**Discrete Structures**
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**Lecture – 33**
**Recurrence Relations (Contd.)**

So, in the last lecture we have read the read how to frame the recurrence relation and today we see how what are the different techniques to solve the recurrence relations and already we know that solving recurrence relation means to get an explicit formula; that means, if the nth term of the sequence that is related to the predecessors; that means, it gives then how it is related to n only that is the explicit formula.

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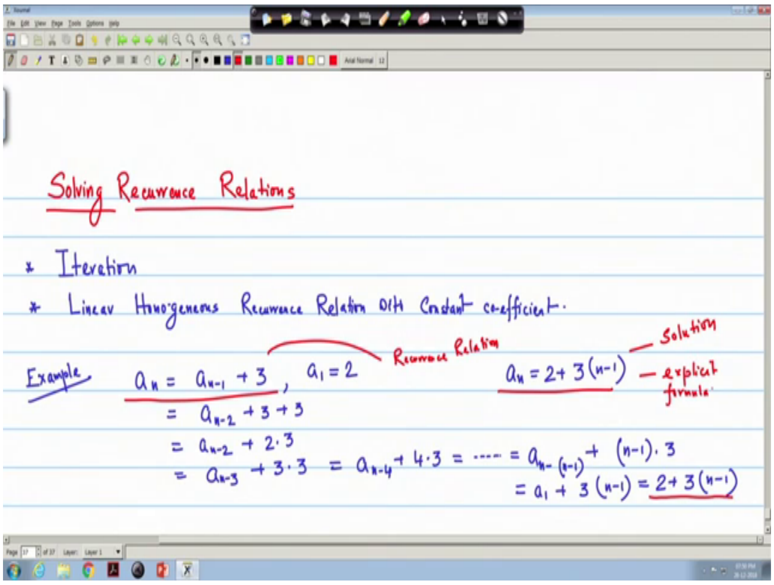


So, the solving recurrence relations what are the different techniques we used. So, first we see very simple type of recurrence relation, we will use first the iteration, first we will see that how we can use iteration to solve that thing. Second, we will see that how to solve the linear homogeneous linear homogeneous recurrence relation with constant coefficient.

So, first we see the how we can apply the iteration to solve recurrence relations we take one very simple examples of sequence say, I have a recurrence relation with an minus n equal to an minus 1 plus 3 and i give that a 1 equal to initial condition is a 1 equal to 2.

What is iteration? That I can again replace an minus 1 by n minus 2 plus 3. So, I get an minus 2 plus 2 into 3.

Again I can replace n minus 2 by an minus 3 plus 3 and so, this becomes 3 into 3. So, in this way if I continue I can write n minus 4 plus 4 into 3 and if I continue, I will be getting a n minus n minus 1 plus n minus 1 into 3, this is a 1 a 1 plus 3 n minus 1. Now I know a 1 equal to 2. So, this becomes this becomes 2 plus 3 into n minus 1. Now see we started with a recurrence relation this is my recurrence relation given a simple recurrence relation, I have taken this is my recurrence relation and if I just iteratively, we proceed we replace each term by it is predecessors the relation, we apply I will get the relation that here n depends on the predecessors n minus 1. Here n depends here, I get the an is here, I get n equal to 2 plus 3 into n minus 1.

So, this is my explicit formula or the solution this is my solution of the recurrence relation because it depends only on n or I can tell this is explicit formula.

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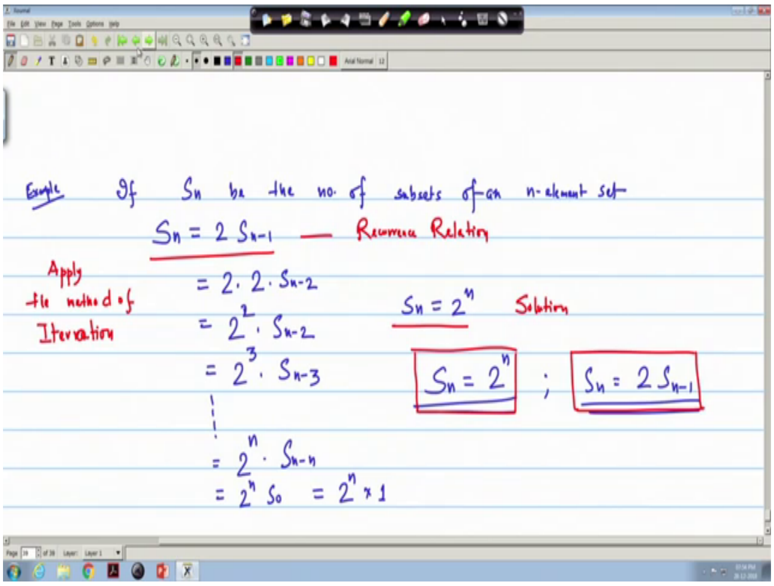

Now, already we have read one example of the number of subsets of a set, if we remember that the example that we have given the if S n be the number of subsets of an n element set then we have framed the recurrence relation like S n equal to 2 S n minus 1 to S n minus 1. Now if I apply iteration. So, this is my recurrence relation again, this is my recurrence relation. Now if I apply a iteration. So, this becomes 2 S n minus 2, 2 S n minus 2 which is 2 square S n minus 2. Again I can apply, 2 square into 2 S n minus 3.

So, this becomes 2 cube S n minus 3, in this way I can apply this is 2 to the power n S n minus n equal to 2 to the power n S 0, which is 2 to the power n into 1, because for 1 element s 0 means empty set, this is only 1 subset the set itself.

So, I gave that I get that a cenacle the solution is the S n equal to 2 to the power n. So, this is my solution and we apply iteration. So, apply the method of iteration. Now, this S n equal to 2 to the power n, this solution we get when my recurrence relation is S n equal to 2 S n minus 1. So, this is a very important thing for very primitive solution that, we will be applying or we will be using this solution, when we will use or we will try to solve by a linear homogeneous equations.

So, we remember we must remember this solution, we must remember this S n equal to 2 to the power n and the recurrence relation is S n equal to 2 S n minus 1, we remember. Now we will give the solution technique for solving linear homogeneous equation with constant coefficient.

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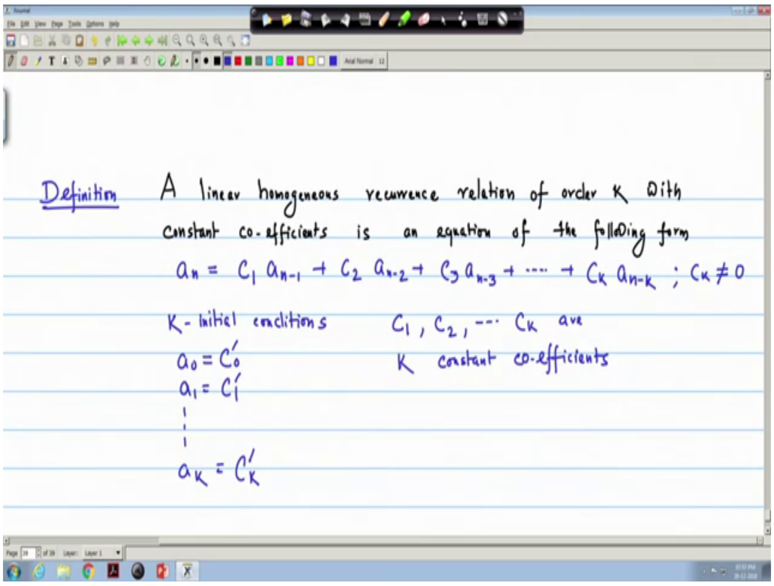


So, first we define the linear to give the definition of linear homogeneous recurrence relation of order K ok.

So, a linear homogeneous with constant coefficients is an equation of the form the following form. Write that n equal to C 1 a n minus 1 C 2 a n minus 2 plus C 3 a n minus 3 plus CK a n minus K, where CK not equal to 0, because it is of equation with order K and here the K initial conditions ,we need the K initial conditions as if a 0 equal to C 0 a 1 equal to C 1 then a K equal to CK and here this small C 0 C 1 I give the is equal to C 0 dash otherwise both are C 0 C 1 C k.

So, these are make a initial conditions and this C 1 C 2 C k or the are constant K number of K constant coefficients. So, we defined that linear homogeneous recurrence relations like that.

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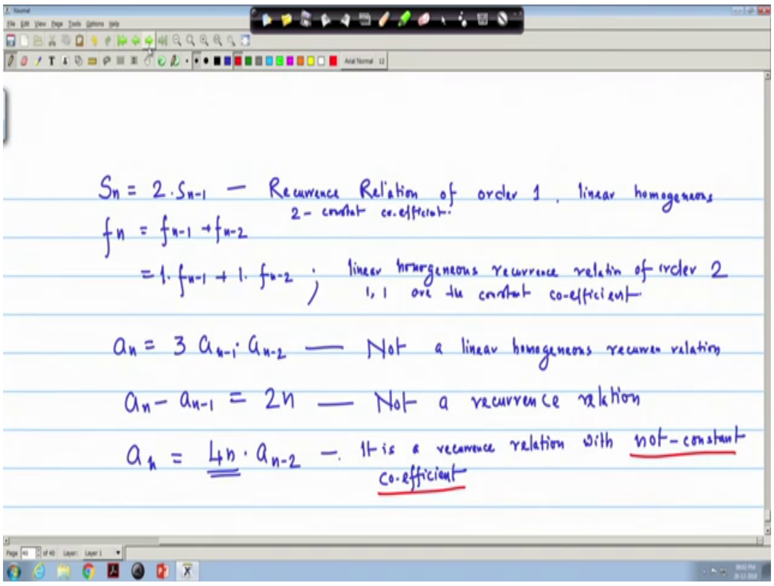


Now already, we have seen that S n equal to 2 to S n minus 1, this is a recurrence relation and I tell this is a recurrence relation of order 1 and this is a linear homogeneous recurrence relation or the Fibonaccisequence f n equal to f n minus 1 plus fn minus 2 here, constant coefficient is 2, 2 is the constant coefficient.

So, as if we can write 1 f n minus 1 plus 1 fn minus 2. So here, this is a linear homogeneous recurrence relation of order 2 and 1 is the coefficient constant coefficients are 1 1 1 are the constant coefficients, it is linear since the power is 1. Now if I write n equal to 3 n minus 1 n minus 2(Refer Time: 15:18). So, this is not a linear homogeneous a recurrence relation or if I write n n minus n minus 1 equal to 2 n. So, this is not a recurrence relation not a recurrence relation. Since there is no relation here that, where the nth term depends on the predecessor like that or if I write n is say 4 n n minus 2 again, this is a recurrence relation, but not with constant coefficient because, here coefficients are 4 n is not a constant. So, it is a recurrence relation it is a recurrence relation with not constant-coefficient, not constant coefficient.

So now, it is clear that normally with linear homogeneous equation with constant coefficient, what we understand. Now we take an example, that for solving linear homogeneous relation.

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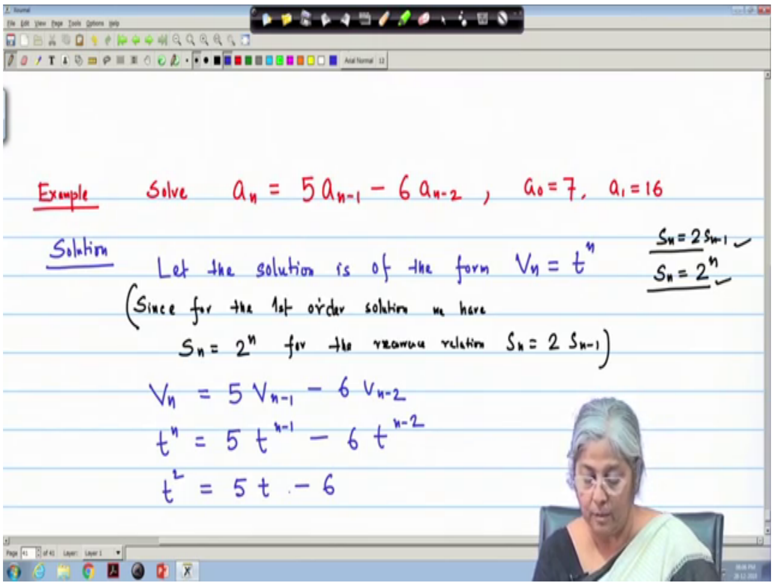


So, we take an example for solving. So, solve a n equal to 5 a n minus 1 minus 6 a n minus 2 initial conditions given is 0 equal to 7 and a 1 equal to 16 ok. So, we see the solution. So, first thing this is the linear recurrence relation homogeneous with constant coefficients 5 and 6 are constants.

So now, let the solution is of the form say Vn equal to t to the power n, see if we remember that recurrence relation of S n equal to 2 S n minus 1, we got the solution that S n equal to 2 to the power n. See this simple form that, we are using here since here, it is of order 1 and it is a linear homogeneous recurrence relation of order 2. Now the solution we have taken as if the of the form is t to the power n ok. So, this is the technique normally, we use to solve the linear homogenous recurrence relation. So, Vn is t to the power n. Since for the first order solution, first order solution we get solution, we have S n equal to 2 to the power n as I mentioned for the recurrence relation S n equal to 2 S n minus 1.

So, we have taken this form. So now, if I put here then I get I get Vn is 5 V n minus 1 minus 6 Vn minus 2, I put the solution since Vn equal to t to the power n this is the form. So, this is my form t to the power n equal to 5 t to the power n minus 1 minus 6 t to the power n minus 2. So, I get t square equal to 5 t minus 6, if I divide the both sides by t to the power n minus 2.

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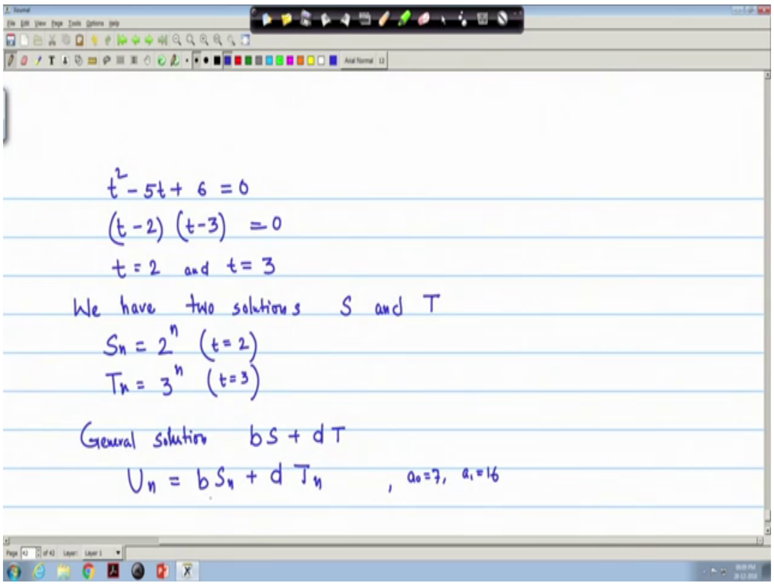


So, I get the equation t square minus 5 t plus 6 equal to 0. So, I get t minus 2 into t minus 3 equal to 0, I get t equal to 2 and t equal to or t equal to 3, I get t 2 solutions that t equal to 2 and t equal to 3. So, we have 2 solutions here say it is an t. So, I can write S n equal to when t equal to 2 2 to the power n that is t equal to 2 and tn equal to we get 3 to the power n; that means, t equals to 3.

So, our general solution that like the general solution form is I get the bS plus Dt. So, I can write the solution U n equal to b S n plus d Tn and we remember the initial condition conditions are a 0 equal to 7 and a 1 equal to 16.

Now, we have to find out the constants b and d.

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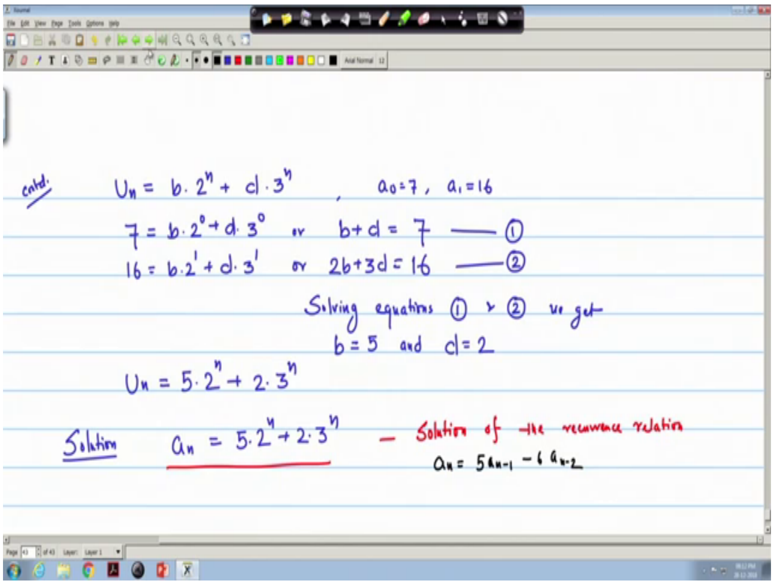


So, it continue this is continued. So, we put the initial conditions the initial conditions are so again, you just write un equal to b 2 to the power n plus d 3 to the power n and a 0 and a 1 equal to 16. So, we put the values. So, 7 equal b 2 to the power 0 plus d 3 to the power 0; that means, b plus d equal to 7 and 16 equal to b 2 to the power 1 plus d 3 to the power 1 or 2 b plus 3 d equal to 16.

Now, we can solve if we solve and we solve. So, solving equation 1 and 2, we get b equal to 5 and d equal to 2. So, we get that U n equal to 5 2 to the power n plus 2 3 to the power n; that means, my solution is solution is we have taken an. So, a n is 5 2 to the power n plus 2 3 to the power n.

So, I get an explicit formula or the solution of a n. So, this is my solution of the recurrence relation, we have taken the recurrence relation, if we see that an 5 a n minus 1 minus 6 a n minus 2 solution of that a n equal to 5 n minus 1 minus 6 a n minus 2. So, this is the solution technique of solving the linear homogeneous relation. Now we have given the techniques actually, we can write the same techniques in a form of a theorem.

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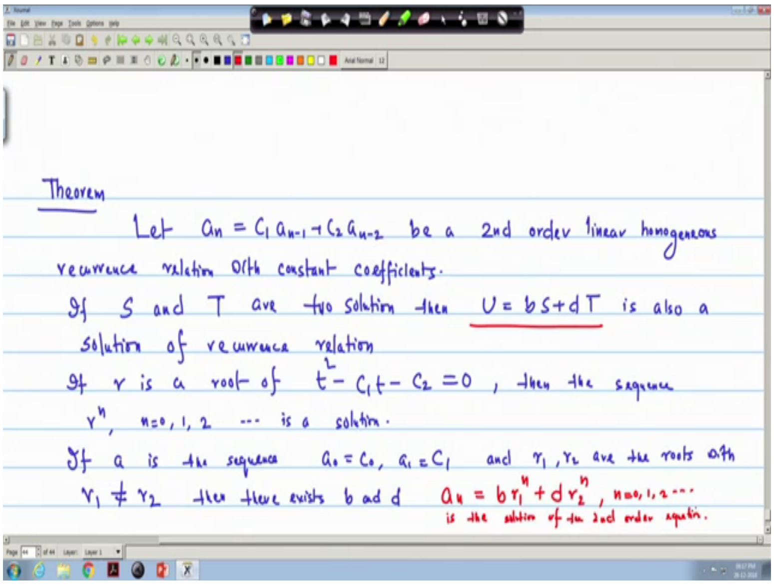


So, I give that if I state the theorem that you can write the statements for second order ok. So, this be a second order linear homogeneous recurrence relation with constant coefficient.

Now, if S and T are 2 solutions then we get then U equal to bS plus dT and b and d are constant coefficient. Now solution of recurrence relation, we get this gibster is this also a solution of recurrence relation. Now if r is a root of t square minus C 1 t minus C 2 equal to 0. Since, we have considered a second order recurrence relation then the sequence r to the power n for n equal to 0 1 2 is a solution.

So, if a is the sequence defined by a 0 equal to C 0 a 1 equal to C 1 and r 1 r 2 are the roots with r 1 not equal to r 2 2 roots are different then there exists b and d earlier, we have taken that bS plus dT that a n equal to b r 1 to the power n plus d r 2 to the power n n equal to 0 1 2 as the solution is the solution of the second order equation.

So, the example I have taken and the theorem statement is the if we follow the example that theorem statement is written as if this is the procedure or the technique that we follow to solve the linear homogeneous recurrence relation. So, this is stated in a theorem and the same thing we have explained with an eczema example. So, next lecture, we will see the other techniques of solving recurrence relations.