Discrete Structures
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Lecture - 17
Sets and Functions (Contd.)

We have read the fundamentals of set theory, mainly the basic definitions and some fundamental properties that we have read on set. Today's lecture will read the basic set operations and some fundamental properties of set. We know the subset of a set, we have defined the subset of a set; first we see the our some basic definitions on set.

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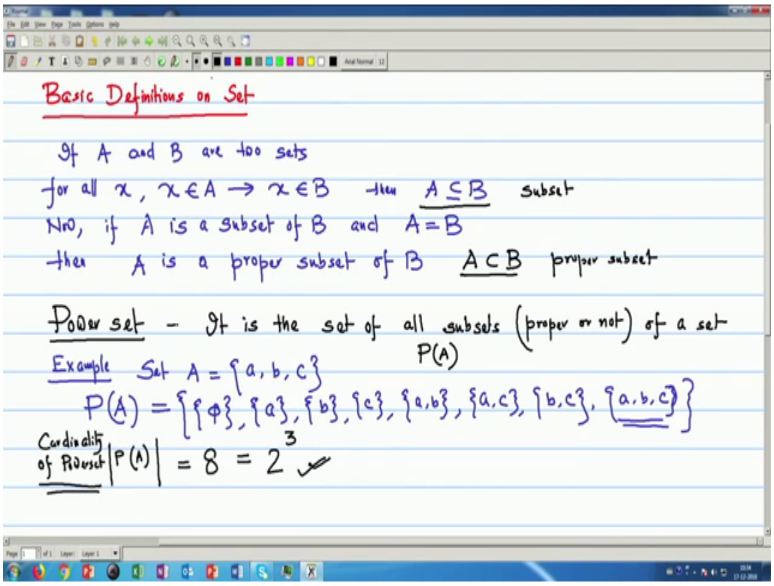

So, if A and B are two sets and for all elements x for all x, x belongs to A if it implies that x belongs to B; that means, all elements of A are also the elements of B, then we have defined that A is a subset of B. Now, if A is a subset of B and A not equal to B; A not equal to B, then we called that A is a proper subset of B and the notation is the A is a proper subset of B.

So, this is our subset notation, this is our proper subset notation. Now, we define another properties of subsets of a set is called the power set. We define power set. So, this is the set of all subsets may be proper or not, proper subset or not of a set. So, this is called the power set. So, set of all subsets of a set. You see one example say we take one, set consider one set A which has three elements say a, b c. Now we define the power set of A and normally we denote as the it is the P of A so, it is the set of all subsets. So, what are the subsets of a b c? First is null set is one subset, then sets have been only one element like a only b c or two elements say a, b a, c b, c and all three elements a, b, c these are the all possible subsets of set.

And other than a b c other than these subset all are proper subset of A, only this subset a b c is the only one subset is equal to A. So, this is my proper or power set. Normally we denote this thing as a notation is P of A. Now what are the cardinality of the power set? The cardinality of power set is P of A the is the here see we that if it is only 3 elements, then this is 8. So, this is actually 2 to the power 3. So, this is my the cardinality of power set. Now, see this is a general rule that if I have a n element set, then the power set cardinality of power set is 2 to the power n. Let us put that thing in a theorem.

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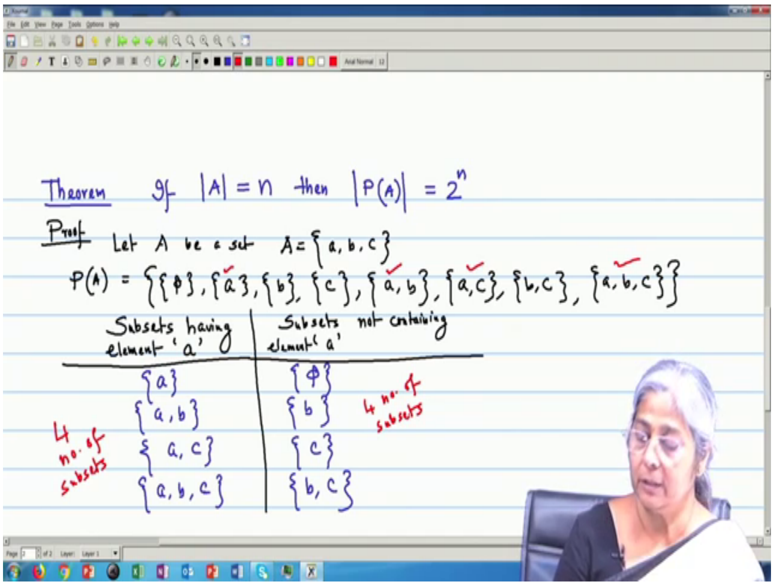

So, I give you theorem, but if cardinality of a set A equal to n then cardinality of power set equal to 2 to the power n. Now how to prove that thing? Before we give the proof so, the we consider the previous example let A be a set that A equal to 3 elements like a, b, c.

Now, just now we have seen that the power sets are, power sets are like phi a b c. Now we see some properties. See these are all the subsets of A. Now if we observe there are two different classes, what are those subsets? One of the subsets having element a say, one particular element a I am considering and subsets not containing element a. So, we see that what are the subsets having element a. This is one a, a b, a c and a b c. So, we put that a subset a, a b, a c and a b c and a b c. These are the four subsets that contain the element a and the remaining they do not contain the element a; one is the null set, then it is b, it is c and it is b c.

So, the property we see that in both the subset both the classes that number of elements are same. Here it is 4 number of elements; this class contains 4 number of subsets. This class also 4 number of subsets. So, we will use this property to prove the theorem. Now we will give the proof by induction. So, we will see the proof by induction.

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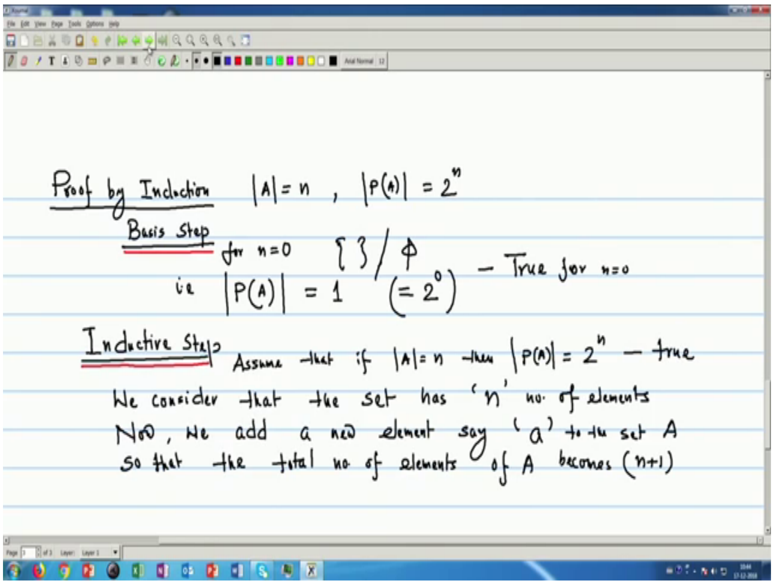

So, the conjecture is that what we have to prove the cardinality is n; that means, in n number of elements we have to prove the cardinality of power set of A is 2 to the power n. So, we consider the basis step. For basis step we consider for n equal to 0; that means, the what is the power set is n equal to 0; that means, it is only one element that is the empty set or phi. So that means, my that is the cardinality of power set is 1 and we know this is equal to 2 to the power 0.

So, the basis step is for the basis step; that means, n equal to 0 the conjecture is true. So, basis step it is true for n equal to 0, it is true. Now what will be our inductive step? What will be our inductive step? For inductive step, we assume that for n for value of n the conjecture is true. So, assume that if cardinality of A equal to n, then cardinality of P A is 2 to the power n; this is true. So, we have to prove that if the cardinality of a is n plus 1, then we have to show that the cardinality of the power set is 2 to the power n plus 1. And, then according to the principle of mathematical induction; it is to be proved.

Now, say I have n element, say here we assume for n it is true; that means, we have considered we consider that the set as n number of elements. Now, we add a new element say a is it is added to the set a to the set A. So, that the total number of elements of A becomes n plus 1; that means, now the new cardinality of A is n plus 1. Now, just now the property we have seen that when we have added a new element a so, we get.

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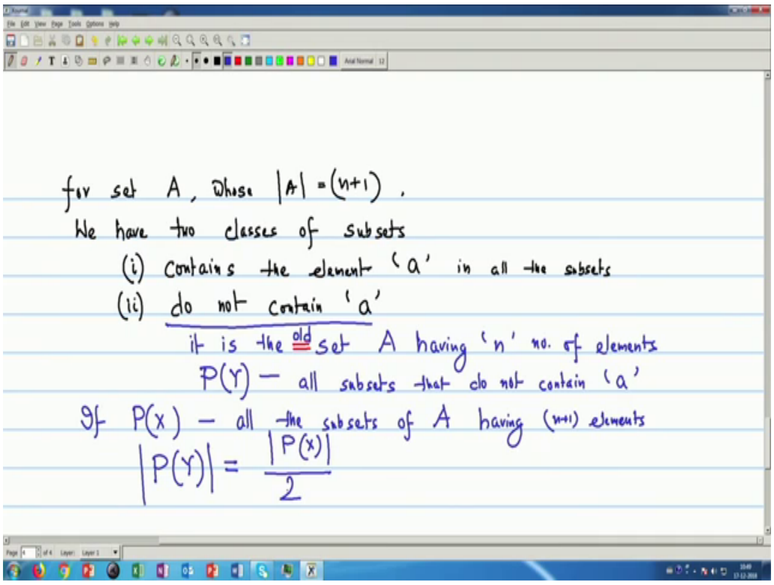

So, for the set A whose cardinality is n plus 1, then we have or we can get 2 classes of subsets; we have 2 classes of subsets. The first one type of class or one category contains the element a in all the subsets. The second category do not contain; that means, all the subsets do not contain the element a.

Now, which do not contain a which is nothing, but the previous set. So, this do not contain a is it is the set A or I should tell the old set a old set A having n element only having n number of elements because the this element we have just introduced. And the first category contains the element a only and just now the property we have seen that always this is the half of the subsets that contains a particular element a and half of the subset that do not contain a particular element a. That means, if we consider that these, say these are old elements set say this is the power set of Y; that means, this is power set of Y having or we write that P Y is the number of all subsets that do not contain a.

And if P X represent the all the subsets of A having n plus 1 elements; that means, after the inserting the new element a then clearly that P Y is or cardinality of P Y is the cardinality of P X divided by 2. Since from the property we have see, this is the property we have seen that always if we consider one specific element the half of the elements contains one particular element and half of the subsets that do not contain that particular element. So, seeing applying that property that P Y is that continuity of P X by 2.

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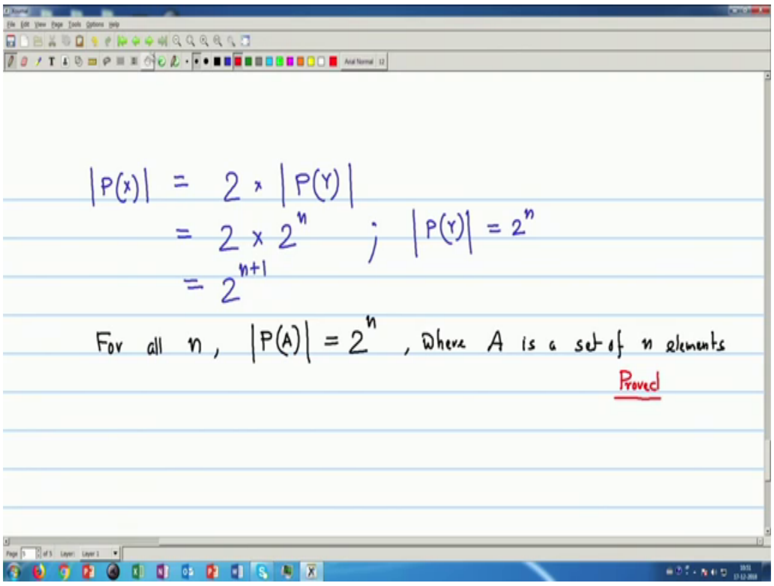

So, the cardinality of P X is 2 into cardinality of P Y. And what is cardinality of P Y? Cardinality of P Y is the cardinality of the power set having n elements. And according to that inductive step that we know we have we know that this is 2 to the power n since the cardinality of P Y is 2 to the power n. So, this becomes 2 to the power n plus 1.

So, X we have consider the P X all the subsets of a having n plus 1 elements and we proved that this becomes the 2 to the power the cardinality is 2 to the power n plus 1. So, we can conclude that for all n we can conclude then for all n all values of n the cardinality of P A is 2 to the power n where a is a set of n elements. So, our theorem is proved. Now, we see some set operations.

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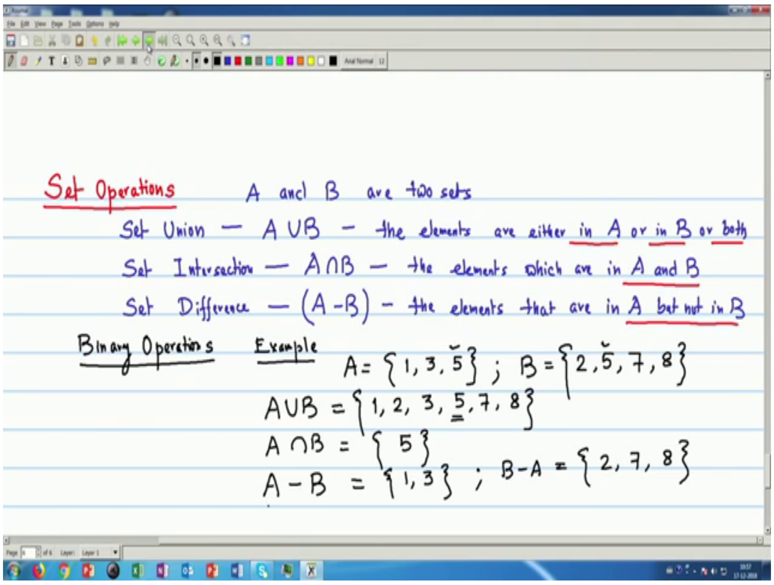

So, we have already defined. So, the different type of set operations that it can be binary operations or it can be unary operations and there can be some special operation. So, we have already read that set union. So, if we consider 2 sets A and B A and B are 2 sets, then set union normally we define this is the notation A union B; that means, it is defined that the elements it contains the elements are either in A or in B or both either in A or in B or both. We have read the set intersection. We denote and the elements this is the set containing the elements which are in A and B and set difference the elements that are in A, but not in B.

So, we see these are all binary operations; these are binary operations. If we take an example so, we take one example say A is 3 elements set 1, 3, 5 and B is another set say 2, 5, 7, 8. So, what is our A union B? So, either in A or both; that means, 1, 2, 3, 5, 7, 8 these are the elements like 5 is element 5 is both in A and B and other elements either in a or B. What is A intersection B? A intersection B is only 5 because the element which is both in A and B is only 5. What is difference A? A minus B which are in A, but not in B and there are only 2 elements that 1 and 3 which are in A, but not in B. Then what is B minus A, what is B minus A? B minus A is 2, 7, 8. So, these are the elements. Now, we have defined that null set.

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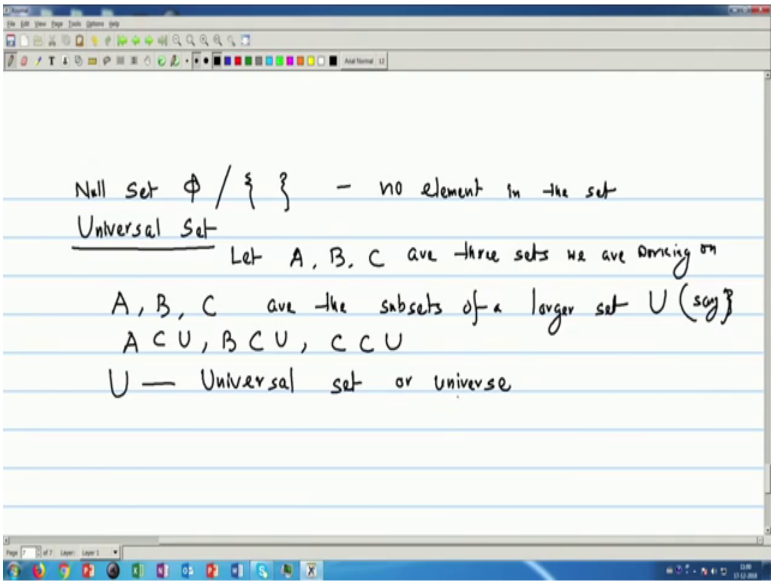

Let me set does not consider normally we define as this notation or empty set; so, no element in the set. Now we define another set called the universal set and the concept is that most of the time we deal with the sets or the subsets of a larger set. Say let A, B, C we are working that a B C are the 3 sets we are working on and A B C are the subsets of A larger set. Say this is larger set is called U say U, then these and these are all proper subset say U B is proper subset of U, C is also a proper subset of U not equal. So, then U is the universal set or sometimes we call universe or universe instead of these 3 there can be any.

So, that what the number total number of subsets under which are the subsets of proper subsets of A larger set we call this is the universal set of universal set. Now, with these basic definitions and some of the fundamental operations we have seen that some binary operations like union, intersection and the set difference. We will again read that some properties and some more operations that are very much important when will be handling some practical problems using set.