Discrete Structures

Prof. Dipanwita Roychoudhury
Department of Computer Science & Engineering
Indian Institute of Technology, Kharagpur

Lecture – 41
Combinatorics

Last few lectures we are discussing about the different counting techniques. And today we will start the Combinatorics which again is a counting techniques and a special type of counting techniques that we will cover under this lecture.

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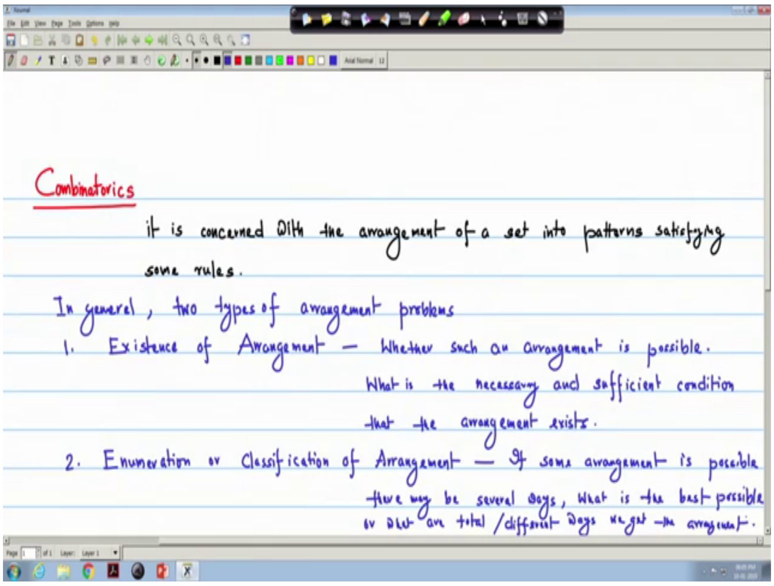

So, combinatorics is normally concerned with the arrangement, arrangement of a set into patterns satisfying some rules. So, I can write that in general, it is concerned with the arrangement of a set into patterns and normally satisfy some of the rules.

Now normally we have two general types of arrangement, we or the type of problems we handle. One we call that existence of arrangement that two types. In general two types of arrangement we consider; one is called the existence of arrangement see when we are concerned with some or we want to make some arrangement satisfying some property. So, first thing is whether at all it is possible or not; that means, whether the arrangement exist. Now if it exist then under what condition or what are the necessary and sufficient condition that the arrangement should exist. So, this is called the existence of arrangement.

So, in short we can write that it is about the whether such an arrangement is possible. And this satisfying rule then tells under or what are the sufficient; what is the sufficient condition necessary and sufficient condition that the arrangement exist. So, that is why it is existence of arrangement.

Another type of combinatorics problem, we called enumeration or classification of arrangement. So, if the arrangement exists then normally many different ways, we can get this arrangement. Now we have to or sometimes we may want to know that which one is the best possible way that we get the arrangement, then we call this is the enumeration. So, we can write that if some arrangement is possible then maybe several ways that we can get this thing, several ways.

Then what is the best possible way or what are the total number of ways or what are the total number of ways we can or different ways we get the arrangement? Now normally most of the arrangement problems that we can divide into mainly two different ways that we make the arrangement.

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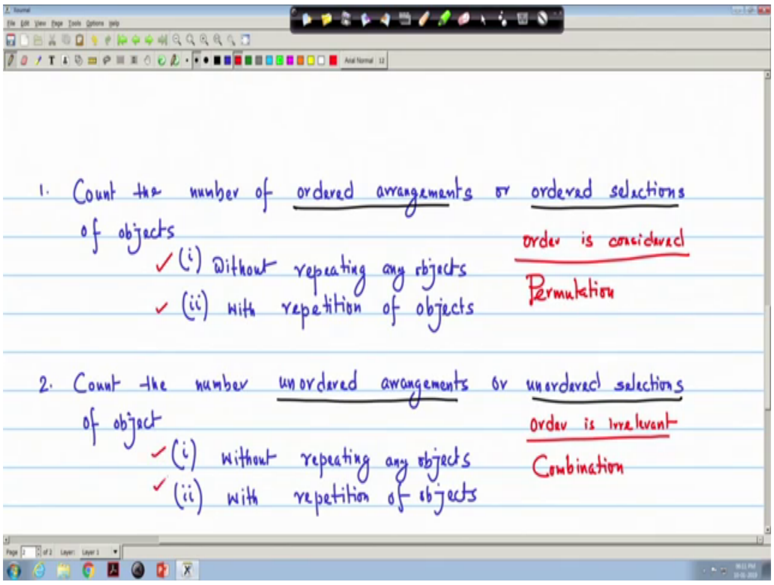

So, what are the two different ways that, one is the count the number of ordered arrangements or sometimes we call the selection so, that is also ordered, so ordered selection. Now this also, so these ordered arrangements or ordered selections that we can do in two different ways; that means, we are selecting some objects from a set of objects and this set can be of two different type.

What are those type? One is that without one is without repeating any objects, other is we consider repetition; that means, with repetition of objects. Similarly, that the another type that we can write the count the number of unordered arrangements or unordered selections of objects and from the set that which have the same property either without repeating any objects or with repetition of objects. Now, normally these arrangements of or selections in which order is taken into consideration; that means, here this first case that order, it is the ordered arrangement; that means, order is considered and order is considered and this type of arrangement we called permutation.

So, while order is considered; that means, the first case we call this is permutation. And the second type, see this is that unordered arrangements or unordered selections, so, here order is irrelevant. So, second here order is irrelevant and we take this is as a we define this thing as a combinations. So, normally we arrangement either ordered or unordered and the problems that we or the counting problems that we will be handling that either these ordering considering the order of the arrangement or we can ignore the order, then it is called the permutation and combination.

So, another point that what we have seen that both the cases whether it is permutation or whether it is combination, we have, we can do to is; that means, it is either it is without repeating any objects or with repetition, both the cases and that we take a set we can define a set or a multi set.

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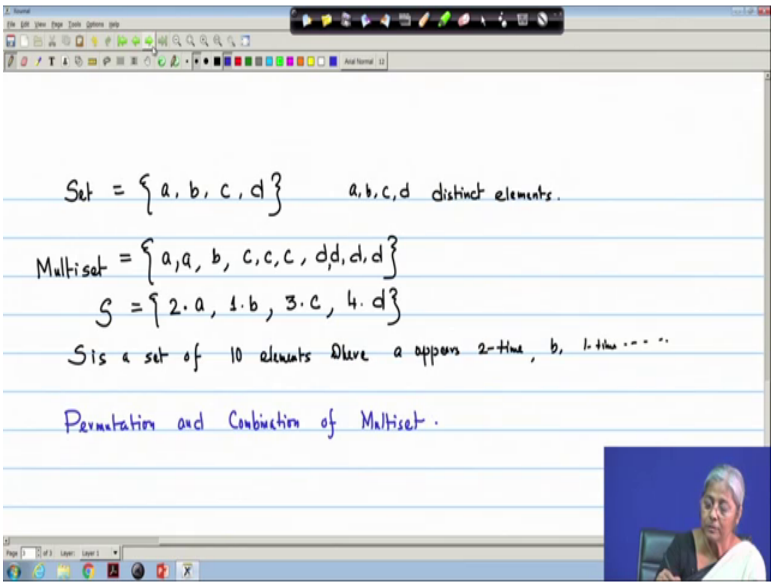

So, now, we said we know that say set say it is a set of say 4 elements a b c d. Normally these are distinct elements so, these a b c d, we take wherever these are distinct elements. Now, we can take this type of set say where these a is appearing more than one, say I have 2 a, say I have 1 b, I have 3 c and say 4 d. Then we define this type of set as the multi set and same set we can represent like that here a appears twice. So, I can give the 2 numbers of a, 1 number of b element, c type of elements that 3 c and 4 d.

So, if I write S; so, here s is a set of 10 elements where a appears 2 times like that the 1 time and this one. Now we will also consider the permutation of multi set and the permutation of combination of multi set. So, we will also consider the permutation and combination of multi set.

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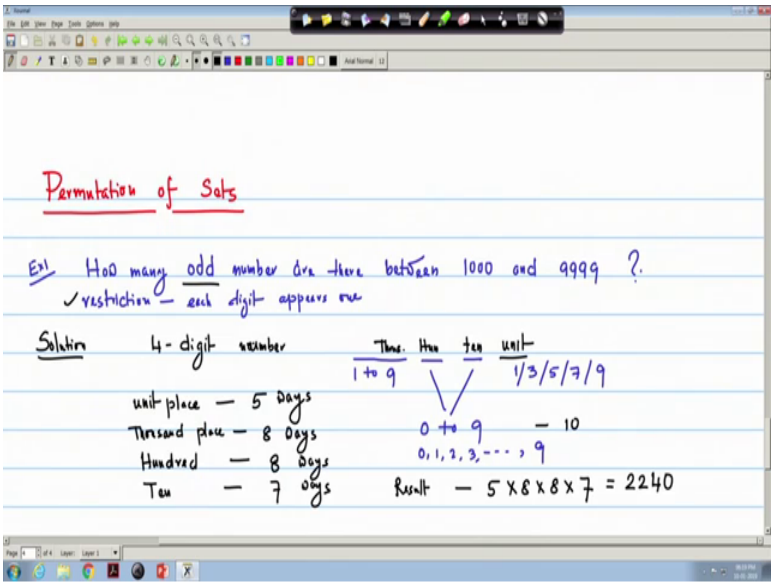

So, first we see the permutation of sets ok. Before we give the definition first we see one simple example. I take a counting example say how many odd numbers are there between 1000 and say in 9999? This is a simple counting technique. Now without considering any formula, etcetera how we can solve this? See there are, I have 4 positions since it is a 4 digit number. So, it is a 4 digit number or numbers are 4 digit. So, it has, there are 4 places say I have unite place, 10, 100 and 1000 place ok. So, since it is odd number. So, this unit place number, unit place number can be only 1 3 5 7 or 9.

So, since it is odd number. So, first, we see that there are five different ways the unit places can be there. So, I take the unit place numbers can be 5 ways. Now if I consider that there will be no repetition; that means, each number appears each digit say restriction what I earlier I mentioned that some properties satisfying some property. Now say I am a giving some restriction that each digit appears one each.

Now what will happen? That unit place; that means, then one odd number that I have given that among 1 3 5 7 9, we have to neglect that thing. And then these thousand numbers they can be this thousand places, number can be only 1 to, 1 to 9, because 0 cannot be there, but these hundred place numbers and tens number they can be anything from 0 to 9, they can be 0 to 9; they can be 0 1 2 3 up to any number from 9.

So, I can put a thousand that number, I have already taken in unit place that I have to discard from 1 to 9. So, there will be some thousand places will be thousand places, place I have 8 8 ways, I can give this. Now then I have to consider either or hundred place then I have to neglect from 0 to 9 there are 10, there are there will be 10 digits. And then already I have consider 1 for unit and 1 for thousand. Since, I have restriction or these are the property to be maintained.

So, I can put that 8 ways 2 since there are 10 2, I have neglected then for ten places, there are 7 ways. So, what are the result that I can get is 5 into 8 into 8 into 7 and these are the different way I can make this number. See I do not know, I have not used any formula, but this is something called counting techniques and some ordering is there and also some property to be maintained here. So, how we can make this thing as a permutation ok? First I give that how what is the idea or how we frame the permutation of numbers?

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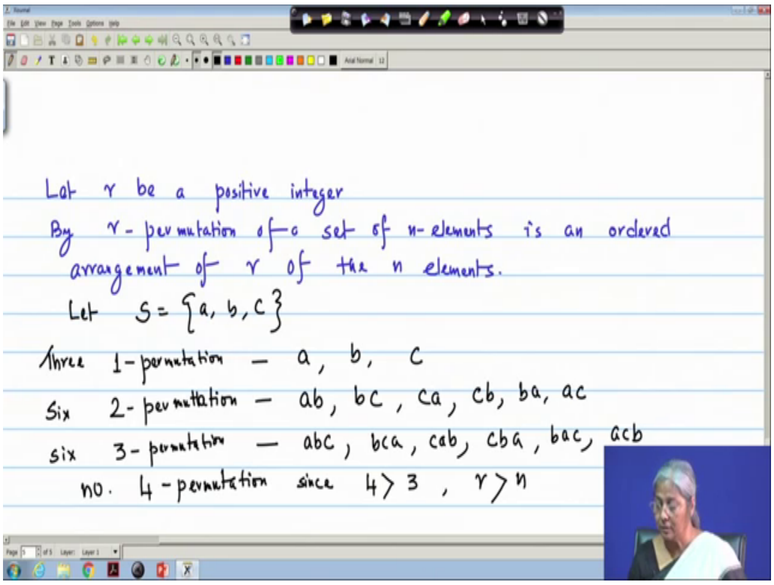

So, I write let r be a positive integer. Then by r permutation of a set of a set S of n elements, set of n elements is an ordered arrangement. Earlier, we have mentioned that thing that permutation is an ordered arrangement of r of the n elements. So, if I take say example let the set S is a three element set a b c. Now what are the different ways I can take? Say if it is a 1 permutation; 1 permutation; that means, how many ways I can take 1 element.

So, there will be three 1 permutation, I can tell there will be three 1 permutation. What are those that a b or c then how many are 2 permutations; that means, from the three elements set I can take two of two elements at a time; that means and some it is ordered arrangement. So, I can take a b c, so, I can take ordered a b, I can take b c, I can take c a. Now since, it is ordered I can take in this direction c b then b a then a c.

So, there are six 2 permutations. And there can be 3 permutations also; 3 permutations also. Then I can take from a three element set 3 permutations is a b c. Since, it is ordered b c a, c a b and again some ordering can be different c b a then b a c then a c b. So, again there are six 3 permutations ok. And no 4 permutation is; no 4 permutation is possible, no 4 permutation; since, 4 is greater than 3 here it is a three element set or I can write that in this case r greater than n.

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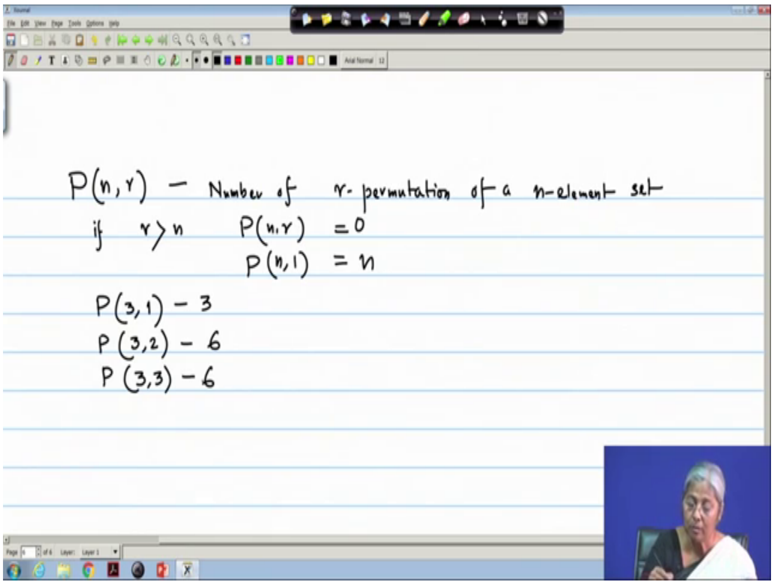

So, now we define normally the permutation, we define this r permutation of n element set, we define as a P n, r. So, P n, r is the number of r permutation of a n element set. If r greater than n then P n, r equal to 0 then we can easily, I can write that P n, 1 equal to n, because one element each time. So, for that example, we have seen that PP 3, 1 three elements set 1 permutation that is 3 P 3 2 2 permutation that is 6 again P 3, 3 that is 6.

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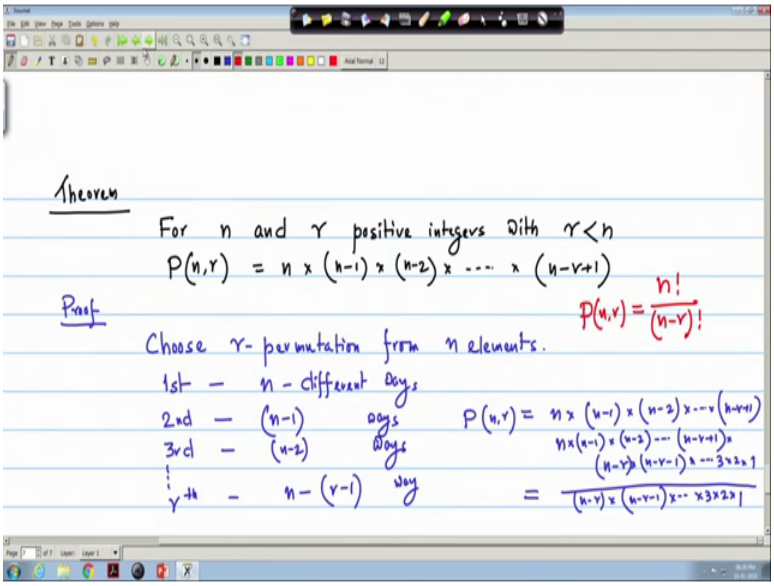

So, now, I can give the theorem, for n and r positive integers with r less than n. We can write the n r permutation of n element set that P n, r is n into n minus 1 into n minus 2 up to n minus r plus 1. Now, how we can prove? The proof is the way the example we have done, say choose r permutation from n elements.

So, I as if, I have that r places. So, for the 1st place I can write the first I can take n different ways, I can choose the n different ways, I can choose my object. Then it is already it is one is chosen. So, for the 2nd I can take n minus ways, for 3rd it is n minus 2, because already 2 have chosen. So, in this way the r element the rth one, I can take n minus r minus 1 ways. So, by multiplication principle I can tell that my that is defined as P n, r this is n into n minus 1 n minus 2 into n minus r plus 1. Though exactly the same, the simple example that we have done from that how many way, how many numbers we can get from 1000 to 9999.

So, now we can write in this way this is n into n minus 1, n minus 2, n minus r plus 1. And I can multiply that n minus r, then n minus r minus 1 up to 3 2 1. Then I did divide this number by this that n minus r what I introduced there n minus r minus 1 into 3 into 2 into 1. So, this is that the from the definition the numerator is nothing, but my factorial n and the denominator is factorial n minus r, because n into up we can multiply n into n minus 1 up to 1.

So, what I get my P n, r I get my I write. So, I get my P n, r is n factorial divided by n minus r factorial what earlier normally we know this is the permutation rule. So, simple for simple set, we get this is my permutation and if P n 0, I can write always this is 1 and P n, n is n factorial.

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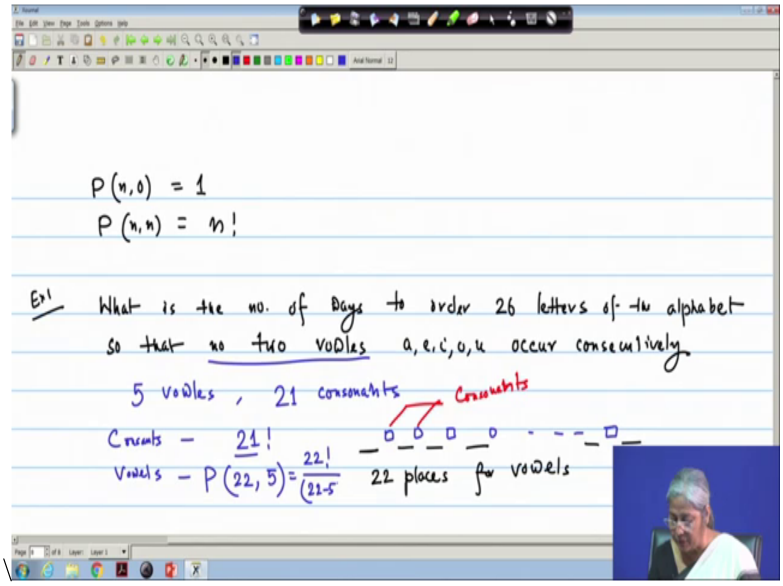

Now, if I quickly see one example that what is the number of ways to order 26 letters of the alphabet so, that no 2 vowels that a e i o u occur consecutively. So, I have I can write that this is nothing, but I have 5 vowels.

So, I have 5 vowels and 21 consonants. So, 21 consonants, I can take for consonants, I can place 21 factorial ways and vowels. Since, this is the restriction for vowels that no 2 vowels occur consecutively. So, as if this 21 consonants if I place, say this is these are the consonants say 21, so, as if these are my black are my places of vowels. So, there are 22 places; 22 places for vowels.

So, 5 vowels I can put in 22 places. So, you can write these are my consonants and just now the formula we read. So, vowels we can put that P 20 to 5, because I have 22 positions and 5 vowels I have to put there. So, they way this is 22 factorial by 22 minus 5 factorial.

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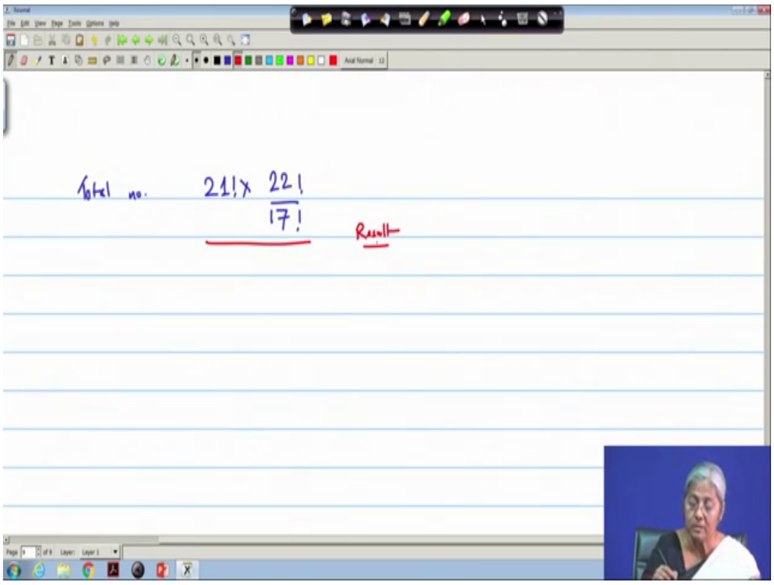

So, if I continue then my total number should be 21 factorial into 22 factorial divided by 17 factorial. So, this will be my, this is my result. So, instead of manual counting now I can, I the permutation formula, I can use to solve this type of problem. So, it is very easy way that are some difficult problems, counting problems when the ordered arrangements are considered, we can use permutation formula to solve those problems.