Stochastic Structural Dynamics
Prof. Dr. C. S. Manohar
Department of Civil Engineering
Indian Institute of Science, Bangalore

Lecture No. # 06

Random Processes-1

We have been discussing, description of random variables. So, we began by introducing the notion of probability - Probability space, conditional probability. The definition of random variable and probability density function, distribution function, expectation, functions of random variables and two random variables and their descriptions, functions of two random variables, generalization to n random variables, nth order probability density function, nth order probability distribution function, sequence of random variables and a few limit theorems.

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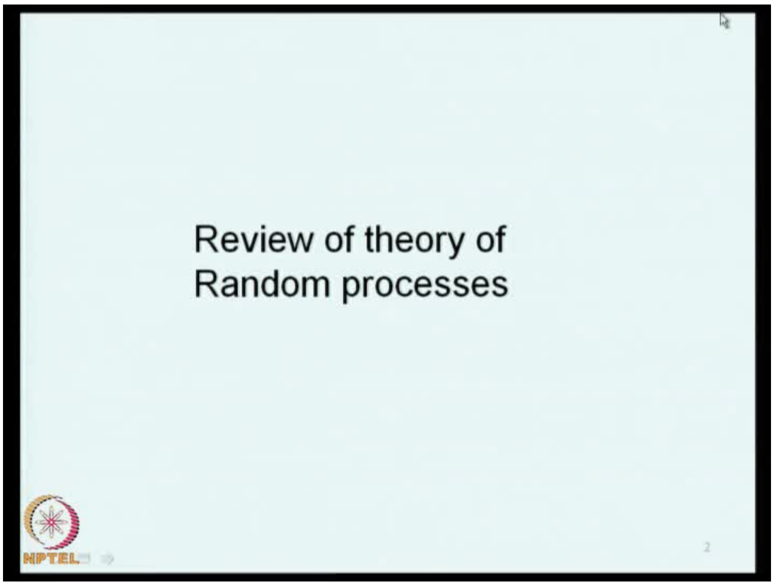

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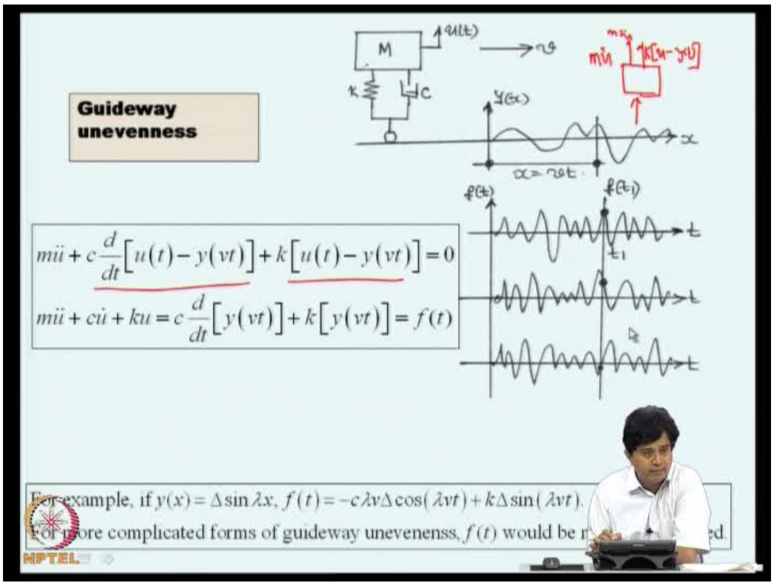

Now, move forward and introduce a notion of random processes. So, in the next few lectures will be reviewing some basics of theory of random processes. We will begin by asking, what is a random process, to explain that we will consider the problem of a vehicle - taxing on a uneven road. So, y of x is a spatial coordinate and y of x is the road roughness. This is a model for the vehicle M is the mass, K is the stiffness ,C is the damper, and we are interested in the vertical displacement of this mass and that is denoted by u of t. The vehicle itself is taken to move with a velocity v. Let us assume that it enters this point at t equal to 0 and moves with velocity v, if you draw the free body diagram of this mass, there will be inertial force. We can sketch here, inertial force, force in the spring that is K into u minus a time t, that is, when x is equal to v t; this will be y of t, because y is the coordinate here. And similarly, there will be a force in the damper and that is shown here. So, if you, right now, you use the (( )) principle we get the inertial force plus the damping force plus. The force in the spring is to be equal to 0. So, this is an equilibrium equation.

If we now collect terms involving u on the left hand side, we get a driving force on the other side, which is c into d by dt of y t plus k into y of vt. Now, if we consider road to be say for example, sinusoidal delta sine lambda x. This forcing function on the right hand side is given by this quantity, but of course roads will not be sinusoidal, there will be quite erratic. And consequently, this forcing function will perhaps look like something like this. This forcing function also depends on properties of the vehicle, namely C and K because C appears here, K appears here and the velocity of the vehicle appears here. Depending on all these parameters, the details of this force would vary, so on. A one ride you may get this kind of force and at another ride, you will get another force, every time you ride on the road, you will not experience the same force; this is another ride.

So, this f of t, now is uncertain. Now, if you look at value of f of t at a time instant t 1 for every ride. This is a ride 1, ride 2, ride 3, ride 4, and so on and so forth; at t equal to t 1. The value of the force here will be this, in the second ride it will be this, third ride it will be this and so on and so forth. So, these set of numbers can be viewed as outcome of a random experiment or in other words, a t equal to t 1 f of t can be viewed as a random variable, clearly as t changes this f of t also changes.

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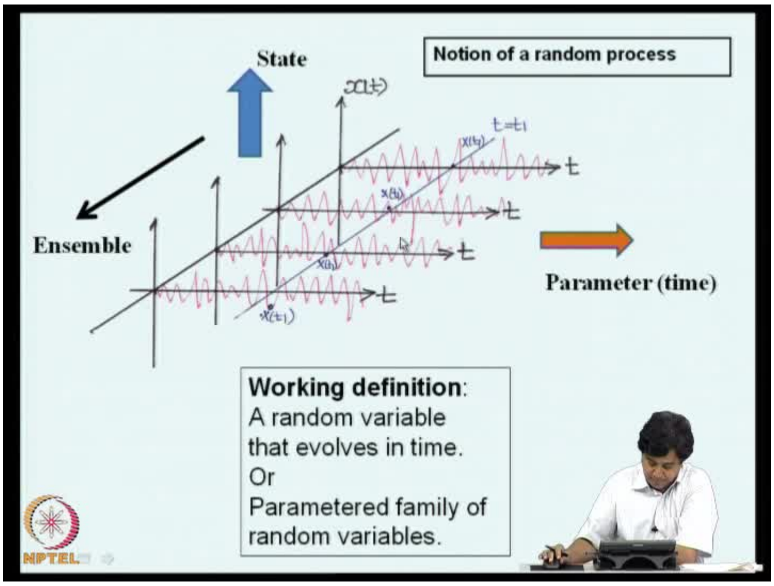

So, we are seeing here, a random variable evolving in time. Let us make this clear, so, let us consider X of t to be a collection of say time histories. So, I am looking at functions of time this could be the force on the vehicle that I will just mentioning it could be wind velocity, it could be guide way unevenness itself, it can have many interpretation ,it could be records of earthquakes in a given side.

Now, if you are talking about say, wind velocity and if you are measuring on different days, the trace of this velocity will not be the same every day. So, at some prescribe time t equal to t 1, you will say X of t 1 on one time x. This the value of X of t that you will see at t equal to t 1 will keep changing. This collection of time histories is known as a random process. So, working definition for this to start with could be that we have seen that for a fixed value of t. These numbers can be viewed as outcomes of a random experiment. Therefore, the, this can be viewed as a random variable and there is a random variable at every t. So, a random process can be taken as a random variable that evolves in time.

We can also call it as parametered family of random variables, because for every t there is the random variable. So, random process can be thought of as a parametered family of random variables. This time t that I am calling as time is called as a parameter, it can be space, it can be space and time, it can be multidimensional. We will see it later and the value that X of t takes, we call it as state and this collection of time histories is called as an ensemble. So, whenever, I mention a random process, I would like that you should carry this mental image of an ensemble of time histories that is what a random process should convey to your mind.

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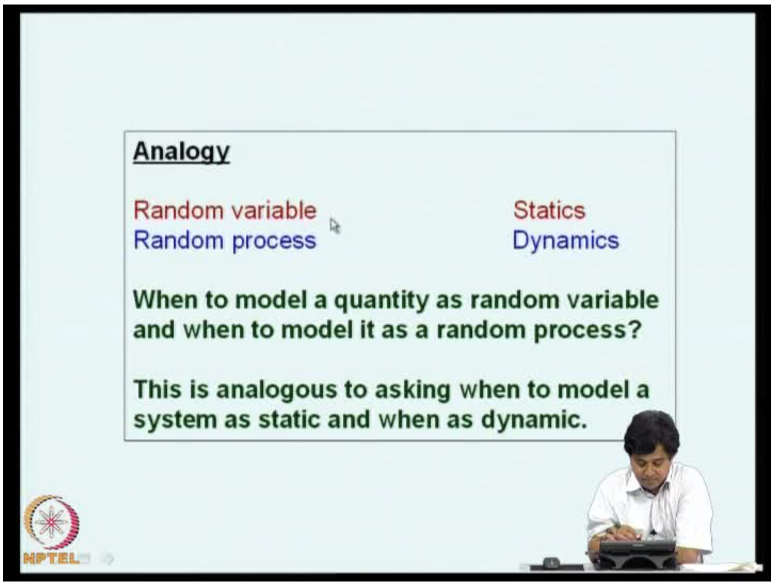

Now, we have discussed so far random variables. Now, we are talking about random process, how do we know, when we should use a random variable and a random process in modeling? This question is somewhat analogous to the question in mechanics, when we should use the statics principles to model a phenomenon or when you should use the dynamics principle.

So, in a way, we can see that random process and random variables share a relation that exists between dynamics and statics. So, if you think random variable is static random processes are dynamic, because there is an evaluation; whereas in a random variable there is no evaluation for example, in a statically load a structure, a displacement at any point there is no evolution, but if there is a time varying load acting on the structure that response variable evolves in time. So, we there is dynamics in it.

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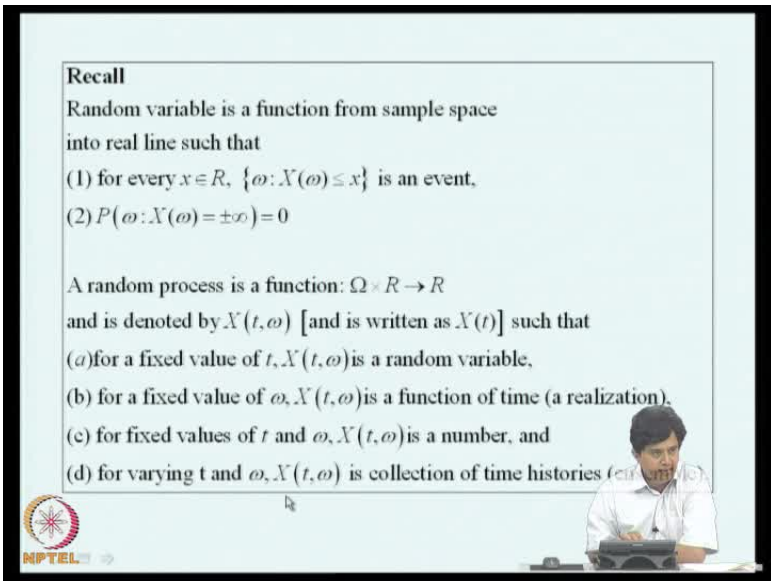

In the same sense, a random variable is static, but moment its starts evolving along a parameter, we call it as a random process. Now, we can try to know offer a slightly better definition, we can recall that when we define random variable, we defined it as a function from sample space into real line, such that for every x omega is a sample point. This set X of omega less than or equal to x was an event on which we assigned probabilities. So, we also impose certain restrictions on the events and we can generalize this definition. Now, a random process is now a function of Cartesian product of sample space and a real line to the real line. And this we denote it as X of t, omega. This omega is a sample point, and t is the element belong into R omega belongs to this. And often we suppress, this omega instead of writing X of t, omega we write it as a X of t. This is perfectly consistent with the practice that we have followed, till now in representing a random variable. A random variable strictly speaking should be written as X of omega, but I write it as x, I suppress writing this omega.

Similarly, we tend to suppress writing this omega in most cases. So, a random process, I denote as capital X of t. Now, what are its properties, if you fix time X of t, omega is a random variable, if you fix omega X of t, is a function of time. It is a realization, an outcome is a realization, a time history for fixed value of time and realization X of t, omega is a simply a number, if you allow both t and omega to vary X of t, omega becomes a collection of time histories.

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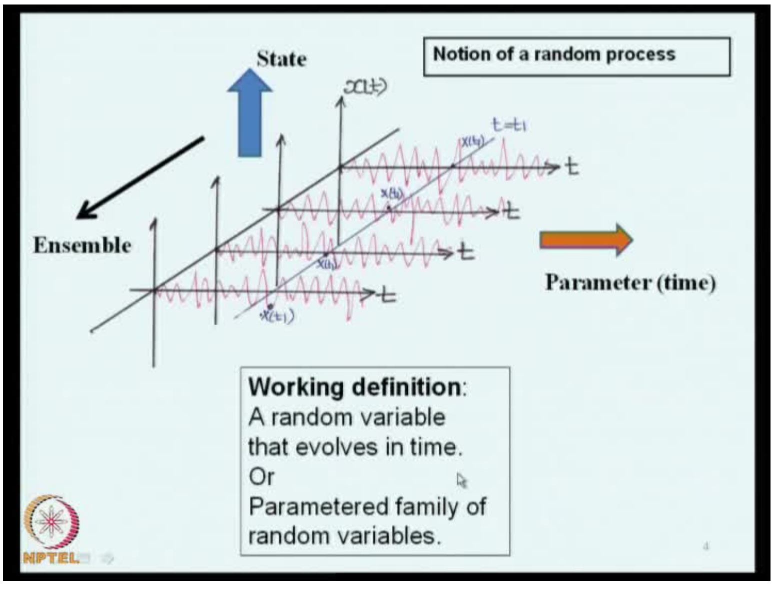

So, that we can quickly see here, if I now, fix t this is a random variable, if I fix a realization, I get a time history, if I fix the realization as well as a time I get a number, but if I now allow t 2 vary and realization also to vary, I get a collection of time histories. This is known as an ensemble and the random process is actually ensemble, just as we defined random variables in terms of outcomes of sample space and subsets of sample space, etcetera. We have to now develop certain strategy to describe this ensemble, there is uncertainties buried here. So, we have to describe that so, that is what the theory of random processes as per to achieve.

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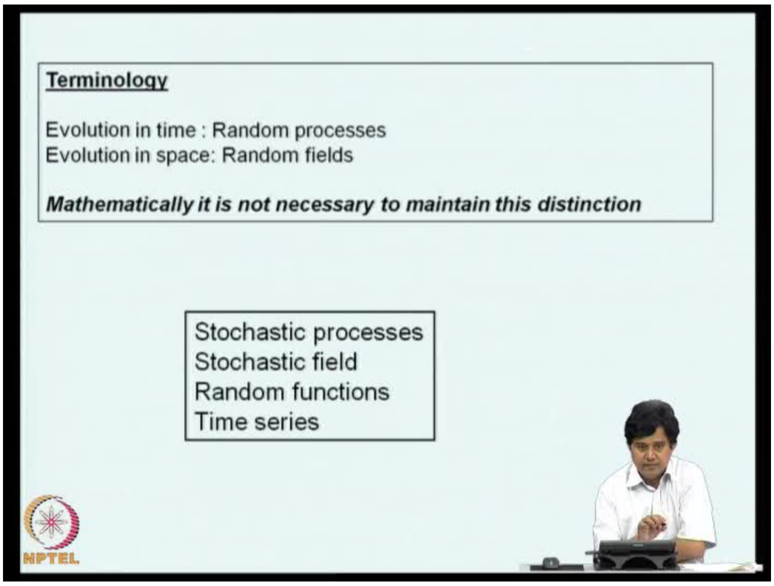

Now, a clarification on terminologies is an order, at the outset in many publish literature, if the parameter is time, the resulting random process is term use is random process; but if evaluation is in space, such as say guide way unevenness or track unevenness in a railway line. We call them as random fields for the thickness of a plate or things like that. Where the evolution is in space, the word random field is often use to denote that but mathematically it is not necessary to maintain a distinction between random fields and a random processes, they can be used synonymously, there are other terms that are used to describe random process that is - stochastic processes, stochastic field, random functions time series, and so on and so forth. So, for the purpose of this course, will take that all these are synonymous although in a given context, we can maintain usefully certain distinctions, but in this course it is not necessary.

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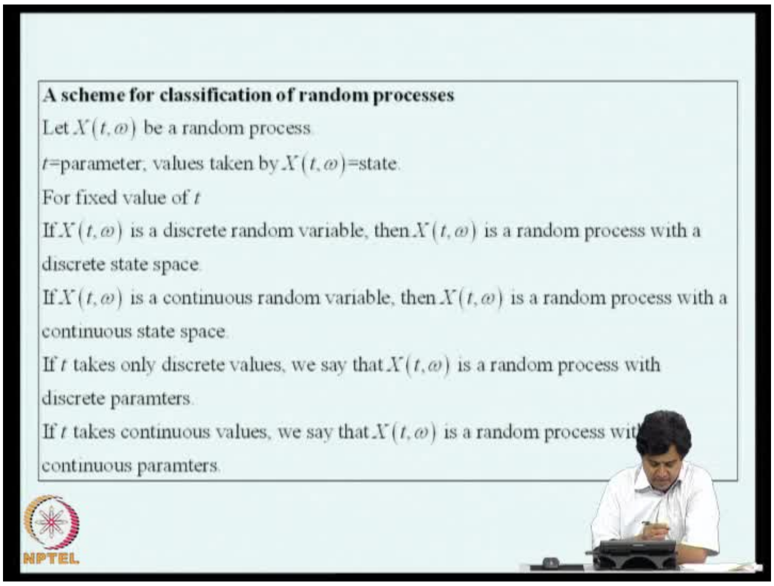

Now, how do we classify random processes?. Now, we have seen that is a collection of time history. So, let X of t, omega be a random process. So, immediately you should think of those ensemble of time histories, as our mental image of this statement; t is a parameter and the values taken by X of omega is the state, that is the numbers on the y axis in that on figure that I show. For a fixed value of t, if X of t, omega is a discrete random variable, then we say the random process, X is a random process with a discrete state space. Similarly, if X of t, omega is a continuous random variable, then we say X of t to be a random process with continuous state space. On the other hand, if t takes only discrete values, we say that X of t is a random process with discrete parameters. Similarly, if t takes continuous values, we say that X of t, omega is a random process with continuous parameters.

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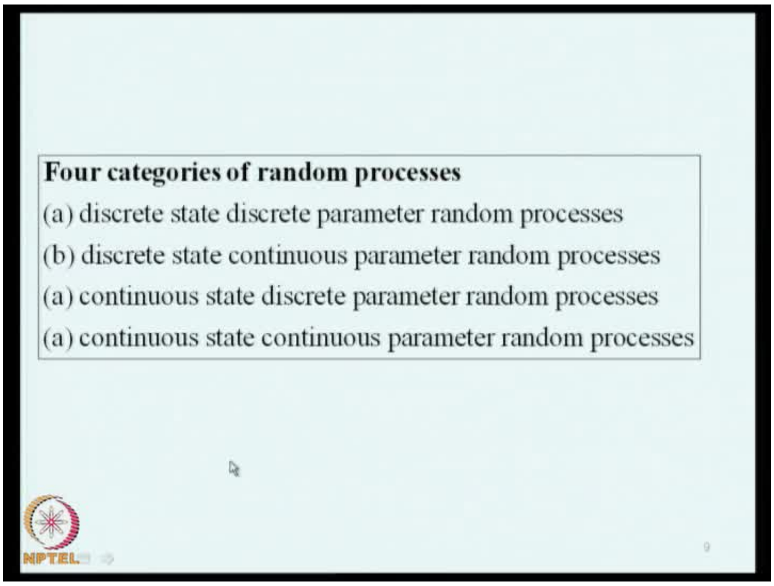

So, the parameter can be discrete or continuous and the random variable can be discrete or continuous. So, this now helps us, to introduce four categories of random processes namely, discrete state discrete parameter random processes; discrete state continuous parameter random processes; continuous state discrete parameter random processes and continuous state and continuous parameter random processes.

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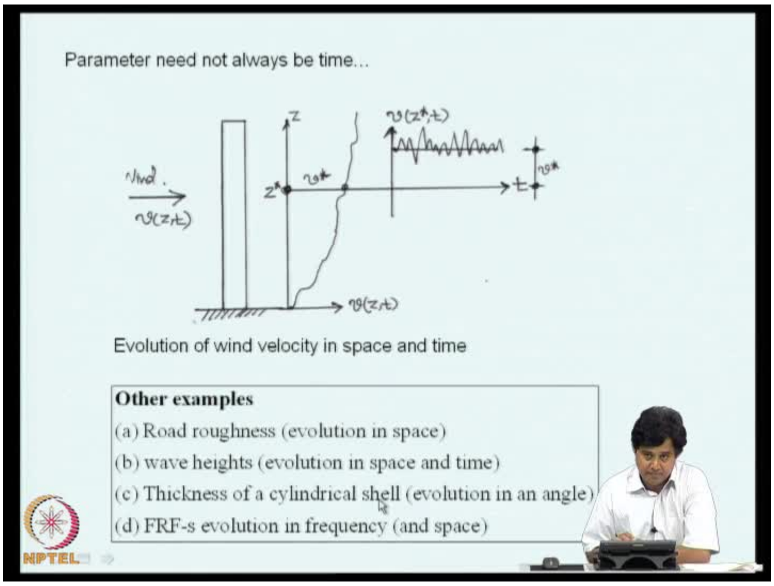

So, in vibration problems, we will be interested in continuous state, continuous parameter random processes and discrete state and continuous time parameter processes.

So, we will see the applications of this, later in the course, as I have mentioning the parameter need not always be time. The parameter along which certain randomness evolves to illustrate that we can consider the distribution of wind load on a say a [fl] suppose, this a wind velocity and v of z, t is the velocity. Now, if I now take a snapshot of the wind velocity at a given time and plot it as a function of space it will look something like this is quite erratic and velocity here is 0, due to boundary layer effect and it reaches the atmospheric (( )) wind velocity as you climb up in the elevation.

Now, on the other hand, if you fix the elevation and take a look at how velocity changes with respect to time. So, this velocity fluctuates around this v star is a value around which it is fluctuating. So, we can see that the evolution of wind velocity, here is in both space and time it varies along the height and also along the time axis. So, other examples road roughness as I was mentioning evolves in space wave heights evolves in space and time it is almost like wind. Thickness of a cylindrical shell for example, it can vary as a function of theta is an angle. Now, neither time nor space but it is an angle.

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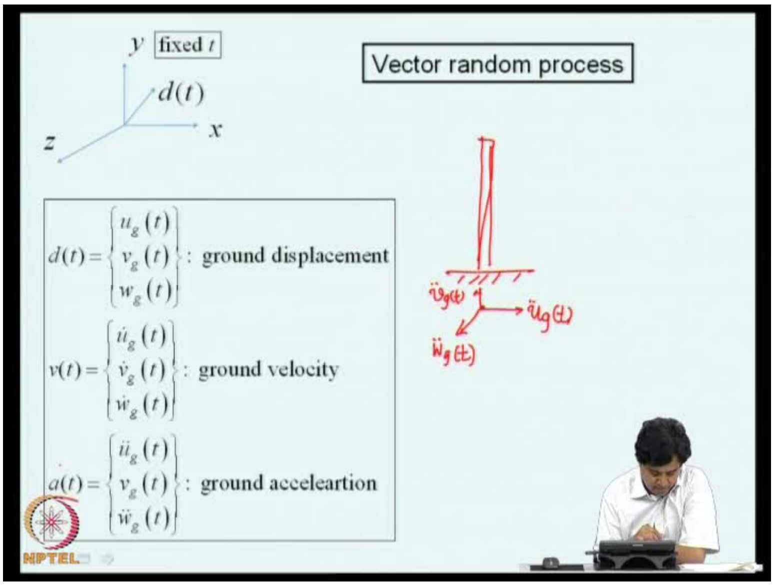

Similarly, if you look at frequency response function of a dynamical system they evolve in omega, which is the frequency. So, this parameter need not be space time it can be frequency as well or it can be combination of them, this time histories that I was mentioning can constitute what is known as vector random process for example, if you consider again the chimney kind of a structure and in the event of an earthquake. The earthquake ground acceleration is a vector at any given point and it can be reserved into 3 components. Say, u g double dot of t, v g double dot of t and say, w g double dot of t. So, if you look at acceleration here, I can describe in terms of 3 components, u g double dot, v g double dot and w g double dot. Similarly, if you look at nearly the displacement, I can look at u g of t, v g of t and w g of t and similarly velocity. So, in x, y, z plane the point the positional vector keeps rotating in a complex manner and it gives rise to the 3 components along x, y and z. so, this d of t we call it as a vector random process, v of t is a vector random process, a of t is the vector random process.

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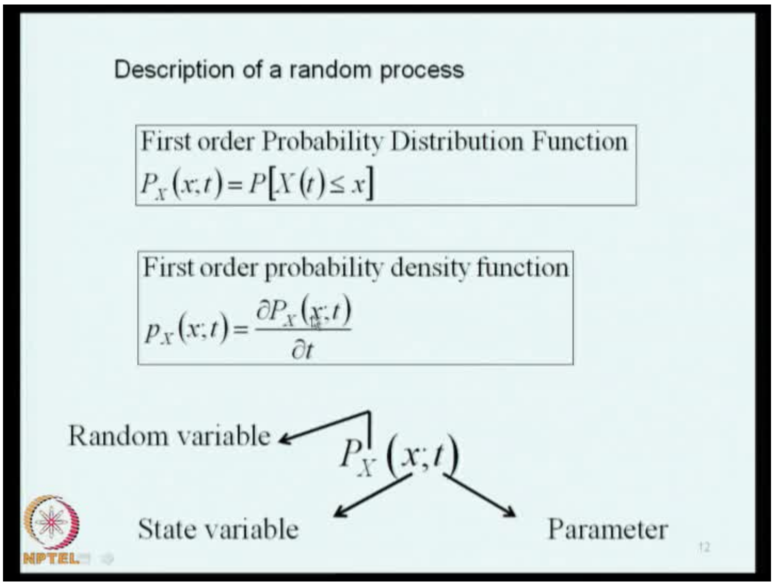

Now, how do we describe random processes? Now, for a given value of time, we have seen that X of t is a random variable. So, I can write down the probability distribution function of that random variable. So, X of t is a random variable. So, I can ask the question what is probability of X of t less than or equal to x? This we call it as first order probability distribution function of random process x and we denote it as P subscript capital X and in the argument, there is a x separated by a semicolon t. So, this is clarified here, this capital X refers to the random variable in question and this is the state and this is the t is a time at which parametered value at which we get this random variable X from the random process X of t and they we use semicolon, because probability distribution function is a function of the states and it is being evaluated at different times. So, to denote that we do not write a comma here it is a matter of convention.

Now, I can, now, associate with this probability distribution function, I can define the probability density function as a derivative. Now, since the function is now function of two independent variables, I use the partial derivative, this distribution function has all the properties of a probability distribution of a random variable, it takes values between 0 and 1, it is monotonically non decreasing and this is the density function, which again is positive area under this density function is 1 so on and so forth.

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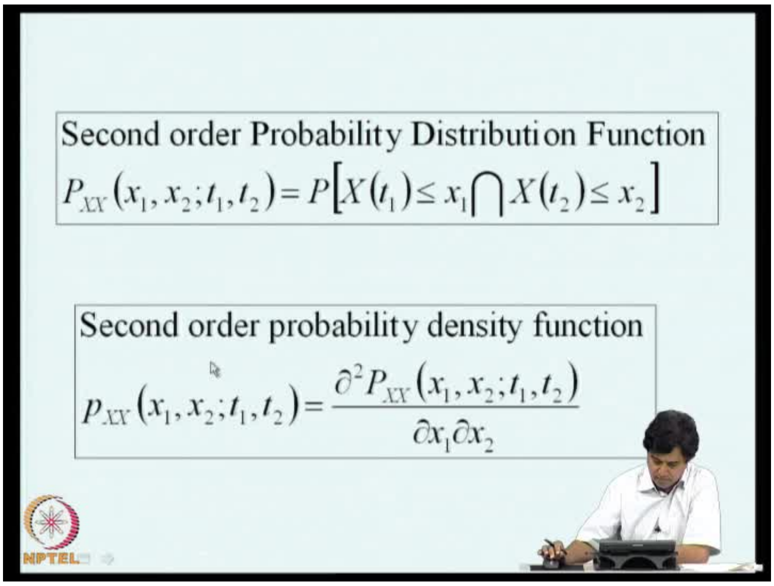

Now, if we consider two time instants t equal to t 1 and t equal to t 2, I will have two random variables. Now, I can write the expression for the joint probability distribution function of these two random variables and that is p of X of t 1 less than or equal to x 1 intersection X of t 2 less than or equal to x 2, this I call it as second order probability distribution function of the random process X of t and x 1 and x 2 are the states and t 1 and t 2 are the times at which I am forming these two random variables associated with this, I can of course define the second order probability density function of the random process X of t and this is denoted as shown here, again there are 2 subscripts XX x 1 x 2 are the states t 1 t 2 are the times and this is the partial derivative on the distribution function.

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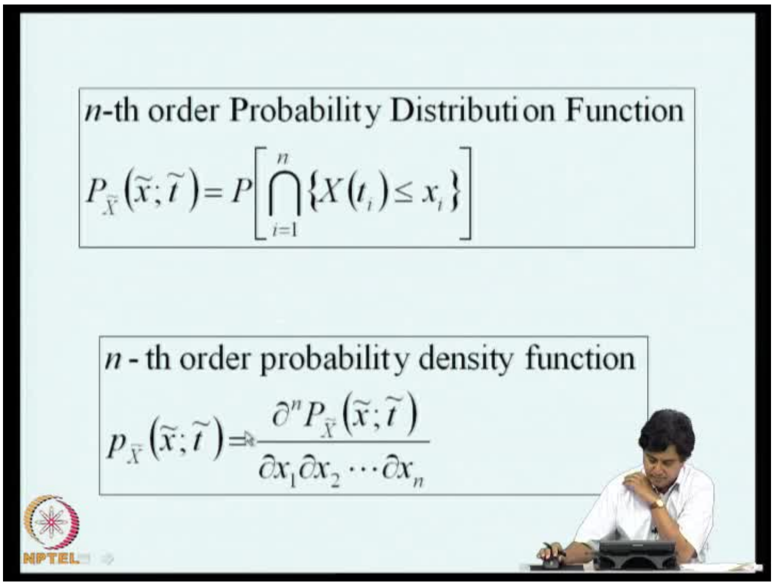

We can generalize this and define the nth order probability distribution function that means, if I select now n time instants t 1, t 2, t 3, t n, I will get n random variables X of t 1 X of t 2 and X of t n and I can consider the joint probability distribution function of these n random variables, which is given by this probability. This we call it as nth order probability distribution function of random process X of t, evaluated at t equal to t 1, t 2, t 3, t n associated with this I can define nth order probability density function defined a t tilde, t tilde is vector of t 1, t 2, t 3, t n. So, this is a nth order differentiation of this distribution function.

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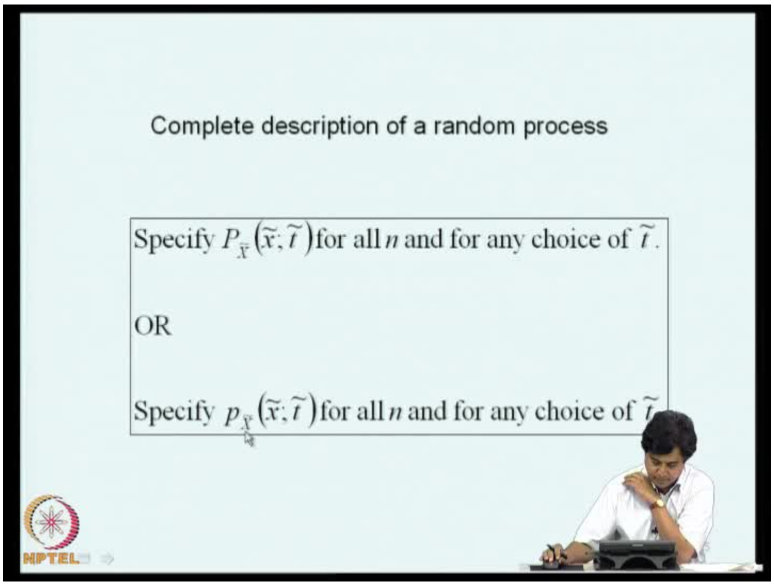

When do we say that a random process is completely specified, clearly we should be able to specify the nth order probability distribution function, for this random process X of t for all n and for any choice of t tilde; for example, n equal to 1 and you can select t at several places. So, no matter where you take the time instant t, you should know what is the probability distribution function? Similarly, if you take two time instants t 1 and t 2 you will get two random variables, no matter where t 1 and t 2 originate from X of t 1 and X of t 2 originate the joint probability distribution function should be known. This you should be able to do for n time instants X of t 1 X of t 2 X of t n nth order probability density function not only that no matter what are these t 1, t 2, t n, you should be able to tell me this nth order density function for any n. So, this is a very tall requirement, because we are dealing with an infinity of random variables the demand on completes specification would also be very large.

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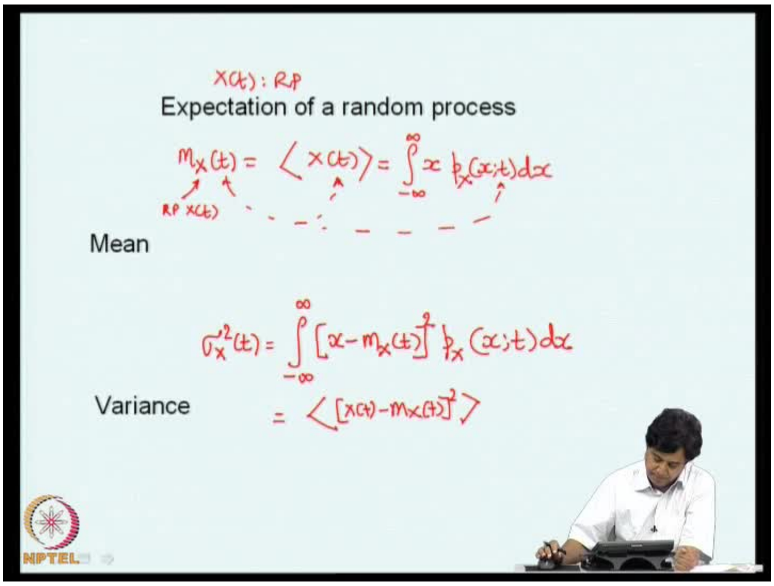

So, this specification can be either in terms of distribution function or in terms of density function both are equivalent. Now, in the context of random variables, we defined expectations, we defined mean and so on and so forth. So, we could introduce that notion for a random process, I will show it now, suppose X of t is the random process. So, X of t is the random process, I defined its mean m X of t as expected value of X of t. This I write it as minus infinity to infinity x p X of x semicolon t dx. This mean has now, a subscript you should notice this. This means, I am taking about random process X of t this time t corresponds to this time t that means, I am considering the random variable at time t and this is the mean. So, mean is now a function of t depends on which random variable, you are talking about; therefore it becomes a function of time. Similarly, the variance can be written as minus infinity to infinity x minus m X of t p X of x t and dx. This lower case x that appears inside the integral the dummy variable, it can be written in any other u or v or whatever it is. So, this, this, X is nothing to do with this X. This is square of this, so, this is the variance. Again, this is the function of time t because I am writing this is nothing but the expected value of X of t minus m X of t whole square. So, at t I have a random variable X of t and I am talking about its variance, its mean is a m X of t. Therefore, all the subscripts and arguments are to be understood the context of this should be understood.

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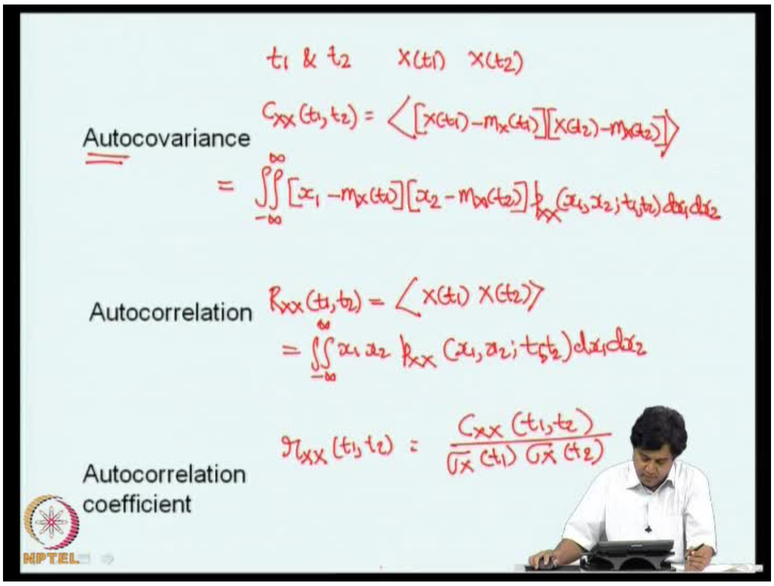

We can define the covariance, now you consider two time instants t 1 and t 2. So, these leads to two random variables X of t 1 and X of t 2 and we can talk about its covariance and I write this as C XX of t 1, t 2. This we write it as expectation of X of t 1 minus m X of t 1 into X of t 2 minus m X of t 2. So, this itself is the double integral minus infinity to infinity x 1 minus m X of t 1 x 2 minus m X of t 2 p XX x 1, x 2 t 1, t 2 dx 1 dx 2; this t 1 and t 2 appear, I mean they basically convert the fact that we are looking at the random variable X of t 1 and X of t 2, the word auto in covariance, auto-covariance refers to the fact that we are getting two random variables and both these random variables are originating from the same random process X of t.

We will see later, that if there are two random processes X of t and Y of t, I can talk about cross covariance by considering one random variable from x, another random variable from y, to be able to do that at a later stage. We are now introducing the prefix auto. Autocorrelation function is defined as R X X of t 1, t 2 is expected value of X of t 1 into X of t 2, this is the double integral. Now, we can also define the correlation coefficient that we have defined earlier for two random variables it can be generalized for random processes as shown here. This is the ratio of covariance to the product of standard deviation.

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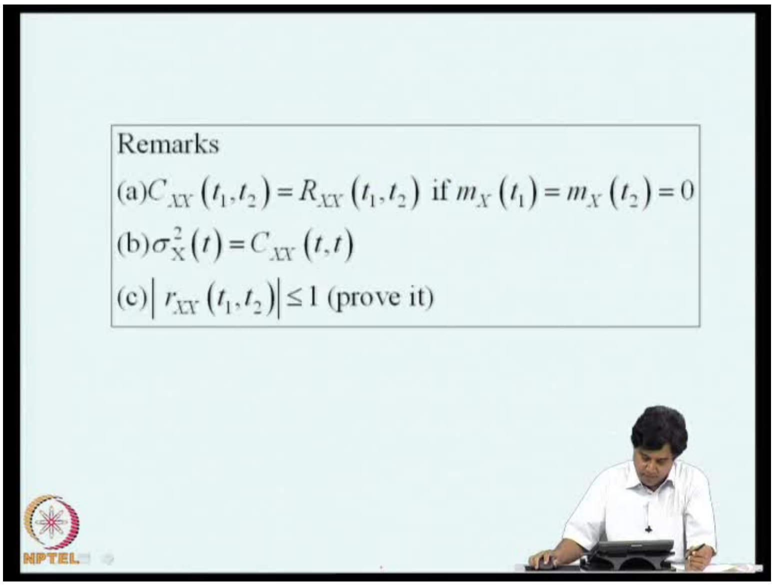

Again the word auto means, I am talking about two random variables originating from the same random processes, we can make some quick observations, if mean of the random processes is 0 or m X of t 1 and m X of t 2 are 0. The autocovariance becomes same as autocorrelation, if you evaluate the covariance function at this t both t 1 equal to t 2 equal to t, what is that I am doing I am finding the variance of the random variable at X of t. So, that is automatically the variance.

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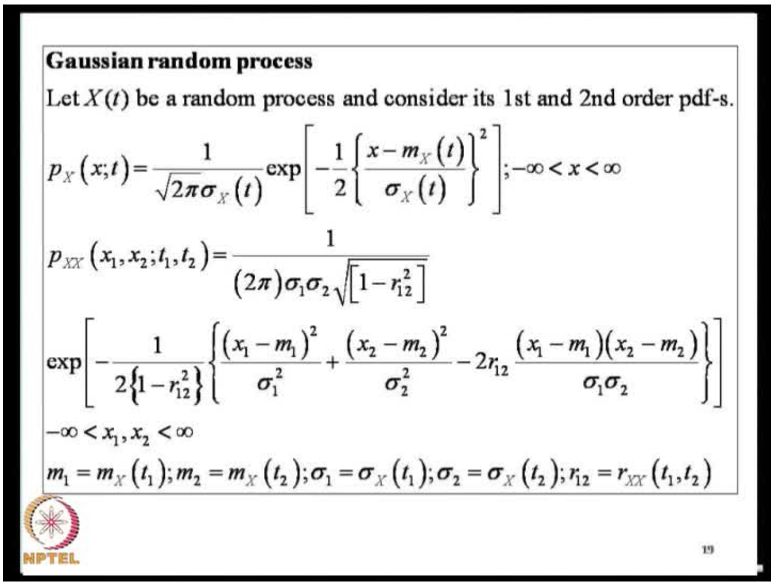

We can show that the autocorrelation coefficient is bounded between plus 1 and minus 1, I leave that as an exercise, again you have to use squares inequality. The way, we did it for two cases of two random variables. As an example, we will consider what is meant by a Gaussian random process? We know, what is the Gaussian random variable. We know, when we say the two random variables are Gaussian and we know, when we say n random variables are Gaussian. That can also be specified. So, let us consider a random process X of t, let X of t be a random and consider its first and second order probability density functions, if its first order density function is in this form or m X of t and sigma X of t are the parameters and they indeed are the mean and its deviation. Let this be of this found and a joint density function, here again there are 5 parameters sigma X of t 1 sigma X of t 2 and r 12 and m 1 and m 2. So, let this is be the joint density function.

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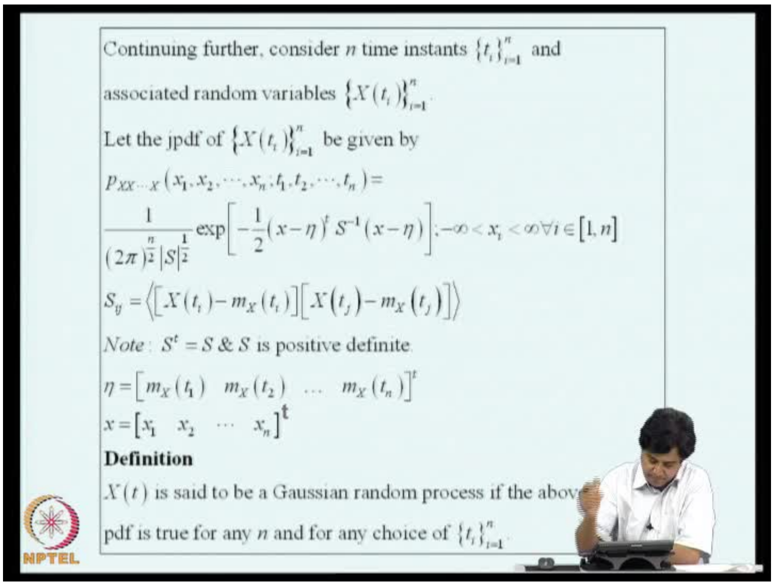

Now, considering this same logic, if we now consider n time instants t 1, t 2, t 3, t n and associated random variables X of t 1 X of t 2, etcetera. And if we say that the joint probability density function of X is given by this form, which we saws a while before where S is now the covariance matrix and it has a mean vector and X is the vector of states, if the joint density function is of this form; let it be of this form. Now, we now offer the definition that X of t is said to be Gaussian random process, if the above form probability density function is true for any n and for any choice of t 1, t 2, t 3, t n, you take say for example, three random variables, no matter where you take it t 1, t 2, t 3, 3 random variables should have the joint density function, which confirms to the joint density function of a Gaussian random variables, 3 Gaussian random variable. This should be true for any n and for any t 1, t 2, t 3, t n, then we say, X of t is Gaussian.

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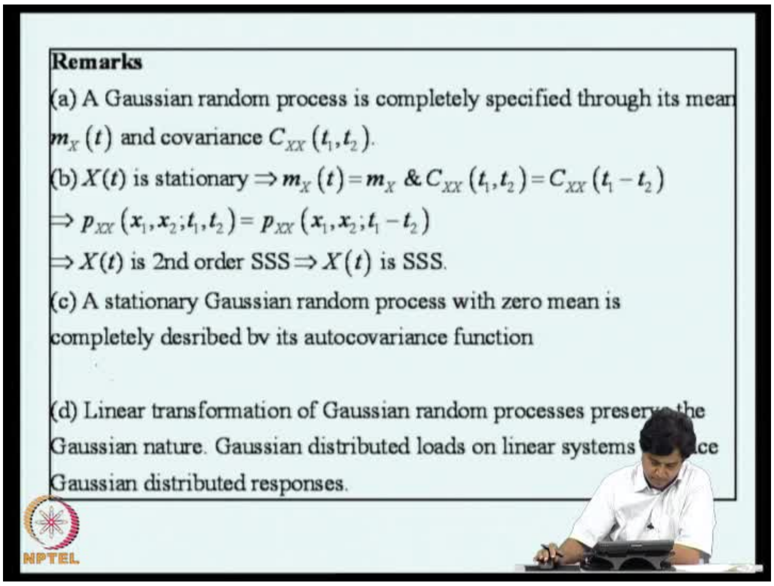

Now, if you are dealing with a Gaussian random process, what constitutes complete description the nth order probability density function, for any t and any vector of t and n. Now, a Gaussian random process is completely specified through its mean as a function of t and covariance as a function of t 1 and t 2.

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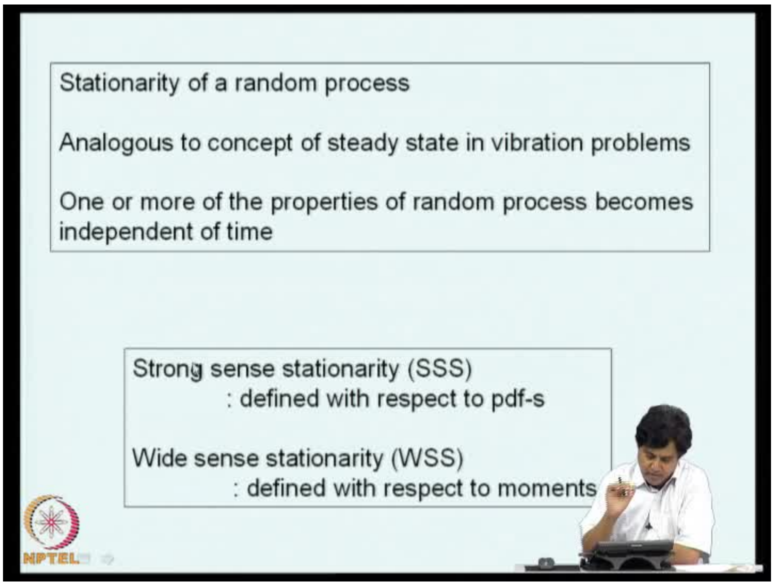

Now, we will come to some of this as we go along, we now introduce an important property of random processes, namely - stationarity. This concept of stationarity is analogous to the concept of steady state that we use in vibration problems; we say that for example, a single degree freedom system driven harmonically would reach a harmonic steady state, what would happen? The response would become harmonic and it will be independent of initial conditions that means certain properties of the response becomes independent of time, then we says system has reached a steady state. Similarly, we say that a random process is stationary, if one or more of the properties of the random process becomes independent of time.

Now, a random process can be described in terms of probability density functions of several orders or in terms of moment mean covariance and so on and so forth. So, depending on which of such properties becomes independent of time there would be different senses of stationarity. So, we say, we use the phrase Strong Sense Stationarity, written as SSS. This is defined with respect to properties of probability density functions or distribution functions, whereas we talk about Wide Sense Stationarity denoted as WSS. These properties are defined with respect to moments like mean, variance covariance and so on and so forth.

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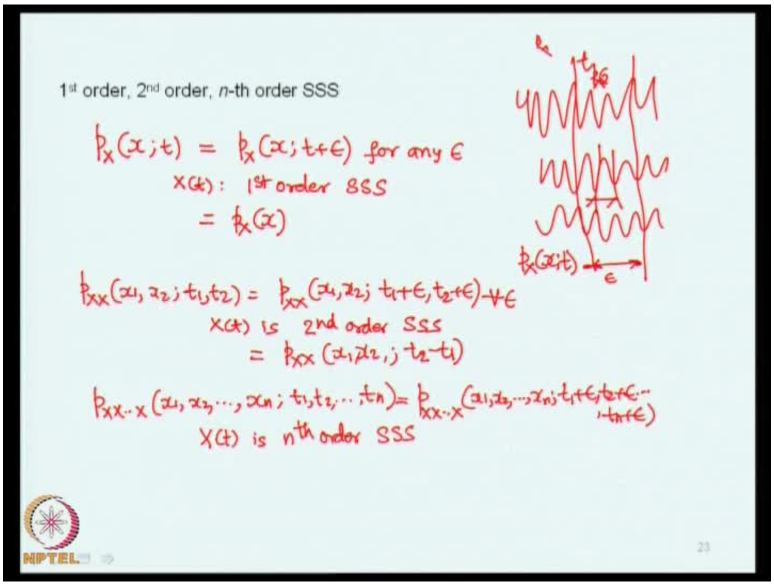

Now, who is start by considering the concept of stationarity, if we consider first order probability density function x : t if this is same as p X of x t plus epsilon for any epsilon, then we say that X of t is first order Strong Sense Stationary. So, what it means is if you have an ensemble of a random process, you consider a time instant t, you have the first order probability density function p X of I will write here p X of x: t. Now, you shift the t by epsilon. The probability density function here should be same as what it was here for any epsilon, if we say, if that happens, we say that the process is first order strong sense stationary, if this is so, what it means essentially is that the probability density function is independent of time, because epsilon itself is arbitrary it means that no matter why are you take your time instant you get the same probability density function.

Similarly, we can talk about second order Strong Sense Stationarity that is p XX of x 1, x 2 t 1, t 2, if it is same as x 1, x 2 t 1 plus epsilon and t 2 plus epsilon for all epsilon. We say that X of t is second order Strong Sense Stationary that means, here we consider two time instants separated by distance epsilon; say for example, these two. Now, you can take different pairs of random variables, which are separated by the same distance epsilon, no matter which pair you take, you get the same joint density function, as long as they are separated by the same distance on the time axis or the t-axis they will have the same joint probability density function. Then we say it is second order Strong Sense Stationary and in this case, what happens p XX would be function of x 1, x 2 and the time difference t 2 minus t 1.

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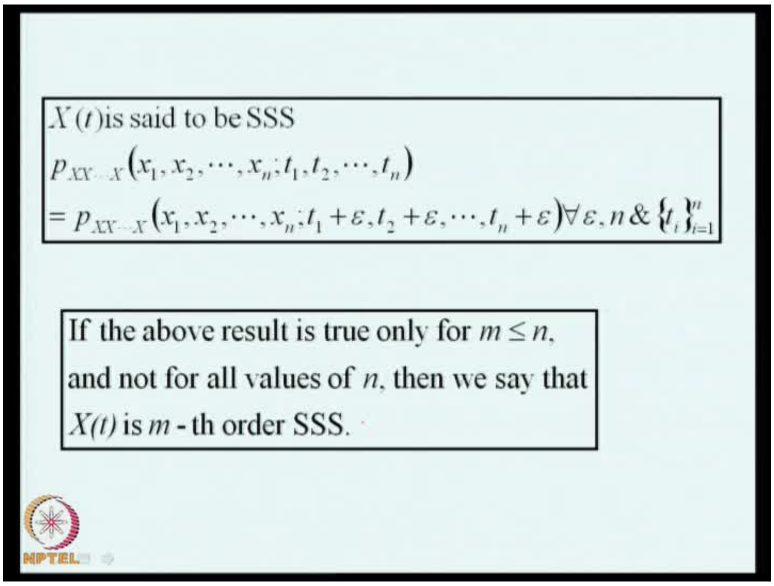

Now, we can generalize this notion and define nth order Strong Sense Stationarity that by considering n time instants, and we shift all these time instants by distance epsilon, if this is true, then we say that X of t is nth order Strong Sense Stationary. Now, we can make few observations. Now, X of t is now said to be strong sense stationary without any qualifier like nth order or not what it means, we say that X of t is strong sense stationary no qualifications just strong sense stationary, if the nth order probability density function p XX X x 1 x 2 x n t 1 t 2 to t n is independent of the shift that we give epsilon that means, if you shift the time axis by epsilon each time instant by epsilon. These two nth order density functions remain the same and this should happen for any epsilon for any n and for any choice of your time instants, this is a very tall requirement. Then we say that it is strong sense stationary, if this property holds good only for some m less than equal to n, then we say that X of t is mth order Strong Sense Stationary, if a process is not stationary it is non-stationary, I only defining what is stationarity a negation of that is non stationarity.

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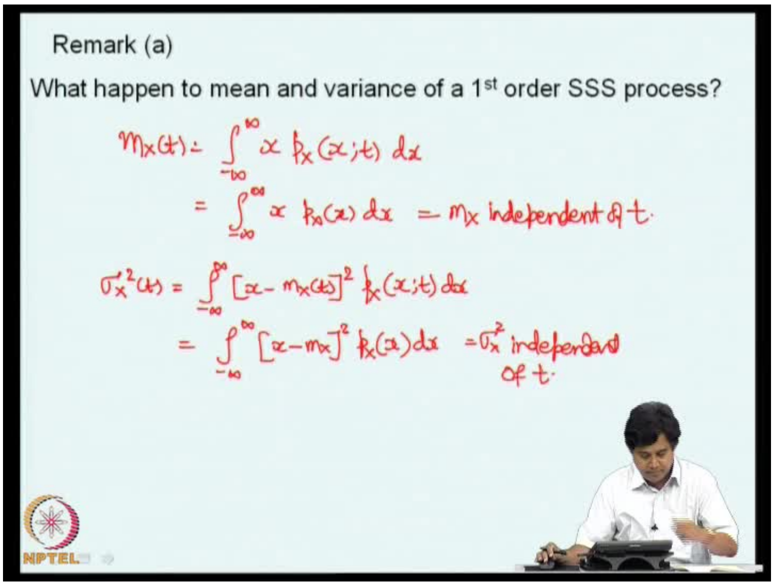

Now, we will consider, now a random process which is first order Strong Sense Stationary and look at its mean and variance. So, m X of t is minus infinity to infinity x p X of x: t dx. Now, this is p X of x: t is independent of time, because it is first order strong sense stationary. So, this become minus infinity to x p X of x dx this is m x which is independent of that means m x becomes constant. Similarly, I can write sigma x square of t which is minus infinity to infinity x minus m X of t whole square p X of x: t dx now minus infinity to infinity m x is a constant. So, I will write it as m x whole square and p X is again a constant with respect to time. So, this is sigma X square independent of time.

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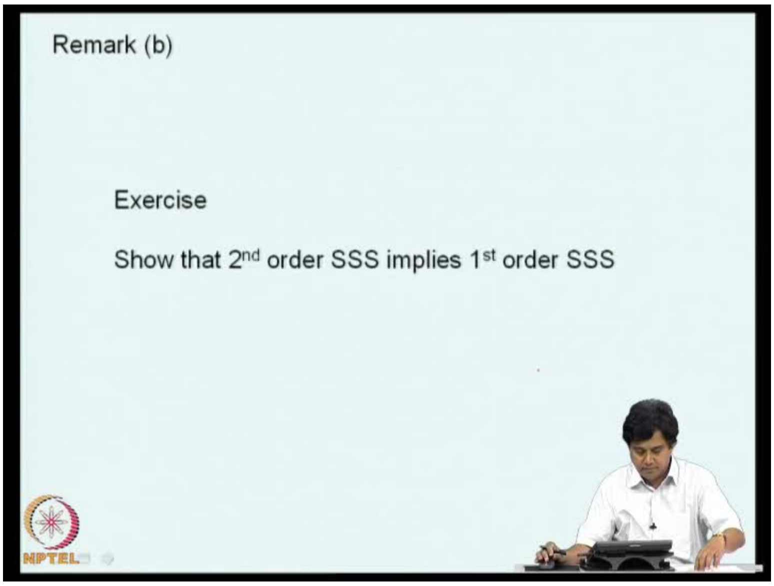

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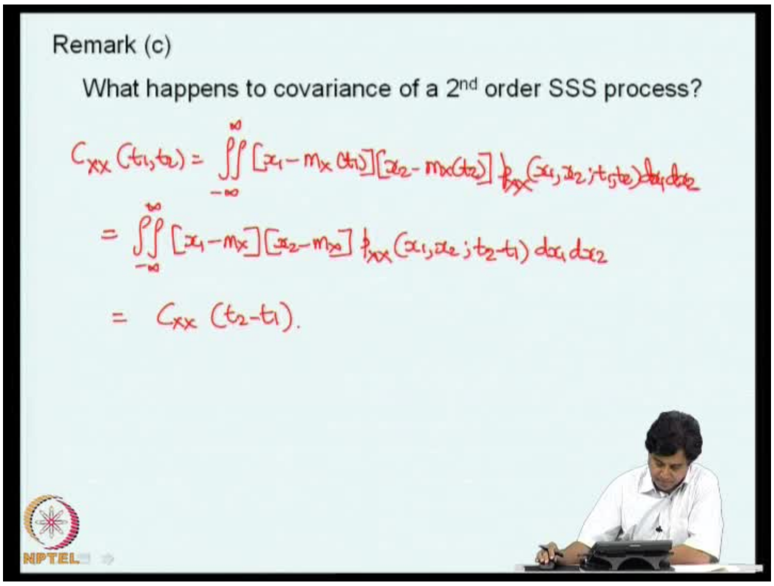

So, in modeling this is a very useful thing, because it simplifies our task enormously. Now, I leave it as an exercise, for you to show that a second order Strong Sense Stationarity implies first order Strong Sense Stationarity or a higher order Strong Sense Stationarity imply lower order Strong Sense Stationarity. Now, if we consider now a second order Strong Sense Stationarity process, we can ask the question what happens to its covariance. So, what is covariance? C XX of t 1, t 2 is minus infinity to infinity x 1 m X of t 1 x 2 m X of t 2 p XX x 1, x 2 t 1, t 2 dx 1 dx 2. Now, this I will now, write a second order Strong Sense Stationarity implies first order Strong Sense Stationarity and that would you mean is independent of time. So, m X becomes m X of t 1 and m X of t 2 are constants with respect to time. So, that first thing we can do this is m X x 2 minus m x and what happens to the probability density function, it will be a function of the time difference.

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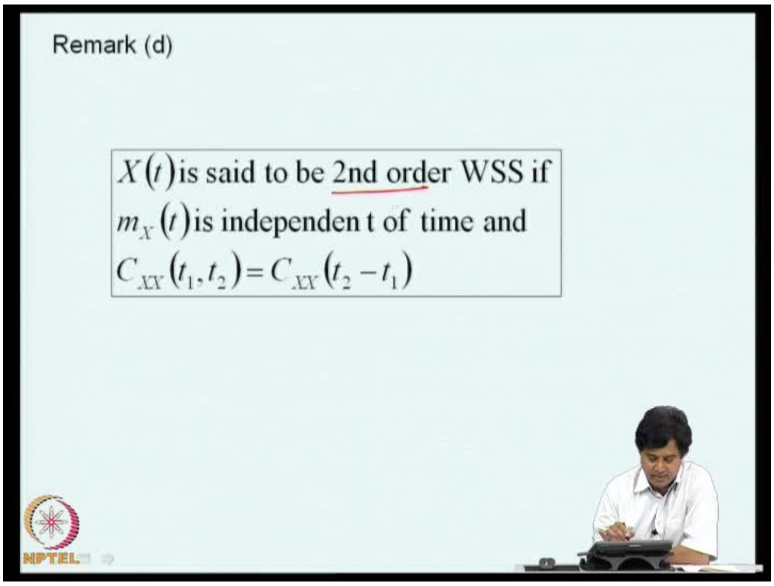

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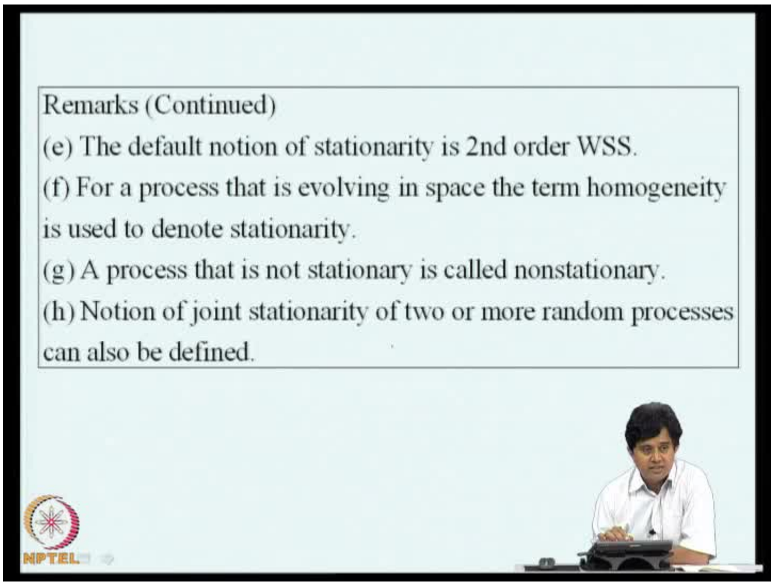

So, this becomes t 2 minus t 1 dx 1 dx 2. So, this is C XX of t 2 minus t 1 that means, the autocovariance function evaluated at the time instant t 1 and t 2. Now, depends only on the difference of the time instant that you are selected, that is a consequence of Strong

Sense Stationarity. Now, we mentioned about Wide Sense Stationarity, we make this definition, let X of t is said to be second order Wide Sense Stationarity, if mean is independent of time and covariance is the function of the time difference clearly a weak sense stationarity does not implies Strong Sense Stationarity, because Strong Sense Stationarity is in terms of probability density functions, whereas these are with respect only moments, if in future, if I say X of t is stationary, what is guaranteed is that it is second order Wide Sense Stationary or Weak Sense Stationary that is the default, notion of stationarity.

If we thinking of a process that evolves in space instead of time, the word homogeneity is used to describe stationarity. So, a stationary process in space whereas, random field in space, if it has stationarity properties, we call it as homogeneous random field in there again you can have strong sense homogeneity, wide sense homogeneity, so on and so forth. So, as I already mentioned a process that is not stationary is called non-stationary.

Now, we will come to another notion, sometime later, if there are two random processes, we can define the notion of joint stationarity. In terms of joint density function functions of random variables emanating from the two different random processes, and in terms of moments of random variables emanating from two different random processes.

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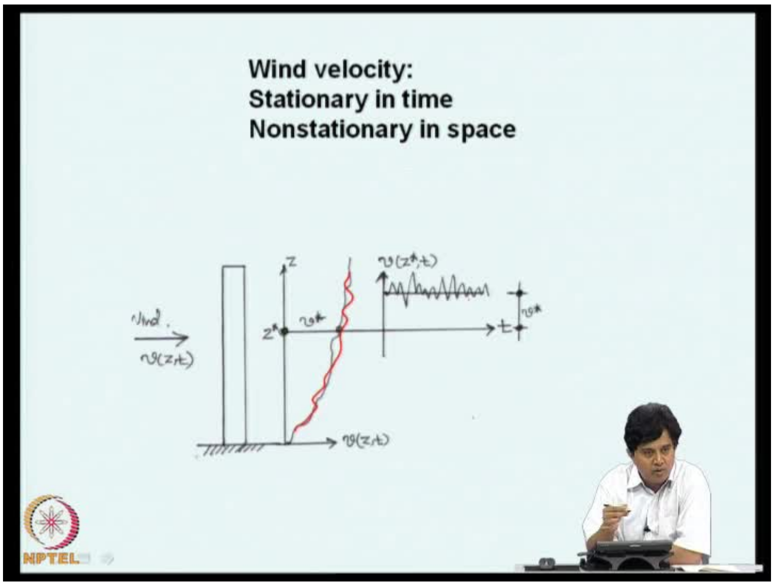

Now, when we talk about using, the notion of stationarity in modeling, there will be certain issues that we should understood, again if you return to modeling of wind velocity: we saw that wind velocity can be conceived as a random process evolving in both space and time. So, if you freeze the time instant, then take the snapshot of the velocity it may look something like this. This is non-stationary, because there is 0. Here it is evolving, whereas if you take a fix point on the structure and look at wind velocity at that elevation. This may be considered as stationary; this is how a sample of a stationary process would look like. Stationarity does not mean that sample realizations will have any smooth or a simple behavior the samples continued to be erratic, just as when we say in harmonic steady state in deterministic context, when we say a response has reached steady state X of t still continues to oscillate, it is only is an amplitude and phase which have become time invariant.

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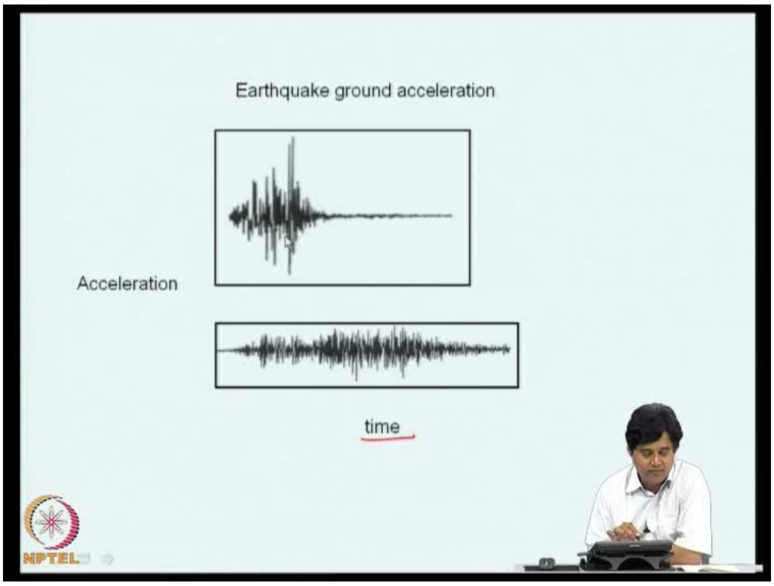

So, the random process continues to appear like this, but some of its property would have become invariant with respect to time. This is some traces of ground accelerations from two different episodes on the y-axis, we have accelerations here, we have time. In certain situations, the earthquake ground accelerations cannot be modeled as stationary, because there is a clear evaluation here it is a transient phenomenon a transient phenomenon cannot be modeled using stationary models, but whereas in certain situations, if the ground acceleration continues to take place, where reasonably long time, then for all practical purposes, we can consider that earthquake ground acceleration is stationary, if we say that it means that earthquake is occurring perpetually in a strict mathematical sense. There is no stop but from the point of view of response of the structure, if the structure is going to fail during an earthquake, it will be during this strong phase of ground shaking, say it does not really matter what happens afterwards.

So, for the purpose of modeling structural behavior, we could assume earthquake ground acceleration as stationary even for this case, the strong phase of shaking takes place. Where a reasonably constant long time interval and if response characterization is limited to this as might be the case, if you are interested only the highest response and so on and so forth. This could as well be modeled as stationary, but it will not be an appropriate assumption it may be a simplification of a given context. The point that is being made is that the choice to modeled a random processes as stationary or not is something that the person who makes the model should take, should make that choice it is nothing inherently hard and fast about this. Although, mathematically there are very clear definitions.

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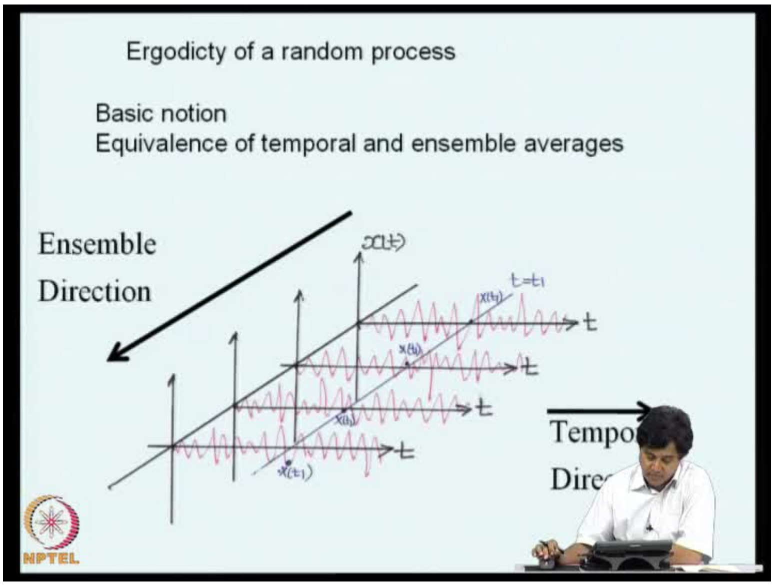

Another important property of a random process is known as Ergodicity. The basic notion of ergodicity is associated with equivalence of what are known as temporal and ensemble averages. To clarify these we can look at again the random process shown as an ensemble of time histories. This axis, this direction that I have shown I call it as ensemble direction, because if you move along that you get different realizations of X of t, this I called it as temporal direction, because if you move along one of the axis you see how the sample fluctuates as the function of time. This is temporal direction and this is ensemble direction.

Again a t equal to t 1 if I look at X of t is a random variable, when I say mean of this random variable and considering an average in the ensemble direction. Now, if I have very few samples or simply one sample of this whatever X of t is this is may be ground acceleration, there is nothing, no ensemble as such which would facilitate you in modeling what you have is a single time history. So, I would be tempted to consider what would be the average of this time history along the time axis. So, as I move along the time I will encounter different values of X of t in the same sense as if I fix t and move along the ensemble I will encounter different values of X of t. So, as for as X of t is concerned there is again a collection of outcomes here a collection of outcomes here. So, I can talk about an average here, as well as an average here.

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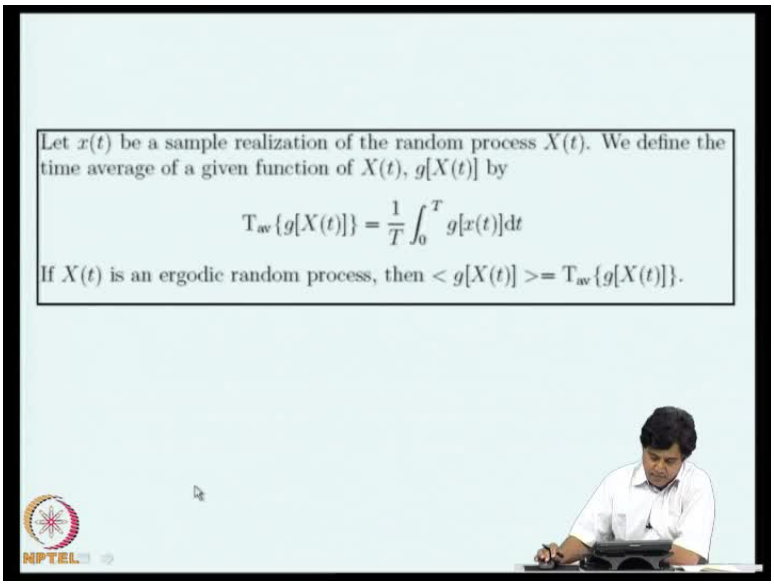

The concept of Ergodicity, basically tells us, the conditions under which a temporal average can be thought of has being a good approximation for the ensemble average. So, as you could easily imagine. This is very valuable modeling tool, if you have limited data you invariably assume Ergodicity. So, what the ergodicity property tells us is this, if we consider X of t to be a sample realization of the random process X of t and we considered we define the time average of a given function of X of t, for example, I consider g of X of t and I call this quantity time average of g of X of t as this integral 1 by T 0 to T g of x t dt if X of t is an ergodic process. What it says is that the expected value of g of X of t is equal to the time average of g of X of t. This is what ergodicity define with respect to this g of X of t and its expectation means.

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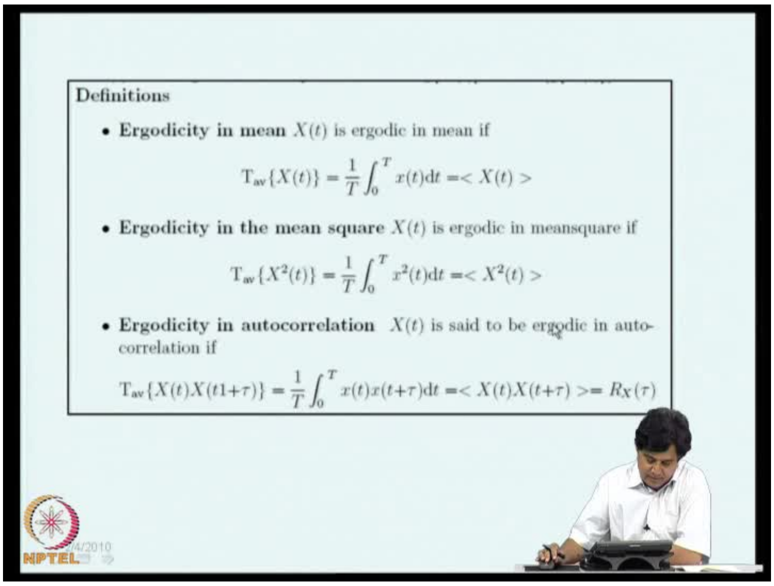

Again, since Ergodicity is being talked about in the context of an average since there exits several averages, mean, variance, covariance, etcetera. A process the definition of Ergodicity invariably gets link to the average that you are talking about, so if you consider Ergodicity in mean - we say that X of t is Ergodic in mean if time average of X of t is same as expected value of X of t; X of t is Ergodic in mean square sense if time average of x square of t is same as expected value of x square of t. So, this definition can be extended to cover for example, Ergodicity in autocorrelation. So, you consider the expected value of X of t and X of t plus tau. This is the time averages that we talk do it from a single sample, if these two averages are the same we say that the process is Ergodic in autocorrelation.

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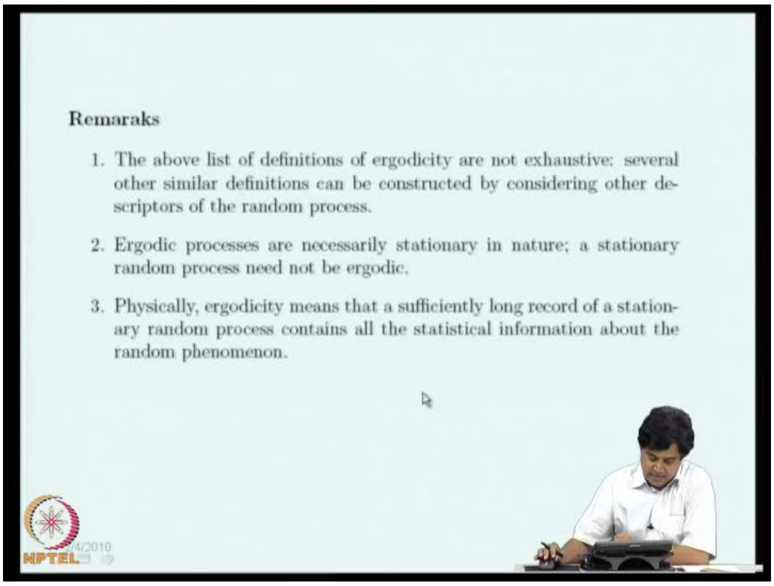

This equality although I am writing it here as an equality, we have to interpret it carefully, what it means? Before we come to that we can make few remarks on what we discuss till now, the list of your definitions for Ergodicity are not exhaustive; you can have similar definitions for other descriptors of random process, you can define Ergodicity with respect to first order probability distribution function, first order probability density function. So on its (( )) so on and so forth

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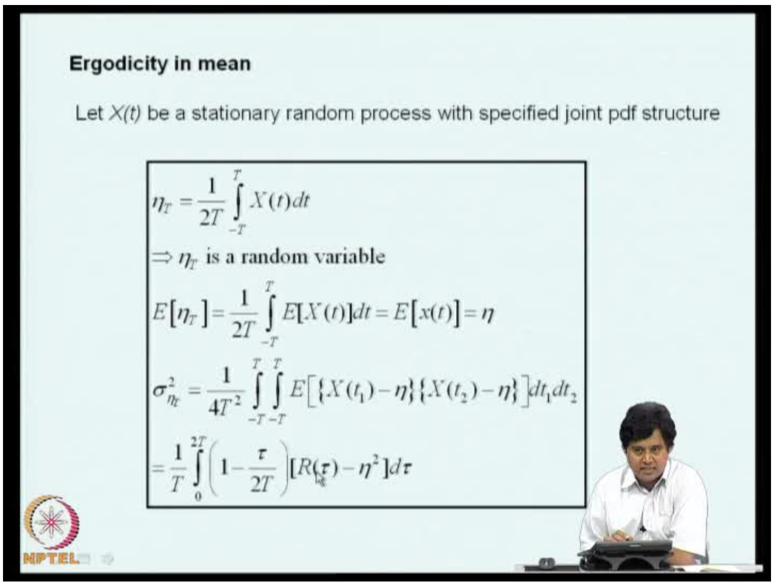

Ergodic processes are essentially stationary in nature; whereas a stationary process need not be Ergodic. We can elaborate on this later, what physically ergodicity means is that if you have a sufficiently long record of a stationary random process, you could deduce its properties from a single record for all practical purposes we will now talk about the equality that I was mentioning. So, let us consider X of t to be a stationary random process with the specified joint probability distribution structure.

Now, the for its sake of simplicity, I assume the time is defined symmetrically from minus T to plus T, the time average I denote as eta of T and this is the definition. Now, imagine we have ensemble of X of t I have different realizations of X of t and associated with each realization of X of t, I will get one number. Therefore, eta of T can be thought of as outcome of a random experiment. Therefore, it is a random variable right it is a transformation on a random process. Therefore, it is a random variable, the expected value of this random variable is itself the expected value of this that means, I have to take the expectation operator inside and I get this as expected value of X of t since X of t stationary this is eta.

Now, what is its variance? We can write the expression for variance and I get this expression. what I am doing when I equate, a random variable to a constant. This is a constant. There is no randomness about ensemble mean whereas this is a transformation on a random process. So, what do we expect? When do you say a random variable can be approximated by a constant. So, in the discussion on (( )) inequality I showed that if variance of a random variable is 0 the random variable can be interpreted has being deterministic.

So, if this assumption that eta of T is equal to eta is to be accepted, then we have to expect that the variance of the eta of T should become 0. So, that is the requirement for a process to be Ergodic in mean, so we will continue with this discussion in the next lecture. So, we will conclude this talk now.