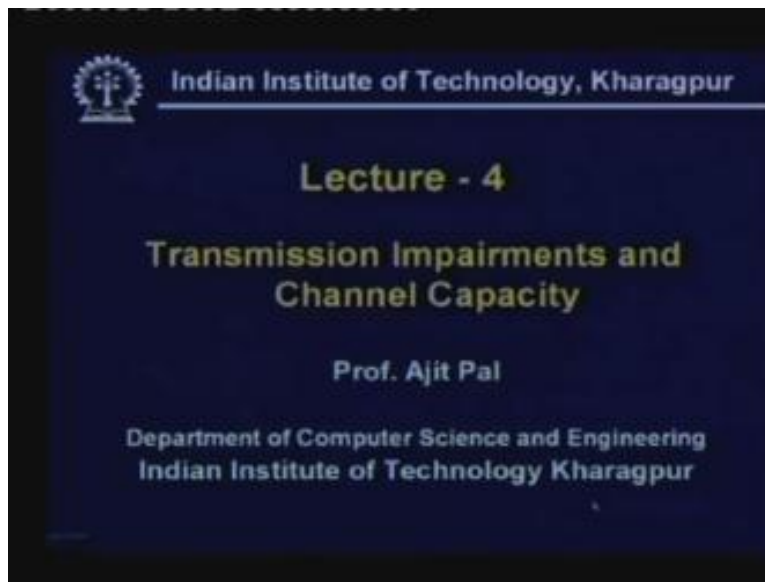


**Data Communications**  
**Prof. Ajit Pal**  
**Department of Computer Science & Engineering**  
**Indian Institute of Technology, Kharagpur**  
**Lecture # 04**  
**Transmission Impairments and Channel Capacity**

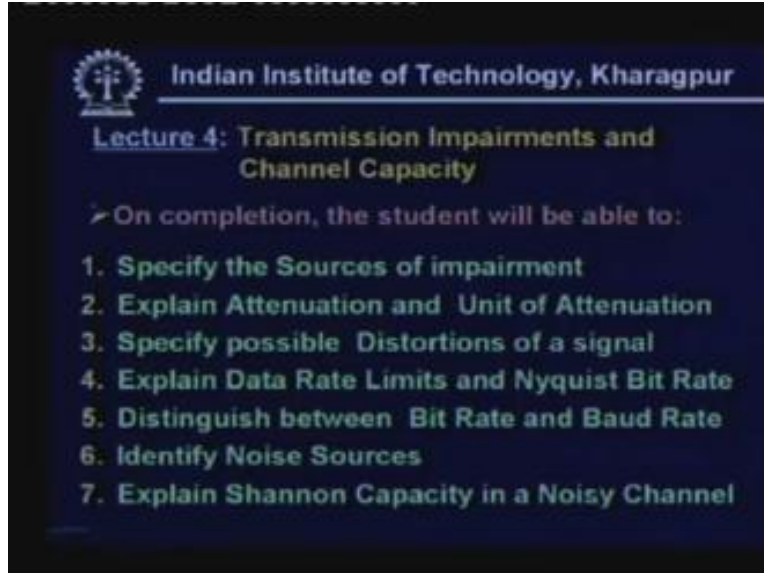
Hello viewers welcome to today's lecture on transmission impairments and channel capacity.

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This is the fourth lecture in the lecture series on data communication. On completion of this lecture the students will be able to specify the source of impairments as the signal passes through a channel, explain attenuation and unit of attenuation the decibel, specify possible distortions of a signal as it passes through the medium, explain data rate limits and Nyquist bit rate because of the bandwidth limitation and then distinguish between bit rate and baud rate then identify noise sources finally we shall explain the Shannon capacity in a noisy channel. That is the maximum bandwidth maximum information that can be passed through a noisy channel.

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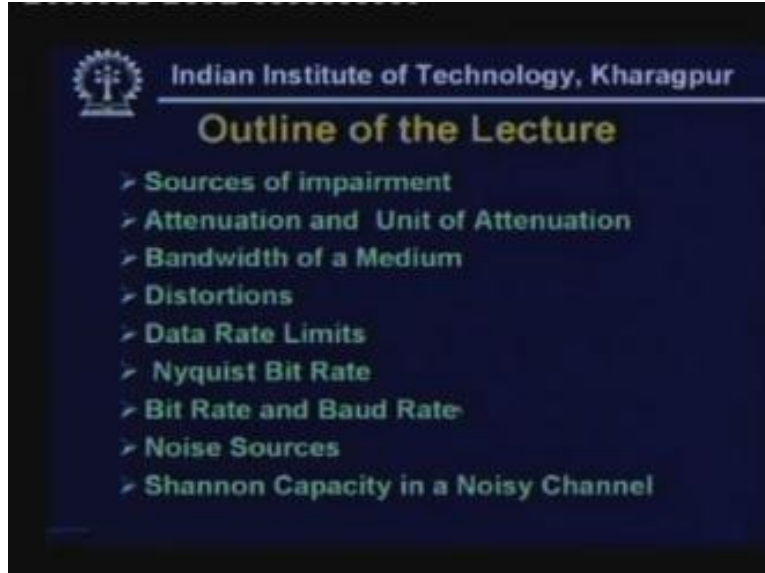
The topics that I shall cover in this lecture are sources of impairment, attenuation and unit of attenuation, bandwidth of a medium, various kinds of distortions that can take place as the signal passes through a medium, data rate limits, Nyquist bit rate, bit rate and baud rate, noise sources and finally Shannon capacity in a noisy channel.

As we know to send data you have to convert it into a signal either analog or digital then that signal has to be passed through a medium it can be a simple medium or a complex communication system. Whatever it may be the transmitter and receiver will be linked by some medium. And unfortunately the medium that we use is not ideal.

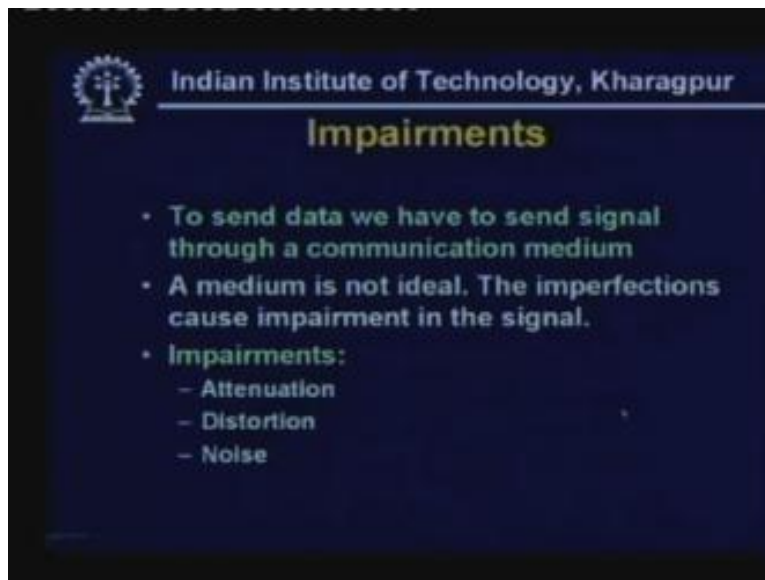
By that what do we mean?

Normally by ideal we mean that whatever is sent by the transmitter should be received by the receiver but because of the limitation of the medium that will not happen there will be some impairments which we shall discuss.

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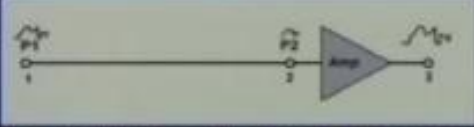
The imperfections of the medium cause impairments in the signal. What are the possible impairments? First is attenuation then distortion and noise. We shall discuss each of these impairments one after the other. First let us consider attenuation.

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### Attenuation


- Attenuation leads to loss of energy expressed in decibel.  
$$\text{dB} = 10 \log_{10}(P_2/P_1)$$
- It decides how far a signal can be sent without amplification.



- An amplifier can be used to compensate the attenuation of the medium.

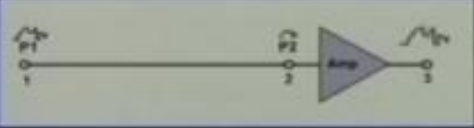
Attenuation can be considered as the loss of energy as the signal passes through a medium. We know from our basic knowledge of physics that as some signal passes through a medium its intensity falls at the rate of  $d$  square distance square. We explain that with the help of suitable unit. Normally the unit that we use is known as decibel dB. Here dB is equal to  $10 \log_{10} P_2$  by  $P_1$  where  $P_2$  is the power at the destination or point 2 and  $P_1$  is the power at the receiver that is at the transmitting end or point  $P_1$ . That means power  $P_2$  is the received power in the destination and  $P_1$  is the transmitted power from the source so this ratio  $P_2$  by  $P_1$  is taken as related measure of the loss of energy and it is expressed as  $10 \log_{10} P_2$  by  $P_1$  and this decides how far the signal can go without amplification. From our basic knowledge we know that whenever the signal level is low we can increase this signal level by amplification by a technique known as amplification and whenever we use the amplification obviously the signal level can be erased.

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## Decibel

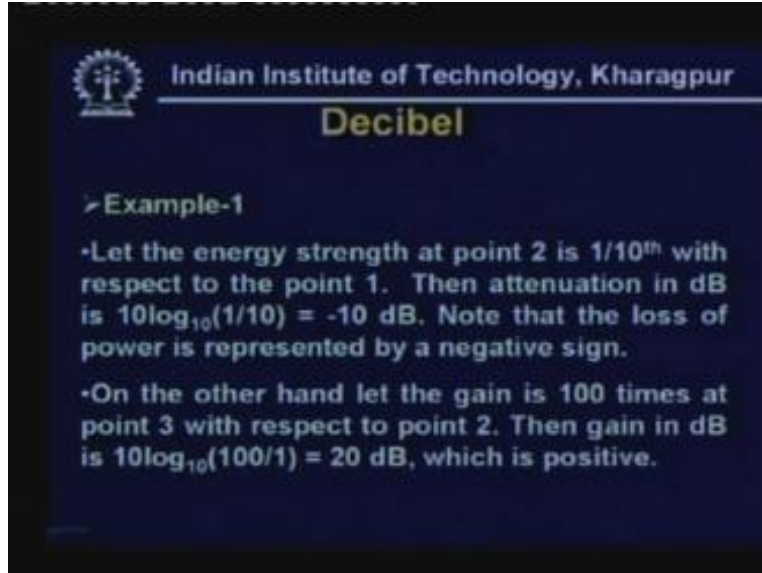
•Decibel (dB) is a measure of the relative strengths of two signals. If  $P_2$  and  $P_1$  are signal strengths at two different points 2 and 1, respectively, then relative strength at the first point with respect to the second point in dB is  $\text{dB} = 10 \log_{10}(P_2/P_1)$



As shown in this diagram for example this is the point P1 from where the signal is passing and it is going to point P2 and as you can see this was the signal sent and at point P2 it is very much attenuated. Now we use an amplifier between point 2 and point 3 and we get an amplified version of the signal.

So an amplifier can be used to compensate the attenuation of the medium. And as I mentioned decibel is a measure of the relative strengths of two signals that means signal at the destination and signal at the transmitter. If  $P_2$  and  $P_1$  are signal strengths at two different points then relative strength at the first point with respect to the second point in dB is **as I explained** dB is equal to  $10 \log_{10} P_2$  by  $P_1$ . So, as you can see here  $P_1$  is the power, here  $P_2$  is the power and here  $P_3$  is the power at this point.

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## Decibel

➤ Example-1

- Let the energy strength at point 2 is  $1/10^{\text{th}}$  with respect to the point 1. Then attenuation in dB is  $10\log_{10}(1/10) = -10$  dB. Note that the loss of power is represented by a negative sign.
- On the other hand let the gain is 100 times at point 3 with respect to point 2. Then gain in dB is  $10\log_{10}(100/1) = 20$  dB, which is positive.

Now let me explain this with the help of an example. Let the energy strength or power at point 2 is 1 by 10 with respect to point 1. Then attenuation in dB is  $10\log_{10} 1$  by 10 that is minus 10 dB. It may be noted that the loss of power is represented by a negative sign. So we find that whenever there is attenuation by looking at the sign of that decibel value we can find out the level of attenuation and also you can identify that it is attenuation.

On the other hand let the gain as it passes through an amplifier is hundred times at point three with respect to point 2. Then the gain in dB is  $10\log_{10} 100$  by 1 that is 20 dB. In this case as we find this is positive.

So whenever we amplify then we get a value which is positive and whenever attenuation occurs we get a value which is negative. Now as it passes through several points say in other words a channel is in cascade with an amplifier then may be say from here there is another channel so in this way it can be cascaded. So what we can do is we can add up the dB values to find out the final attenuation or gain at a point 3 with respect to one. For example, in this case say the signal strength at point 3 with respect to one can be obtained by adding the attenuation between P1 to P2 then amplification between 2 to 3 is equal to minus 10 plus 20 is equal to 10 dB.

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## Decibel

It may be noted that signal strength at point 3 with respect to 1 can be obtained by adding the two values;  $(-10) + 20 = 10$  dB.

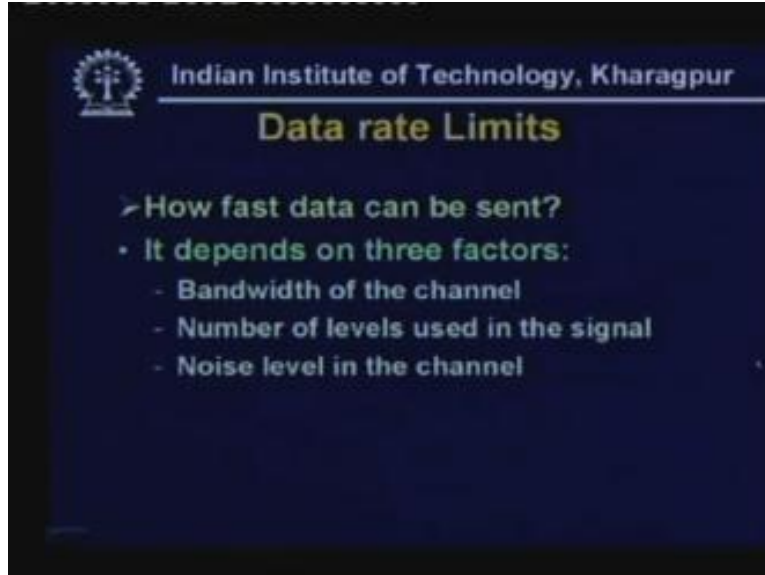
10 dB

So here with respect to point one the amplification is 10 dB. So we find that finally from here to here if we consider then it is not attenuation but a gain of 10 dB. So in this way the decibel values can be added whenever we have a number of devices or channel in cascade to find out the final value of attenuation or amplification whatever it maybe.

Now, as we know we are interested in sending data through a medium and we want to send it as fast as possible. Or in other words we want to send it at a high speed. Obviously we want the maximum possible speed. But at what speed we can send will depend on several parameters of the medium. What are the parameters of the medium?

Let us see.

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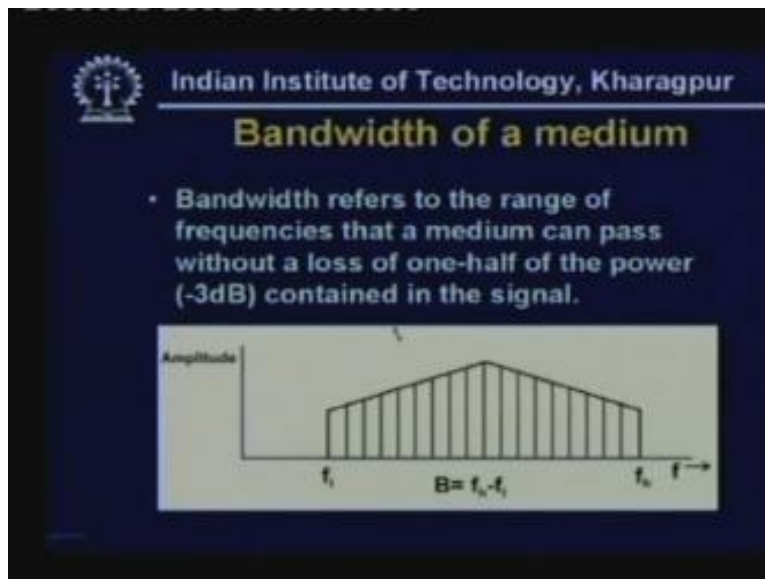
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## Data rate Limits

- How fast data can be sent?
- It depends on three factors:
  - Bandwidth of the channel
  - Number of levels used in the signal
  - Noise level in the channel

First one is the bandwidth of the channel, number of levels used in the signal, noise level in the channel. So these three factors we shall consider one after the other to understand how fast data can be sent through a medium or channel and because of these parameters it is restricted. First of all let us consider the term bandwidth.

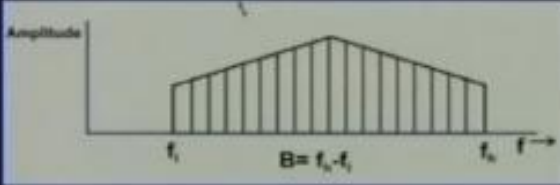
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## Bandwidth of a medium

- Bandwidth refers to the range of frequencies that a medium can pass without a loss of one-half of the power (-3dB) contained in the signal.



Amplitude

$f_1$   $B = f_2 - f_1$   $f_2$   $f \rightarrow$

In the last lecture we have considered the bandwidth of a signal. What is the bandwidth of a signal?



Bandwidth of a signal is the signal frequencies where most of the energy lies. Here it is somewhat different. Here bandwidth refers to the range of frequencies that a medium can pass without a loss of 1/2 half of the power that is minus 3 dB contained in the signal. So you see that the bandwidth term is used to refer to the bandwidth of a signal, it also refers to the bandwidth of a channel. So, whenever we refer to the bandwidth of a signal we refer to the major frequency components. On the other hand whenever we refer to a medium we mention the range of frequencies that can be sent without much attenuation through the channel.

For example in this case this is the amplitude (Refer Slide Time: 12:03) and that reflects essentially the attenuation or gain. So here we find that the attenuation is less in this part and on both sides there is higher attenuation that means amplitude is less and half power is at frequency  $f_l$  and another half power is at frequency  $f_h$ . So on both sides of this mid point there is the power label goes down to half and so the bandwidth here for the channel can be considered as  $f_h$  minus  $f_l$ . This is how the bandwidth of the channel is represented.

And as you know the frequency components of a digital signal varies from zero to almost infinity. However, at higher frequencies the signal levels are gradually lower and lower. And as you know a digital signal not periodic in nature but usually aperiodic in nature requires a bandwidth from 0 to infinity so it needs a low pass channel low pass means starting from 0 to  $f$ . So we require a channel which will not attenuate starting from 0 frequency that means dc to almost say we can consider that this is somewhat like infinity a very high frequency if we want to send all the frequency components of a digital signal. So the bandwidth of a medium decides the quality of the signal at the other end. If we restrict the bandwidth somewhere here then some of the frequency component will not reach (Refer Slide Time: 14:06).

On the other hand whenever we send analog signal then we can send it through a band pass channel  $f_1$  to  $f_2$ .

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Digital signal requires low-pass channel

- Bandwidth of a medium decides the quality of the signal at the other end.
- A digital signal (usually aperiodic) requires a bandwidth from 0 to infinity. So, it needs a low-pass channel.

Amplitude

Low pass channel

0 f

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Analog signal requires band-pass channel

Amplitude

Band pass channel

$f_1$   $f_2$

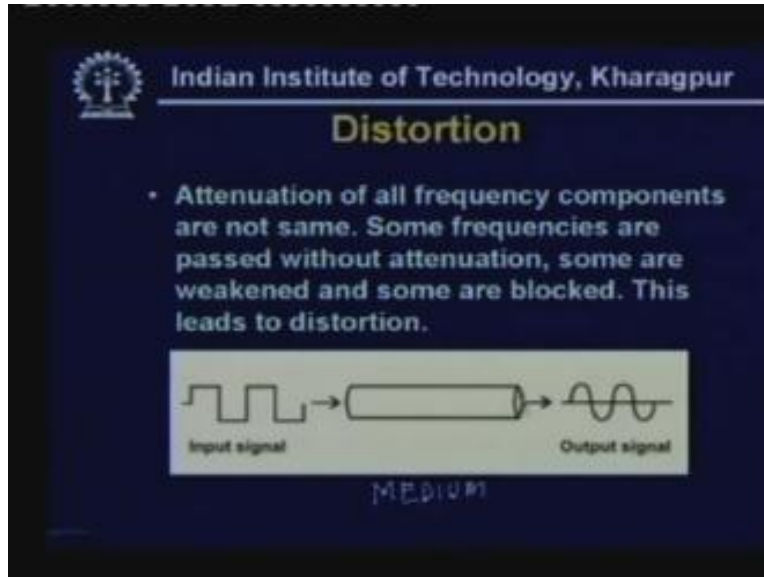
So this is the frequency range of a band pass channel and this band pass channel can be used to send an analog signal because the analog signals have bandwidth within certain range say from lower frequency to upper frequency.

Later on we shall see that these will generate signals similar to a band. That means it can be passed through a band pass channel and it has other consequences as I mentioned in the last lecture. This will help us to send several signals through one channel. You can send one signal between  $f_1$  and  $f_2$ , another signal between  $f_3$  and  $f_4$  and so on. Several

signals can be simultaneously send which you may call frequency division multiplexing. We shall discuss about it in more detail later on.

Now let us consider distortion.

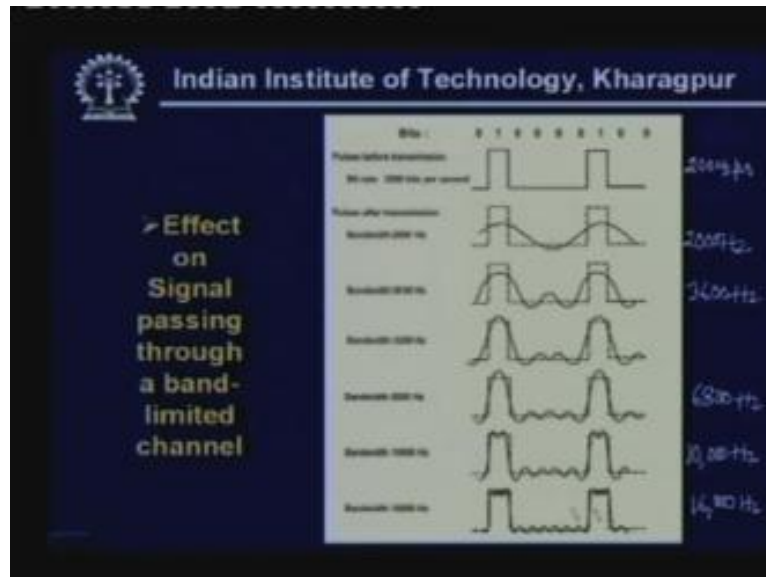
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As we have seen one important phenomenon that occurs is attenuation, apart from attenuation what occurs. Whenever attenuation occurs it is assumed that all frequency components are attenuated uniformly. However, that does not happen in practice. It has been found that attenuation of all frequency components are not same, some frequencies are passed without attenuation, some are weakened and some are completely blocked so we can see that there are three situations; some are passed without attenuation, some are weakened and some are blocked this leads to what is known as distortion. That means what the transmitter is sending at the other end of the medium the receiver is not getting the same thing they are not same. So in such a situation we can say that signal is distorted or it has suffered distortion.

For example this is an input signal (Refer Slide Time: 16:36) and this is the medium through which the signal is being sent. And as it is sent through the medium at the other end of the output we get the output signal which is much different from the input signal and this will be decided by the bandwidth of the channel or medium. Let me take an example and explain the effect of signal passing through a band limited channel.

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Suppose we are sending a digital signal 0 1 0 0 0 1 0 0 which has bit rate of 2000 Bps, here the bit rate is two thousand bits per second Bps (Refer Slide Time: 17:25) it is the 2000 Bps signal we are sending. So this is the fundamental frequency of the digital signal. Obviously apart from two thousand fundamental signal it will have the various other harmonics like 3000 Hz, 4000 Hz, 5000 Hz all the frequencies will be present however they will be of lower and lower amplitude.

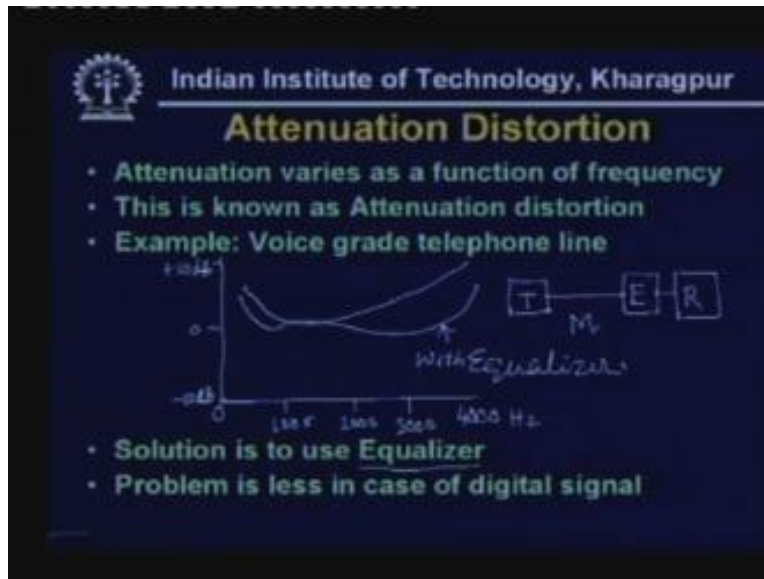
Now as we pass this signal through a transmission medium having bandwidth of exactly 200 Hz we find we get a signal like this. So this is completely different from what was sent. It was a rectangular wave but what we are getting here is a sine wave. That means only the fundamental is passing through the medium. The other frequency higher frequency components are not passing. So here the bandwidth is two thousand hertz that is the bandwidth of the medium.

Now we implicitly little bit say to 3600 Hz then we find that signal is somewhat like this. That means another harmonic has passed through the medium, 3000 harmonic has passed through the medium. To increase the bandwidth to 5200 some more harmonics have passed. We increase the bandwidth to 6800 Hz we find it somewhat very close to the original signal but definitely not sent.

Now we increase the bandwidth to 10000 Hz. We find a signal which is very close to the original signal. And now when the bandwidth is 16000 Hz we get a very good quality signal that means this will pass up to eighth harmonic and as a consequence the signal we receive at the other end after passing through the medium will be very close to the original signal that we have sent. So with this diagram I have explained how a signal gets distorted as it passes through a band limited channel.

Now, apart from attenuation and in bandwidth limitation there are other kinds of distortion the signal will suffer. One of them is known as attenuation distortion. Why attenuation distortion occurs? If the attenuation of the medium varies with frequency this leads to attenuation distortion. Let me explain with the help of a voice grade telephone line. Let me draw a diagram for a voice grade telephone line.

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Here the voice grade telephone line has bandwidth from 0 to maybe 4000 Hz. This is your 2000 Hz, this is your 1000 and this is your 3000. And on this side we draw the attenuation and this is 0 say this is minus 10 dB in decibel and this is plus 10 dB. The bandwidth of the medium that means the voice grade telephone line in this case can be represented like this. Here it is shown as 100 KHz that means with respect to hundred KHz it is 0 so it will be somewhat like this.

Therefore with respect to thousand hertz attenuation is increasing at lower frequencies and attenuation is increasing at higher frequencies. So the higher frequency components will be attenuated more than the frequencies around this region. That means lower frequency component as well as higher frequency components will be attenuated more than this central part. What is the way out of this? How can we overcome this problem?

We can overcome this problem with the help of a device known as equalizer. So you can put an equalizer to change the characteristic to somewhat like this.

For example, this is the transmitter and this is the medium through which the signal is sent (Refer Slide Time: 22:38) so at the end of the medium we put the hardware known as equalizer and then the output of the equalizer is fed to a receiver. Now we see that medium and equalizer together is giving a bandwidth like this. And obviously in this case whenever the bandwidth is like this then the distortion is much less compared to this one. That means this particular bandwidth we get only with equalizer.

Therefore in this particular case whenever we use equalizer the problem is less in case of digital signal and problem is more in case of analog signal. The reason for that is for digital signal we have seen that most of the energy is concentrated near the fundamental and lower harmonics. At higher harmonics lesser and lesser energy is present. On the other hand an analog signal can have frequency components over the entire range.

So whenever it is attenuated at some frequency range we get more distortion. On the other hand in case of digital signal problem is less severe. That's why this kind of equalizer is used in case of voice grade telephone line to correct the bandwidth of the medium to get a better signal at the receiving end.

Now let us consider another type of distortion that occurs that is known as delay distortion. And this delay distortion arises particularly in a guided media but not in air. Later on we shall discuss about the different types of transmission media. We will be having two types of transmission media guided and unguided. So in case whenever the signal is passed through a guided media like twisted pair cable, coaxial cable or optical fiber it leads to delay distortion.

What is delay distortion?

Delay distortion arises because velocity of propagation varies with frequency. That means this signal components that we are sending will have different velocities for different frequency components as it passes through a guided media and this leads to delay distortion. And again let us take up the example of voice grade telephone line and in this case we consider the delay.

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### Delay Distortion

- Arises in case guided media
- Velocity of propagation varies with frequency
- This leads to delay distortion
- Example: Voice grade telephone line

Delay

4000 Hz

0

4000

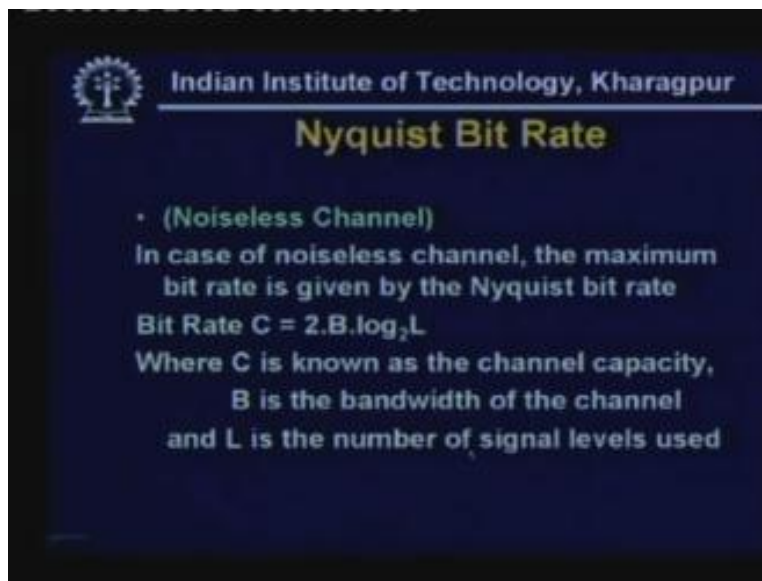
With equalizer

- Effect can be minimized using equalizer
- Digital signal is more affected

Earlier we considered the attenuation, here we consider the delay, on this side it is the delay and delay is varying from 0 to 4000 microseconds. And on this side this is the frequency that means hertz. So zero to four thousand is the bandwidth of the medium (Refer Slide Time: 26:05). Now it has been found that in the middle part the velocity delay is less that means velocity is more. That means on either side of the frequency range delay is more or velocity is less. So what will happen is the lower or upper frequency component will reach later then the middle frequency components. That means the frequency components in this range will reach earlier than the frequency component in this range and in this range. So, it may so happen that the lower and upper frequency components of the previous signal will fall on the middle frequency component of the present signal, this will lead to distortion. Again this effect can be minimized with the help of using an equalizer and after using an equalizer the characteristic can be somewhat like this so this is with equalizer.

With equalizer the bandwidth is much better. That means relay distortion is much less and bandwidth characteristic with respect to delay we find is much better. And in this particular case digital signal is more affected. As we know digital signal will be having many high frequency components so we find that the delay distortion affects a digital signal more than an analog signal. This is opposite to that of attenuation distortion. Now let us consider the Nyquist bit rate.

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### Nyquist Bit Rate

- (Noiseless Channel)

In case of noiseless channel, the maximum bit rate is given by the Nyquist bit rate

$$\text{Bit Rate } C = 2.B.\log_2 L$$

Where C is known as the channel capacity,  
B is the bandwidth of the channel  
and L is the number of signal levels used

As we already mentioned that the signal we send to the other end is dependent on the bandwidth of the channel, noise of the channel because of these parameters. Now let us see that in case of a noiseless channel we assume that there is no noise although it is not true in practice but for an ideal channel that means without noise the maximum bit rate is given by the Nyquist bit rate and it is equal to C that means Nyquist bit rate C is equal to  $2 \text{ into } B \log_2 L$  where C is known as the channel capacity, B is the bandwidth of the channel and L is the number of signal levels. Here we are taking bandwidth and number

of signal levels into consideration. Later on let's see what we really mean by number of signal levels. So it depends on the three parameters. Rather two parameters in this case the channel capacity depends on bandwidth and the number of signal levels that we are sending.

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## Baud Rate

- The *baud rate* or *signaling rate* is defined as the number of distinct symbols transmitted per second, irrespective of the form of encoding.
- For baseband digital transmission  $L = 2$

Maximum Baud Rate =  $\frac{1}{\text{Element width (in seconds)}} = 2B$

Handwritten notes on the right side of the slide:  
0V  
1V  
0V  
0.5V  
1V  
2V  
0.5V  
L=2

Actually to understand the number of signal levels another parameter known as baud rate has to be understood. Let us see what we really mean by baud rate.

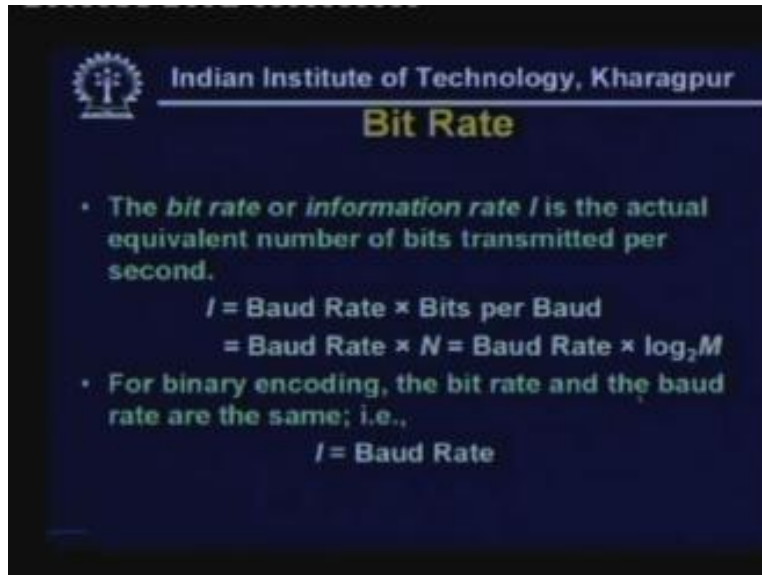
The baud rate of a signal or the signaling rate is defined as the number of distinct symbols transmitted per second irrespective of the form of encoding. Whenever we are sending a digital signal we know the bit rate and bit interval. Now in a single bit interval it is possible to send the number of distinct symbols that can be transmitted per second. Actually it will depend on how much we are sending within the bit interval. In a bit interval multiple levels can be sent then we can send more information.

For example in case of baseband digital transmission where the number of levels is two that means within a bit interval we can send either 0V or 1V. So we have got two distinct values or two levels; zero level and one level. So in this case it is two. Suppose it is possible to send 0V, 0.5V, 1V and then 2V like that. Let's take these four values we can send 0, 0.5, 1 and 2 so in this case the number of levels is not 2 but 4. That means within that bit period we can send one of the four values 0V, 0.5V, 1V or 2V so in that case L is equal to 4. So, depending on how many levels are present the maximum baud rate will be dependent on that.

So, for digital transmission as we know it is equal to 1 by element width in second is equal to 2B because here L is equal to 2. So this is the case for digital transmission with only two levels.



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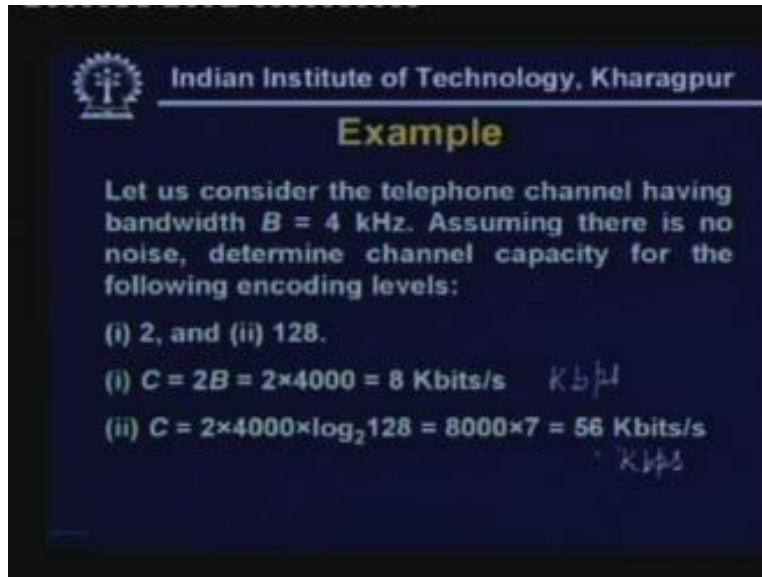
## Bit Rate

- The *bit rate or information rate I* is the actual equivalent number of bits transmitted per second.  
$$I = \text{Baud Rate} \times \text{Bits per Baud}$$
$$= \text{Baud Rate} \times N = \text{Baud Rate} \times \log_2 M$$
- For binary encoding, the bit rate and the baud rate are the same; i.e.,  
$$I = \text{Baud Rate}$$

The bit rate or information rate is the actual equivalent number of bits transmitted per second. That means this allows us whenever we are able to put more number of levels within a particular bit interval then we can send more information for a given bandwidth. For example, the bit rate or information rate is the actual equivalent number of bits transmitted per second. That means  $I$  is equal to baud rate into bits per second. Baud rate means the number of possible values that we can send and bit rate is actually the number of bits we are sending, bits per baud. So it is baud rate into  $N$  or baud rate into  $\log_2 M$  so here  $M$  is the number of levels.

We find that for binary encoding the bit rate and the baud rate are same because here the value of  $N$  is equal to 2 so  $\log_2$  is essentially 1. That means the information rate or bit rate is same as the baud rate. So information rate is same as the baud rate in case of binary encoding or digital encoding. But for maximal use of the channel we can use multi level encoding. That will allow us to send more information through a communication medium.

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### Example

Let us consider the telephone channel having bandwidth  $B = 4$  kHz. Assuming there is no noise, determine channel capacity for the following encoding levels:

(i) 2, and (ii) 128.

(i)  $C = 2B = 2 \times 4000 = 8$  Kbits/s    Kbps

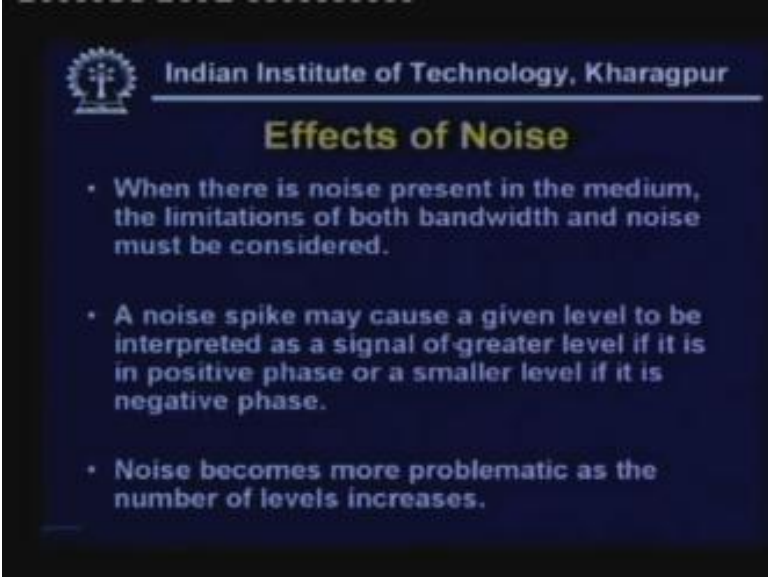
(ii)  $C = 2 \times 4000 \times \log_2 128 = 8000 \times 7 = 56$  Kbits/s    Kbps

Let me consider an example of the telephone channel having bandwidth is equal to 4 KHz **as we have already discussed.**

Assuming that there is no noise determine the channel capacity for the following encoding values. Say we are considering encoding level two and encoding level 128. So, when the encoding level is two we get the channel capacity equal to  $2B$  that means 8 Kbps or we can write it as Kbps. On the other hand when we are using 128 different levels for encoding how multilevel encoding can be done, **this we shall discuss in details later on.**

Then we can send two into 4000 that is the bandwidth into  $\log_2 128$  that means it gives us 8000 into 7 or 56 Kbps that means Kbps 56 Kbps. So you see by suitable multilevel encoding through a channel having bandwidth of only 4 KHz we can send a very high data rate 56 Kbps that is precisely what is done in the case of modem. Modem can send 56 Kbps which is the maximum baud data rate. So we find that this is how we can send more information.

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### Effects of Noise

- When there is noise present in the medium, the limitations of both bandwidth and noise must be considered.
- A noise spike may cause a given level to be interpreted as a signal of greater level if it is in positive phase or a smaller level if it is negative phase.
- Noise becomes more problematic as the number of levels increases.

So far we have neglected the effect of noise but ideally that will not be so and the channel will always have some noise. So, when there is noise present in the medium the limitations of both bandwidth and noise must be taken into consideration. So far we have only taken into consideration the limitation due to bandwidth. Now let us take the limitation due to the noise.

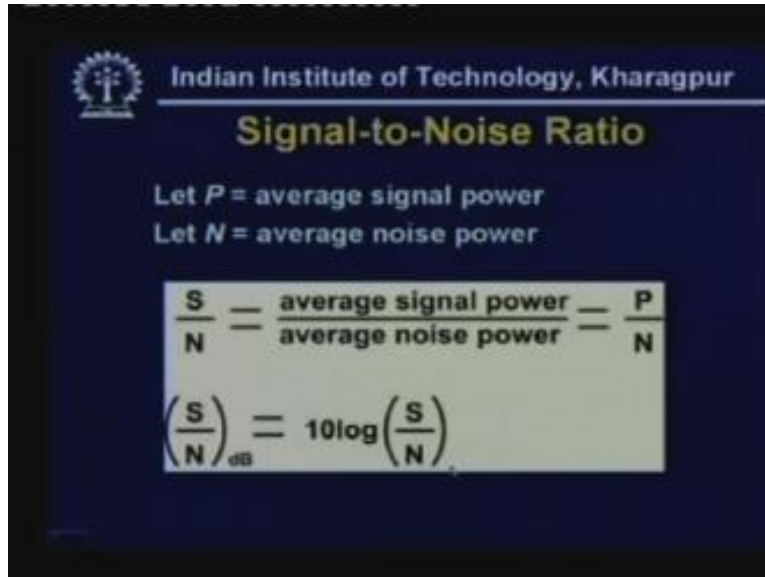
A noise spike may cause a given level to be interpreted as a signal of greater level. If it is in positive phase or a smaller level if it is a negative phase that means suppose you are sending a voltage level 0.5V and there is a noise of plus 0.5V that will make the signal level equal to 1V although 0.5V was sent now we are receiving 1V at the receiving end. So at the receiving end it will be interpreted as a signal of different level so we shall get incorrect data.

On the other hand if it is negative phase say plus 5 and the received signal say noise is minus five volt then together it will make a 0V so we shall get another signal level which is also incorrect. So by this way because of noise we may not get correct data. Particularly noise becomes more problematic as the number of levels increase. so if we have only two levels zero and one which is represented by zero volt and plus five volt then problem is less, we can tolerate up to maybe plus minus two volt of noise.

On the other hand when we are using four levels we divide 5V into four equal parts there it will be more susceptible to noise. Or if the number of level is 128 then we can see that say 0 to 5V is divided into 128 different values and a small noise will change the transmitted noise level so at the receiving end we shall get incorrect voltage signal level so it becomes more problematic as the number of levels increases. To quantify the noise level a parameter known as signal to noise ratio is used.

Let  $P$  be the average signal power and  $n$  the average noise power. Then signal to noise ratio is equal to average signal power by average noise power  $10\log(S \text{ by } N)$ . So in decibel this is the signal to noise ratio.

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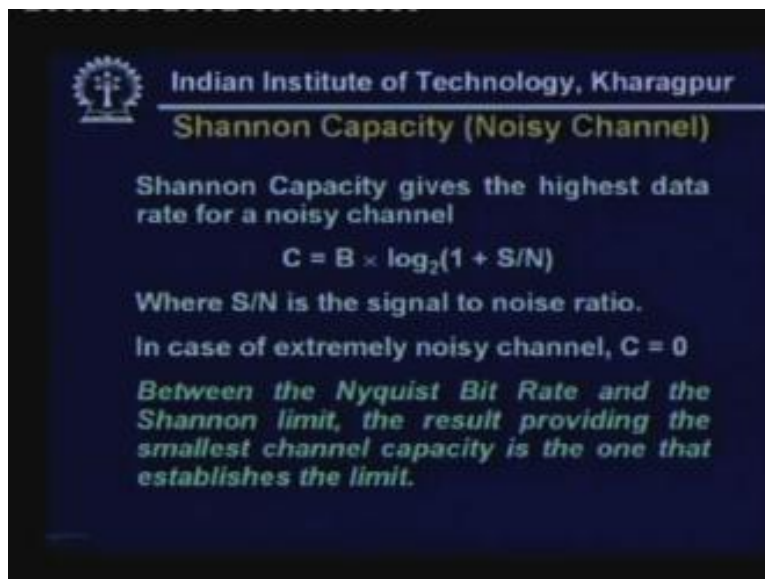
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### Signal-to-Noise Ratio

Let  $P$  = average signal power  
Let  $N$  = average noise power

$$\frac{S}{N} = \frac{\text{average signal power}}{\text{average noise power}} = \frac{P}{N}$$
$$\left(\frac{S}{N}\right)_{\text{dB}} = 10\log\left(\frac{S}{N}\right)$$

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### Shannon Capacity (Noisy Channel)

Shannon Capacity gives the highest data rate for a noisy channel

$$C = B \times \log_2(1 + S/N)$$

Where  $S/N$  is the signal to noise ratio.

In case of extremely noisy channel,  $C = 0$

*Between the Nyquist Bit Rate and the Shannon limit, the result providing the smallest channel capacity is the one that establishes the limit.*

Now for a given signal to noise ratio the Shannon capacity for this noisy channel gives the highest data rate represented by  $C$  is equal to  $B$  bandwidth part is there and also it takes into consideration the signal to noise ratio. So the Shannon capacity gives the capacity equal to  $B \text{ into } \log_2(1 \text{ plus } S \text{ by } N)$  where  $S \text{ by } N$  is the signal to noise ratio. So

we find that in case of extremely noisy channel  $C$  is equal to 0 that means  $\log_{21}$  is equal to 0. That means whenever we try to send data through a very noisy channel irrespective of its bandwidth we may not be able to send any data because of this fact. That means if the noise level is very high compared to the signal level then we cannot send any data through it that means channel capacity is 0.

Now we have two different parameters; one is Nyquist bit rate which gives us a channel capacity based on bandwidth and number of levels that we used for encoding and another is based on bandwidth and the signal to noise ratio. So between the Nyquist bit rate and the Shannon limit the result providing the smallest channel capacity is the one that establishes the limit. That means we may compute the Nyquist bit rate and Shannon capacity and find out which one is the lower and the lower one has to be taken as the channel capacity or the maximum information that can be send through the medium.

Let me take few examples here particularly in case of noisy channel.

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### Example

- A channel has  $B = 4$  KHz. Determine the channel capacity for each of the following signal-to-noise ratios: (a) 20 dB, (b) 30 dB, (c) 40 dB.

(a)  $C = B \log_2 \left[ 1 + \frac{S}{N} \right] = 4 \times 10^3 \times \log_2 (1 + 100)$   
 $= 4 \times 10^3 \times 3.32 + 2.004 = 26.6 \text{ kbps/s}$

(b)  $C = B \log_2 \left[ 1 + \frac{S}{N} \right] = 4 \times 10^3 \times \log_2 (1 + 1000)$   
 $= 4 \times 10^3 \times 3.32 + 3.000 = 39.8 \text{ kbps/s}$

(c)  $C = B \log_2 \left[ 1 + \frac{S}{N} \right] = 4 \times 10^3 \times \log_2 (1 + 10^4)$   
 $= 4 \times 10^3 \times 3.32 + 4.000 = 53.1 \text{ kbps/s}$

So here the channel has got bandwidth of 4 KHz as earlier. Now determine the channel capacity for each of the following signal to noise ratios: 20 dB, 30 dB and 40 dB. So we find 20 dB is essentially 100 signal to noise ratio so it gives you Shannon capacity of essentially which is equal to 26.6 Kbps. For 30 dB it is actually signal to noise ratio of 1000 it gives you a channel capacity of 39.8 Kbps and for signal to noise ratio of 40 dB we get a channel capacity of 53.1 Kbps as the signal to noise ratio is higher and higher we can achieve higher channel capacity we get higher channel capacity for higher value of signal to noise ratio.

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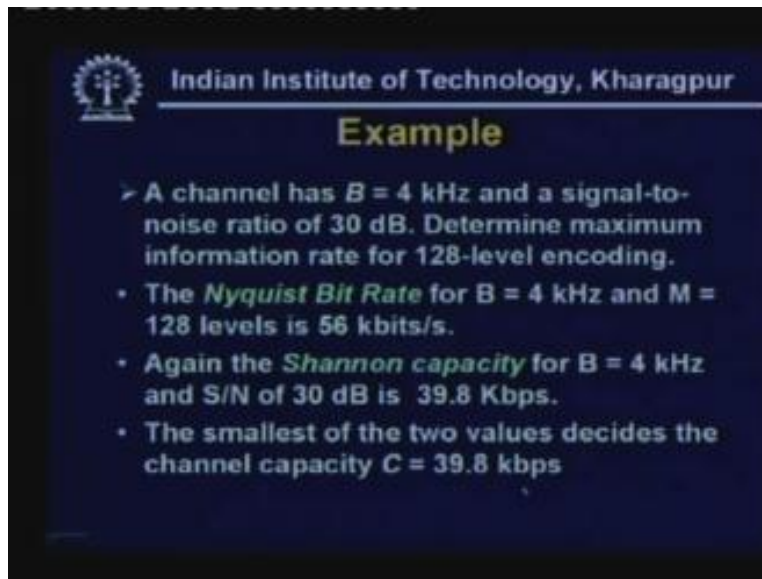
### Example

- A channel has  $B = 4$  KHz and a signal-to-noise ratio of 30 dB. Determine maximum information rate for 4-level encoding.
- For  $B = 4$  KHz and 4-level encoding the *Nyquist Bit Rate* is 16 Kbps.
- Again for  $B = 4$  KHz and S/N of 30 dB the *Shannon capacity* is 39.8 Kbps.
- The smallest of the two values has to be taken as the Information capacity  
 $I = 16$  Kbps

Now let us consider a situation where we have got a channel with a bandwidth of 4 KHz, a signal to noise ratio of 30 dB so we have to determine maximum information rate for four level encoding. Thus here the value of  $N$  is equal to 4 so we are using four level encoding. Therefore based on our Nyquist bit rate we can calculate the value for  $B$  is equal to 4 and four level encoding we get 16 Kbps. That means  $2$  into  $B$   $8$  into  $\log_2 4$  that means we get  $2$  so  $2$  into  $4$  into  $2$  we get 16 Kbps.

On the other hand if you take into consideration the signal to noise ratio of 30 degree then we get a Shannon capacity of 39.8 Kbps so we find two values; one value is 39.8 kilobits per sec second that is the maximum capacity that is possible. However with four level of encoding we can achieve only 16 Kbps. So smallest of the two values has to be taken as the information capacity. So in this particular case when we are using  $B$  is equal to 4 KHz, signal to noise ratio of 30 dB then using four level that is based on these three values for four level encoding the information capacity will be equal to 16 Kbps. so in this way we can find out what is the information capacity in a particular situation. Let us take another example.

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### Example

- A channel has  $B = 4$  kHz and a signal-to-noise ratio of 30 dB. Determine maximum information rate for 128-level encoding.
- The *Nyquist Bit Rate* for  $B = 4$  kHz and  $M = 128$  levels is 56 kbits/s.
- Again the *Shannon capacity* for  $B = 4$  kHz and S/N of 30 dB is 39.8 Kbps.
- The smallest of the two values decides the channel capacity  $C = 39.8$  kbps

Here a channel has bandwidth  $B$  is equal to 4 KHz, signal to noise ratio of 30 dB we would like to find out the maximum information rate for 128 level encoding. So here it is based on Nyquist bit rate  $B$  is equal to 4 and  $M$  is equal to 128 levels we get 56 Kbps. On the other hand because of the signal to noise ratio of 30 dB we get Shannon capacity of 39.8. So here we are using a large number of levels 128 which may provide us 56 Kbps. But because of the lower signal to noise ratio the information capacity will be restricted to 39.8 so you won't get 56 Kbps but we shall get 39.8 Kbps. So here we have to take the smallest of the two. And here it is limited by the signal to noise ratio but in the previous case it was limited by the limited by the Nyquist bit rate.

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### Example

- The digital signal is to be designed to permit 160 kbps for a bandwidth of 20 KHz. Determine (a) number of levels and (b) S/N ratio.

(a) Apply Nyquist Bit Rate to determine number of levels

$$C = 2B \log_2 M \text{ or } 160 \times 10^3 = 2 \times 20 \times 10^3 \log_2 M$$
$$M = 2^4 = 16 \text{ meaning 4bits/ baud}$$

(b) Apply Shannon capacity to determine the S/N ratio

$$C = B \log_2 \left[ 1 + \frac{S}{N} \right]$$
$$\text{or } 160 \times 10^3 = 20 \times 10^3 \log_2 \left[ 1 + \frac{S}{N} \right]$$
$$\frac{S}{N} = 2^4 - 1 = 255 \text{ or } \left( \frac{S}{N} \right)_{dB} = 24.07 \text{ dB}$$

Now we can consider another example. The digital signal is to be designed to permit 160 Kbps that means we have to send 160 Kbps for a bandwidth of 20 KHz. Determine the number of levels and signal to noise ratio. Here the problem is given in a little different way. Here the bandwidth is given 20 KHz and our desired data rate is given. We have to find out the number of levels and signal to noise ratio. So applying Nyquist bit rate to determine the number of levels we get C is equal to  $2B \log_2 M$  so we get 16 that means 4 bits per baud that means a single signal element has to represent four bits so that the number of levels is 16.

Applying Shannon capacity we get signal to noise ratio based on this formula 24.07. So you see which one is the signal to noise ratio of 24.07 and number of levels equal to 16 so we can achieve 160 Kbps data transmission through a channel having bandwidth of 20 KHz. So this example tells us how we can design a transmission system to provide suitable number of encoding levels and a suitable value of signal to noise ratio so that we can achieve a desired data rate.

I believe our discussion will not be complete if we don't discuss about the types of noise that is present in the medium. There are several types of noise that may corrupt a signal and the most common noise types are discussed here.



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## Noise

- Several types of noise may corrupt the signal
- Common Noise Types:
  - Thermal:  $N = k.T.B$   $f_1 \quad f_2 \quad (f_1 + f_2)$
  - Intermodulation: Occurs when signals of different frequencies share the same medium
  - Crosstalk: It is due unwanted coupling between two media.
  - Impulse noise: Arises due disturbances such as lightning, electrical sparks.
    - Digital signals are more affected than Analog Signals

First one is thermal noise. What is thermal noise?

Normally if we look at a conductor a copper wire apparently we find that it is a very common tool there is no movement nothing is taking place. However, if you look at the molecular level we know each molecule is vibrating within a range and all the free electrons are moving around the conductor so there is a lot of movement which leads to noise because of the movement of the electrons which are responsible for conducting current known as the thermal noise and this thermal noise is dependent on three parameters where the parameter  $K$  is the Boltzmann constant,  $T$  is the absolute temperature in Kelvin and  $B$  is the bandwidth of the channel so we can see that higher the bandwidth higher is the noise so  $N$  is equal to  $k, T, B$ .

Another very important point we must notice is that the thermal noise increases with temperature. As the temperature increases more and more noise increases. That is the reason why during summer the quality of signal we receive is poorer than in winter.

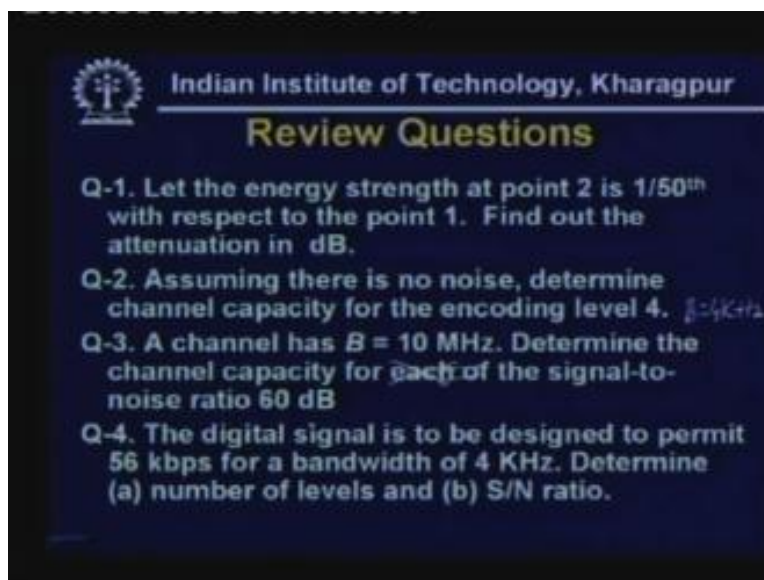
Then the second type of noise is known as intermodulation. intermodulation occurs when signals of different frequencies share the same medium. Suppose a medium is sharing a frequency  $f_1$  it is also sharing a frequency  $f_2$ . Now because these two signals are present due to **non**linearities in the transmission system it may generate signal like  $f_1$  plus  $f_2$ . This signal  $f_1$  plus  $f_2$  will add if it is in the same place if there is a signal of  $f_1$  plus  $f_2$  in the original signal. That means the noise generated because of intermodulation will be added to the signal if it is having a frequency component  $f_1$  plus  $f_2$ . So we find that this intermodulation occurs when signals of different frequencies share the same medium. This is an example of how different signal components are generated from two frequency components.

Then the third type of noise that we commonly encounter is known as crosstalk. This crosstalk is due to unwanted coupling between two media. In the next class we shall discuss the various types of transmission media. There we will see particularly in telephony and many other applications a number of cables are bunched together and sent from one place to another. So whenever we are sending a number of cables side by side the signal passing through one cable induces signal in other cable and this leads to crosstalk. In our telephone network the crosstalk is a daily phenomenon. We have seen that in telephone network we frequently encounter crosstalk. We hear some unwanted talk going on in the background. This is because of this unwanted coupling between two transmission media that means cables.

Impulse noise: This arises due to disturbances such as lightning and electrical sparks. This is not getting generated in the medium but the environment is generating it. For example, in the rainy season there is lightning or in an industrial environment there are electrical sparks like welding, turning on and turning off switches so various types of things can occur that leads to radiation of electromagnetic magnetic signals and that may corrupt the signal that is passing through the medium. This affects more severely the digital signal not the analog signal that much. Because the 0 may become 1 or 1 may become 0 if it is a digital signal. On the other hand in case of analog signal it will not affect much. Signal level will possibly change. If we are listening to voice we will here some disturbances but that error is tolerable. But in digital domain this error will lead to corruption of data.

With this we complete our discussion on the transmission impairments and channel capacity because of the limitation of the channel. Let us give some review questions which will be answered in the next lecture.

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The slide features the IIT Kharagpur logo and name at the top. Below the title 'Review Questions', there are four numbered questions (Q-1 to Q-4) related to signal attenuation, channel capacity, and digital signal design. The text is white on a dark blue background.

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### Review Questions

Q-1. Let the energy strength at point 2 is  $1/50^{\text{th}}$  with respect to the point 1. Find out the attenuation in dB.

Q-2. Assuming there is no noise, determine channel capacity for the encoding level 4.  $\log_2 4 = 2$

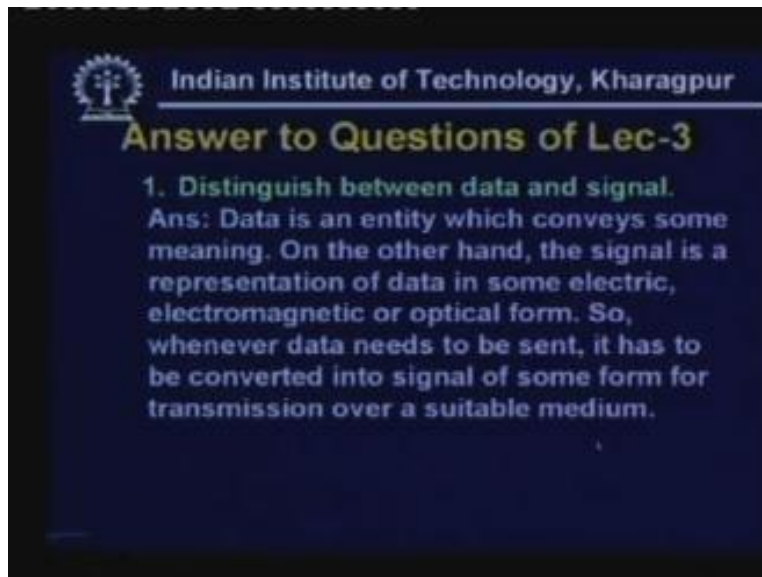
Q-3. A channel has  $B = 10$  MHz. Determine the channel capacity for  $\text{pract}$  of the signal-to-noise ratio 60 dB

Q-4. The digital signal is to be designed to permit 56 kbps for a bandwidth of 4 KHz. Determine (a) number of levels and (b) S/N ratio.

- 1) Let the energy strength at point 2 is 1 by 50th with respect to point 1. Find out the attenuation in dB for point 1 with respect to point 2.
- 2) Assuming there is no noise determine channel capacity for the encoding level 4 and bandwidth is equal to 4 KHz.
- 3) A channel capacity has bandwidth of 10 MHz. Determine channel capacity for signal to noise ratio of 60 dB.
- 4) The digital signal is to be designed to permit 56 Kbps for a bandwidth of 4 KHz determine the number of levels required and the signal to noise ratio.

These are the four questions. We have to answer and answer will be given in the next lecture. And here is the answer of the previous lecture.

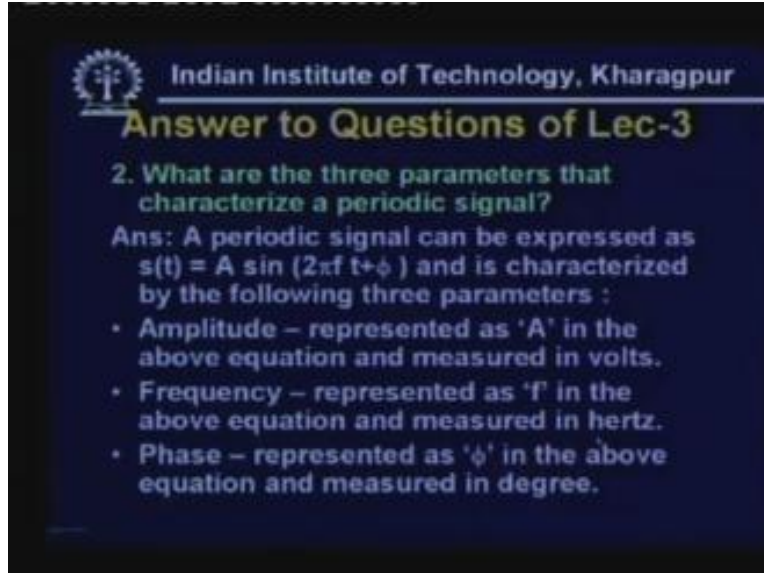
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Distinguish between data and signal.

Data is an essential entity which conveys some meaning. On the other hand the signal is a representation of data in some electromagnetic and optical form. So whenever data needs to be sent it has to be converted into signal of some form for transmission over a suitable medium.

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The slide features the IIT Kharagpur logo and name at the top. Below that, the title "Answer to Questions of Lec-3" is displayed in yellow. The main text is in white on a dark blue background. It asks for three parameters of a periodic signal and provides the answer with a sine wave equation and a bulleted list of amplitude, frequency, and phase.

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**Answer to Questions of Lec-3**

2. What are the three parameters that characterize a periodic signal?

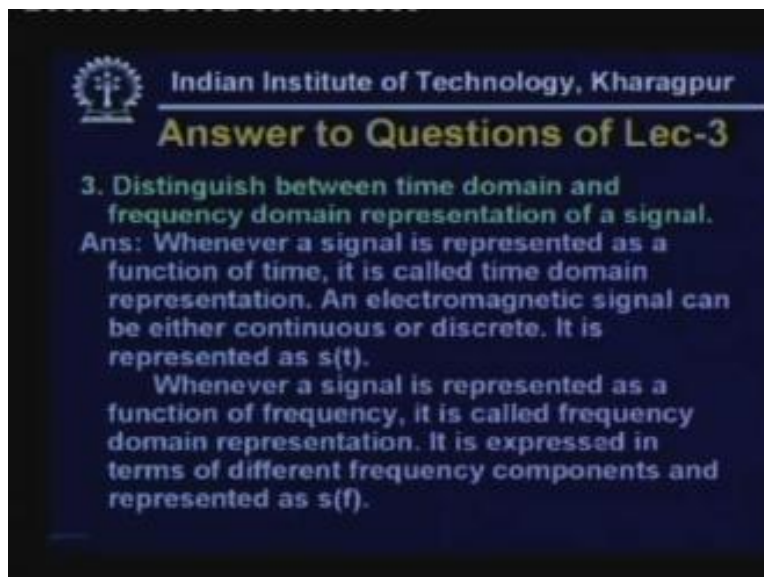
Ans: A periodic signal can be expressed as  $s(t) = A \sin(2\pi f t + \phi)$  and is characterized by the following three parameters :

- Amplitude – represented as 'A' in the above equation and measured in volts.
- Frequency – represented as 'f' in the above equation and measured in hertz.
- Phase – represented as ' $\phi$ ' in the above equation and measured in degree.

2) What are the three parameters that characterize a periodic signal?

As we know the three parameters are amplitude, frequency and phase. Amplitude is represented by A, frequency is represented by F and phase is represented by phi.

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The slide features the IIT Kharagpur logo and name at the top. Below that, the title "Answer to Questions of Lec-3" is displayed in yellow. The main text is in white on a dark blue background. It asks to distinguish between time and frequency domain representations and provides a detailed answer.

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**Answer to Questions of Lec-3**

3. Distinguish between time domain and frequency domain representation of a signal.

Ans: Whenever a signal is represented as a function of time, it is called time domain representation. An electromagnetic signal can be either continuous or discrete. It is represented as  $s(t)$ .

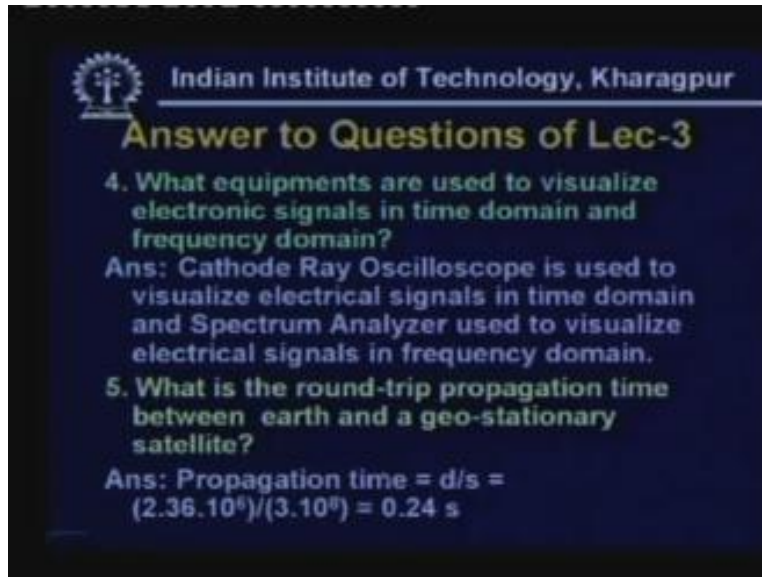
Whenever a signal is represented as a function of frequency, it is called frequency domain representation. It is expressed in terms of different frequency components and represented as  $s(f)$ .

3) Distinguish between time domain and frequency domain representation of a signal.

Whenever a signal is represented as a function of time it is called time domain representation. An electromagnetic signal can be either continuous or discrete it is

represented by  $s(t)$ . Whenever a signal is represented as a function of frequency it is called frequency domain representation, it is expressed in terms of different frequency components and represented by  $s(f)$ .

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### Answer to Questions of Lec-3

4. What equipments are used to visualize electronic signals in time domain and frequency domain?  
Ans: Cathode Ray Oscilloscope is used to visualize electrical signals in time domain and Spectrum Analyzer used to visualize electrical signals in frequency domain.

5. What is the round-trip propagation time between earth and a geo-stationary satellite?  
Ans: Propagation time =  $d/s = (2.36 \cdot 10^8)/(3 \cdot 10^8) = 0.24 \text{ s}$

4) What equipments are used to visualize electronic signals in time domain and frequency domain?

For time domain representation we use cathode ray oscilloscope and for frequency domain representation we use spectrum analyzer.

5) What is the round trip propagation time between earth and a geo-stationary satellite?

As we know distance is 36000 Km and speed of light is  $3 \times 10^8$  meter per second so from that we find that the round trip propagation delay is about quarter of a second, it's get a propagation time of 0.24 second so a long time. **Later on we shall discuss about it more. For the time being thank you.**