Introduction to Engineering Seismology
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Lecture No -24
Fourier Spectrum

So, Vanakkam, we will continue our lecture on engineering seismology So we have been discussing about the Fourier domain parameters calculation by the Fourier analysis, So we have seen that the acceleration time history, what we are getting is the consist of amplitude and duration you can interpret directly. So if you want to understand the structural response at different frequencies or a particular structure will have a unique natural frequency, how the amplitude varies with the frequency you have to know.

So in order to know that basically the acceleration time history data has been transformed into the Fourier spectrum by converting into the Fourier transformation. So you can do with the two kind of Fourier transformation, so one is that discrete Fourier transformation and then the first Fourier transformation where that acceleration time history data will be, transformed by doing this Fourier transformation calculation. So you get here, Fourier amplitude spectrum and power spectrum. So we have been seeing that.

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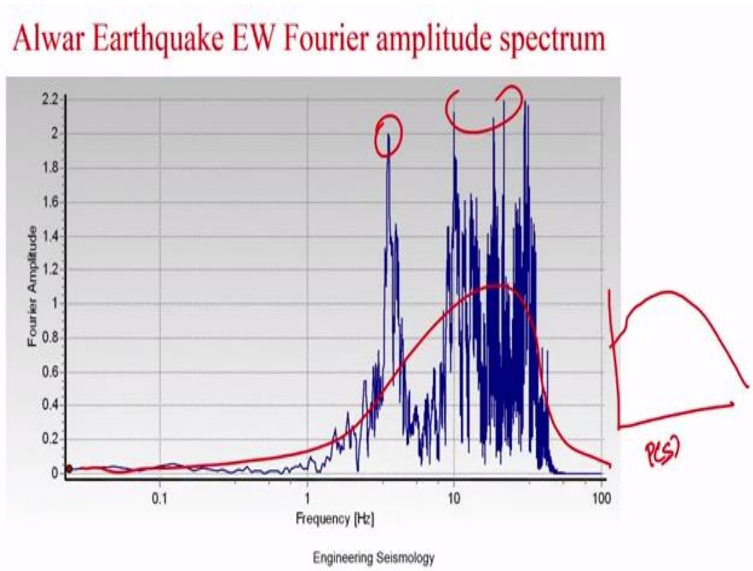

The Fourier spectrum; how it is for the typical data which we have discussed even the time. So this was the same Alwar earthquake east west Fourier component. So you can look at basically the amplitude of the, this data, you can see the amplitude of the data and also if you get a some kind of shape you can see like this. So if you convert this into period, so you can see what we have discussed in the response spectrum.

So response spectrum, we have seen that this goes like this and like this. You can see this is actually the inverse shape. So here you are using the natural period. So here the frequency, this is a basically the inverse of the same.

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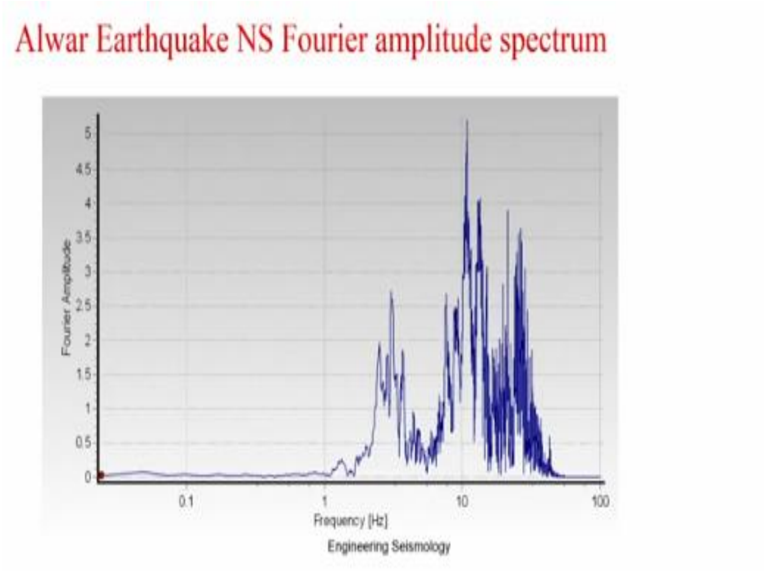

So this is similarly at the Alwar earthquake component of the north south Fourier amplitude spectrum. So here also you can observe a similar trend, so you can observe a similar trend, but here you can see that amplitudes. So here basically the Fourier amplitude is 2.5 the highest one will be the 5 the previous one if you see the 2.2 is highest one and then the 2 is the lowest one, So that scale you can clearly see how it is different.

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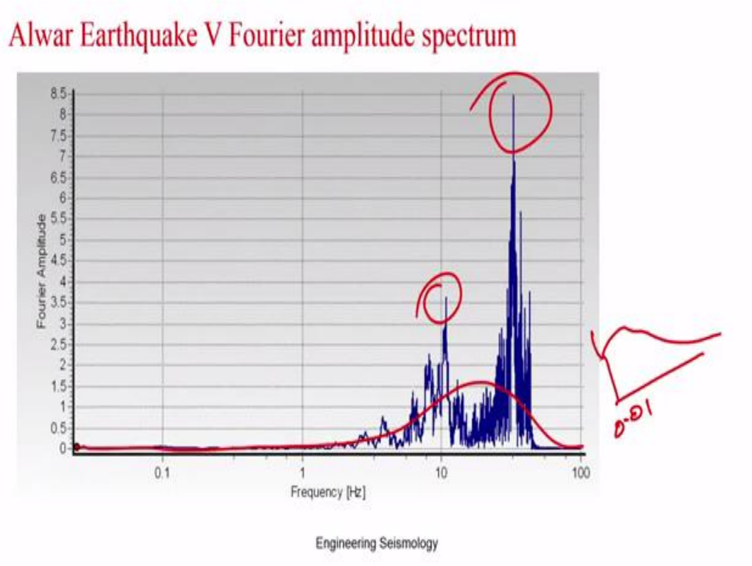

So this is the vertical component; you can see the vertical component here as well as here and the similar kind of shape you can also get here. So this is how you get. So when you plot this again as I told you that so 1 by 100 is 0.01 or 0 spectral isolation where you get like this. So this is a response spectrum.

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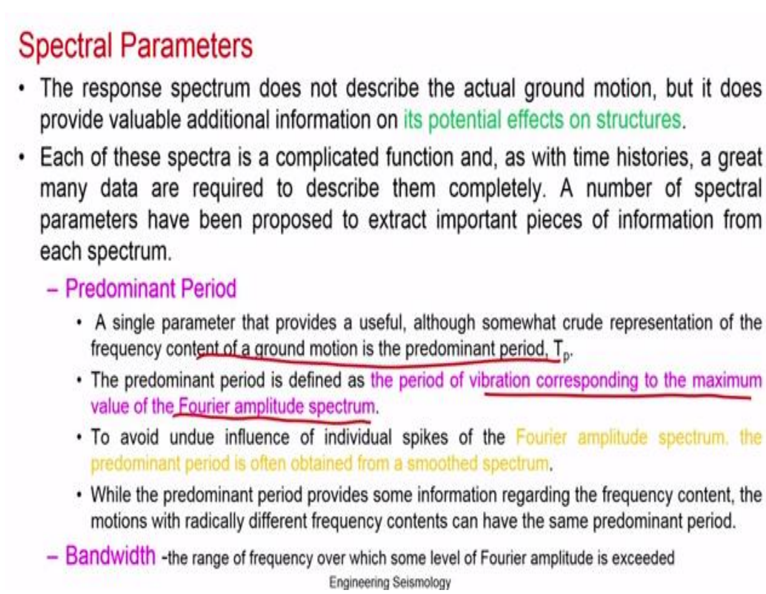

So this Fourier spectrum has a different application different spectral parameters, you can derive from this Fourier spectrum the response spectrum does not describe the actual ground motion, but it does provide a valuable additional information of the potential effect of the structure for the given input. So each this spectrum is complicated function and with the time history the rate many are the data required to describe then completely.

So, number of spectral parameters have been proposed to extract important piece of information for the each spectrum is a predominant frequency. See a predominant frequency is basically a single parameter that provide useful although somewhat crude representation of the frequency content of the ground motion in the predominant period. So there is the frequency content. So the predominant period is defined as the period of vibration corresponding to the maximum value of Fourier amplitude spectrum.

So if we get the Fourier amplitude spectrum, the maximum value corresponding period is called as a Fourier, so the predominant period. To avoid undue influence of the individual spikes of the Fourier spectrum, the predominant period is often obtained by smoothed spectrum. While the predominant period provide some information recording the frequency content the motion readily different frequency content that can have the predominant period.

The bandwidth the range of frequency over which the some level of Fourier amplitude is exceed. So, we are now discussing about the predominant period so what is the predominant period? If we take a Fourier amplitude spectrum if you smooth that, then a particular frequency where the peak is obtained or having the maximum value of the Fourier amplitude then that period is called as a predominant period or predominant frequency of the system, so what you should do for that actually to get the.

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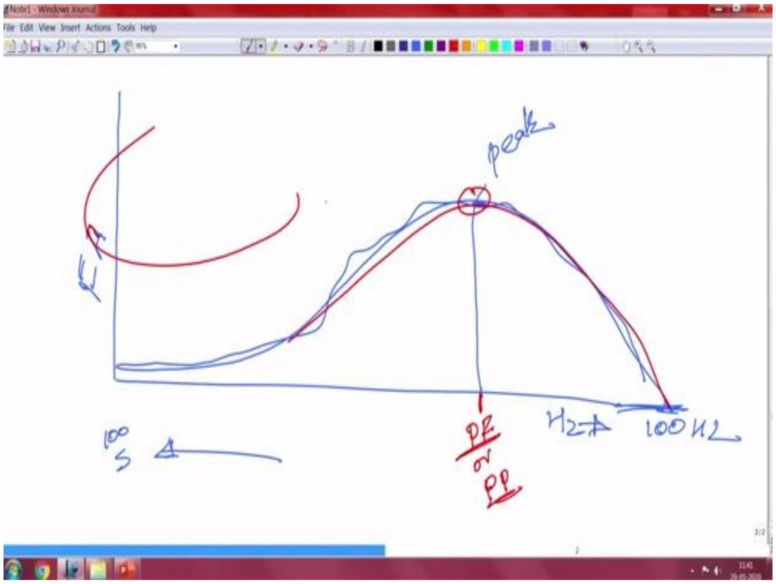

So you basically you can get a Fourier spectrum, so this will be we have seen that this is varies like this, like this, like this, like this. So you can see this is basically your 100 hertz, so this will be the 100 seconds, so this will be the frequency hertz. So this will be the period. So at one point if you make the smooth of this basically is the, where you are attaining the peak. So this is basically the Fourier amplitude.

So this is the Fourier amplitude. So this smoothening, of this data further this point is P. So this corresponding frequency is called as a predominant frequency or predominant period of the system, so this is what is a described in the predominant period which is basically the peak value of the Fourier amplitude spectrum smoothed plot where you get a peak response. So this, for the frequency of the; so single degree freedom.

So, you will get each earthquake will have its own predominant frequency of the system, So here when we are talking about the earthquake, so there is a each structure or system also as it own natural frequency or natural period. So that part we will be discussing now.

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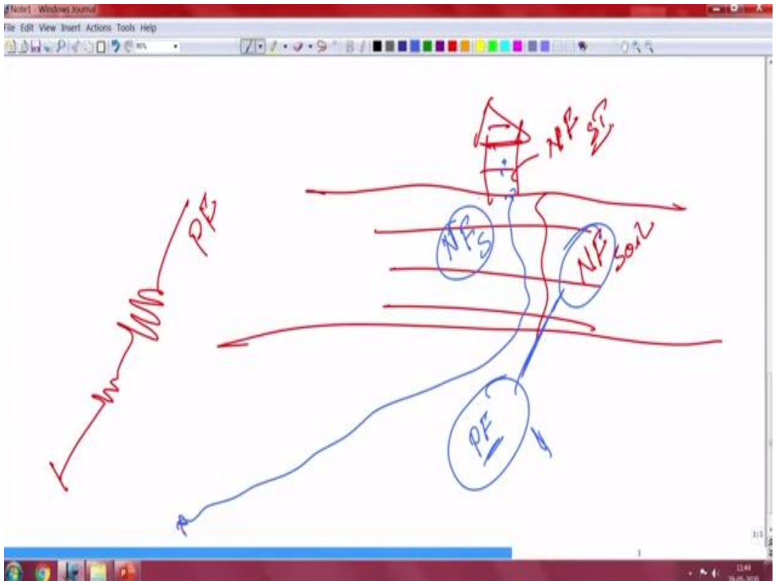

So we get a for example, whatever acceleration time history you are getting. So this will have a predominant frequency. So when you are having in the ground your structure standing like this. So this will have its own natural frequency and then you have the soil. So this will have its own natural frequency, this is the soil. So, this is the structure, so then the earthquake whatever it comes from the distance.

So whatever comes from the distance will travel like this and affect goes like this. So here you can see that if this natural predominant frequency of the. So this you see that you have the predominant frequency of the input motion and then natural frequency of soil. So in some of the cases this frequency and this frequency will be coinciding. If that coincide you will get a maximum response of the ground vibration at this particular place.

So all this combined frequency will reach a structure it has a frequency close to the structure then this will have the maximum response and failure. So when we are designing we should see that the natural frequency of the soil column does not go closer to the natural frequency of the input motion expected in the region and even the combined frequency of these two should not be matching with the, your structural frequency.

So you as we know we do not know that what nature of frequency the input may come but it is easy to find out what is the natural frequency of the soil, natural frequency of the structure and we see that this 2 are not matching we keep far away. So when you keep far away basically, even if this frequency modifies because of the input motion, you will not get so much effect to the structure.

So that is what in the design they will take care when they analyze the this kind of Fourier data and analysis.

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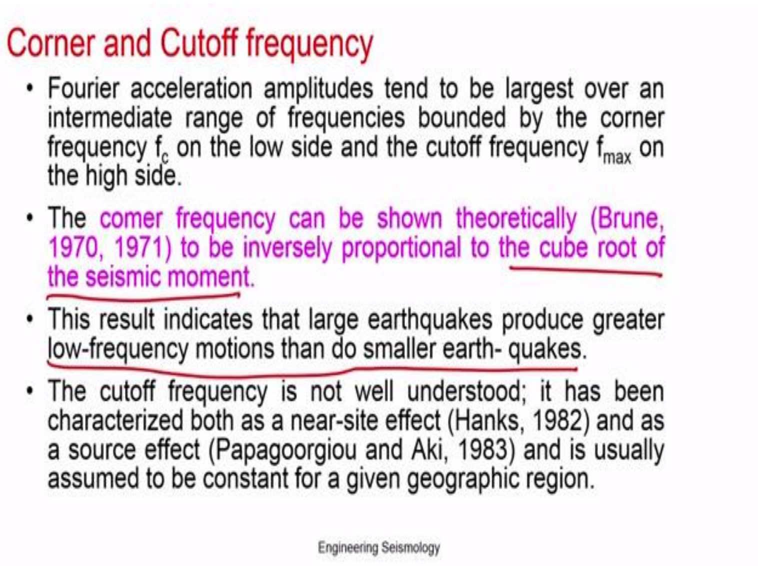

So the Fourier data will help you basically to get all those things. So the Fourier spectrum further can be used to get a, so other parameters which is useful for the seismology part. So the Fourier acceleration amplitude tend to be largest over a intermediate range of frequency. So bound for a corner frequency fc on the low side and the cutoff frequency Fmax at high side the corner frequency can be shown theoretically the Brune 1970, 1971 he designed that to be inversely proportional to the cube root of seismic moment.

So the result indicated that large earthquakes produces a greater low frequency motions than the smaller earthquakes. So the cutoff frequency is not well understood it is been characterized that both as a near-site effect and as well as the source effect and it usually assumed it to be constant for a given geography region, which is the cutoff frequency.

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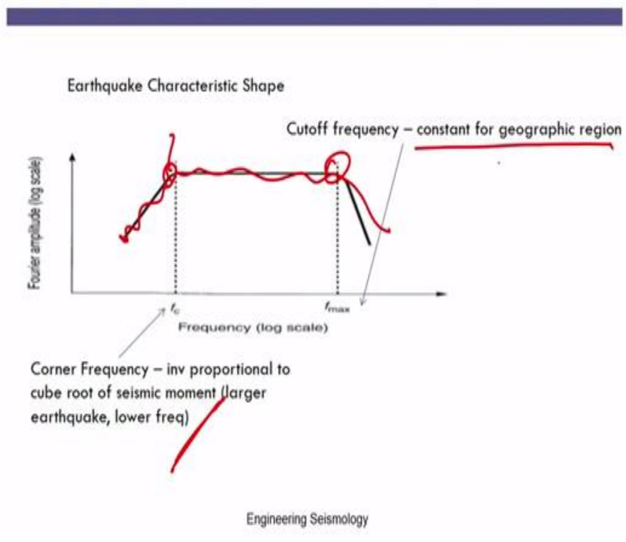

So basically when you plot a Fourier amplitude log scale and the frequency in the log scale you get typically the plot like this. So after you smooth you can find that your data's are like this after smoothing you can get this point and this point. So, these point is basically your corner frequency. So it is the basically a proportional to the cube root of the seismic movement, larger earthquake and low frequency component.

So this is very difficult to define but however, it is constant to the geographical region. So this will be the earthquake based one this will be the region based one. So, this corner cutoff frequencies are used in the simulation part and the synthetic ground motion in the seismology, this one.

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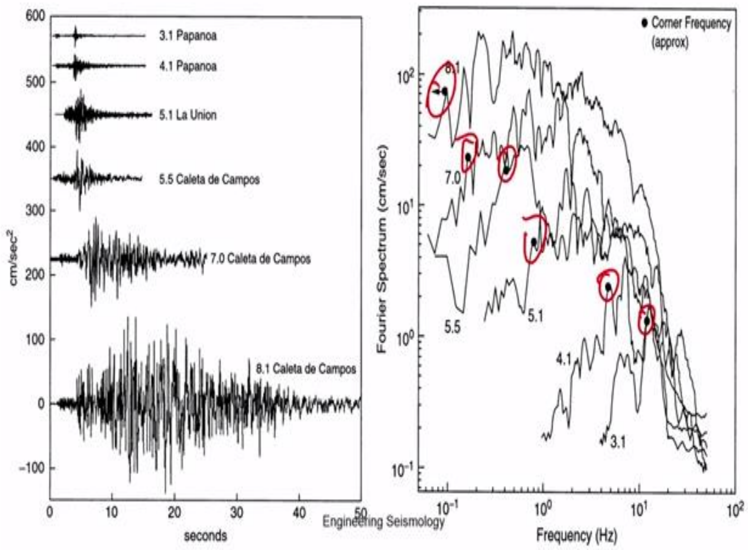

So you can see this is the typical earthquake, so recorded from the different part and you can see how the your corner frequencies are changes. So it is a function of basically your seismic movement, so the Fourier spectrum and the frequency where you get your frequencies are keep basically reducing with increasing in the magnitude. So this is the typical acceleration history of the data which is used to get this Fourier spectrum values.

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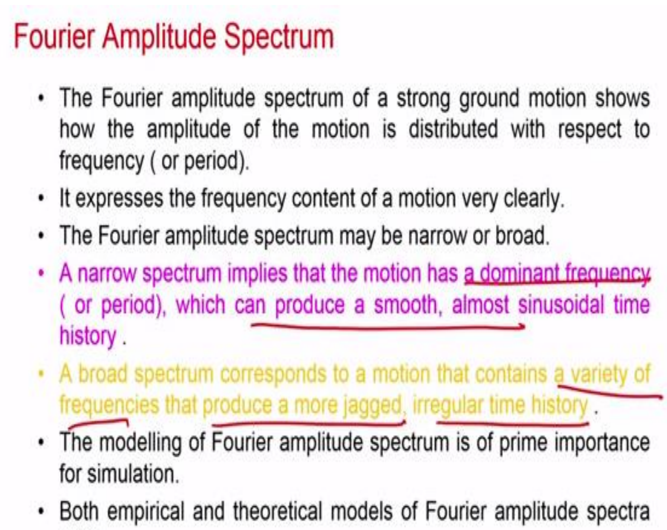

So the Fourier amplitude spectrum the Fourier amplitude spectrum strong ground motion source how the amplitude of the motion is distributed over here, respect to the range of frequency and period is expressed as a frequency content of a motion, very clearly. Fourier amplitude spectrum may be narrow or broader. So the narrow spectrum implies that the motion as a dominant frequency or short period which can produce a smooth and almost sinusoidal time history.

So a narrow spectrum can be expected for the where you can you can see that the produce smooth and the sinusoidal time history. So implies the motion as a dominant frequency and the period, the broad spectrum corresponding to the motion contain a variety of frequencies not particular frequency and produce more jagged and irregular time history. So the modeling of Fourier amplitude spectrum is of prime importance for the simulation of ground motion data.

Both empirical and theoretical models of Fourier amplitude spectra are exist, that can be used for the modeling your, so simulations of the ground model.

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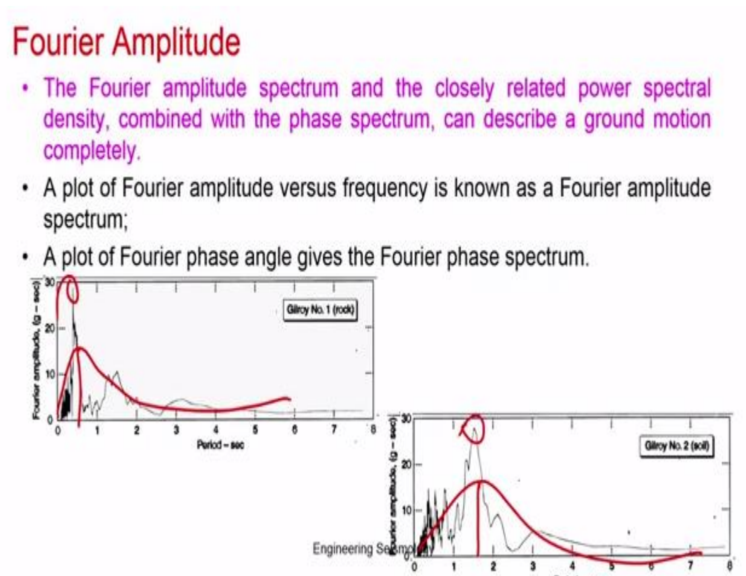

So, this is the typical Fourier amplitude spectrum, Fourier amplitude spectrum and the closely related to the power spectral density combined with the phase spectrum or can be described the ground motion completely. The plot of Fourier amplitude spectrum is frequency known as a Fourier amplitude spectrum and the plot of Fourier phase angle gives a Fourier power spectrum. So the plot are the Fourier phase angle and Fourier amplitude with respect to the frequency will give you the Fourier amplitude spectrum.

And Fourier phase spectrum definitions, so you can see here this is actually the typical ground motion, recorded at a rock and the soil side, you can see when you take a Fourier spectrum you can see where it reaches here peak. So here also you can see where it is a peak. So you basically find out the you can see here your period, how it varies even you can also verify your amplitude.

You can see how this values are changes, so it is influenced by the medium which travels as well as to the your input Fourier content of the input, peak values of the input.

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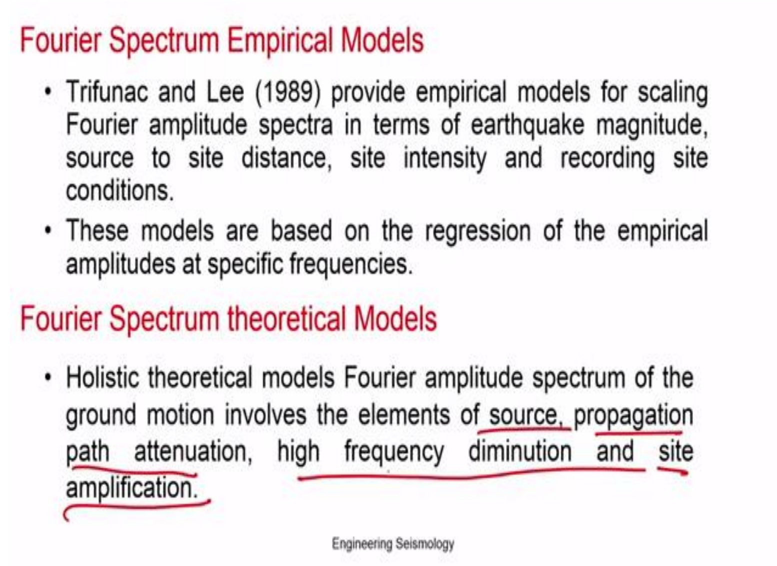

So these are the, so Fourier spectrum empirical models, so as you said that the Fourier spectrum has a empirical model to derive this so those things are like the theoretical and say empirical model, so we will discuss those things in short form, so detailed discussion if you want to basically if you are interested to continue your career as a seismologist. So then you need to study all those things.

So as per this class concern will give only a small definition and introduction what is the Fourier spectrum empirical model and the theoretical model. So Trifunac and Lee 1989, provided empirical models for scaling Fourier amplitude spectrum in terms of earthquake magnitude source to site distance and site intensity and recording site condition, so this models are based on the regression of the empirical amplitude of the Fourier spectrum.

So the empirical model basically they got lot of Fourier amplitude spectrum and then each frequency they taken a component and try to give a model, empirical model to fitted with the data that is a Fourier spectrum empirical model. So that model you can use to get a Fourier spectrum in your region if the seismicity and the seismo tectonics are similar if the developed data.

So another well known model what the use is actually Fourier spectrum theoretical model, so holistically theoretical model of warrior amplitude spectrum of the ground motion involves the element of the source propagation path attenuation, and high frequency domination on the side amplification, so these are all the factors which basically a theoretical model controls.

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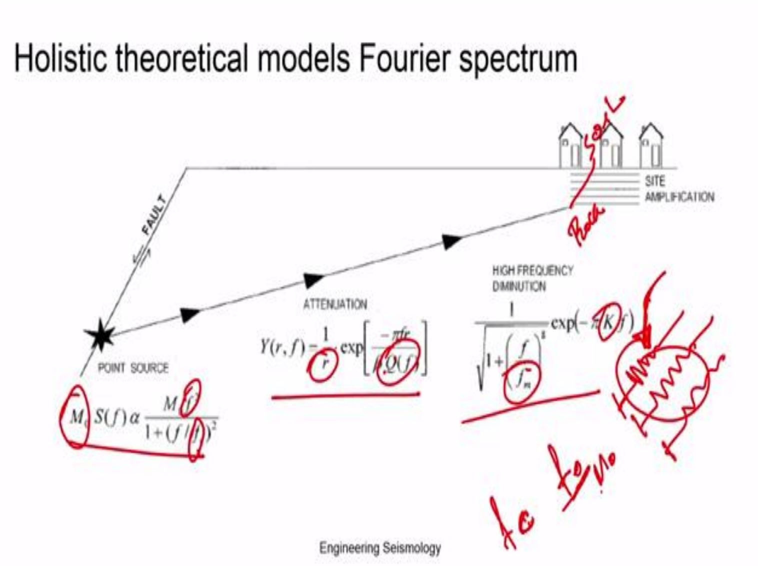

So the basically a theoretical model of Fourier amplitude spectrum combines your source, path and the so the attenuation characteristic and amplification and so this will be like you will have the rock here, this is the soil. So this site condition when you want to get taken to soil you need to get the site condition otherwise up to rock this derived parameters basically solves the theoretic, so here you can see that the points shows theoretical model expressive that the seismic moment.

So then the frequency content at which you are looking the corner frequency, f is the corner so attenuation is expressed as the function of your frequency and then you are the travel path, the r, so that is even expressed as a quality factor Q. So again, the attenuation characteristics of the path expressed the frequency content of the motion and then the particular frequency what you are look for and then the kappa factor those kind of things are used.

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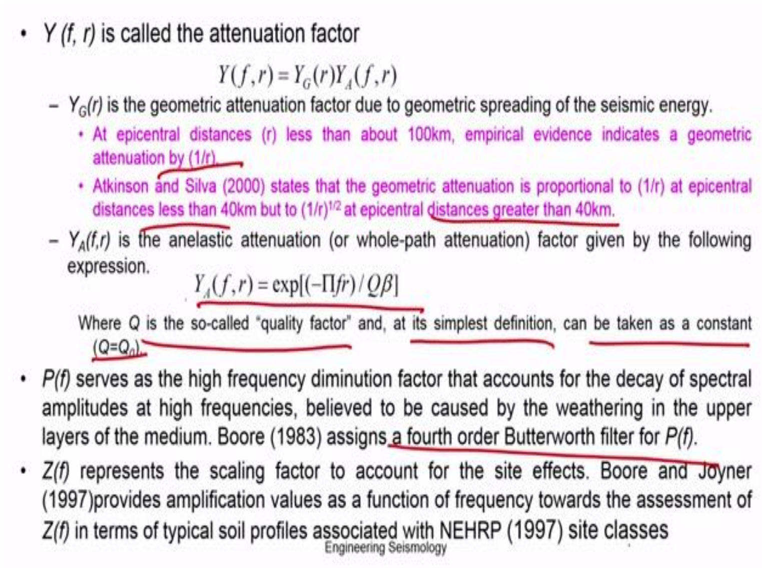

So this parameter knowing this basically you can try to get here spectrum for a given site. So the y is, f and r is called as attenuation factor, which is the function of the y, g, r is actually geometric attenuation factor due to the geometric spreading of the seismic waves, at the epicentral distance r less than 100 kilometer empirical evidence indicates that geometric attenuation is the function of 1 by r.

So that Atkinston and Silva state that geometric attenuation proportional to the 1 by r at epicentral distance less than 40 kilometer but to the 1 by r square root to the empirical distance greater than 40. So this basically depends upon the region to region. So Atkinston and Silva given for the western US. So it is not constant throughout that. So recently we have been derived this kind of factors for our Indian Himalayan data.

You can refer our papers publishing the soil dynamics and the earthquake in journal; we used data and try to derive all the attenuation characteristics occurring at a Himalayan region that was the first unique work what we have done. So we have been keep working on this kind of areas.

So where you get more information, but here only the concept is sufficient for the course point of view.

Because if you want to generate this synthetic ground motion simulation, you have to go in detail to these kind of things. So the YA, f and r are anelastic attenuation the world path attenuation factor given by the following expression you can see. So here basically so the Q is the so called quality factor it is a simplest definition can be taken from the constant Q equal to Q0. So this quality factor also one can derive from the recorded earthquake data.

So you have to estimate recorded earthquake data, which will be unique for a particular seismo tectonic region. So the Pf serve as a high frequency dominant vector accounts a delay decay of spectral amplitude at high frequency, believed to caused by the weathering of the upper layer of the medium Boore 1983 assigned fourth order of the Butterworth filter to find out a Pf. Zf expressed is scaling factor to account site effect Boore and Joyner provides amplification values of the functional frequency towards assessment of the Zf.

And typical soil parameter as associated with the NHRP classifications. So this is a way a theoretical model is defined. So here you should understand that, so if you have the recorded data, so you can find out your fc is a cutoff frequency and f0 is the corner frequency. Then so from the data you can also find out your seismic movement or if you have the corner frequency, your seismic movement is actually the cube root of the corner of inverse of the cube root of the corner frequency.

So by knowing this and you can estimate here, your source point Fourier spectrum. Similarly the attenuation characteristics where the quality factor again you need a recorded data. So again the I frequency domination factor K kappa factor that is also is the function of data. This parameters generally if you have the large number of recorded earthquake in the region. So you can get all those data and try to process and get the this kind of factor estimated.

So as per your so the class concern you should know what is the this happens in the this entire Fourier theoretical spectrum, that is sufficient you do not need to mug up any of this equation the definition of corner frequency cutoff frequency what is mean the quality vector Q is our important. So as per India concern very less researcher working on estimating this kind of factors using Indian data.

So first of all, the data available in Indian regions are very limited. So even among the limited data. So this kind of analysis are very less carried out. So most of the people who you use synthetic ground motion simulation are used this kind of factors in their analysis, they simply take factors derived from the elsewhere and use it here as a input parameter for that. So the first time actually we have did a very regressive analysis of the data collected from the Himalayan region.

And then try to estimate all these factors for the Himalayan data and which we also published and used to generate a synthetic ground motion and then that ground motion we used to get here predictive equation for the ground motions, at different frequency or period level. So the simulation of the synthetic ground motion and the models which used for the simulation of synthetic growth motion would be discussed in the next class.

So today class what we have seen basically we have seen that the spectrum, whatever data you are obtaining. So from the acceleration time history, you can transfer that to the Fourier spectrum so by form of Fourier amplitude versus the natural frequency, or frequency content of the your ground motion. You can see that, this Fourier amplitudes are weak at a particular frequency. So that is called as a predominant frequency or predominant frequency.

Predominant period or predominant frequency of the given input motion that data you can get from this analysis. So then when you plot a Fourier spectrum you can also notice that it follows a inclined portion, the horizontal portion and vertical portions. So that data is basically will give you the idea corner and a cutoff frequency the corner is termed as fc and cut off frequency is called the same fc and corner is named as F0. So this corner frequency is the function of seismic moment at your source.

So that means if you know the corner frequency of the particular place, you can try to get what is the seismic movement? So from there you get here Fourier amplitude for a given frequency. So if by generating ladder Fourier amplitude, you can also see that how this amplitude to the varies with the time and then the quality factor and then the kappa factor which is the function of basically your regional seismic data, which you need to be estimated.

And above we also see that the attenuation character, 1 by root R or root r. So which is basically is the function of even regional level data. So we have observed that it has been reported in US where you have to like less than 100 kilometer of 1 by root r and but that it has been proved that it is not constant. So up to 40 meter only with 1 by root r other place, it is different. So that is a derivation.

So this kind of parameters are individually region wise it keep varying. So you have to estimate those parameter and try to get a Fourier theoretical model which will be useful for simulation of the ground motion at a site. So basically this parameters Fourier amplitude spectrum, Fourier analysis and Fourier parameters of corner frequency, cutoff frequency and the predominant frequency.

All those data are useful basically to get here simulations of the ground motions which will be discussing in the next class, so with this we close this class. So we will be discussing the source model simple source model and other things in the next class. So far whatever we have seen that so we try to understand the earthquake, what is the earthquake hazard? What are the hazard it is caused?

So how this, you can measure the earthquake measured earthquake how you can interpret? How to identify the earthquake. So with this actually the complete seismology part with respect to data is over. So here after what we are going to discuss is how this data will be used for the getting a application site. How this parameters whatever the frequency domain parameter and time domain parameter will be used to get a earthquake simulations or earthquake hazard estimations or structural analysis point of view.

So we will be basically studying how this data's are to be converted whatever magnitude intensity so the duration peak amplitude, so then the Fourier amplitude, so all this data should be how useful for getting the simple model parameters and simulations and useful to arrive a design parameters for the practical design application, that will be discussed in the coming classes. So with this we will close here today class, thank you very much for watching this video.

So we will see you in the next class. So this mathematical form explained in this class basically is a slightly more advanced stage but you should know that, that is what because since the this course also offered to the B.tech students or Civil or Any other branch of the Civil were interested, so those people will be willing to proceed to their career on the engineering seismology part and simulation part. As I also tell you the research gap.

Though somebody want to work on seismological part of Indian data and all, so for them this is the introductory part it will make them to study further in the master level and PhD level. So most of my students basically, they do research on this area, so we have been working to develop a synthetic ground motion since my PhD, so we are done a synthetic ground motion generation for the Bangalore region which is Peninsular India.

We have done for the Lucknow region we have done for entire Himalayan region as well as the Peninsular Indian region where large amount of the synthetic ground motion data has been used in the absence of the real earthquake record that has been further used to derive ground motion prediction equation and then understand how the spectral acceleration changes with the different period of the structure.

Those are all the research are linked with that so this mathematical form, you know need to worry only you need to know the definitions so how it varies and all those things particularly the wordings which is given in the different color underlined portions will be important with the exam point of view which you have to pay attention, so remaining is formulas under you no need to mugg up anything there is no formula will be ask in the exam point of view.

Only the simple and the straight forward formulas which you can remember will be asked in the exam point of view, so with this we close the mostly the seismology part of the course; understanding earthquake, measuring the earthquake and interpreting the earthquake recorded data, how to quantify earthquakes, scaling the earthquake all those things what we are completed as on today class.

So the next class onwards we will be using all the studied whatever information we are studied will do by the application of the different part, one is the prediction of the earthquake hazard another one is a simulation of ground motion, these two major application will be looking at in your course, so for that application, whatever the background knowledge we have learned will be helpful to you.

I request you to closely monitor the classes whenever you have doubt on particular things you can go back to the video and check in case even if it is not clear sometime you can all talk to TA or approach me so that we will be interacting and try to understand better, so many of you are mathematical background maybe may not be that level advanced as the master level and PhD level it does not matter only you should know how these things are working.

I always believe that all whatever theoretical models like we have wave propagation theory and this automation theoretical models and all with a lot of assumption which practically does not match with the practical condition only to help you to understand how it works, that understanding you are making it is okay, no need to make a derivation and all. Derivations you can find out in the standard textbooks or even you can find out in the website.

These days everything is available with the mat lab coding but you should know fundamentally how it was, that is what very important not the mugging up the equation. So with this remark of all our so far classes so we will close today class. So I request you to go through all the material part and so that next application side will be very clear. So thank you very much for watching this video, I will look forward meeting you in the next class, thank you very much.