Discrete Structures
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Lecture – 43
Combinatorics (Contd.)

So, we are discussing the Combinatorics; that means, mainly the counting problems with arrangements and in particular the ordered arrangements that means, the permutation and the unordered arrangement that means, the combinations. Now the last lecture, we have read the permutations; mainly, the permutation of a set with having n elements or a combinations of a set having n elements.

Now today, first we will see that permutations of multi set; that means, r permutations of a set, where the elements and not distinct. Normally the set we define where the elements are distinct and we define that type of set as a multi set. So, we will read the Permutations of Multi sets.

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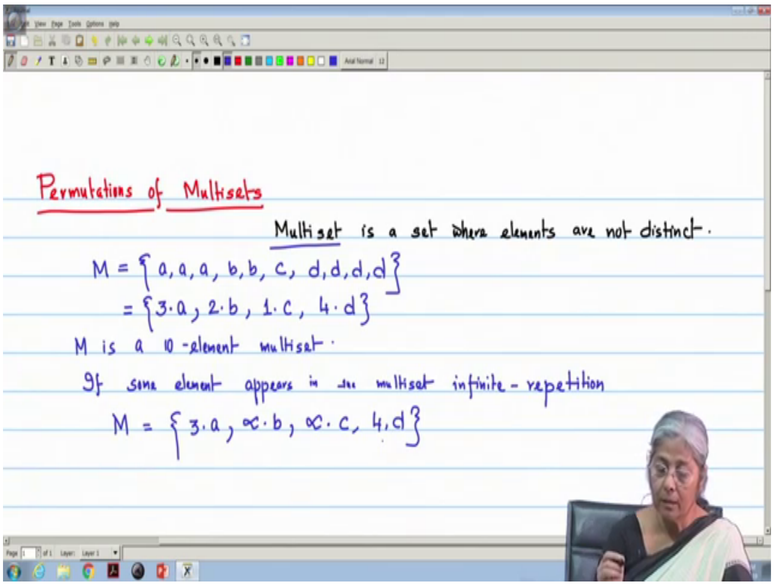

So, a multi set is a set where elements are not distinct. So, if elements are not distinct that in other way, I can tell that elements are with repetition; that means, set with the repetition of elements that we can tell as a multi set. So, I take one example that say M is a multi set and M, I can write say I have a; there are 3 a's; 2 b's; then, c 1; I have saved 4 d.

So, it has 10 elements set. Now I can write this thing since a b c a b c d they are not there are four different types, but they are not distinct and the same thing, I can write as if a with 3 repetitions. So, normally we define or we denote this thing as a 3 dot a; that means, a of with repetition 3; 2 dot b, b with repetition 2 repetition; 1 c and 4 d. So, there are 10 elements; this is a 10 element multi set. So, M is a M is a 10 element multi set. Now it may happen that there is no restriction of some particular elements; that means, that we can tell that infinite elements.

These are finite. Finite number of elements exist. Now if infinite, if some element appears in the set or I should write the multi set infinite time with infinite repetition; then, I can write the set M say this is a with 3 repetition; b with infinite repetition that I can write infinity dot b; again c can be infinite repetition and then this is a. So, this and infinite number of elements multi set. There is no restriction. Now, with these introduction, we try to define that the permutation of a multi set. So, as we have defined already that permutation we have to defined that this is an ordered arrangement. So, we will be counting the some objects when the arrangements in order.

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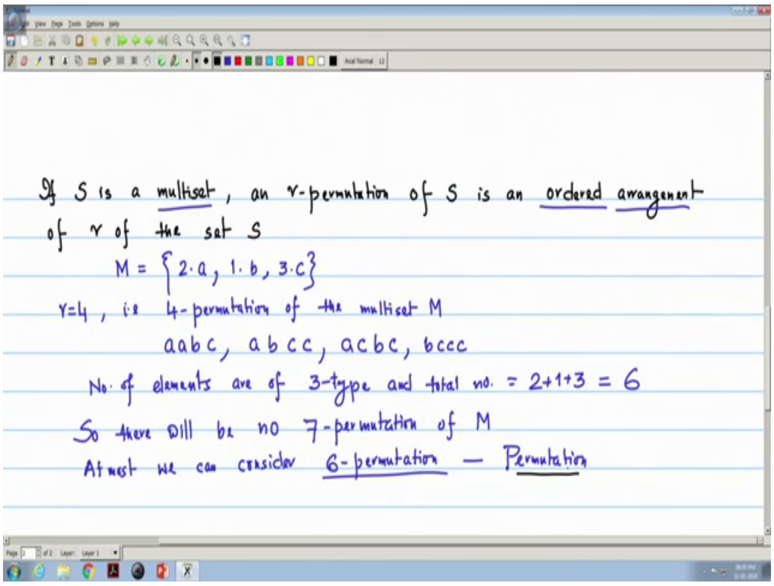

So, I can write that if S is a multiset and r-combination this is a we are discussing permutation, this will be r permutation; r - permutation of S is an ordered arrangement of r of the set S having n elements. So, this is same; only this is again ordered arrangement only we have taken this is an multiset. So, if I take an example say M is a multiset and I have taken say a with repetition 2; then b with repetition 1 and C with repetition say 3. Then, the if I if I want the r equal to 4 say that means, 4 permutation; that means, that is 4 permutation of a set M some of the 4 permutations at be I can write a a b c or a b cc; these are some of the some of the 4 permutation.

It can be that since it is ordered I can take a c b c, I can take this type of ordered permutation permutation or it may be b c c c that is again another order permutation. Now I have here the number of elements or in M number of elements of 3 type a b c elements are of 3 type and total number equal to 2 plus 1 plus 3 equal to 6.

So, there will be no 7 - permutation of M. So, at most we can take the upto 6 permutation we can take. So, at most we can consider 6 - permutation and normally if it is and then, it is called permutation only because if it is in permutation of a set having n elements, then we call this is nothing but the permutations of all elements. So, this is called the Permutation. Now, we will see a property or so, I give a theorem.

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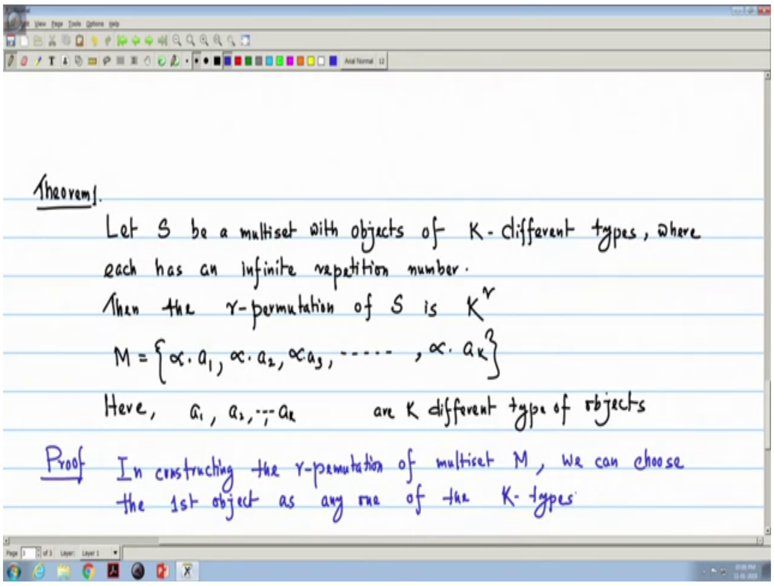

How to compute the a r-permutation of a multiset having n elements when it is finite or we will see later if it is infinite. So, let S be a multiset with objects of K different type; like last example we have to seen a b c 3 different types. So, here K different types, where each type each has an infinite repetition. Then, the r r - permutation of S is K to the power r. So, the first theorem we consider that the multiset having infinite number of or having elements with infinite repetition number and they are of K different types; that means, I can write that like M is a multiset, if I write that infinite of if a 1; then infinite number of a 2; infinite number of a 3 and in this way it is infinite number of a K.

That means, here a 1. Here a 1, a 2, a K are K different type of objects; are K different. Now, how to prove this thing; how to prove this theorem? So, in constructing the r – permutation, we have to construct the r - permutation of this type of multiset, we can choose of multiset M since every element appears or may appears infinite time. So, we can choose the first element or the first object; first type of objects, any one of the K types; is it as any one of the K types.

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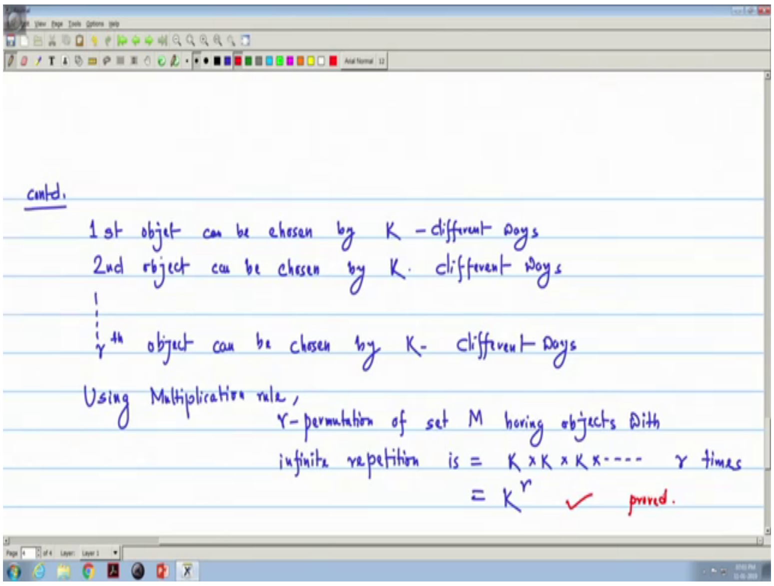

So, since I have r r - permutation. So, I take that first object can be chosen by K different ways because any one of the K type second object again that can be chosen by K different ways and similarly if it is rth because it is r - permutation. So, rth object can be chosen by K different.

Since there is no restriction and any type of objects appears infinite with infinite repetition. So, the multiplication rule; using multiplication rule, I can tell that r r - permutation of set M having objects with infinite repetition is equal to K into K into K r times. So, this is equal to K to the power r. So, result is proved. Now, we see that application of this theorem. I take one example one problem.

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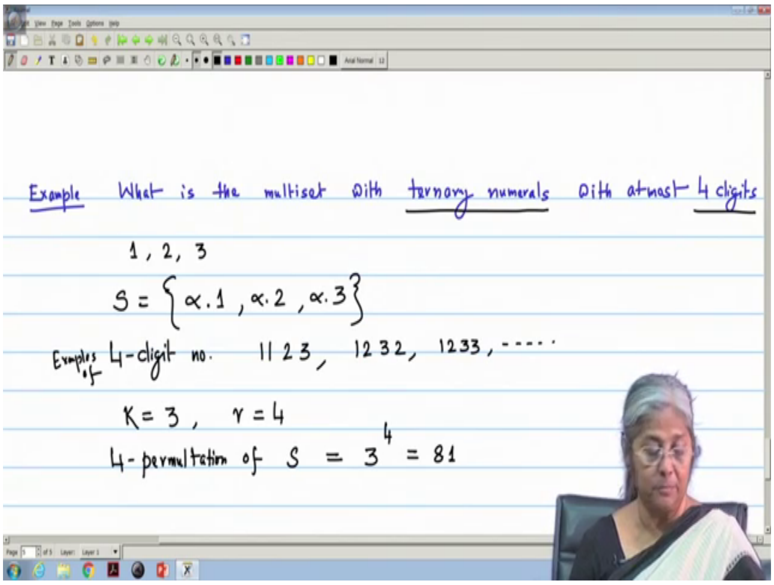

How we can apply this one. So, I give one problem that what is the multiset with ternary numerals with at most 4 digits; what does it mean? So, with ternary numerals means my numerals can be only 1, 2 or 3, but obviously, that set has the 3 type of elements. So, if I consider a set S 3 type of elements, but order of infinite with infinite repetition because there is restriction. Now, I have to form a 4 digit number on the with I have the 4 digit number like examples; examples of 4 digit number only with 3 literals.

So, that may can be 1 1 2 3; it can be 1 2 3 2; it can be 1 2 3 3; all this type of 48 numbers. Now, mainly we have to find out that what is the multiset; that means, how many this type of numbers are possible? So, directly we can apply this result of this theorem because here it is K to the power r. So, K of 3 because I have 1 2 3 only; three different types. So, K equal to 3 and r equal to 4 because mainly I have to take the 4 permutation, since I have only 4 digits. So, 4 permutations of S and this is equal to 3 to the power 4 is 81.

So, I have only these many numbers possible. Now, I give that finite if the theorem want a set if the multiset have objects with infinite repetition. Now it take if it is of finite number of repetition.

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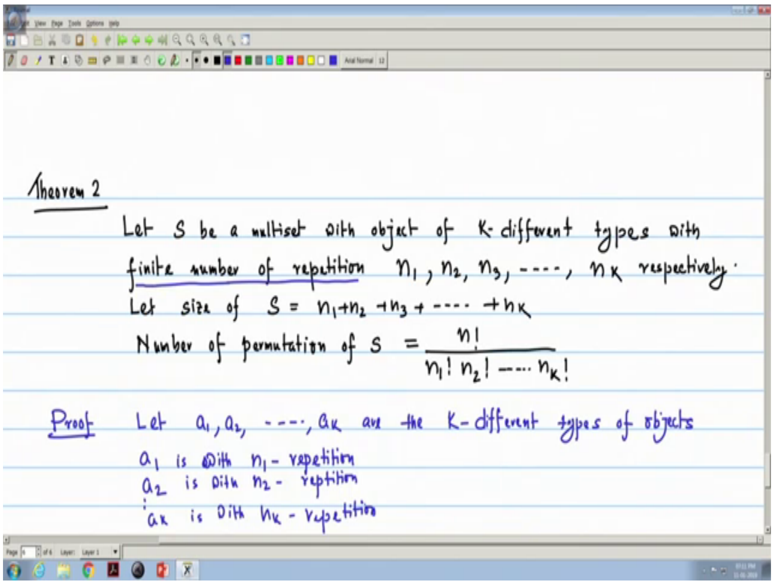

So, I write Theorem 2. Write the statements of the theorem that similarly let S be a multiset with object of K different types with finite number of repetition and I am writing the repetition numbers are n 1, n 2, n 3, K different types so n K respectively. So, the I can write or we consider that let size of S is n 1 plus n 2 plus n 3 plus n K. So, number of permutation of S, we can write is n factorial divided by n 1 factorial into n 2 factorial up to n K factorial.

So, first thing is here Theorem 2 tells about finite number of finite number of repetitions. So, let the K different types of the elements are a 1 to a k. So, let a 1, a 2, a K are the K different type of objects and a 1 is of is with repetition finite repetition with n 1 repetition; that means, n 1 number of a 1. Similarly, a 2 is with n 2 repetition and a K is with n K repetition, then say I have to take the I have K different types. Now, let first we choose the a 1 objects.

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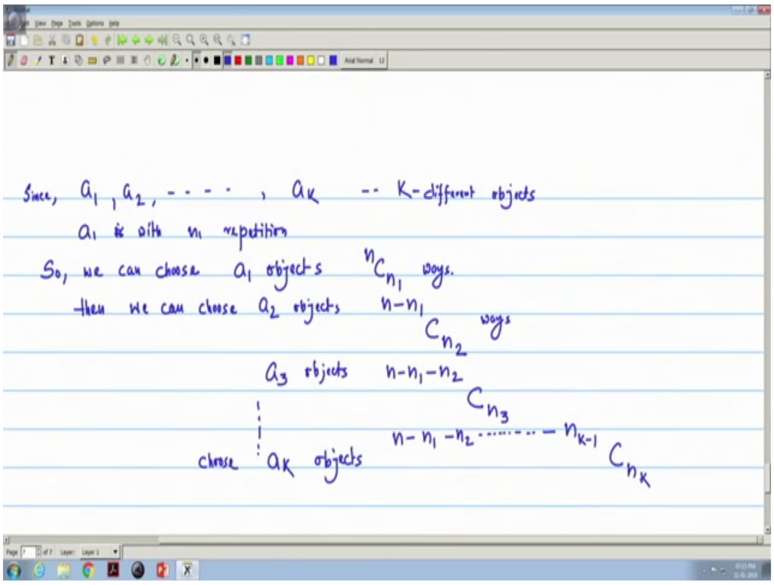

Since I have I have a 1, a 2 a K; these are the objects we have a K, the K different since I have a 1 into a K; K different objects. Then, an a 1 appears a 1 is with n 1 repetition. So, we can choose; we can choose a 1 objects by n C 1 ways n C 1 because n C because a 1 is of is with repetition n 1 ok. Now, once I have chosen n C n 1 ways of a 1, then we can choose a 2 from remaining for remaining objects. That means, remaining objects are a 2 objects; remaining objects are n minus n 1 and my a 2 is with repetition n 2. So, this many ways I can take.

So, the same thing if it is for 3 objects, then now my objects remaining n minus n 1 minus n 2 C 3 a 3 is with n 3 repetition; a 3 with n 3 repetition. So, I have n 3. In this way if I continue, then I can tell my a K if I choose a K objects; a K type of objects that with n K repetition. So, n minus n 1 minus n 2 minus n K minus 1 C n K. So, now, we can use the use multiplication rule.

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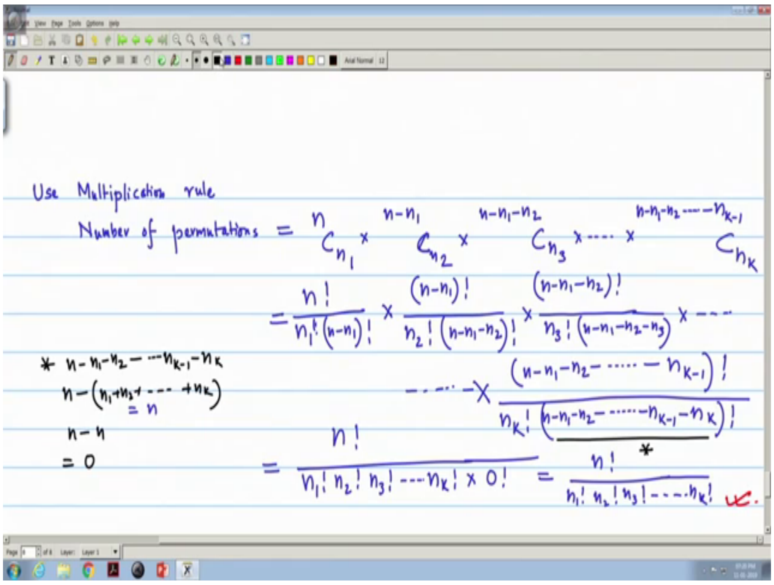

And we can get the number of number of multiplication, number of this is number of permutations; number of permutations equal to the further of a 1 objects n C n 1; then, it is n minus n 1 C n 2, then n minus n 1 minus n 2 C n 3 and in this way are get the for a K n minus n 1 minus n 2 minus n K minus 1 C n K. So, if we put the values the n factorial by n 1 factorial into n minus n 1 factorial, then again n minus n 1 factorial divided by n 2 factorial n minus n 1 minus n 2 factorial.

Then, n minus n 1 minus n 2 factorial; then n 3 factorial n minus n 1 minus n 2 minus n 3 and in this way if I go, then the last term will be n 1 minus n 2 minus n K minus 1 factorial divided by n K factorial and n minus n 1 minus n 2 minus n K minus 1 minus n K factorial. So, the numerator and the previous denominator, they will cancel because it has same term n minus n 1 here; n minus n 1 minus n 2 n minus n 1 minus n 2.

So, they will cancel and or be getting the n factorial divided by n 1 factorial, n 2 factorial, n 3 factorial; then, n K factorial and the last denominator will be there. Now, what is this last denominator? See the last denominator is if I write here, the last denominator is n minus n 1 minus n 2 minus n K minus 1 minus n K.

So, if you remember the size of S that is n is n minus this size of n is n 1 plus n 2 plus n K which is nothing but equal to this is equal to in only; the size of S. So, this is n minus n is equal to 0. So, I get for the last one this 0 factorial. So, which keeps S that or n factorial divided by n 1 factorial, n 2 factorial, n 3 factorial; then, up to n K factorial. So, this is my that the proof. So, quickly if we see one example.

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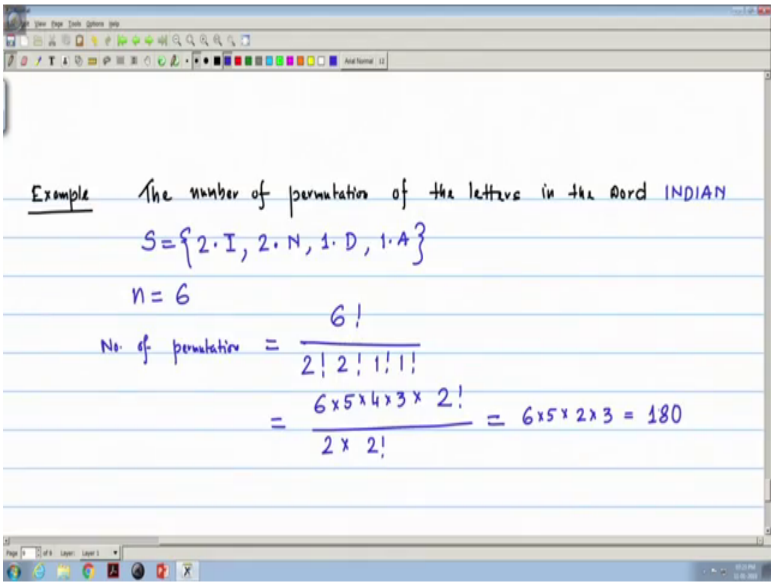

So, we want to compute the number of permutation of the letters in the word say word is INDIAN. So, here if I consider the set S, then I repeats 2 times; N repeats 2 times; then D repeats only 1 time; A repeats 1 time. So, this is my multiset and I have total number is the size of S is 6. So, my n equal to 6 and I can write the number of permutation. So, the number of permutation is 6 factorial divided by there are 4 types I is of this is my n 1 2 factorial, n 2 2 factorial, n 3 1 factorial and n 4 1 factorial; that means, I with 2 repetition; N with 2 repetition; D with 1 repetition; A with 1 repetition.

So, I can write 6 into 5 into 4 into 3 into 2 factorial and this will be 2 into 2 factorial on this becomes 6 into 5 into 2 into 3 this 180. So, in this way, that we can apply this theorem of to count the number of permutations for multiset, where the elements appear with finite repetition or infinite repetition and we can apply to solve the counting problems with permutation.