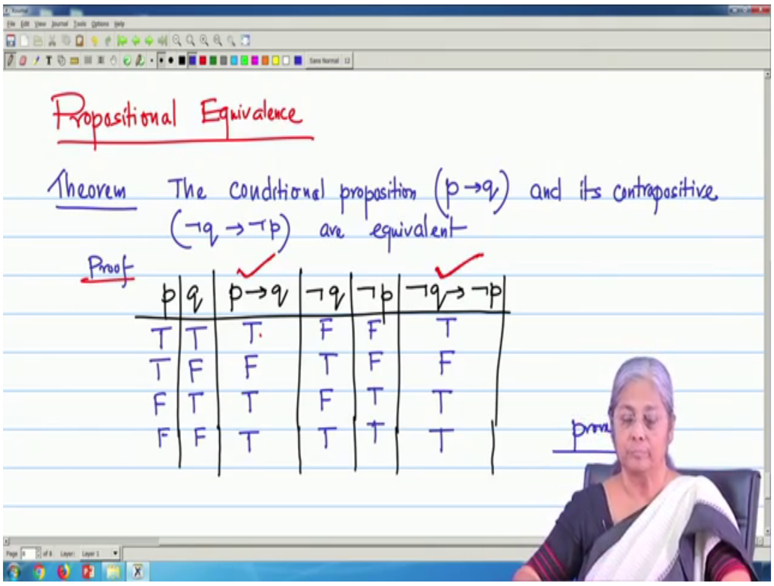
On Discrete Structures
Prof. Dipanwita Roychoudhury
Department of Computer Science and Engineering
Indian Institute of Technology, Kharagpur

Lecture - 04
Introduction To Propositional Logic (Contd.)

So, we have defined the propositional equivalence, and the De Morgan's law we have read. Now, we see some more laws that are very useful to derive or to check the equivalence between two compound propositions. So, first we see one theorem.

(Refer Slide Time: 00:46)

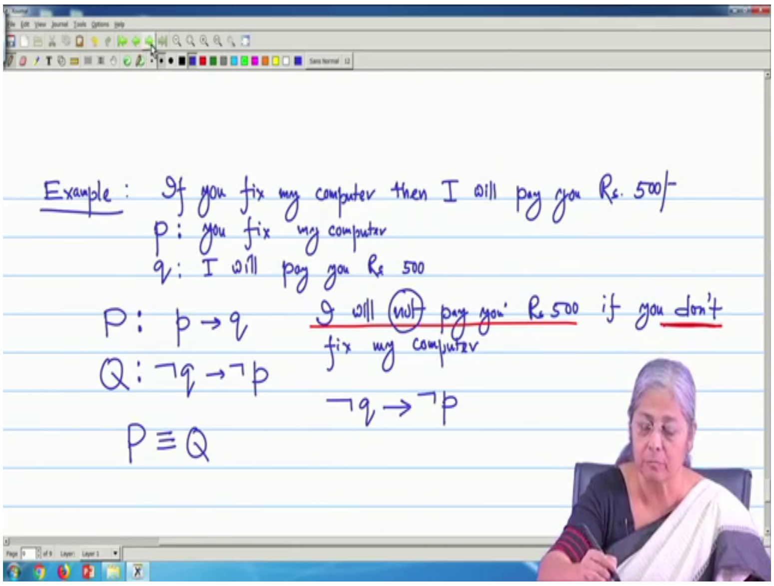


We continue the propositional equivalence continue. So, first we read one theorem, though it gives a relation between the conditional proposition and the contrapositive of the conditional proposition. So, the conditional proposition say p implies q and its contrapositive that is negation q implies negation p are equivalent.

We prove by my truth table. So, if we draw truth table, p implies q, and then negation p, or first we give that negation q because that will help us to write the truth values, then negation q implies negation p all possible assignments we of p and q. So, evaluate if we remember the we read in last class that what is p implies q if p, q both are true. So, this is true, but true implies false we have taken these as a false result remaining cases, they are true.

Now, negation q, so this negation q becomes F, T, F and T; negation, p there F, F, T, T. So, what is negation q implies negation F F implies F true T plus F, again this is F F T, this is true. T T this is true. So, we see that p implies q takes the truth values T F T F. And negation q implies negation p T F T F that means both are taking the same truth values for the same set of p and q values. So, they are equivalent. So, they are equivalent. So, it is proved that they are equivalent.

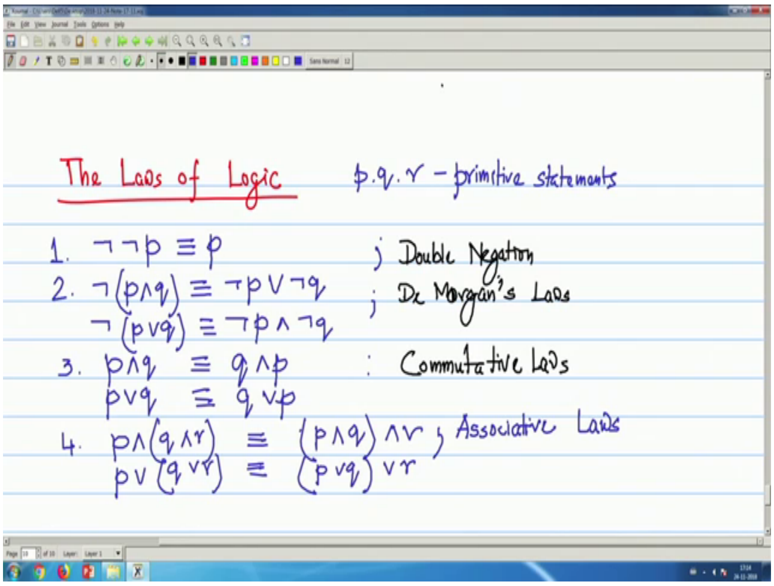
(Refer Slide Time: 05:20)



We take one example. If you fix my computer, then I will pay you rupees 500, then I will pay you rupees 500. So, this is the compound proposition because this is a statement. So, what are the primitive statements here? Primitive statements say I take p is you fix my computer ok p is you fix my computer. And q is I will pay you rupees 500. So, this statement, the example statement tells that p implies q. So, my if it is my compound statement p, then it is telling that p implies q.

Now, I will not pay you if I write that I will not pay you rupees 500, if you do not fix my computer, if you do not fix my computer. So, I will not pay you. This tell I will not pay you this five 500. This tells that it is negation q not not. So, it is negation q implies if you do not fix my computer. Again if it is do not fit, if you do not fix my computer do not that means, again it is negation p. So, if I tell this compound proposition is negation q implies negation p, then these two step statements are same that means, p is same. So, p is equivalent to q, that means, if you fix my computer then I will pay you rupees 500 or I will not pay you rupees 500 if you do not fix my computer, that are same that are logically equivalent.

(Refer Slide Time: 09:09)

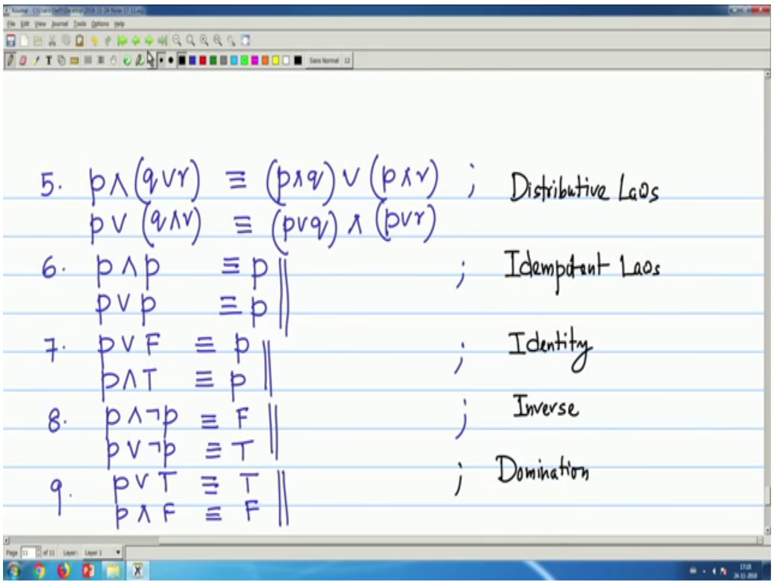


Now, we see the some other rules of although we call that laws of logic ok. So, only one law we have read that is De Morgan's law. Now, we will see that there are many other similar type of laws that we can apply on the compound propositions and the primitive statements that are that makes this compound state propositions. So, first to it tabulate that this type of laws or first we see that the law is the very primitive laws that we can tell that the, it is double negation.

Since we know the De Morgan's law by how to negative one compound statement, so we write that negation of negation p is equivalent to p itself we call this is double negation. Now, already we have seen that single negation which is nothing but De Morgan's law I can write negation. Now, we see that these propositions that they follow the our basic algebra algebraic laws the law of algebra like commutative associative distributive laws, so that is why sometime we call these are the laws of propositions ok.

So, first we see that it is it if I consider the connectives at the conjunction that means, p and q, this is equivalent to q and p. Then p OR q this is equivalent to q OR p, this we called our commutative laws. Then associative, so this is our associative laws. And here our p, q, r are primitive statements.

(Refer Slide Time: 15:20)

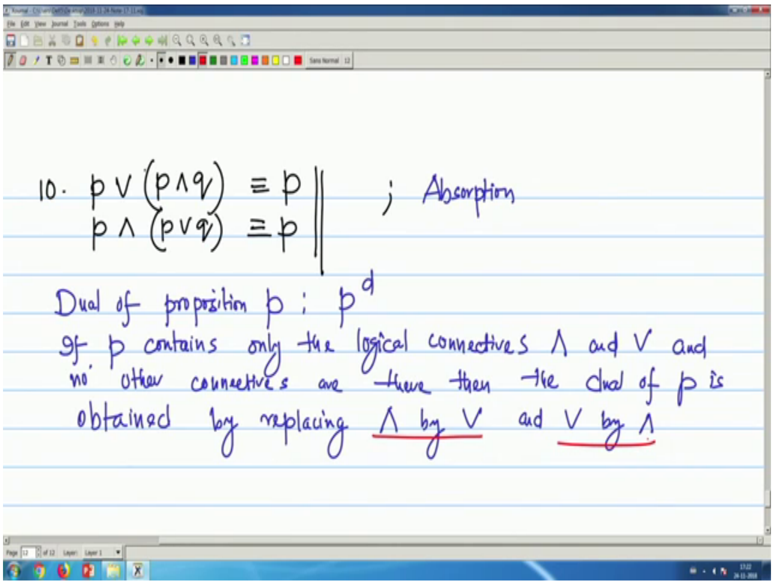


Now, we see we read some more rule. So, we read the associative to commutative and associative, we see the distribute distributive. We have p, q, r. So, we take p AND q OR r which is equivalent to p AND q OR p AND r. Similarly, if we replace AND by OR and vice versa or then this is similarly I get p OR q AND p OR r. So, these are my distributive laws. Now, there are some other some more basic laws that we tell one is if I take p and in conjunction with p itself, that means p AND p would take p AND p, then it is p only. Similarly, if I take P OR P, it is p these two are called the rule the idempotent this is of idempotent laws.

Now, if I take the conjunction and disjunction with the true and false only that means if I take the p OR F or p AND T, then also p OR F because we know that whatever the p has the value true or a false, it will be it will take that value. So, it is p only. Similarly, p AND T, this is again p. So, this is called the identity, this two our rule of identity.

Now, we have some other rule, we call that inverse. So, if I take again the conjunction of negation p, we have already seen that this will be false. If I take p on negation p this is always true. So, these two our inverse rule ok. Now, I have if I take the p OR T, we will get always T we give p AND false, we get always false. So, it is our domination loss. It will dominate the true and false always dominate. So, this is our rule of domination.

(Refer Slide Time: 20:08)

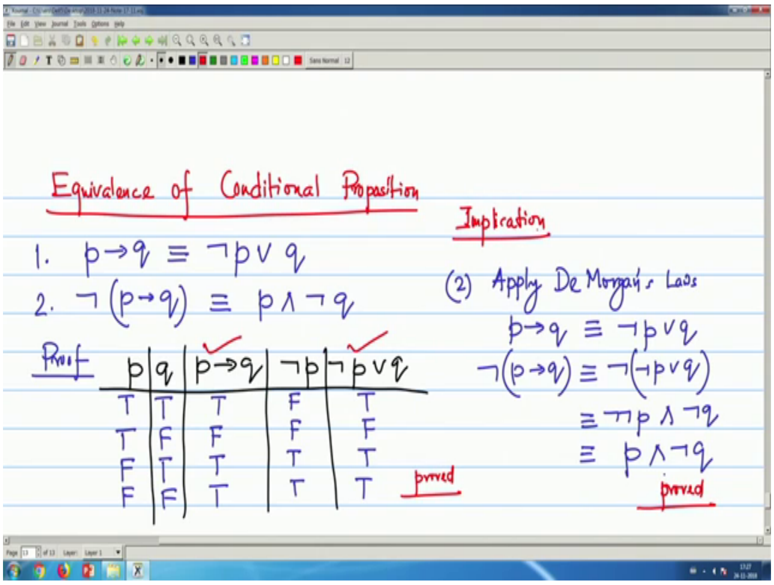


Now, another important is the absorption. This is very much required when we try to simplify the compound proposition. So, absorption is that if I do the p OR p AND q, then it is p only and p in conjunction with p OR q, then also it is p. Why, see here it is p OR. So, whatever be the value of this p AND q, it will take the truth value of p. Similarly, here p AND p OR q. So, p OR q is p. Just the logic we give, and then p AND p is p. So, this is the rule of absorption rule of absorption ok.

Now, we have seen the negation where the proposition becomes the when we take negation in De Morgan's truth that proposition becomes the negation of that particular proposition and the conjunction disjunctions they are actually exchanged. Now, one is called the dual and if that that we dual of proposition p.

Normally, we denote this thing as p d. So, here if how we define the dual if the proposition p, p contains only the logical connectives only the logical connectives AND and OR, and no other connectives are there, then the dual of p is obtained by replacing or I can take by replacing AND by OR and OR by AND, then it is dual. Please note that here there is no negation concerned because my proposition contents only the connectives AND and OR.

(Refer Slide Time: 23:58)

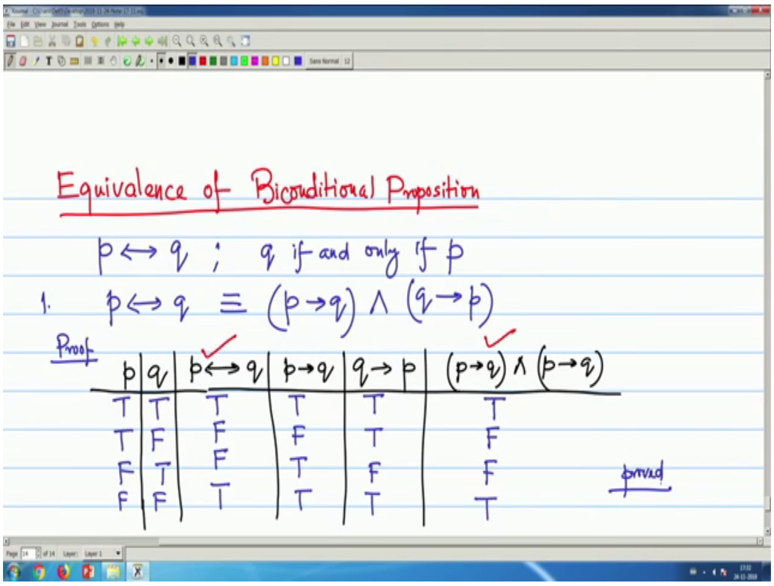


Now, we see that some of the now we have read the laws and now applying these laws, how we can check whether the two compositions are compound statement propositions are equivalent or not. So, first we see one conditional proposition, the equivalence of the conditional propositions. The equivalence of conditional proposition, the conditional proposition we know that p implies q. And if p implies q that it is equivalent to negation p or q, this is one I can write that negation of q is equivalent to p; and negation q this is nothing but if we apply the De Morgan's law on both the side.

So, how we can prove that thing if we try to prove always we will prove by making the truth table. So, we draw the truth table we take all possible assignments as usual then p implies q it is true only this is false, true, true. Now, negation p negation p is false, false, true, true. Now, negation p or q, so this is my negation p and OR q, so T OR F, this is T; F OR F this is only F; T or T, T F T this is T. So, if we see p, p in condition p implies q and negation p OR q and they have taking the same two values. So, it is proved.

And in the second case if we just apply for two, if we apply De Morgan apply De Morgan's laws, then the left hand side that it is p implies because already we have proved that p implies q is equivalent to negation p OR q. Now, you apply De Morgan this is negation of p OR q. So, this is equivalent to negation of negation p, q this is negation of negation p AND negation q. Negation of negation p is p only double negation rule, and this is p implies p AND negation q which is the second one. So, it is also proved. So, this is equivalent equivalence of conditional proposition that is that we consider the, we consider implication we consider the implication.

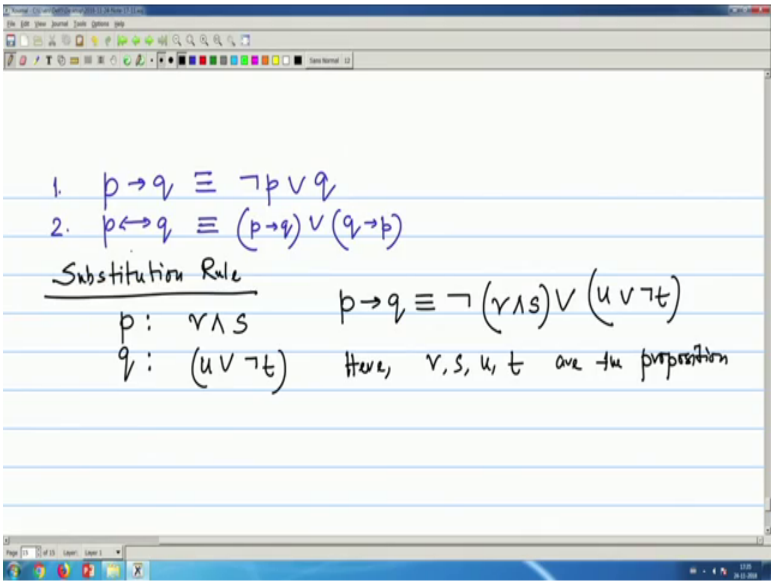
(Refer Slide Time: 29:08)



Now, we see that if we consider the our equivalence of biconditional proposition. So, we know our biconditional proposition is if p AND q, two proposition, it is if and only if q if and only if p. Now, we give that biconditional proposition is equivalent to p implies q, and q implies p. Then the proof will be we take all possible assignments then it is biconditional if p then an if q if and only p or if p only n if only if q.

So, whenever both are taking the same assignments then only it is true. So, T T – true, F F – true, and the remaining cases it is F, we know p implies q is T F T T. q implies p, so it is T T F T and this is and. So, T T T F T F T F F, and this is T. So, again you see that this is biconditional. So, our LHS that it is T F F T, and the RHS this is T F F T, that means, these are equivalent. So, there it is proved; they are proved.

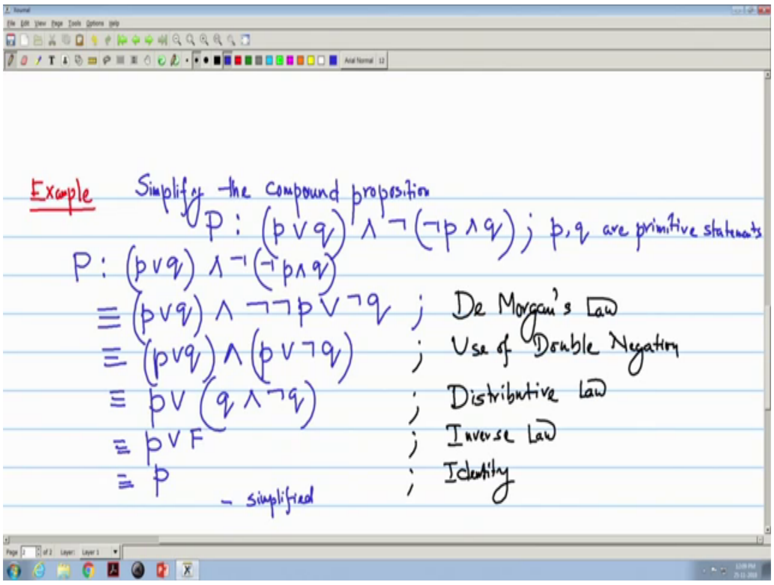
(Refer Slide Time: 33:14)



Now, if we see that what we have seen that to equivalence of two conditional proposition, one is p implies q is equivalent to negation p OR q. And is equivalent to p implies q AND q implies p. Now another is their substitution rule that means, if substitution is see if the see for we take that p the p AND q is the small p it is equivalent to r conjunction s r conjunction S. And q is say u OR negation T then in one what we can do that some by substitution rule that p implies q this can be equivalent to the if I p, I can instead of p, I can write s r AND S, then r instead of q, I can write that u OR negation t. Here, here p q as well as r s rs ut here r s u T are the propositions are the propositions ok.

So, I can write it actually if I can take that p is p is same as that of r and s, then of this substitution we can apply. So, in this way, we can similarly we can apply the bi equivalence of the biconditional a proposition also and we can evaluate the compound propositions for it and we can simplified, and then we can check whether two propositions are equivalents or not. Now, we give an example that how the laws of logic can be applied to simplify the compound proposition.

(Refer Slide Time: 36:54)



Take one example, that simplify the compound proposition p which is p OR q AND negation, negation p AND q. Here p q are prim primitive statements primitive statements. So, we start from the given compound proposition say p which is p OR q AND negation, negation p AND q. And we apply the laws of logic. So, this is equivalent to p OR q, and we apply first De Morgan's law because it is we have to negate this expression. So, it will be negation of p, this n becomes OR, and this becomes negation q then immediately we should write that what law we have applied here this is De Morgan's law. Now, this will be double negation.

So, this becomes only p p OR negation q. So, we write this is use of use of double negation. Now, see this is p OR q AND p OR negation q. So, this is the distributive law. So, we can use the distributive law, or we can apply the distributive law of or or the con disjunction over conjunction that means, I can write this thing p OR q AND negation q. So, I write this is the distributive law.

So, q OR negation q q or negation q is q AND negation q is F always false. So, this is my inverse law, and this becomes p OR f. So, this becomes only p which is identity which is identity. So, my compound proposition that capital P becomes only the the primitive statements p, this is simplified to small p p is simplified to small p. So, it is simplified. So, this examples tells that how the laws of logic that we have read the different laws that can be applied to simplify the compound propositions. So, we have now we can conclude this lecture with this example.