

Introduction to Engineering Seismology
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Lecture No -23
Frequency Domain Characteristics; Response Spectrum

So Vanakkam. So we will continue to lecture on engineering seismology. So, last class we have been discussing about the interpretation of the earthquake record. So particularly, so how we can get a time domain parameters. We have started like noting down the peak values, which is basically each component of the record will have the acceleration, velocity and displacement. You can get integration and differentiation, look at one recorded data, the other two are basically derivative by integration or differentiation.

So, we have seen that basically each seismic station will have the 3 components of the record and each component of the record can be converted into 3 forms such as acceleration time history, velocity time history, displacement time history. So in each record, we will have the acceleration peak values which is called as a peak ground acceleration or peak horizontal acceleration. Similarly the velocity forms the peak values or peak horizontal velocity or peak ground velocity.

Similarly the displacement, the peak horizontal displacement or peak ground displacement values. So these values we have seen that the accelerations are very sensitive to the high frequency range, velocities are intermediate frequency range, displacement are the low frequency range. In general, the accelerations are predominantly used for the design of structures since it is related to the inertial force calculation.

So the velocities are used for the tall structures and the intermediate frequency range of structures is being used, displacement has actually a lot of issues with the noise and is very difficult to get a proper displacement plot. But now because of the digitalization it is maybe possible, but generally the displacement based approach structural design they use a displacement time history. We have also seen that this peak value is sometimes; very high when

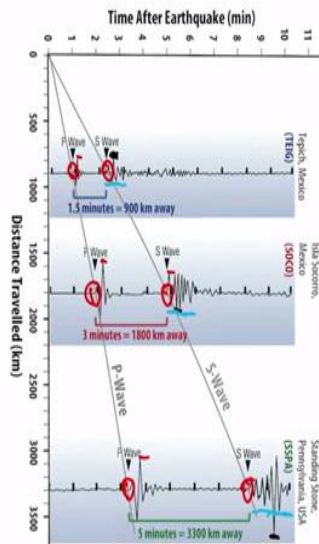
compared to the other amplitude in the same record or sometimes slightly higher than the other record.

In that case, considering one peak may not be sufficient, so then there is an effective acceleration, those kind of terms also we have seen. Then we have seen that this amplitude, how many times it repeats, depends on the damage of the structure is defined particularly perpetration generation, landslide liquefaction related phenomena. So the scientists start using basically the duration of the record, that is like the strength of the record how much it is basically, amplitude is repeating for a given period.

So we have been start seeing at the effective duration, significant duration, things like that, which is basically relate a amplitude and the energy released by the earthquake, how that energy, how much time duration it released or it impacts the structure, based on that, we have seen. We also discussed that the arrival time picking up for the P and S wave will help you to identify basically your epicentral location of the earthquake.

If you have the 3 stations and if you have three and more stations you can also get the hypo central or focus point; the depth of the earthquake in that. Today we will be starting again with the epicentral kind of things, for the given earthquake if it is recorded at three places at least need to precisely identify an earthquake, how these three play stations are your P and S wave changes with the distance, that is what you are going to see.

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Using P-wave and S-wave travel times to determine how far seismic waves have travelled. Grey curves show the distance travelled by P-waves and S-waves after an earthquake occurs. P-waves are faster than S-waves, and the gap between them increases with time and distance.

The delays between P-wave and S-wave arrivals on seismograms are matched to the curve to find the distances of seismic stations from the source of the seismic waves.

Source: Karla Panchuk (2018) CC BY-NC-SA 4.0 modified after IRIS (n.d.) "How Are Earthquakes Located?"

<https://openpress.usask.ca/physicalgeology/chapter/12-2-measuring-earthquakes/>

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So, if you look at this picture very carefully, you can see that this is basically the earthquake record recorded at a different place. This is basically a distance in the kilometer, you can see this is the distance in the kilometer. So the distance in kilometers, this is basically the origin of the earthquake. This is actually the time after the earthquake in minutes. You can see that this is like TEIG; it is one of the stations in Mexico, the other one is SOCO; the other station, another is SSPA; is another station.

So you can see that this station records a P wave arrival, since this station is very close, roughly about 800 kilometer from this one. So the first P wave then the S wave arrival is there, you can also see the amplitude and then the next station is close to 1800 kilometer from the epicenter, you can see the amplitude. Then the S wave arrival time, then again the other station was around close to 3300 kilometers away, you can see them again S wave and the P wave arrival time.

You can see that, this S wave arrival time peak and then the arrival time of this point is basically this portion, this portion, this portion, this portion, this portion and this portion will give you the arrival time. So you get the amplitude from here, you can see. The P completed, I mean the peak marking actually. So, if you look at this very carefully; this is the peak point, basically this will be the peak point here, this will be the peak point.

If you look at that with the distance, the arrival time keeps increasing, that is one information you can get. Second; the amplitude, the P and S wave generally the P wave we do not consider as an amplitude required for the peak ground and velocity or displacement part, only the S wave part we consider generally the P wave will be lower than the S wave amplitude. So you can see here this amplitude, this amplitude, this amplitude you can see these values of the amplitude also keep changing.

Second; you can also look at the component of the, how much time the S waves are repeating, that also you can see. You can see here basically this is repeated up to this, here you can see it is repeated slightly larger, here you can see it is repeated up to this. So, your duration, then the amplitude keeps increasing with the distance so, that is one observation we can make from here and then the gap between the P and S wave arrival time basically, this here 1.5 minutes, 3 minutes and 5 minutes.

This time also keeps increasing, these details are useful basically to show where exactly the origin of the earthquake basically located. As we have known that the P waves are faster waves which arrive at first part and the second, the S wave which arrives later part, the S and P are time determine, how far the seismic wave has traveled from the seismic station. The delay between the P and the S wave arrivals on seismograms are matched to the curve to find the distance of seismic stations from the source of the seismic waves.

Basically this particular plot has been taken from the website given here basically so this is the website this been has been taking. You can also verify that this website details more information kind of things.

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Locating Earthquake using Wave form



Locating earthquakes by drawing three circles with radii of lengths determined from P-wave and S-wave travel times.

Station names (SOCO, TEIG, SSPA) correspond to seismograms in Figure 12.11. Source: IRIS (n.d.) "How Are Earthquakes Located?"

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PWA
V
A

So you can see the plot of this distance, basically from the origin 900 kilometers and the another one was 1800 and 3300, these are all the locations of your stations. By plotting all this curve basically you have located the place where this meets like last we have discussed last class, where it meets basically. This part basically is your epicenter location or the earthquake location. This is the way we will locate a earthquake, last class I drawn in the free hand and explained that so this class I shown you how the waveform is differ.

So, if you have to find the epicenter, you need a three record and you have to pick up P wave and arrival very clearly from that, like here what we are given, then the time travel will be taken and then you can know. In the physical cases if you know basically here you are using that time of arrival and velocity in the region to estimate a distance. So, in the physically that velocity in the region like P wave velocity and S wave velocity we have shown the range.

Generally what in the region if you have the data recorded from like this, for the known distance then you can actually estimate what is the average P wave velocity you can use, average S wave velocity you can use for this kind of calculation. For example, this station and this station, these stations are far. This in between this, this velocity will be different; this will be your V_p and V_s . Similarly this velocity may be different, it no need to be necessarily the same, this you have to find out.

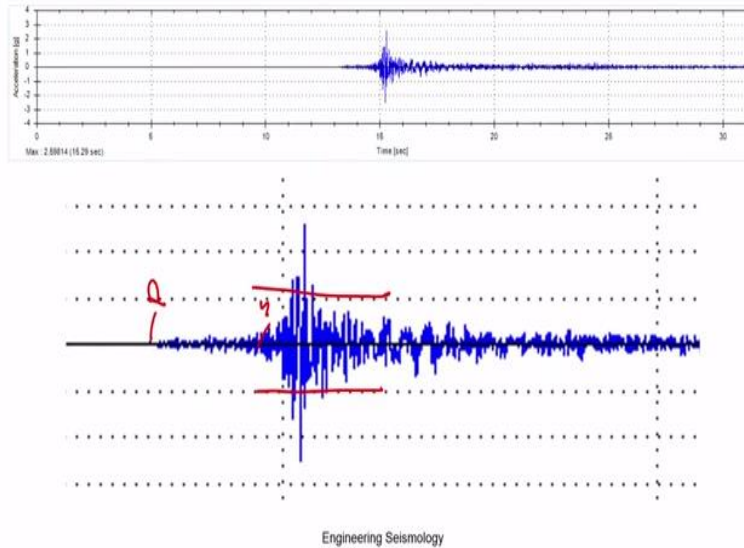
Similarly this velocity will be different as the geological terrain varies, the wave characters are function of the stiffness density of the geological material like P wave is the function of the constraint modulus, S wave is the function of the polar moment of inertia and then basically the shear modulus of the property. So that varies, then this value also varies. So, if it is, you can find out what is the average value by getting a lot of this kind of measurement at a particular place or doing the deep geophysical surveys on those locations.

This particular graph is also available on this website shown here. If you want more information you can refer the same. So, this is the direct wave where we can expect basically the P G A and P G V and P G D and then we have duration which we have seen here, this is the part where duration depends upon the type of duration your threshold value keep changing but this is the data we will be predominantly using to arrive the duration part.

So, this details, I will be help you to identify the infer from the earthquake data, which is useful for the engineering application. So another form of data, so whatever we discussed which is directly the time domain parameters, where you just look at a acceleration time history of the data and you will take that data basically to get a direct observation from the acceleration velocity displacement as well as the duration part.

So, this data basically you can convert in the frequency domain, then you can interpret a parameter which is useful for the simulation as well as a structural analysis. So, that kind of interpretation is a frequency domain, characteristics analysis of the frequency domain analysis. So, we will see what are those parameters and how we can interpret in the, this one. As we have shown that this is the typically one component.

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Where the acceleration and then the time record. You can also see that these records are basically, this is actually the extracted portion of the entire record. You can see that this record basically dominated at this place. So predominantly this data's is basically, this is a P wave, this is somewhere here the S wave will be there, if you zoom in then you can see that the strength of the earthquake lies somewhere here.

These portions are more impact energy to the structure, understanding of this we need to deal in a very properly and systematic manner. That is what we do in the frequency domain analysis and estimate the frequency domain parameters.

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Frequency Domain Characteristics of Strong Ground Motion

- Earthquakes produce complicated loading with components of motion that span broad range of frequencies.
- Only the simplest of analyses are required to show that the dynamic response of compliant objects, be they buildings, bridges, slopes, or soil deposits, is very sensitive to the frequency at which they are loaded.
- The frequency content describes how the amplitude of a ground motion is distributed among different frequencies.
- Since the frequency content of an earthquake motion will strongly influence the effects of that motion, characterization of the motion cannot be complete without consideration of its frequency content.

So the earthquake produces complicated loading with the component of the motion that span broad range of frequencies. Only the simplest analysis are required to show the dynamic response of the complaint objective by the buildings, bridges, slopes, or soil deposits, is very sensitive to the frequency at which they are loaded. Basically the different structures as it wants natural frequency, the earthquake frequency and natural frequencies are very close then you will get a response is very big and it will also get a resonance.

In that case, so depends upon the building what we design, it is natural frequency, you need to identify what frequency range you have the data, you will be interested while using the analysis. In that context only the frequency content describes, how the amplitude of the ground motion is distributed among different frequencies? So earlier, whatever we have seen is basically the amplitude distributed over a time.

But here we are basically looking at how the amplitude of the ground motion distributed among different frequencies depends upon your frequency of the structures and what amplitude you need to consider for the design, you will be taking from this frequency domain characteristics. Since the frequency content of the earthquake motion will strongly influence the effect of that motion, characterization of the motion cannot be complete without considering the its frequency content. Only using the time domain parameters may not be always good.

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- In the frequency domain:
 - Fourier amplitude ✓
 - Phase spectrum, ✓
 - Power spectrum and ✓
 - Several definitions of response spectra are used in the quantification of strong ground motion.
 - Five types of response spectra are defined:
 - Relative displacement (S_d), ✓
 - Relative velocity (S_v), ✓
 - Absolute acceleration (S_a), ✓
 - Pseudo-relative velocity (PSV), and ✓
 - Pseudo-relative acceleration (PSA). ✓

To get analysis, you should look at how the amplitude of the frequency content varies. The frequency domain parameters are basically the Fourier amplitude, Phase spectrum, Power spectrum several definitions of the response spectra are used in the quantification of the strong ground motion. 5 types of response spectra used are: relative displacement, relative velocity, absolute acceleration, pseudo velocity and the pseudo-relative acceleration.

These are all the terms and described in the frequency domain. We will be discussing the important terms which you will encounter in engineering applications of this kind of things.

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Response Spectra

- It is used extensively in earthquake engineering practice.
- The response spectrum describes the maximum response of a single- degree-of-freedom (SDOF) system to a particular input motion as a function of the natural frequency (or natural period) and damping ratio of the SDOF system.
- See Figure
 - The frequency contents of the two motions are reflected in the response spectra.
 - Rock motion produced higher spectral accelerations at low periods than did the soil motion and lower spectral accelerations at higher periods.
 - The higher long-period content of the soil motion produced spectral velocities and displacements much higher than those of the rock motion.

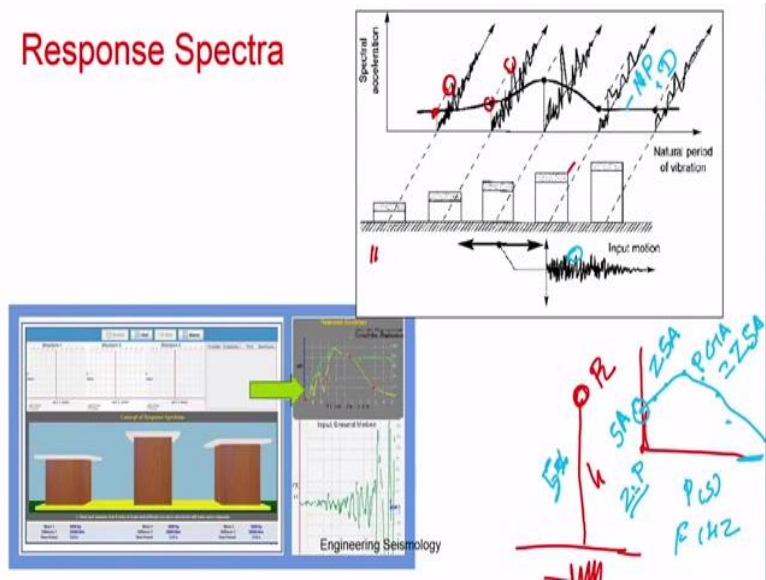
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First we will discuss the response spectrum, the response spectrum is basically a plot of frequency versus a amplitude. There is the spectrum are the frequency also can be plotted as the natural frequency or natural period by making an inverse. This is itself written as a spectrum, the response. It is basically the response of something plotted as a spectrum is called as a response spectrum.

It is used extensively in earthquake engineering practice, the response spectrum describes the maximum response of this single degree of freedom system to a particular input motion as a function of natural frequency like a natural period and damping ratio of the single degree of freedom system. Basically, the spectrum is described as the maximum response of the single

degree of freedom system to a particular input motion as a function of natural frequency or natural period and the damping ratio of the single freedom system.

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So here it is basically clear that it is a response of a single degree freedom system. Basically a single degree freedom system is generally assumed as a single pendulum with the mass, this is a single degree freedom system. The damping and other things you can incorporate, this is the simplest form you can assume. So maximum response of this for a given input by changing its natural period, so the response, so then this height will decide your frequency and you will give a typical input and then you measure a response.

That response only is called a response spectrum. The frequency content of the two motions are reflected in the response spectrum, the rock motion produces higher spectral acceleration at low period than did the soil motion and lower spectral acceleration at higher periods So this is the sum of the observation. The higher long period content of the soil motion produced spectral velocities and displacements much higher than those of the rock motions.

What are the cases, how these amplifications are different is actually described in the statement actually. If you look at that as I told you that you take a single degree freedom structure and take some input motion, then you start vibrating. You will get basically the response of this which varies with the time based on this, so the response first you give, it will respond, respond then

you keep on responding then you can see that there is a point where you get a maximum response, that point is noted down.

Now you know, what is your period of this structure which you are estimated for, that period for this given motion what is the response? Similarly now you change a period of the structure by varying a height, then again the same input you measure a response, that response is this one which is plotted. Similarly you will keep on altering that where the period is changes from, for example, 1 second natural period or natural frequency whatever you want, so 1 second, 2 second, 3 second like that or 1.5, 2.5, like that.

You keep changing your height to vary the period and the same input you try to observe a maximum response. That response basically plotted as a the time, this time is actually the period. This period is actually your natural period of the structure, then you show in the zero period, you will get some acceleration. Zero period you will get some response, then the 1, 2, 3, 4, 5 like this, this connecting of this is basically a response spectrum.

This will be the spectral acceleration because you get a response, this is the response spectral acceleration or response spectral velocity, this will be your period basically, this is the natural period of the system or structure where you will be using the, this is done for the single freedom system so that the multi freedom system you can take it as a multiplication of the level. This is basically a period, this is the period in second or frequency in hertz. Then here you can observe that, this is actually a zero period, the period here equal to 0, correct?

Then this keeps changing up to where the response plot and kind of things, this acceleration is called zero spectral acceleration. If you look at this acceleration this is a maximum response, that means if the zero the period the structure basically zero natural frequency means 0. Basically your peak acceleration of the input, the peak ground acceleration equal to the zero spectral acceleration, why because at 0 period there is no structure, your input motion is your response.

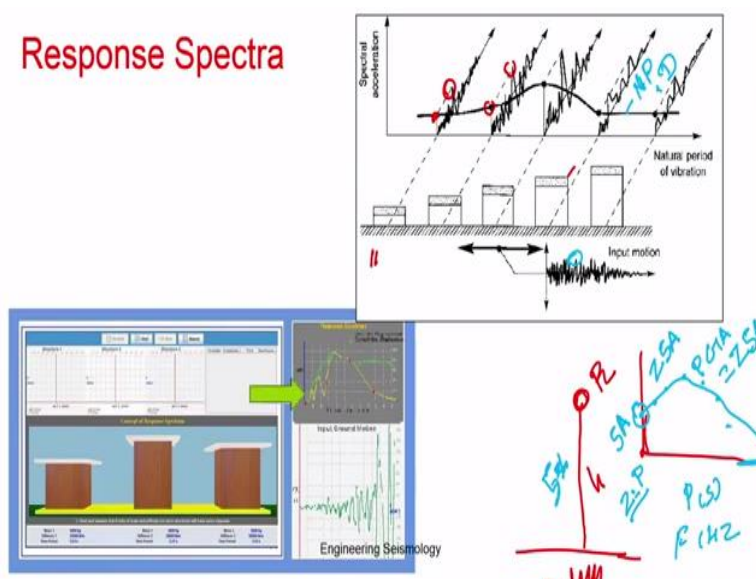
Then when you have a period this response changes, that response only plotted here. This kind of analysis, basically the response spectrum analysis, involves solving a mathematical equation for

the single degree freedom system, this is basically you can see how the response you will get like this. Since this course I told you that it is also taken by them plus two level or degree course people with the geology or geophysics.

We are not going very detailed in the response spectrum analysis, generally this response spectrum analysis you taught in the structural dynamics, people who study about the structural dynamics in B.Tech or M.Tech courses, they know about what is the response spectrum. So that like single degree freedoms structure response for the given input at a different natural frequency and with the node damping level. Generally the 5% damping is generally used, if you have a higher damping level you can also do the different damping level and get a curve.

This curve is basically a function of your natural period and damping. So the single degree freedom system, then when you are going for the multi degrees, this response can be extrapolator and modified for the design, this kind of spectrum given in the code, the response spectrum is used to get from the several input motion and try to accumulate that and then try to process in the smoothed manner and get finally the design spectrum or normalized spectrum, that is given in the code for the design of structures. That basis for the design of structures is what you are getting basically a response spectrum.

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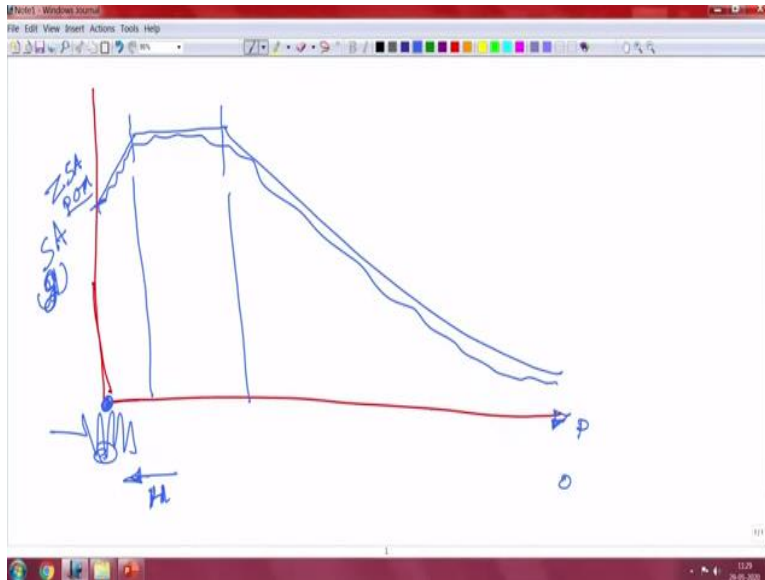
So the response spectrum may be plotted individually to arithmetic scale and may be combined in a tripartite plot. The tripartite plot displays spectral velocity on the vertical axis and natural frequency on the horizontal axis and acceleration and displacement in the inclined axes. This response spectrum individually you can take and then you can combine with the several response spectrum and try to plot in the tripartite plot.

And then you will understand which part of the component comes to the velocity part, which component part comes in the displacement part, which component comes in the acceleration part, that we can analyze. So the acceleration and the displacement axes are reversed when the spectral values are plotted against the natural frequency rather than a natural period. The shape of the typical response spectra indicates that the peak spectral acceleration, velocity, and displacement values are associated with different frequencies or periods.

Low frequencies the average spectral displacement is nearly constant; at high frequencies the average spectral accelerations is fairly constant. In between lies a range of linearly constant spectral velocity. Here you can understand that the low frequency range, basically the displacement is nearly constant. The high frequency range where the spectral accelerations are fairly constant.

The intermediate frequency range basically spectral velocities are nearly constant which indicates that basically if you have the plot, this is what I am making as a frequency, the high frequency component, low frequency component and then the intermediate frequency component. I assume that my frequencies are, it is a 0 here and highest here, this frequency increases here.

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I just use this board to explain the response spectrum. This is basically your plot, so we will call the period is increasing this and the frequency is increasing here, the frequency is 0 here and period because it is inverse know. This one will be your spectral acceleration, in the g per centimeters per second, the unit spectral acceleration g per centimeter per second squared. As we told that the displacements are constant at the low frequency range, so the fairly constant is basically your high frequency ranges or the acceleration.

The fairly constant means it will be like this kind of slope, then there is an intermediate frequency, there is a velocity constant, the displacement constant will be in this way. This is how the response spectrum look. So basically what you will get, like this you will get. So if you carefully watch that you can see that it is starting here and going then it is coming like that and ending here, this portion basically the constant for the displacement the remaining is the velocity, this place for the acceleration.

This is basically like this, we have discussed that it is a Z spectral acceleration or PGA when the 0 is your structural period, your input motion will have the same signal that maximum input motion is the GPA. This is basically a response spectrum, this can be also plotted in the tripartite plot, this is for the one ground motion. So we will be taking a large number of ground motions. So try to plot all the ground motion data in the single graph, then by taking the mean value and smoothing then you will get a smoothed tripartite plot.

That tripartite plot will be normalized then you will get a spectrum called design spectrum, which we will discuss in detail of the design spectrum in the later stage of the class. Right now, we are to know about the response spectrum of your this one. Because of this behavior, response spectra are often divided into acceleration-controlled or high frequency range, velocity-controlled or intermediate frequency, and displacement-controlled or low frequency portions. This portion comes in the tripartite plot and design spectrum development portion, you can discuss this in detail.

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Fourier Spectrum

- Since ground motion data is an array of equally spaced time-sample points, *Discrete Fourier Transform (DFT)* is used.
- Since it deals with a finite amount of data, it can be implemented in computers by numerical algorithms.
- Analysis in Frequency domain describes clearly how the amplitude of ground motion is distributed among different frequencies.

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This is about the response spectrum, the other spectrum is actually Fourier spectrum. Since the ground motion data is an array of equally spaced time-sample intervals, the discrete Fourier transform is used. Since it deals with a finite amount of the data, it can be implemented in computers by numerical algorithms. So what you do, the time history data is what you get actually, you convert that into the Fourier transformation.

Fourier transform FFT, fast Fourier transformation or the discrete Fourier transformation DFT you can do. Once you do that basically you get.

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Fourier Spectrum

- A periodic function (for which an earthquake history is an approximation) can be written as

$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \sin(\omega_n t + \phi_n)$$

- Here, n corresponds to n^{th} harmonic.
- A spectrum is a function of frequency.
- The spectrum shows frequency-dependent characteristic of the ground motion.
- The **Fourier amplitude spectrum** is a plot of c_n versus ω_n .
- The **Fourier phase spectrum** is a plot of ϕ_n versus ω_n .

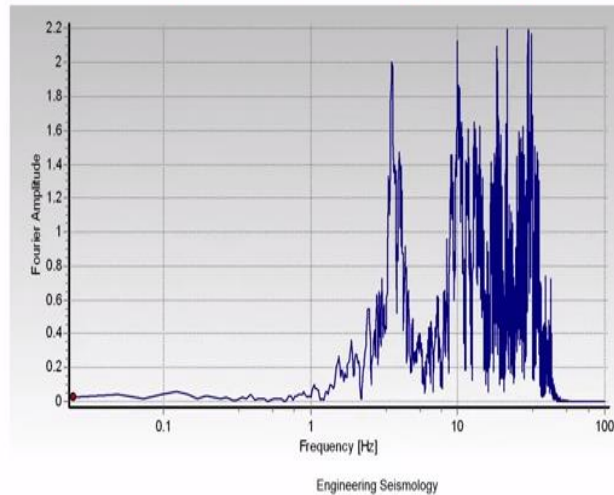
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A spectrum called Fourier spectrum. The Fourier spectrum is basically the conversion of acceleration time history data or velocity time history data to the Fourier spectrum acceleration or Fourier spectrum velocity by doing a discrete finite transformation or fast Fourier transformation you can do or discrete Fourier transformation. Here the periodic function for which an earthquake history is an approximation can be written as this is the one.

When the n is corresponding to the n^{th} harmonic, the spectrum is the function of frequency, the spectrum shows the frequency dependent characteristics of the ground motion. The Fourier amplitude plot is the plot of the c_n versus the natural frequency of the system. Fourier phase spectrum is a ϕ_n is the natural frequency of the system you can see where the angular and the ϕ_n values in this one. This plot will be generated.

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Alwar Earthquake EW Fourier amplitude spectrum



That plot is called a Fourier spectrum. This is the typical Fourier spectrum for the different natural frequency, the lowest frequency you can see that the displacement amplitudes are constants, the higher air frequency where the accelerations are constant, intermediate frequency, the velocities are constant you can get the smoothed curve of this, you will see that constant kind of things. This is how you will get a Fourier spectrum, this Fourier spectrum further will interpret get the Fourier domain characteristics of the earthquake.

We have seen that the Fourier domain characteristic is one of the response spectrum is a Fourier and the another one is a Fourier spectrum. This will be discussing the corner cut off and predominant frequency of the system using the Fourier amplitude spectrum plot. With this we will close today class, next class we will be discussing the corner, cut off frequency and Fourier amplitude spectrum, how it varies for the different components of the data.

How this will be useful for the other applications in engineering seismology. Thank you very much for watching this video, so we will see you in the next class, thank you.