Discrete Structures

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Lecture – 10
Predicate Logic (Contd.)

We have read the fundamentals of logic, mainly the propositional logic, the predicate logic, the difference between propositional and predicate logic. Then, the laws of logic, the laws of inference and some examples also we have seen that how they are used for some for solving problems.

Today we will see all the rules whatever we have read that how they are actually used to represent symbolically or with logical symbols the statements of a problem and to infer something that means, to conclude or to prove given a conclusion whether that is true or false. And in that way we can tell whether it is a whether it proves a statement to be correct or false.

So, first we see how the laws of inference is applied to practical problems. We have read the quantifiers last lecture, and today we will see that how these laws of inference are used for quantified statements for practical problems.

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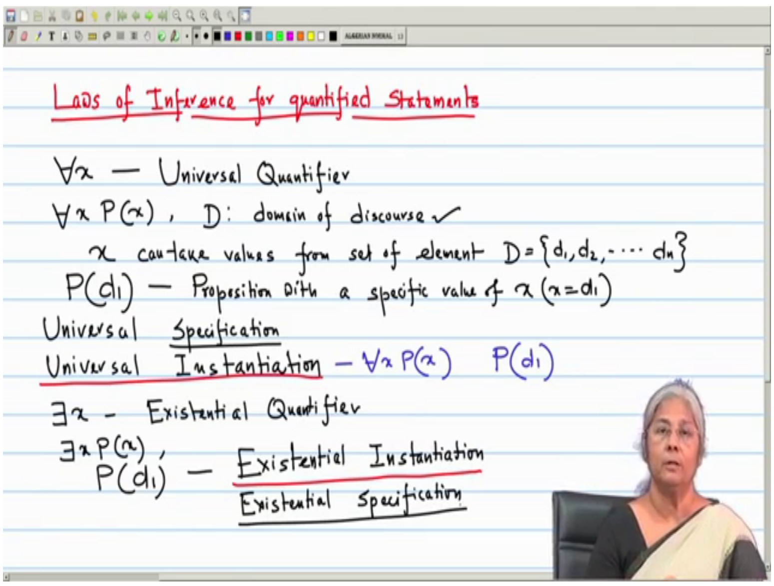

Laws of Inference. So, the quantifiers we have read are of two types, one is the universal quantifier we have given the name for all x, P x that for all x is the or I can write only the for all x, the universal quantifier. If I write for all x P x then immediately I have to mention a domain of discourse D. It means that x can take values, values from the set of element D, they can be a set say d 1, d 2 like d n.

So, if we remember that the main difference between the propositional logic and the predicate logic I mentioned that predicate logic mainly that involves a variable. And the variable can take any values and this is some specific values it take that means, when x can take a value d 1 then I can write P d 1. So, this is when I give this P d 1 then it is called the sum the proposition because P x is the proposition, so the proposition with a specific value of specific value of x say here x equal to d 1. So, this is called the specification. So, for all x P x if I write P d 1 this is called the universal specification or universal instantiation; this is I write that for all x P x if I replace this thing by for I replace P d 1, this I replace as P d 1, ok.

Similarly, if I use the o for existential quantifier, there exist x which is my existential quantifier then there exist x P x domain of discourse I take the same thing the D then I can write that it is P d 1, similarly I can write this is P d 1 and it is written as the existential instantiation. So, very simply I can tell that both P d 1 when there exist x P x or P d 1 for all x P x that means, for a specific value of x that is why it is called specification I can tell this is also existential specification, that one for a specific value of x that what will be the proposition.

Now, in our real life problem that most of the cases that it will take some instances that means, that variable can take a value for a particular problem. We see with some example this thing.

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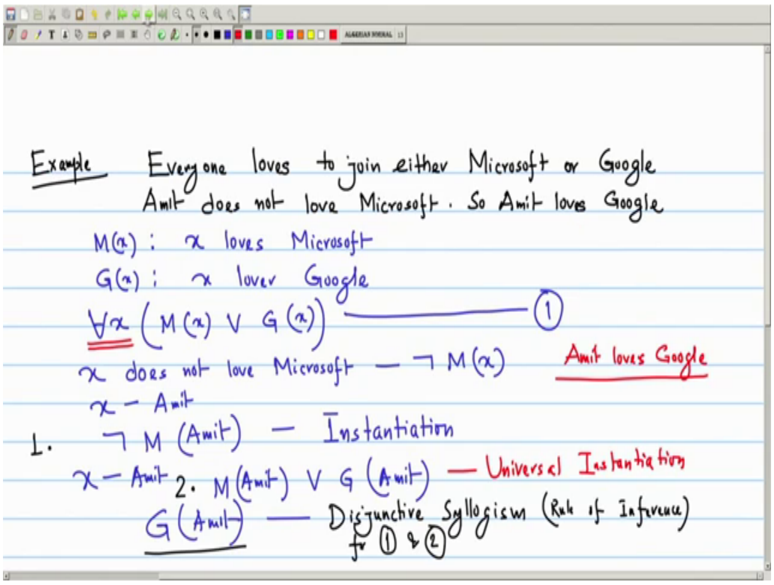

So, we take one example very simple example, we take that everyone loves to join either Microsoft or Google, Amit does not love Microsoft, so Amit loves Google. So, we have to check whether this inference is correct or not or whether this statement is correct or not. Now, we apply the proposition or we see that from predicate logic or the how we can use the laws of inference for this quantified statement.

So, first we see we assume some proposition. So, proposition I take that M x is x loves Microsoft and G x is x loves Google. So, everyone loves to join either Microsoft or Google; so for everyone means for all x. So, what is my? These are my hypothesis. So, what is; that for all x everyone is that that means, everyone that for all x hypothesis is M x or G x. Now, Amit does not love Microsoft. So, if someone does not love Microsoft if it is general variable for x then x loves Microsoft, so x does not love Microsoft that I can write as negation M x.

Now, x can take the value that if I put x is Amit because it is for everyone that. So, if x is Amit, so this is something called the instantiation that means, if I write that negation M a, M negation M Amit. So, this is something is called that instantiation or specification. This is something called instantiation or specification.

Now, if I reflect see this here that x is Amit with this instantiation then we can get that M x, M a M Amit or G Amit. See here we note that whenever it is instantiated by some value specific value then we omit this quantifier, that here it is a universal quantifier that we have replaced. So, it is called that universal instantiation, it is universal instantiation.

Now, I have negation M Amit and M Amit or G Amit. So, I can write with, from this again I write this one is, if I give some numbering say this is my 1 and this is 2 that. So, from 1 and 2 we can write that it is M Amit M Amit negation, negation M Amit. So, it is from (Refer Time: 16:05) I write from 1 and 2 we can write that it is G Amit, and this rule of inference it is called the disjunctive syllogism. So, this is one rule of inference, this is my rule of inference one rule of inference that if we apply for 1 and 2, for 1 and 2 then we get that it is that means, G Amit that means, Amit loves Google. So, we can tell that, this is Amit the conclusion that Amit loves Google.

So, first we have taken the propositional function then actually the predicate then the variable, takes what particular value what just now we have defined that this is a instantiation or some specification. And when specification if we put then we omit the notation of that universal quantifier, we write and then only we can apply the rule of inference and from there we get the we check whether my conclusion is correct, that means our inference is correct, that means given hypothesis these are my hypothesis and this the conclusion is true, ok.

So, now we see another example.

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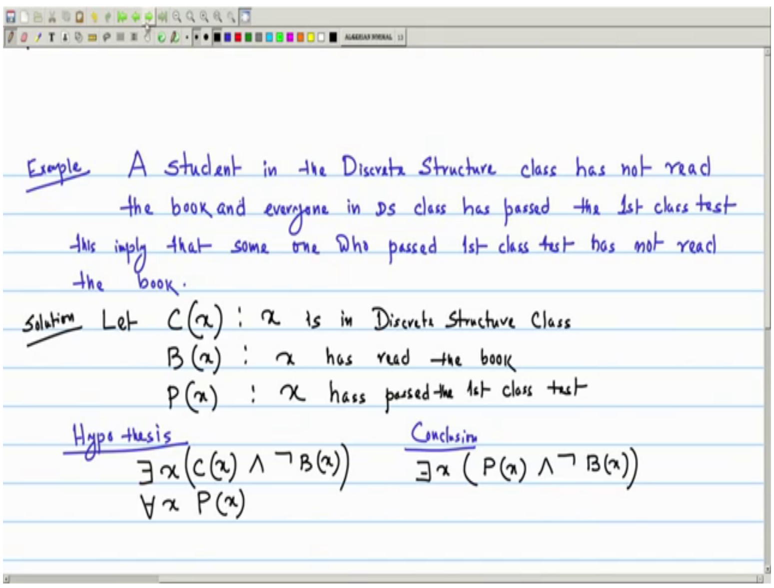

Say, a student in the discrete structure class has not read the book and everyone in discrete structure class has passed the first class test. So, this imply that someone that someone who passed first class test has not read the book. So, whether our conclusion is correct or not that we have to check.

So, first thing is that we consider some proposition. So, let C x is the student is in the class, discrete structure x is in the discrete structure class, x is in the class, x is in discrete structure class. Then B x that x has read the book and P x is the proposition that tells that x has passed the first class test on DS.

So, now what are the hypothesis given? So, the hypothesis are student in the discrete structure class has not read the book, so there exist x is student, so someone; there exist x that in the discrete structure class C x and he has not read the book, so C x and not read the book so it is negation B x. This is one hypothesis. And then everyone in DS class has passed the first class test. Everyone, so for all x for all that means, every everyone that has passed the class first class test that means, for all x P x is true.

And then we have to check that whether someone who passed first class test has not read them that means, with the conclusion should be, that someone who passed first class test has not read the book first class test means that it is P x that means, there exist x P x and who has not read the book that means negation B x. Now, we have to check whether given these hypothesis whether the con conclusion is true or false. Now, we apply the rules of inference and for this quantified statement.

Now, there is some procedure to do this thing. So, the procedure is that we have to write the, for the solution we have to write the, for this solution we write the steps and the reason.

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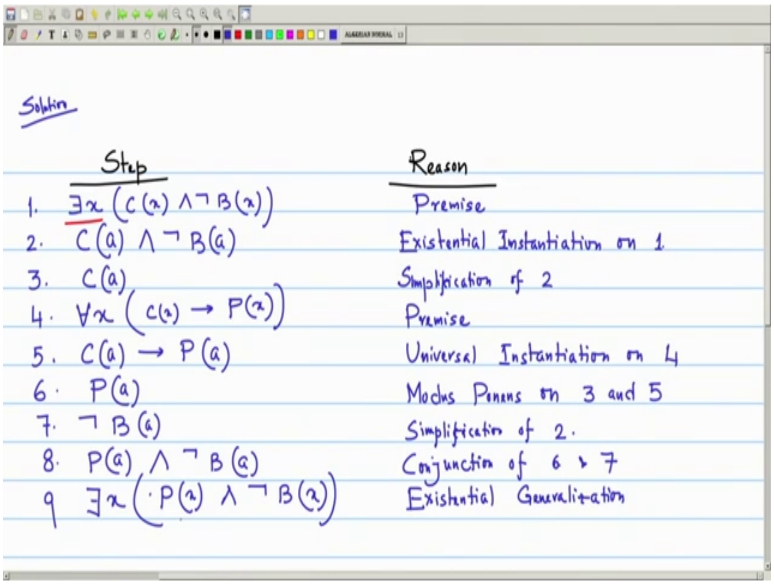

So, we start with one the hypothesis there exist x, C x and negation B x and this is the hypothesis or premise. So, we write that this is a given premise. Now, we take one existential instantiation that means, x takes a value say a, that means it I give C a and negation B a. So, this is existential instantiation on 1, step 1.

Now, we note that we have omitted that whenever we have done this instantiation we omit this quantifier this exist existential quantifier, now we can we can actually apply the laws of inference. So, we can take that only C a which is nothing, but the simplification because if it is a conjunction then we can write any one of this of the proposition. Then we have another hypothesis that for all x, C x implies P x. This is, this was a premise that mean, if he is in discrete structure class he has passed the class test.

Now, again you need universal instantiation if we do then this is C a implies P a. I write that universal instantiation on 4. Now, all are passed. So, I can write that it is P a or even I can apply some rule which is actually modus ponens the rules rule of inference is the modus ponens on C a and C a implies P a on 3 and 5. Now, I have also I can from 2, I can also write it is 2 also tells it is negation B a is also true again this is simplification because these are conjunction. So, again this is simplification of this is simplification of 2, this is also simplification of 2.

So, now I can write that P a and negation B a this is conjunction of 6 and 7. So, now, I can write that there exist x, P x and negation B x which is the existential generalization because in step 8 it was a instance for x equal to a is one instance and then we can generalize and we can tell that, this is existential generalization. And, so it is true that means, for all x that, someone who there exist some x there exist some x that who has passed, but not the read the book.

So, if we can summarize that thing that rules of inference for quantified statements, we write whatever we have read if we just do the summary that rule of inference for universal quantifier and the existential quantifier.

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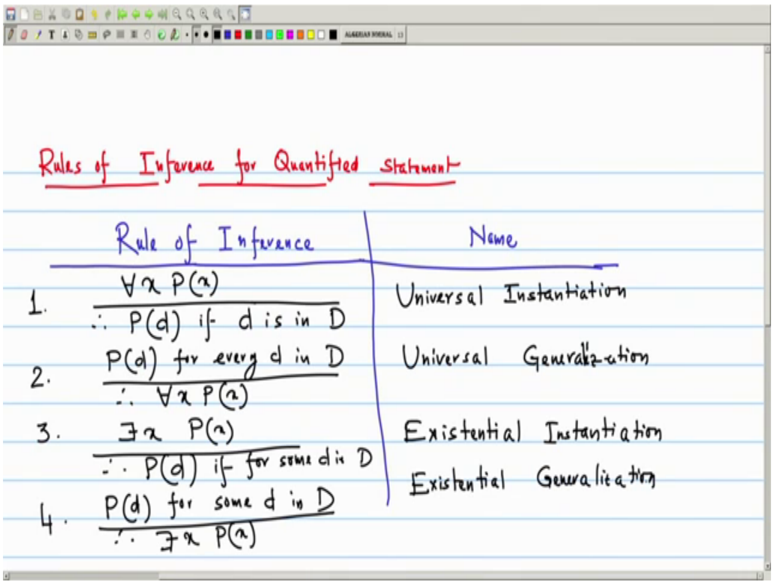

So, these are the rule of inference and we give the name one, for universal quantifier we can write for all x, P x. So, we can tell P d if d is in the domain of discourse capital D and we call this is the universal instantiation. Second one that reverse that means, if P d for every d in D for all D then this means for all x P x this is universal generalization.

Now, the same thing we can write for there exist x P x, some P d if for some d for some d in D the domain of discourse which is existential instantiation. And the reverse that P d for some d in D and there exist x P x which is existential generalization. So, whenever the rule of inference we will apply we have to remember this two quantification, that is the universal quantification and the existential quantification.

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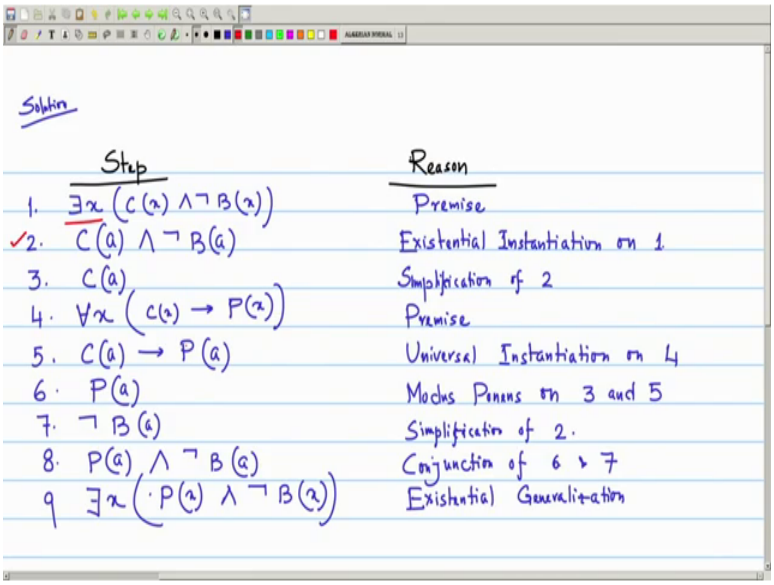

And the one point we must remember this is very important that whenever we will use some laws of inference that it must be the some quantified statement that means, that it must be that some instantiation must be done before that. Like here in step 2, we have done that x equal to a that means, there is no quantifier here there exist x of for all x then only we can apply. So, this is something we must maintain that whenever we will be using laws of inference to solve some problem that we have to write in these steps, and whatever the reasons that means, that what which laws of inference we have applied that we must mention here. And this is some convention we must follow.

So, with this we finish the lecture of the Foundations of Logic and how the logic rules the mainly the laws of inference, the laws of logic, the negation, how to find negation, the De Morgan's rule; they can be applied on the proposition.