorphism. So, we can take one quick example.

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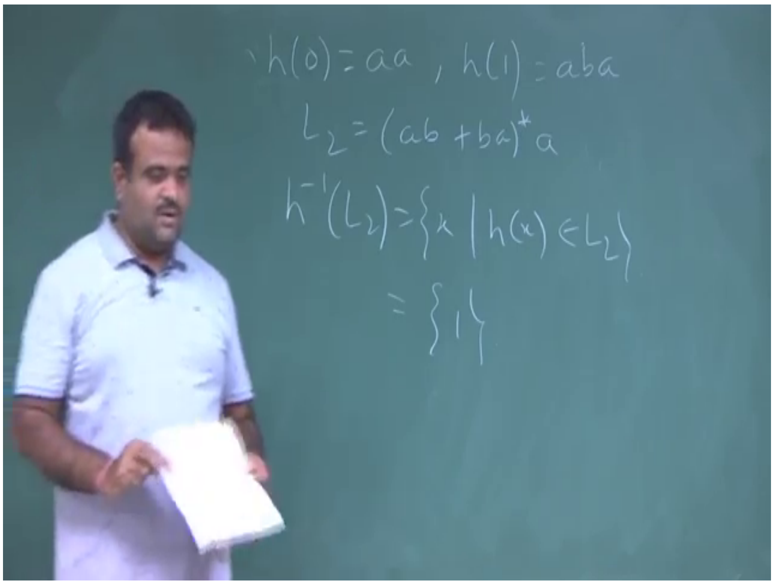

So, the same example if h of 0 is a a, h of 1 is a b a, then suppose L 2 is a b plus b a star, then a then what is inverse of L 2? Inverse of L 2 is set of all string which are mapping to L 2 ok. Now we can verify that this set will be just 1 we can do it this exercise and we can verify that this string is just one.

Thank you.