Stochastic Structural Dynamics

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Lecture No. # 34

Probabilistic methods in earthquake engineering-3

We have been discussing application of principles of random vibration analysis and stochastic modeling, to problems of earthquake engineering; I will continue with that discussion. In today's class, we will talk about evolutionary random process models and models for multiple components of earthquake ground motions.

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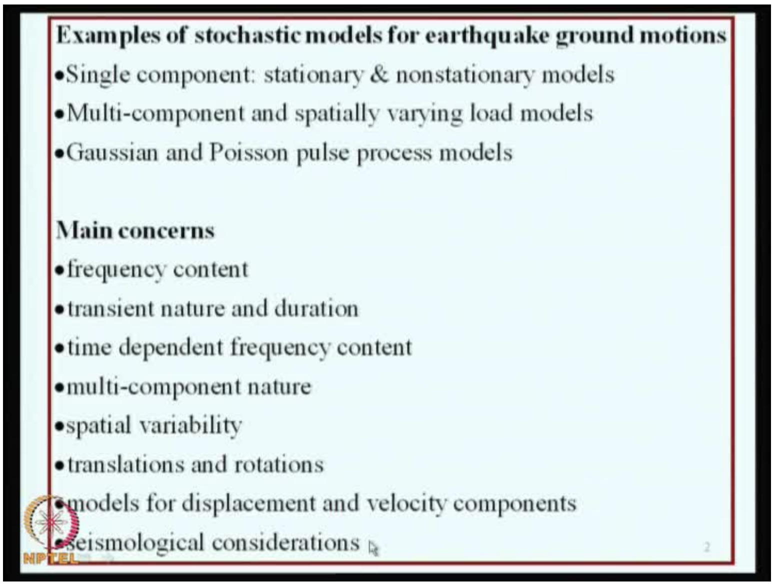

So, we will quickly recall what we have been discussing. The discussion is on stochastic models for earthquake ground motions; basically, earthquake ground acceleration displacement and velocity. We could model either the single component or multi-component and spatial variations. For single components of earthquake ground acceleration, we could use either stationary random process models or nonstationary random process model.

For multi component earthquake ground acceleration modeling, we need to use vector random process modeling. Spatially varying load models would include, again vector random process models, but with the additional index which denotes the space, point in space where the accelerations are modeled. We could use Gaussian random process models or Poisson pulse process models, to model the components of earthquake ground accelerations.

The main concerns in modeling are to capture correctly the frequency content, the transient nature and duration of the earthquake ground acceleration, and the time dependent frequency content, which is caused due to arrival of different waves at different times; each wave carrying different kind of frequency content. Then the multi-component nature of earthquake ground acceleration, earthquake ground acceleration is a vector at any point, it can be resolved into three translations and three rotations. So, at any given point, we need to model three translations and three rotations.

Spatial variability is another issue, that we will be discussing in due course, where the details of ground acceleration could vary from point to point, within the domain of a structure. For example, in the case of a long span bridge or a large dam, different points which are in touch with the ground may suffer different earthquake induced ground motions.

Now, we also need to model displacement and velocity components, in addition to the acceleration components. We could either adopt a data based approach or a heuristic engineering approach to model the ground accelerations or we could also take into account certain seismological consideration like, source mechanism, propagation of waves through geological medium, scattering of waves, so on and so forth.

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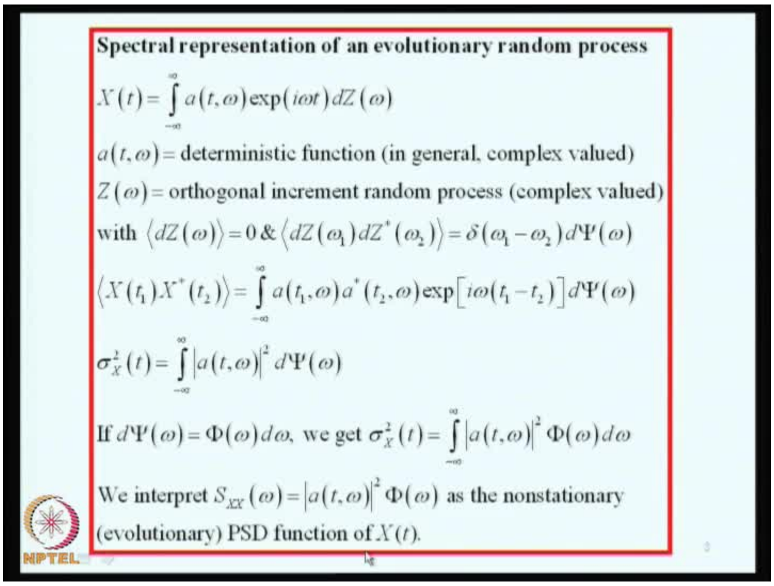

Now, we briefly discussed about Poisson pulse process model in the previous lecture. We will continue with that, to start with. So, I would like to discuss to start with the topic spectral representation of an evolutionary random process. So, let us consider a random process X of t, and consider its representation as integral a of t comma omega exponential i omega t dZ omega, where a t comma omega is a deterministic function; in general, it could be complex value. This Z of omega is a orthogonal increment random process, which is complex valued. With expectation dZ omega is 0, and expectation of dZ omega 1 into dZ conjugate omega 2 is a direct delta function multiplied by a function d psi of omega.

So, with this in mind, if we now consider expected value of X of t would be 0, because expected value of dZ omega is 0. Now, if you consider expected value of X of t 1 conjugate into X star of t 2, we can write this as write in this form. There is a direct delta function here, which enables us to carry out one integration; at the end of that, we get this integral.

Now, a t 1 equal to t 2, we get sigma variance which is sigma x square of t is a of t comma omega modulus whole square d psi of omega. Now, if this psi of omega is differentiable, we can write this as d psi omega, phi of omega equal to phi of omega into d omega. We get the variance to be modulus a of t comma omega whole square phi of omega d omega.

Now, we know that area under power spectral density of a stationary random process, it is variance. So, in analogy with that, we can now interpret the quantity a of t comma omega whole square into phi of omega as the non-stationary, for the evolutionary power spectral density function of X of t. If a is independent of time or a equal to unity, then the definition coincide with the definition of a power spectral density function, for a stationary random process.

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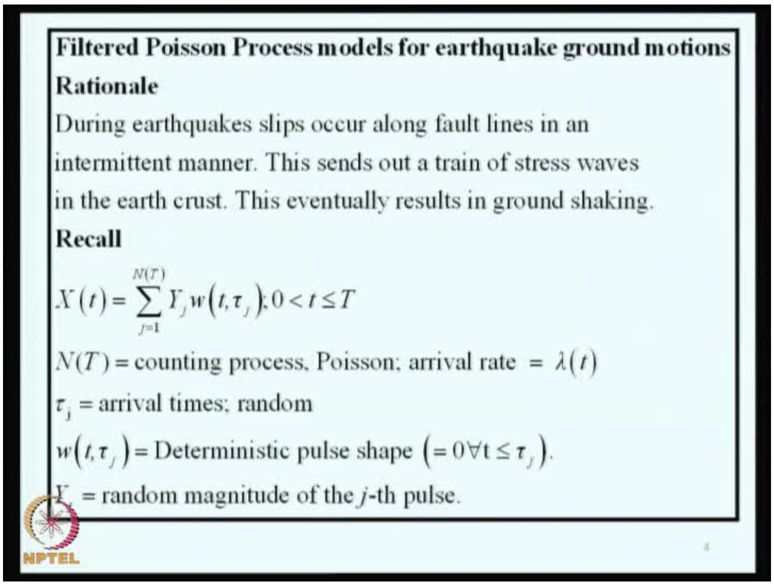

Now, we will recall the filter Poisson process models for earthquake ground motions, which are briefly mentioned in the previous lecture. Now, the rational here is, during earthquake, slips occur along fault lines in an intermittent manner; this sends out a train of stress waves in the earth crust, which eventually results in ground shaking.

So, what we do is, ground shaking at any point, we model as, summation of j equal to 1 to N of T Y j w t comma tau j, where N of T is an Poisson process which is the counting process, which counts a number of wave trains arriving at this given point, with a arrival rate of lambda of t; tau j are arrival times, which are random in 0 to capital T; this w is a deterministic pulse shape, which is 0 for t less than tau j and Y j is a random magnitude of jth pulse. So, this is the basic assumptions of this model.

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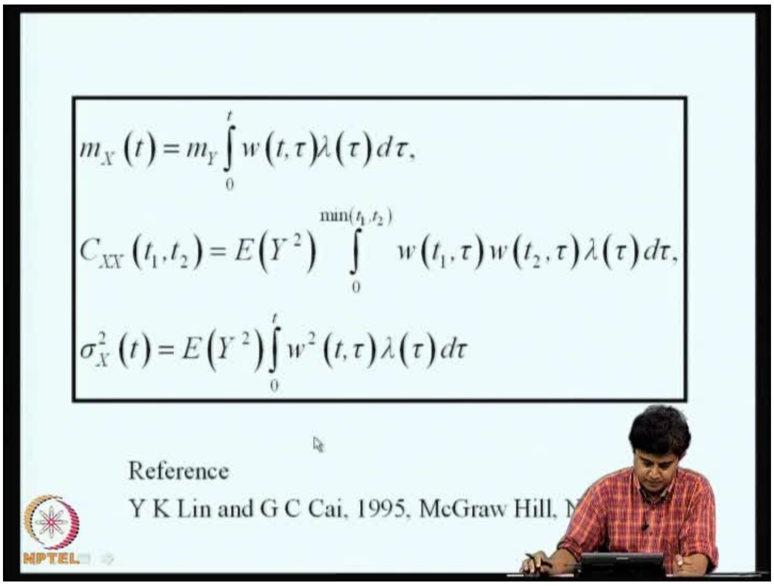

Now, I have shown in the previous lecture, that the mean of this such a random process is given by this; under co variance is given by this expression. This we have seen in the previous lecture, and variance, therefore, from the covariance, we can write it as E of Y square 0 to t w square t of tau lambda of tau d tau. So, details of this discussion are available in the book by Lin and Cai, which I will be following to some extent in this lecture.

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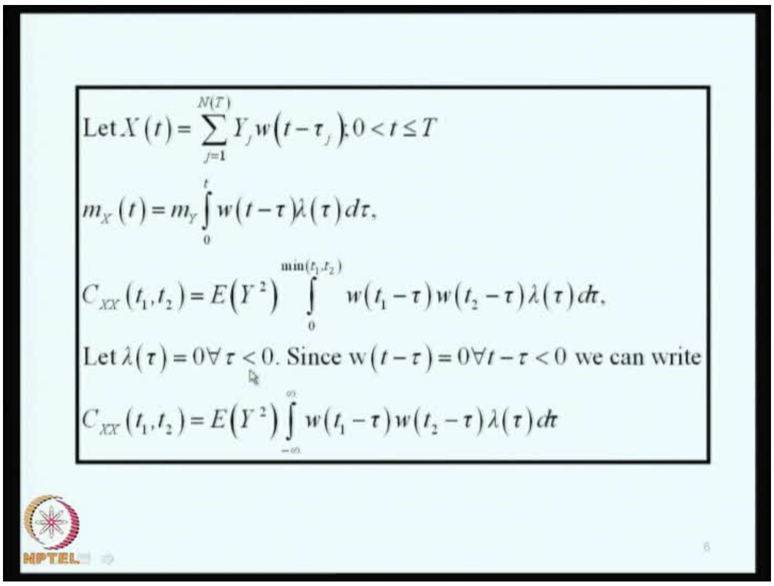

Now, let us consider again X of t as this random process, where now this pulse I am now assuming to be, instead of w being a function of t gamma tau, I will assume that it is a function of time difference t minus tau g. This as you can imagine, now we will be relating this to, the pulses will be interpreted as output of linear time invariant systems; that is the eventual goal in discussing this. But to start with, we will discuss the nature of moments of this random process. So, mean is given by m y 0 to t w of t minus tau lambda of tau d tau; that means, mean m X of t is given by the convolution of this pulse w and the arrival rate lambda tau.

The covariance is given by, 0 to minimum of t 1 comma t 2 w t 1 minus tau t 2 minus tau lambda of tau d tau. Now, if we now said lambda of tau to be 0, for tau less than 0, and this pulse we have already taken to be 0, for t minus tau less than 0, we can replace this limits from 0 to minimum of t 1 comma t 2 to minus infinity to plus infinity.

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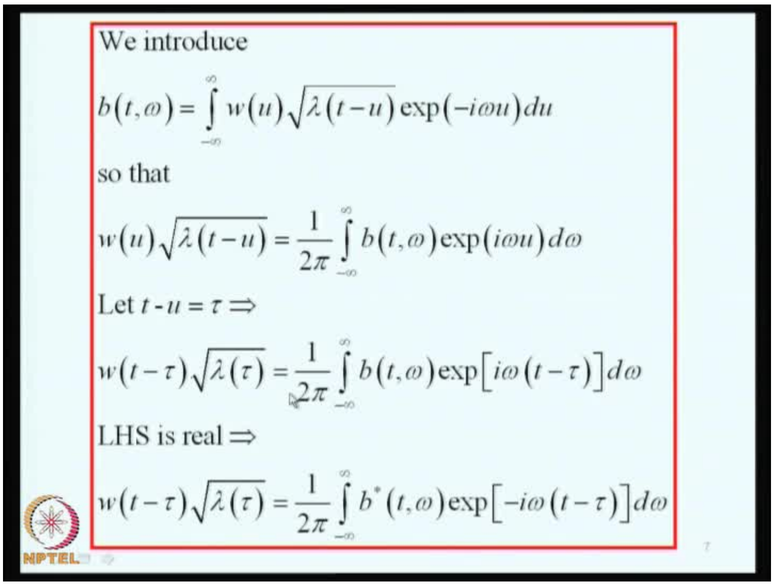

Now, we introduce the quantity b of t comma, which is minus infinity to plus infinity w of u square root of lambda t minus u exponential minus i omega u du, so that w of u into square root lambda t minus u is given by the inverse transform of this representation. And now in this integral, if we put t minus u equal to tau, we can simplify this, and the left hand side is real, therefore, we can also write the, this should be also equal to the conjugate of this.

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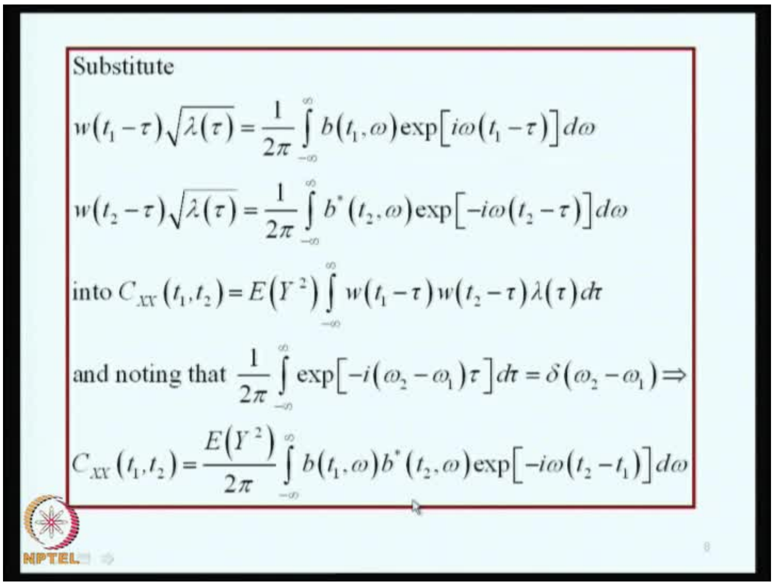

Now, based on this argument, if we now substitute w of t 1 minus tau square root lambda of tau equal to this and w of t 2 minus tau lambda of tau equal to this conjugate, and then put these representations into the expression for the covariance. We can show that this covariance is now given by, E of Y square 2 pi b of t 1 comma omega b star t 2 comma omega exponential minus i omega t 2 minus t 1 d omega, limits from minus infinity to plus infinity.

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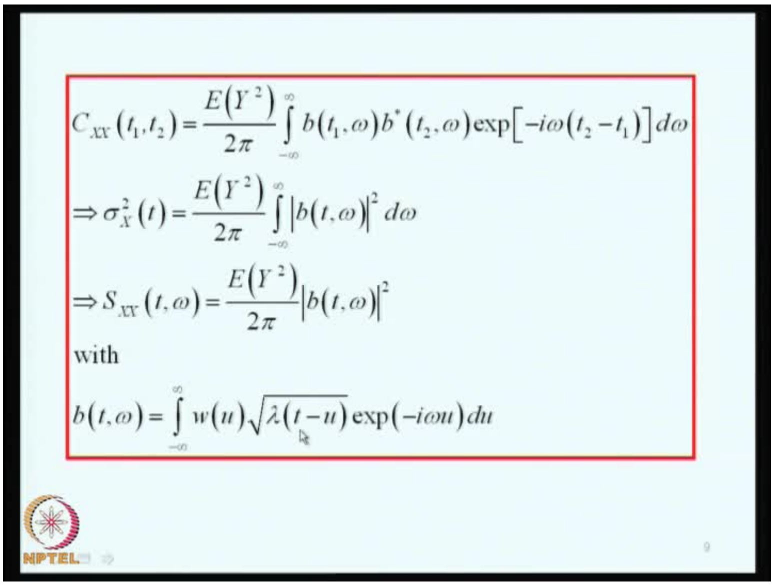

From this, if you now take t 1 equal to t 2, then I would get variance to be sigma X square of t is equal to E of Y square by 2 pi into modulus of b t comma omega whole square d omega integral minus infinity to plus infinity. Therefore, the power spectral density function of the pulse process that we are considering is given by, expected value of Y square by 2 pi into square of the absolute value of b of t comma omega, where b of t comma omega is related to the pulse shape and the arrival rate, through this relation.

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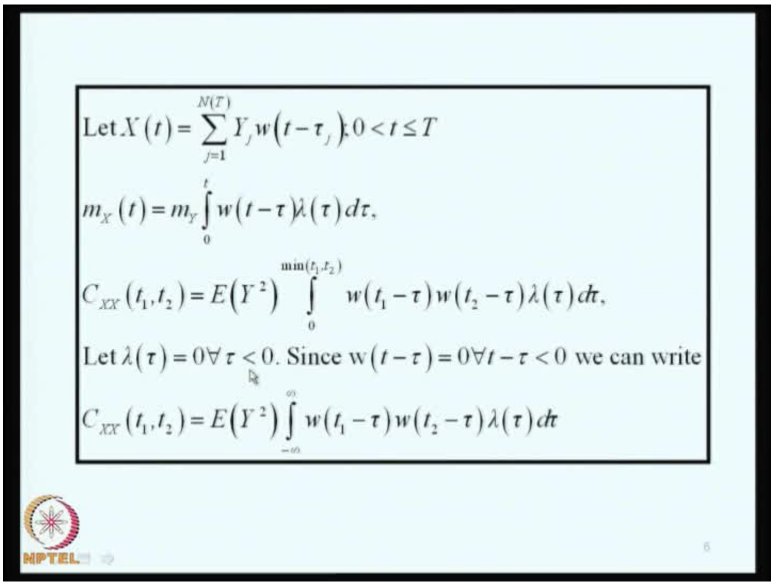

Or in other words, the point that is being made is that, this random process has the evolutionary power spectral density function. So, it can therefore be used for modeling the time varying nature of frequency content in earthquake ground acceleration.

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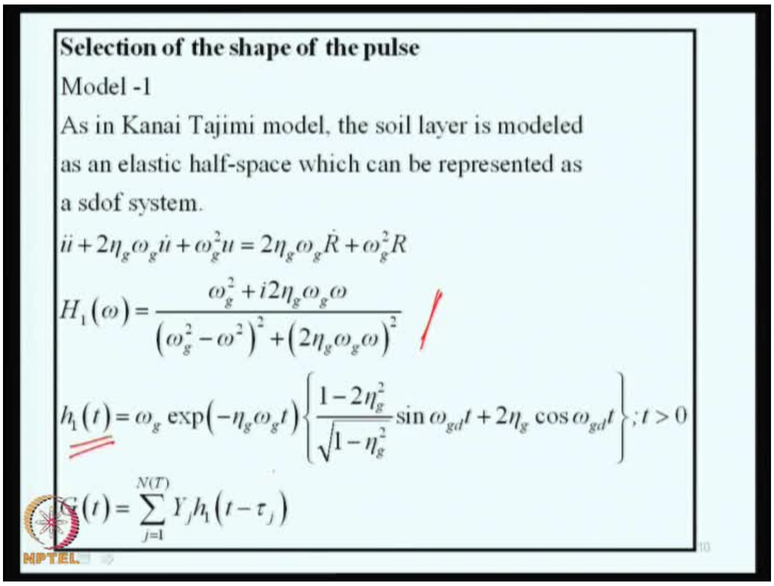

Now, so far I had not yet discussed how to select the pulse shape w. Now, there are various possible choices for this; one simple model would be, we will assume that, as in Kanai Tajimi model, the soil layer is modeled as an elastic half space, which can be represented as a single degree freedom system. So, we write the equation for the response of the soil layer; this is the u of t is absolute displacement, and we get this expression; and the transfer function for this system is, H 1 of omega is given by this expression.

Now, the Fourier transform of that, is the impulse response associated with this and that we can show by inversion to be this function. Now, one of the simple models for evolutionary random process model for earthquake ground acceleration would be, to use this h 1 of t in the Poisson pulse model; that means, at the bedrock level, impulse is arrived randomly in time and they propagate through the soil layer, and each pulse that is observed at the ground surface is the response of the soil layer, to the unit impulse applied at the bedrock level.

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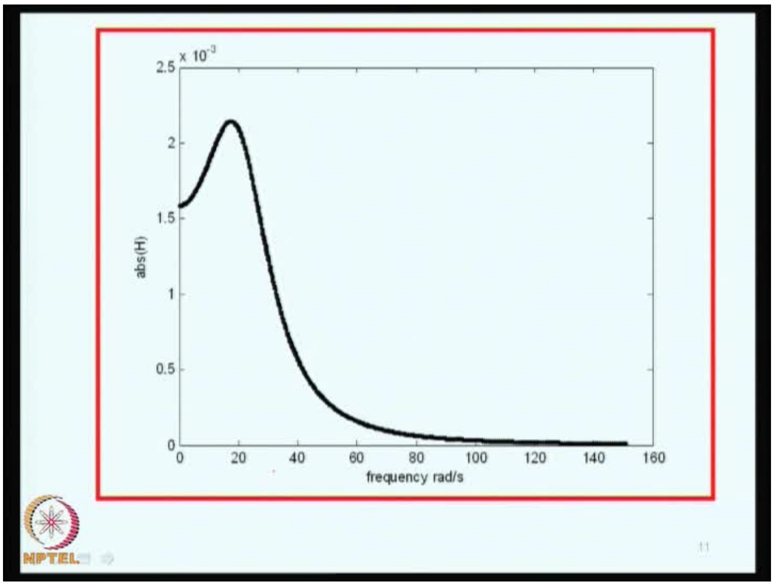

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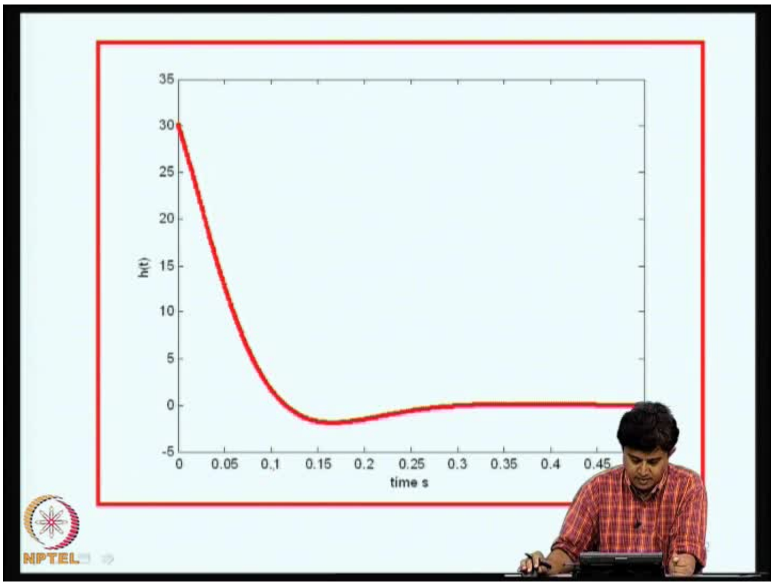

So, this is the transfer function and this is the Fourier transform in time, is typical plot of that.

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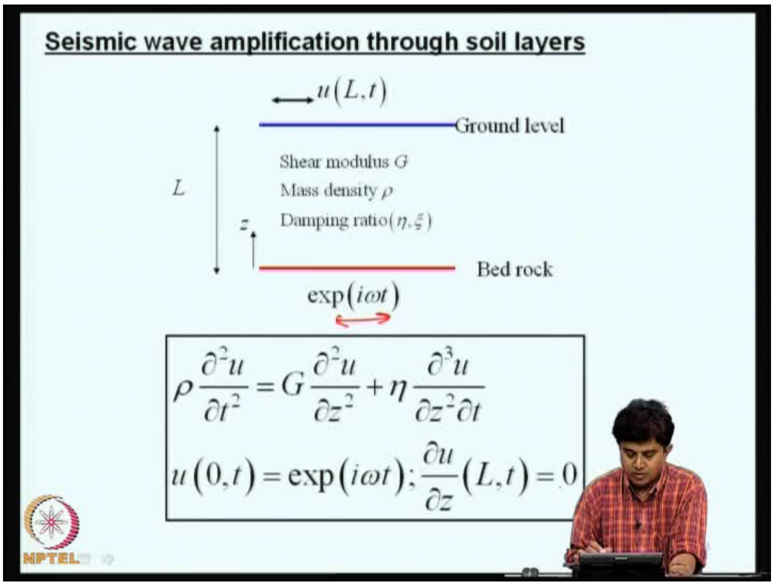

Now, we have use single degree freedom system representation for soil layer. This assumption can be relaxed, we can as well represent the soil layer as an elastic medium; to start with as a one-dimensional elastic medium, we can apply a harmonic excitation at the bedrock level, and see how much amplification is possible at the top. So, this will give us a transfer function, and the Fourier inversion of that, would give us the pulse shape, that we can use in Poisson pulse process model.

So, this we have discussed earlier, when we discussed continuous systems. So, the governing equation here can be written as a wave equation, rho dou square u by dou t square is equal to g into dou square u dou z square plus eta into s; this as dependent damping - viscous damping - and at z equal to 0, we are applying harmonic displacement, and the top is stress free, therefore, the boundary condition dou u by dou z at l comma t equal to 0.

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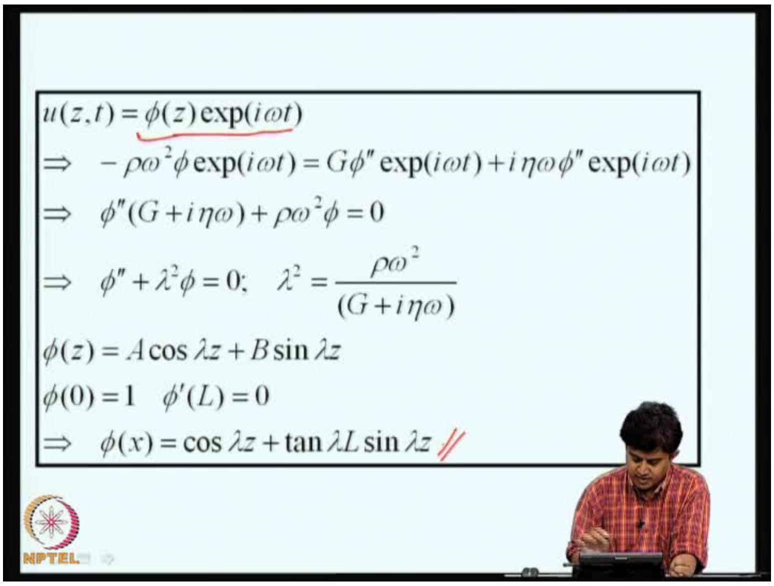

Since the system is linear and the actions are harmonic in time, we can expect the response also to be harmonic with an unknown amplitude. So, if we substitute this into the governing equation, we will get an ordinary differential equation for this phi of z, with the boundary condition phi of 0 equal to 1 and phi prime of L equal to 0; and this quantity lambda square is a complex quantity, rho omega square divided by g plus i eta omega.

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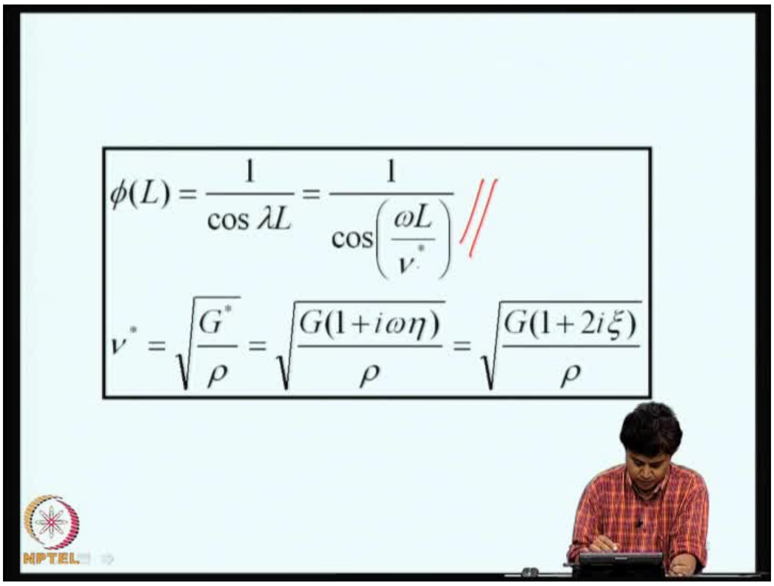

So, we will get phi of x to be this. And if we evaluate this at x equal to L, we get the transfer function to be 1 by cosine of omega L by nu star, where nu star is this where propagation velocity.

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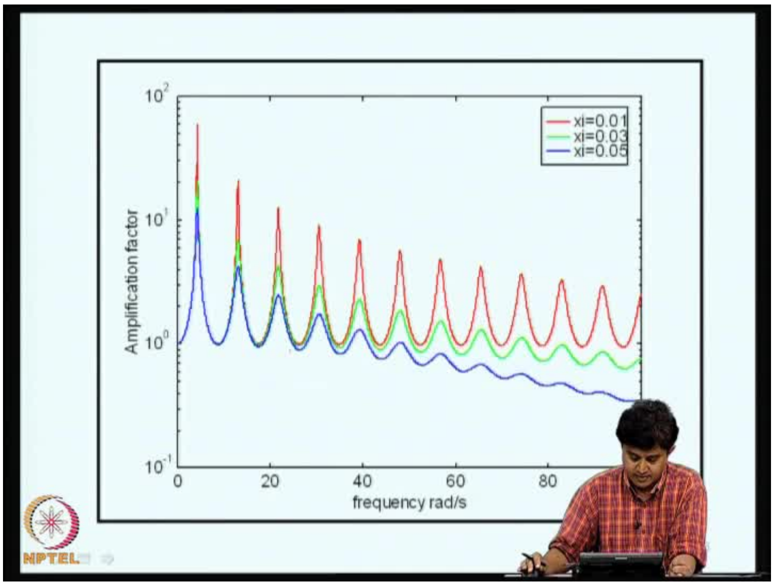

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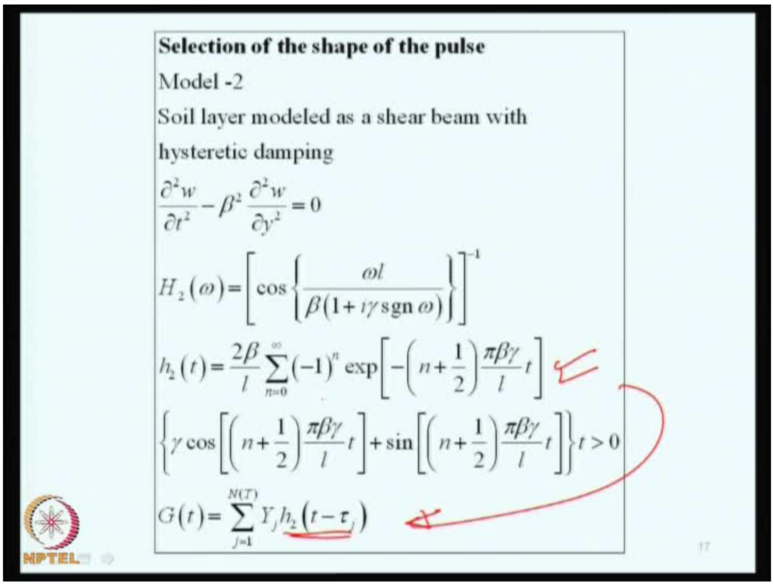

So, this is how the amplification factor looks like. So, what we could do now is, we can select the pulse shape our Poisson process model, to be the Fourier transform of this transfer function, right. So, we get this as or h of t.

Now, that can be used in my definition of Poisson pulse process; this h 2 of t minus tau is the pulse shape, that I am deriving by treating soil layer as a one-dimensional shear beam. That damping it could be hysteretic or it could be viscous, it could be train rate dependent or velocity dependent; those variations can be easily introduced into this model.

So, this is a generalization of a Kanai Tajimi power spectral density function model. This is evolutionary Kanai Tajimi power spectral density function model, and not only that, the soil layer now is modeled as a one-dimensional shear beam, as a continuous system than as a single degree freedom system.

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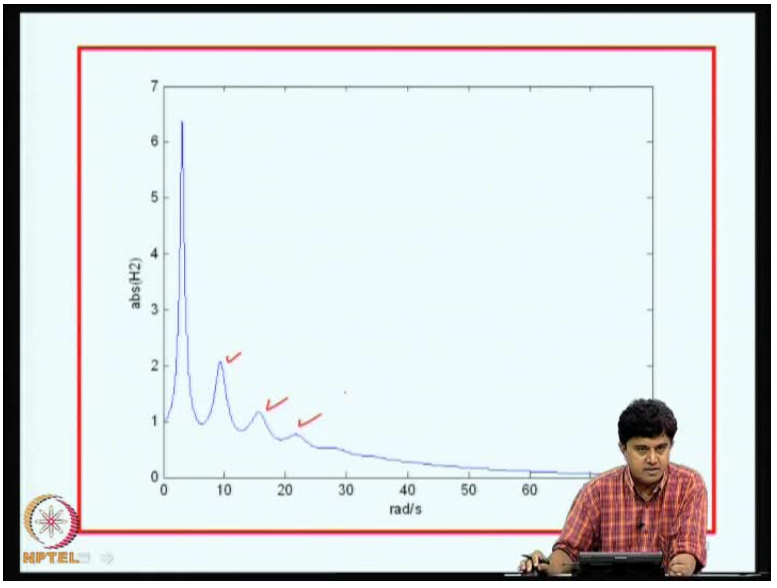

So, this is the absolute value of the transfer function for illustration; this is how it typically looks like. Kanai Tajimi power spectral density function model will assume only one peak, whereas this model permits additional peaks, that may be present in the soil layer, where the frequency range of interest.

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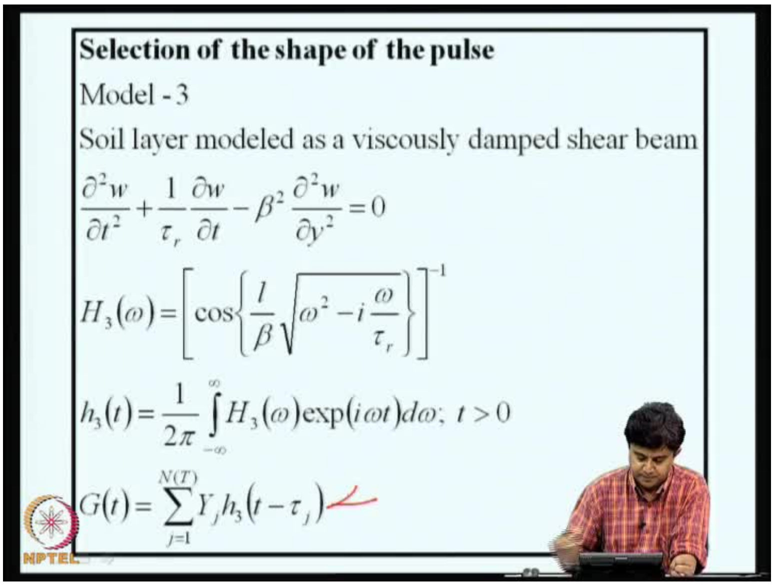

Now, we can exchange this argument. And the analysis that I showed just now was for hysteretic damping; now, we can use viscously damped shear beam model. Again, we get H 3 of omega to be this, this is the transfer function, and the Fourier transform of this is given by this, and we can substitute that and get this model.

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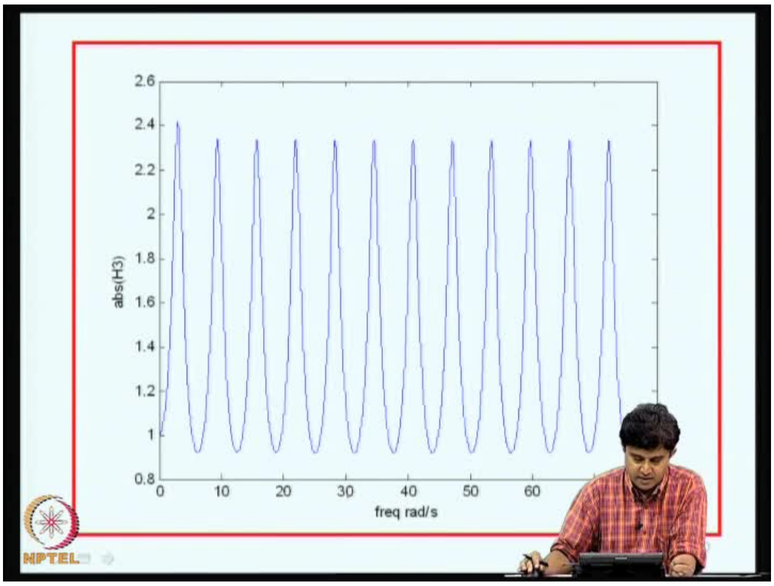

Now, this transfer function has this appearance. In this model, the model bandwidths will be equal for all modes, and if the damping property is like this, we could use this model.

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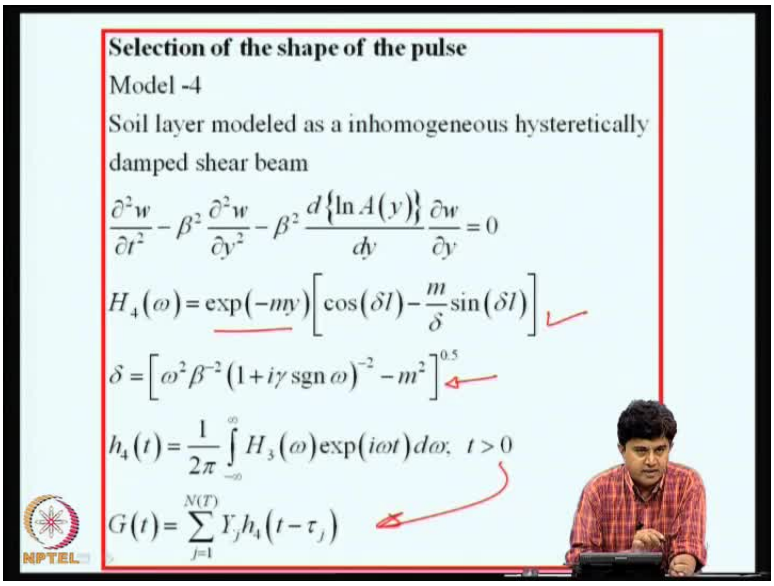

Now, as we go deeper into the soil layer, the density of the soil layer increases; there will be a inhomogenities, and that can be represented as a soil layer, modeled as a inhomogeneous, hysteretically damped shear beam. Again, in this particular case, certain close form solutions are possible, and we can show that, the transfer function is given in terms of cosine and sine functions, which are exponentially decaying, and delta is this quantity; and we can take the Fourier transform of this and substitute into this, and get a more sophisticated model.

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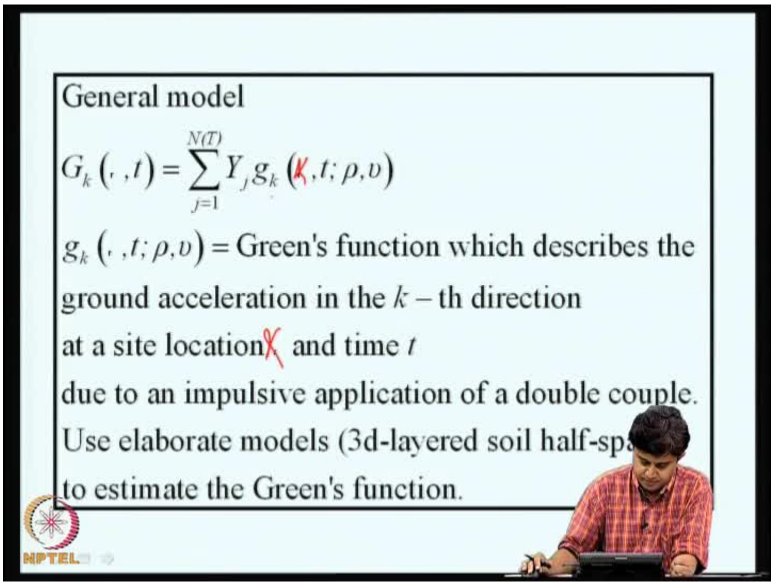

So, one can a generalized this; in fact, we can use a three-dimensional velocity model if you wish. For example, this g k of t rho nu can be used as a greens functions, which describes the ground acceleration at the kth direction at a site location, I think there is some symbol missing here - call it as chi if you wish, and time t, due to an impulsive application of a double couple.

Now, we can use elaborate models 3d-layered soil half space to estimate this greens function; so, the detail to which would like to model this greens functions, is the matter of choice and matter of accuracy that is needed from these models.

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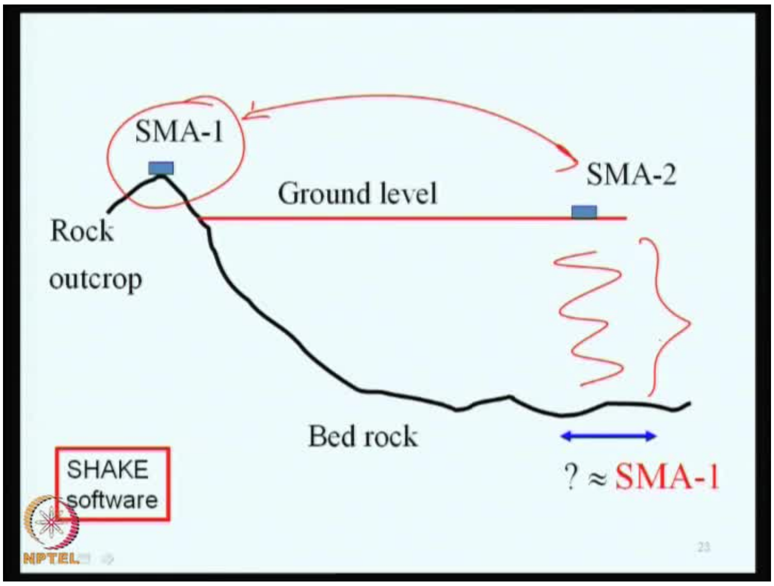

Another way of empherically arriving at the pulse shape would be to, if we have a setting like this, SMA is a strong motion auxiliary graph. Suppose, if we have strong motion graph on a outcrop and on a soil layer, we would like to know how waves amplify here, but we will not have details of ground bed rock displacements here; so, we could use this record that we have. And if you take, for example, the cross PSD between this observations and this observation, we will get details of the transfer function here. So, this is, some of this could be used to derive this pulse shapes.

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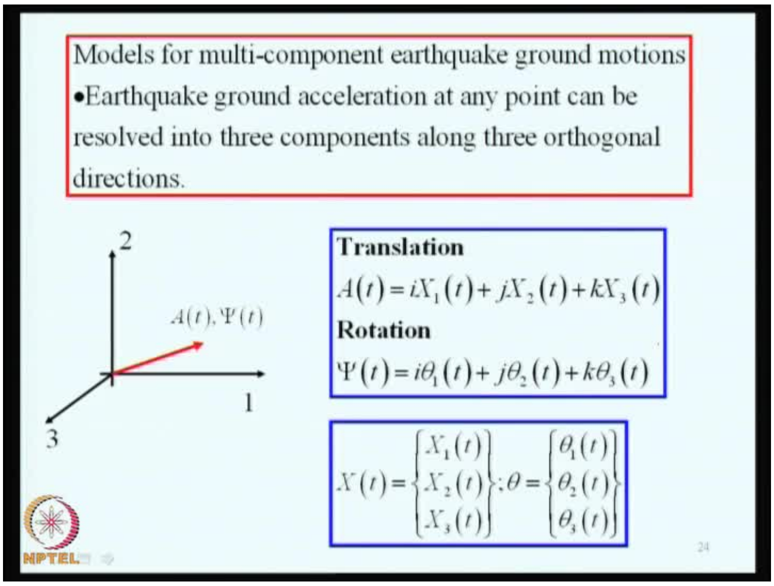

So far I have discussed about modeling a single component of earthquake ground acceleration. As I already mentioned, earthquake ground acceleration at a point can be resolved into three components along the three orthogonal directions, so if A of t represents the acceleration vector, it can be resolved along direction 1 2 and 3. And I can write a of t as, iX 1 of t plus jX 2 of t plus kX 3 of t; so, X 1, X 2, X 3 are the translations along 1, 2, 3 translation acceleration.

Similarly, rotation also can be written as a vector psi of t; it can be resolved into three components, theta 1, theta 2, theta 3, and we can write psi of t as i theta 1 plus j theta 2 plus k theta 3. So, i j k are unit vectors along 1 2 3. Alternatively, I can assemble these components X 1, X 2, X 3 into vector X of t, and theta 1, theta 2, theta 3 into vector theta of t, and we try to model X of t and theta of t as vector random processes. We will focus our discussion on modeling of X of t, that is acceleration associated with translations.

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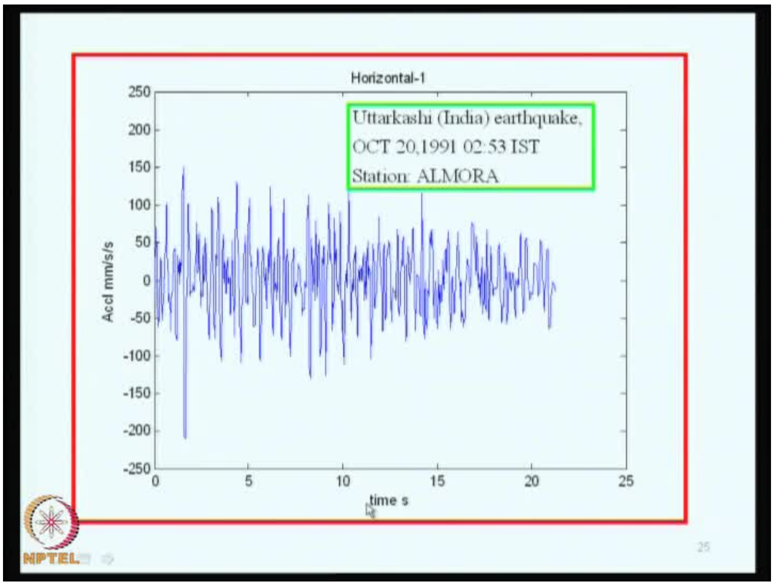

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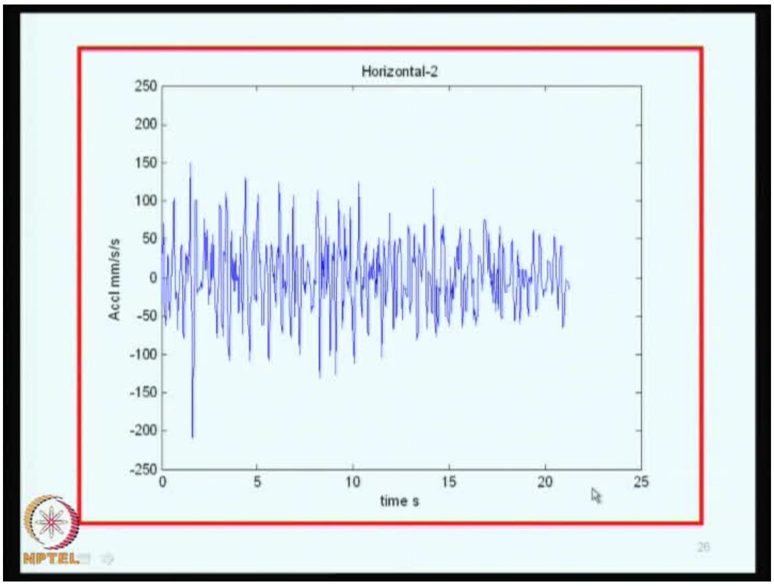

And I am just showing for the sake of illustrations, instrumentally recorded ground acceleration in one of the events in India. This is one of the components in the horizontal plane, y axis is accelerations in millimeter per Second Square and this is time.

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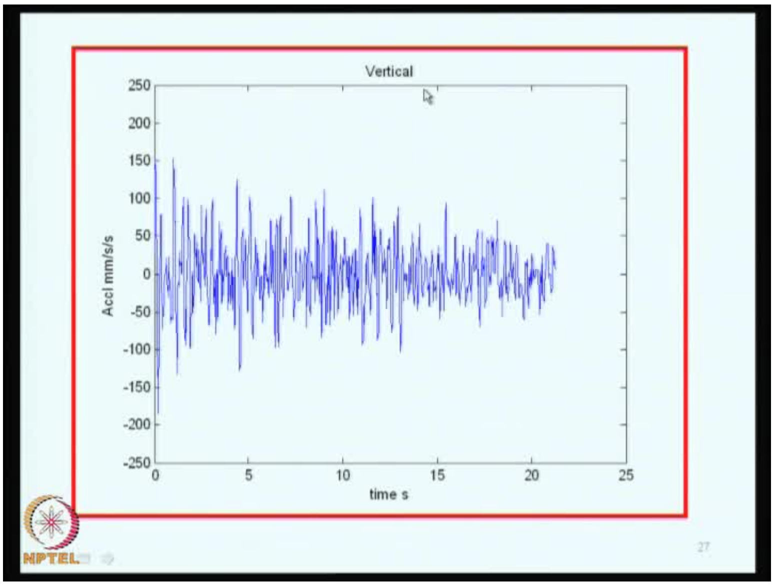

This is the second component, this is the vertical component. So, these are instrumentally recorded ground accelerations using triaxial accelerometers. So, we can expect that, we will be having this type of data, in the event of an earthquake accelerations earthquake event; and our objective is to models these three components as a vector random processes.

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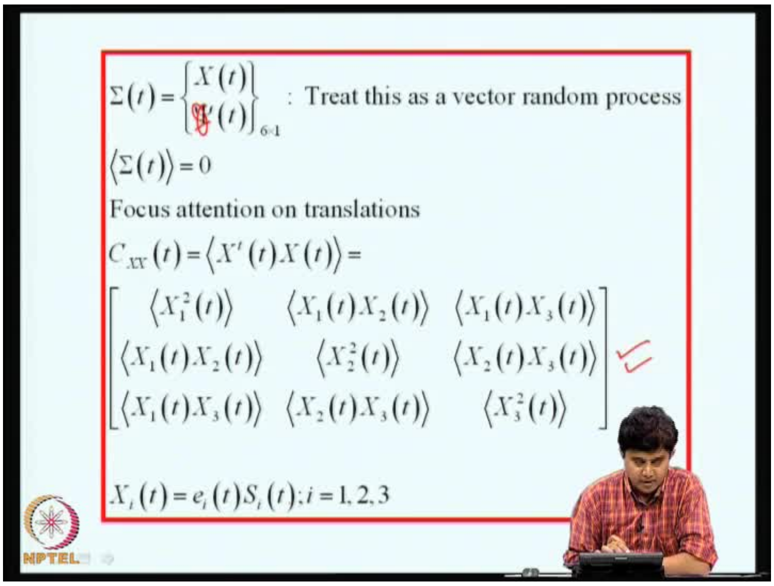

So, a general approach would be to take X of t and psi of t to be a vector, say. sigma of t and treat this as a vector random process. The mean of this process, we take it to be 0, and suppose if we focus attention on only translations, we can consider for instance the covariance - covariance at same time t of these three components - and we get this 3 by 3 matrix which is symmetric.

We could model each component X 1, X 2, X 3 as a nonstationary random process e 1 of t into S 1 of t, e i of t into e S i of t, where I runs from 1 to 3; e i of t is the deterministic envelope, which imparts the non stationarity trend to the signal, and S i of t is a stationary random process, which has the requisite frequency contents observed in ground accelerations.

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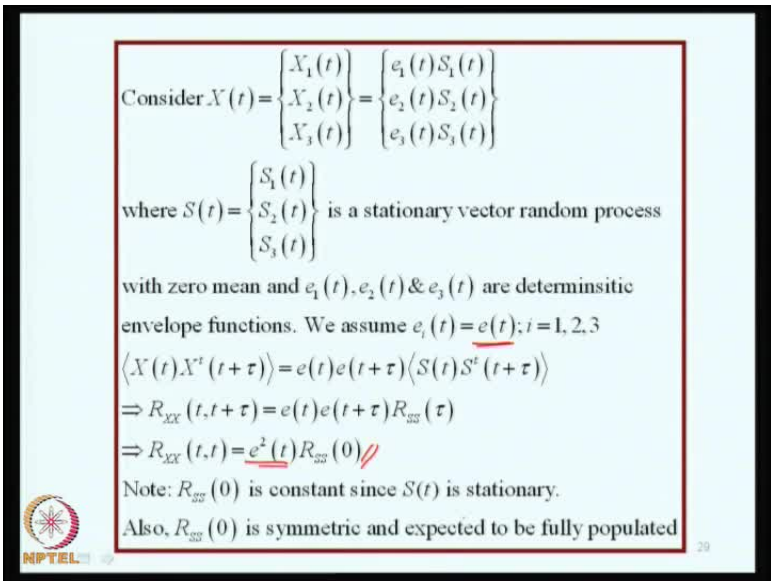

So, let us now consider for sake of discussion, X of t to be given by the vector e 1 of t e 2 of t e 3 of t into S 1, S 2, S 3. Here S of t is a stationary random process with 0 mean; e 1, e 2, e 3 are deterministic envelope function. We assume that, all these non stationarity trends for the three components are identical, that would mean e 1 of t equal to e 2 of t equal to e 3 of t and I denote this is e of t.

Now, if we consider the expected value of X of t into x transpose t plus tau, I get e of t into e of t plus tau into S of t S t of t plus tau; that would mean R XX of t comma t plus tau is e of t into e of t plus tau into R SS of tau. If you take tau to be 0, that would mean you are considering covariance of X of t, that is expected value of X of t into x transpose X of t, we get this function to be this.

So, here, this R SS of 0 is constant, because S of t is the stationary random process; it is not function of time; the time variation is impacted only through the envelope. Now, this is the 3 by 3 matrix of constants; now, it is symmetric and expected to be fully populated.

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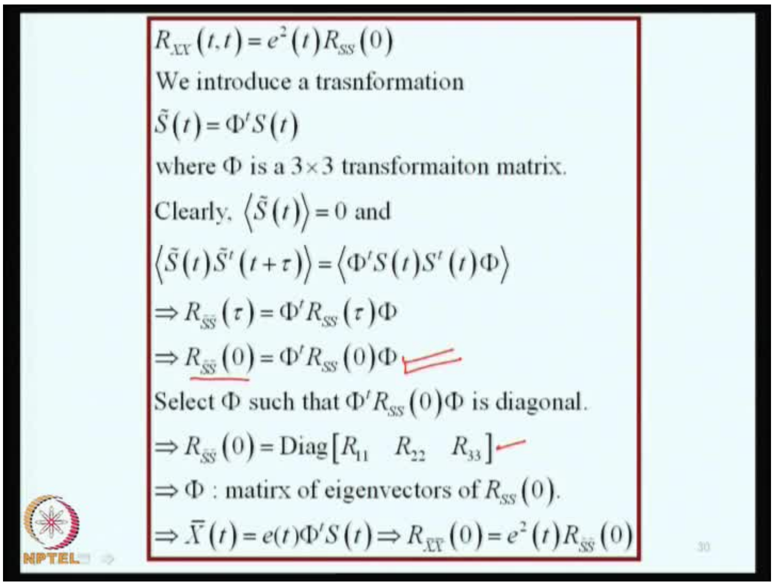

Now, what we could do now is, we can transform S of t to a new coordinate system through this transformation matrix phi of phi transpose. So, S tilde of t is phi transpose S of t, where phi is a 3 by 3 transformation matrix. Now, clearly, if expected value of S of t is 0, expected value of S tilde of t is also 0; and if we now compute the covariance, S tilde of t into S tilde of S tilde transpose t plus tau, this is given by phi transpose S of t and S transpose phi. And this expected value of S of t into S transpose t is nothing but R SS of tau, and if tau equal to 0, I get this expression R of S tilde 0 is phi transpose R SS 0 phi.

Now, how to select this phi matrix? We will select phi, such that, upon coordinate transformation, this matrix becomes diagonal. So, we want R S tilde S tilde is to 0 is to be a diagonal matrix. So, how do we select this? We need to select phi to the matrix of eigen vectors of R SS of 0; R SS of 0 3 by 3 matrix. This is quite similar to the way we find principle stresses, in a problem in three-dimensional sorry mechanics; if we are given in the state of stress at a point, we find the eigen values of the stress matrix. We solve the eigen value problem associated with the stress matrix at a given point, and determine the principle axis and principle stresses.

So, in the same sense, we are finding now orientation for the axis X 1, X 2, X 3, in which the covariance matrix of the stationary component, I evaluated at the same time t is diagonal.

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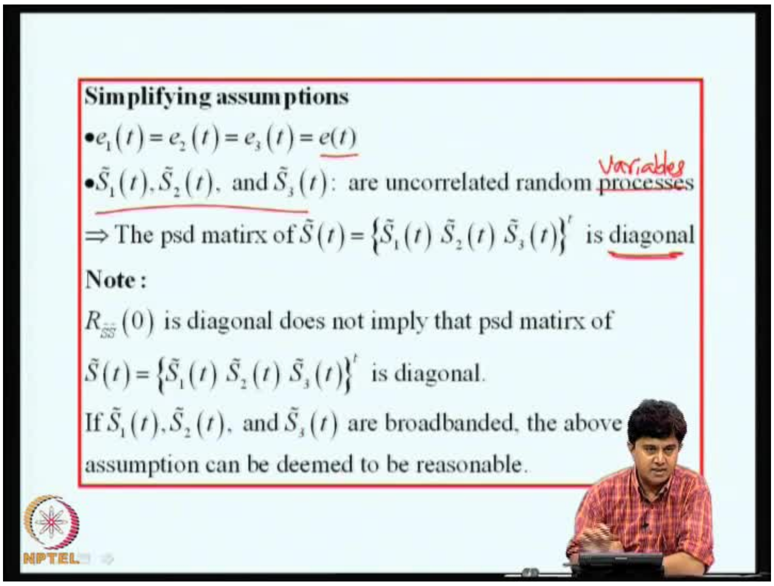

Now, we recall that e 1, e 2, e 3, we have taken it to be e of t. And if we are in the so called principle axis, where the covariance of S tilde of t process is diagonal, then the random variables S tilde 1 of t S tilde 2 of t S tilde 3 of t are uncorrelated random variables.

Now, we make a further assumption that, at same time t, these three random variables are uncorrelated, but we now extrapolate this model to assume that S 1 tilde S 2 tilde S 3 tilde are uncorrelated. This is an assumptions; consecutive that assumptions is acceptable, the PSD matrix of S tilde of t is now will be diagonal. Or in other words, we are now establishing coordinate system, in which this three components of ground accelerations are mutually uncorrelated; in that sense, these axis are orthogonal, but we must bear in mind that, the fact that, R S tilde S tilde 0 is diagonal does not implied that PSD matrix of S tilde of d is diagonal; this is an assumptions.

However, if these processes, S 1 tilde, S 2 tilde and S 3 tilde are broad banded, one could consider the above assumptions to be reasonable.

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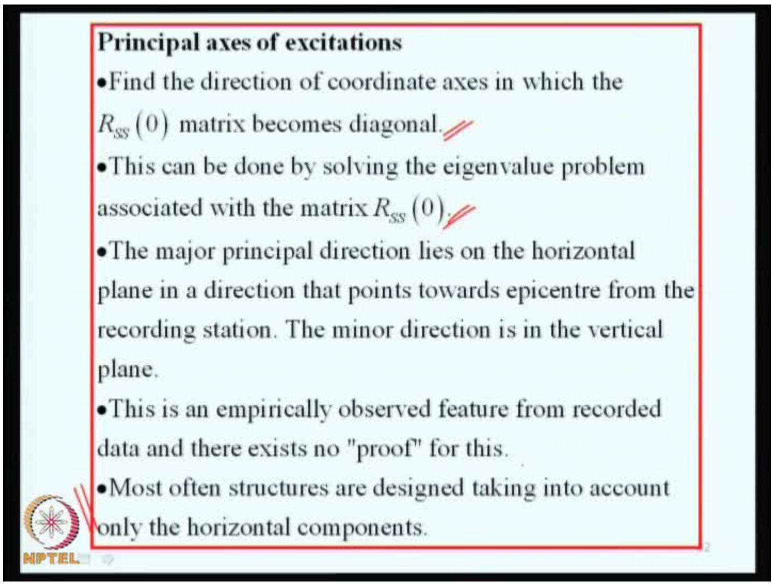

So, we find principle axes of excitations. How do we do that? Find the directions of coordinate axis, in which R SS of 0 matrix is diagonal. This can be done by solving the eigen value problem, associated with the matrix R SS of 0. It is found by analyzing recorded ground accelerations, that if we were to find this principle axis, the major principle axis lies in the horizontal plane and connects the observation point to the epicenter.

The minor axis, it is the third axis, will be in the vertical direction; the intermediate axis will be in the horizontal plane normal to the line joining the observations station to the epicenter. This is an empirically observed feature and there is either no theoretical proof or a proof based on theory of wave propagation, simply an empirically observed feature in recorded strong ground motions.

Now, we must bear in mind that, in most of the design problems, the structures are often designed only for the horizontal component. Now, this seems to be justified, because the principles axis is in the horizontal plane. And if we consider the only the horizontal components in design, it seems reasonable assumption; although in reality, there will invariably the vertical component will be also present.

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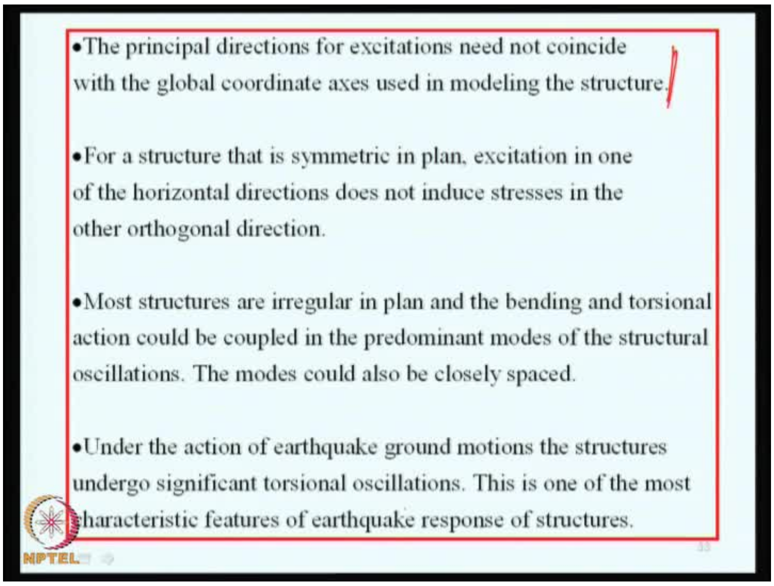

Now, the excitations have a set of principle direction, but the coordinate system in which we model the structure - that is a global coordinate system for the structure - there is no reason why it should coincide with the principle excitation directions. In fact, for a structure that is symmetric in plan, excitation in one of the horizontal direction does not induce stresses in the other orthogonal direction. So, in that sense, the structure also can be part of having certain principle axis. So, the principle axis of the structure, if it exist, need not coincide with the principle axis of the excitations. But however, most structures are irregular in plan, and the bending and torsional actions could be coupled in the predominant modes of the structural oscillations; the modes also could be closely coupled.

Therefore, under the action of earthquake ground motion, the structures undergo significant torsional oscillations. This is one of the most characteristic features of earthquake response of structures. So, to be able to model this correctly, we need to take into account, the multi-component nature of the ground accelerations and the orientation of principle axis of excitations, with respect to the axis of the structure, what that we choose to model the structure.

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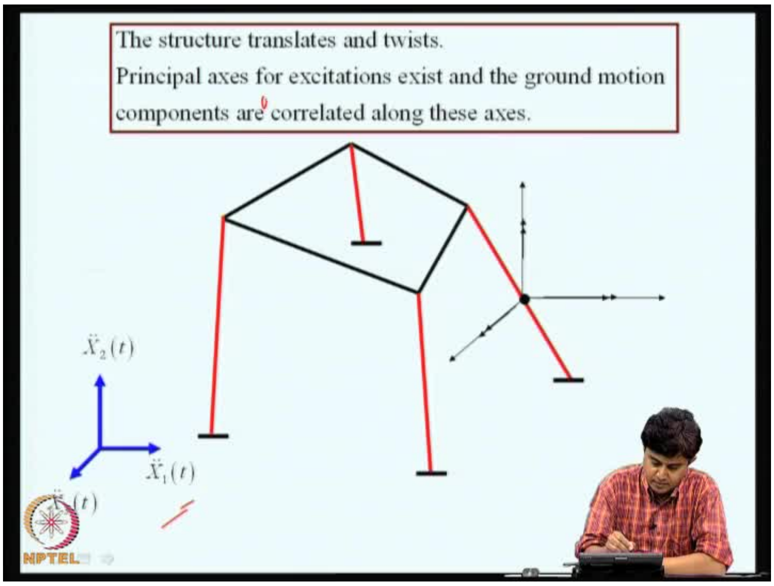

So, to emphasize that, if this blue lines represent the principle axis of excitations, a structure like this under the action of these excitations would not only translate, but also would twist. So, here, the principles axis for excitations, we are assuming that they exists and the ground motion components are uncorrelated along these axis.

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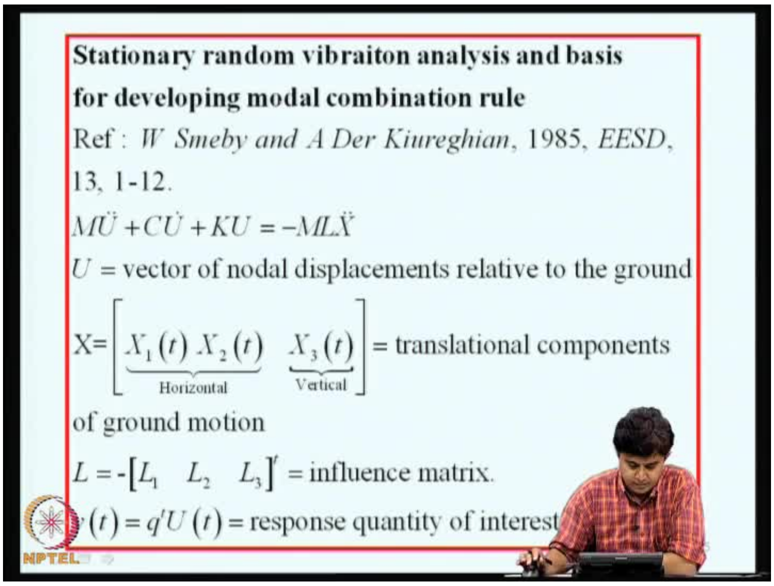

Now, let us consider the problem of stationary random vibration analysis and basis for development of model combination rules. This discussion is based on the results presented by the Smeby and der Kiureghian, in the earthquake engineering and structural dimensional journal.

Now, we write the equation of motions as, MU double dot plus CU dot plus KU equal to minus MLX double dot, where U is the vector of nodal displacements relative to the ground; and X is X 1 of t X 2 of t and X 3 of t, where X 1, X 2 are horizontal components and X 3 of t is the vertical components. So, these are the transitional components of the ground.

L is the influence matrix; these we have encounted in our early discussion on structural dynamics. Now, suppose if we are interested in typical response quantity, say v of t, which is q transpose U of t; that means, a linear function of the basic states of the system, if this is what we are interested in. It could be, for example, reaction transpose to the support or interest to a drift, and so on and so forth.

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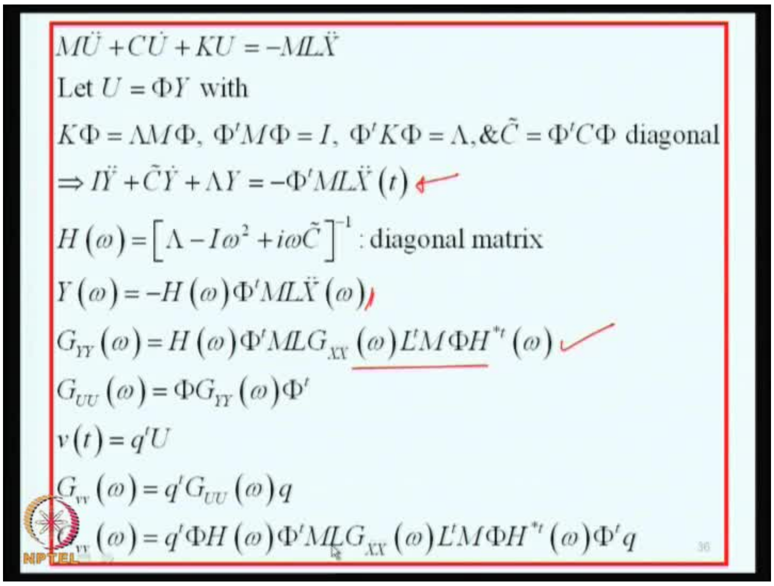

Now, we use the normal mode decomposition method; we transform the displacement vector, U is equal to phi Y y, where phi is the solution to the eigen value problem, K phi is equal to lambda M phi, and phi is taken to be mass normalized - that is phi transpose M phi i - phi transpose K phi is diagonal matrix of squares of the natural frequencies, and C tilde is diagonal matrix phi transpose C phi; that means, we are assuming the damping to be classical.

So, upon this transformation, we get this set of uncoupled equations for the new variable Y; here, I is the identity matrix, C tilde is diagonal, capital lambda is diagonal. So, the transfer function matrix, H of omega will be a diagonal matrix. So, Y of omega, we can write notionally in this form; we are assuming, we are writing Fourier transform of X double dot of t, but this is being done with explicitly to derive the power spectral density function. So, we follow the, this Fourier transform is taken to mean, the Fourier transform of a windowed sample of X double dot of t.

Using the principles of the definition of power spectral density function, we get the expression for the matrix of power spectral density functions, in the generalized coordinates in this form here. And in the original coordinate system U, we get, once we get G YY, we can get phi into G YY phi transpose. And for the quantity of interest, v of t is q transpose U, the power spectral density of this scalar quantity can be given as q transpose G UU into q. So, we can express G vv of omega; in this form, there are several matrices here, all of these are known and this is the power spectral density matrix of the input ground acceleration, and this is a diagonal matrix.

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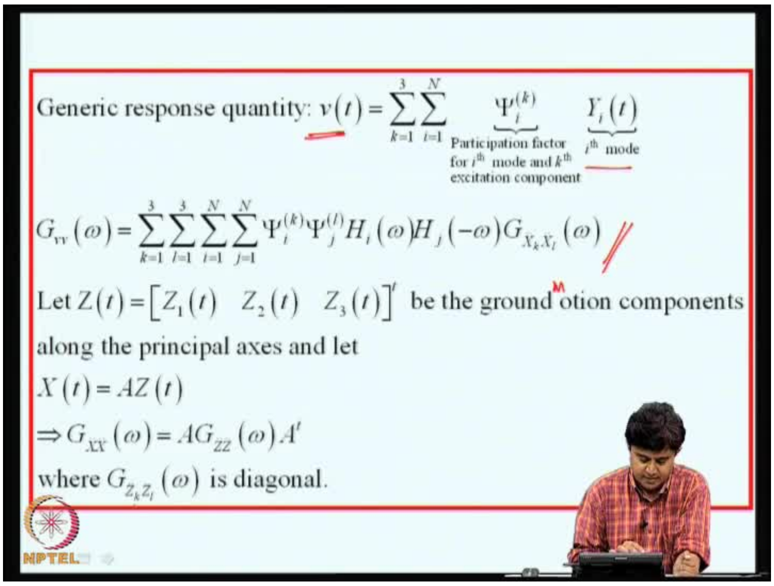

This of course can also be written in a form of double summation - multiple summations. Suppose, if we take generic response quantity v of t, we can always write this as psi i of k into Y i of t, where psi i of k is the participation factor for ith mode and kth excitation component.

Now, if the structure is acted upon by only one component of ground acceleration, we have already known what are model participation factors for each mode. Now, there are three excitation component, therefore, these participation factors should have further index k, which indicates a components. So, these are the generalized coordinates - this Y i of t.

Now, power spectral density function in terms of this, in the form of a double summation can be given in this form. Now, if Z of t equal to Z 1 Z 2 Z 3 is with a ground motion components along the principle axes and if you write X of t is a Z of t, we can write G X double dot X double dot omega as a G ZZ of omega a transpose. Now, if X of t is not already in the principle directions, we have to make these transformations. So, we get G ZZ k Z double dot l omega is diagonal.

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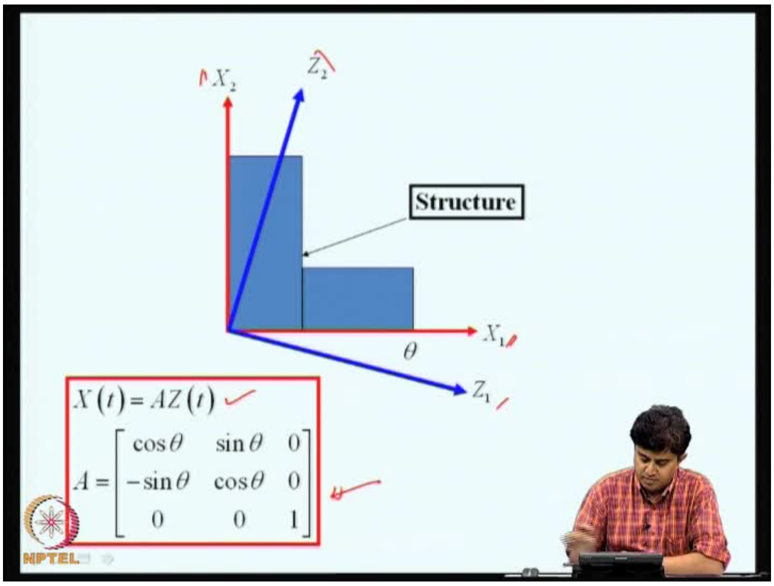

And how do we find these a matrix? Suppose, there is a l shaped structure as shown here, with coordinates X 1 and X 2; and if Z 1 and Z 2 are the principles axes for excitation, then X of t can be related to Z of t through this coordinate transformation, where a is the matrix of direction cosines, which is well known orthogonal matrix.

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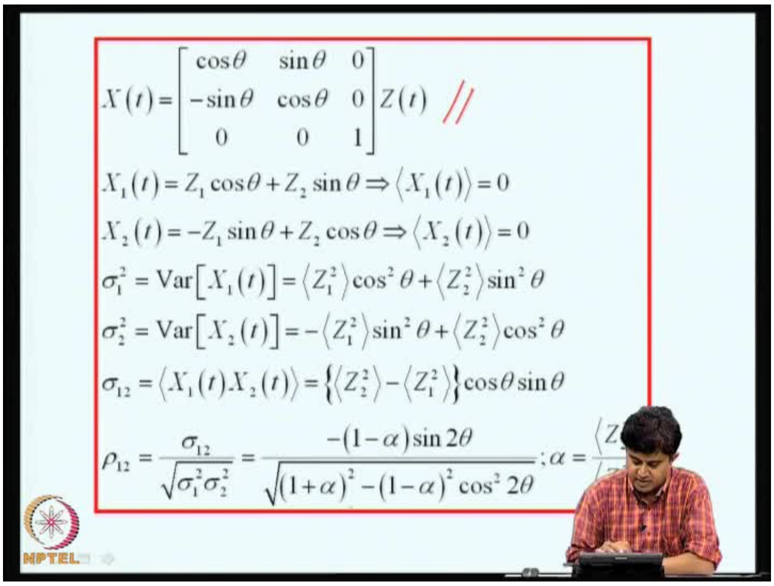

Therefore, X of t is given by this, and consequently, I can write X 1 and X 2, where assuming that the vertical axes of coincide, only there is a rotation in the horizontal plane. So, we can make these transformations, and examine the variances in the X 1 X 2 X 3 system and we get this, and we define rho 1 2 as a correlation coefficient associated with X 1 X 2; and in terms of theta, we get this in this form.

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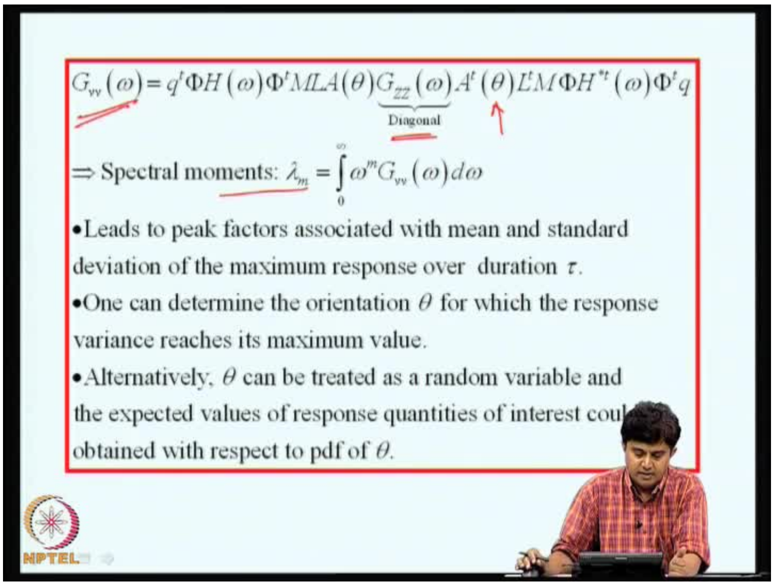

Now, if we substitute all this, we get this final expression for the power spectral density function of the required response variable. This is the diagonal matrix, that is sitting inside this parallely long expression. And once we know the power spectral density function, we can evaluate respected moments, the peak factors, and carry out any elaborate study that we would like to perform.

Now, this expression has one variable theta, which is which has to be specified; that means, you need to know the direction of in which the waves arrive, we saw v the direction that you have to chosen to model the structure. But if there are several seismic sources surrounding a building , surrounding a structure, we could proceed in different ways. One option would be to determine the orientation theta, for which the response variance reaches its maximum value; that means, find out the worst value of theta and use that in your reliability of response analysis studies.

Alternatively, theta can be treated as a random variable, and the expected value of the response quantity of interest could be obtained with respect to the probability density function of the theta. So, if you have a some idea of theta is uniformly distributed between certain range of angles, that can be factored in, and we can take the expected values with respect to the probability density function.

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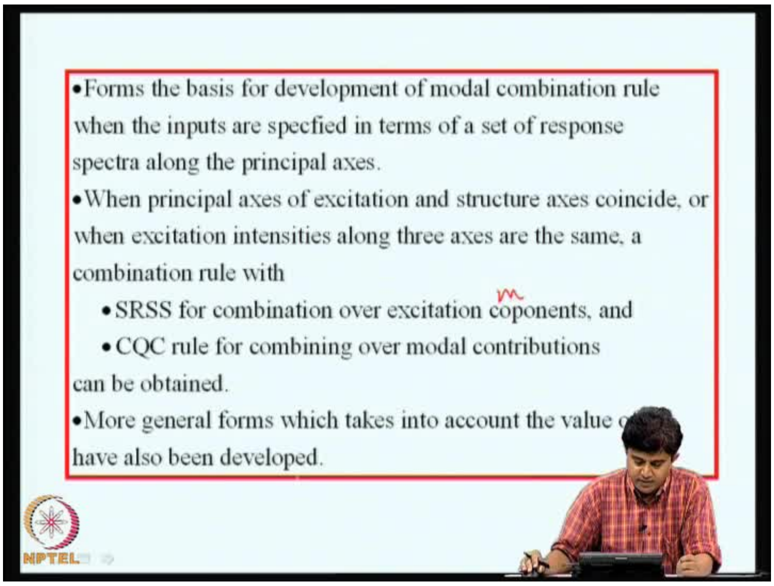

Now, this is a standard linear random vibrations analysis. Now, this will form the basis for development of modal combination rules, when the inputs are specified in terms of a set of response spectra, along the principal axes. The discussion so far has been under assumption, that we are specifying the ground motion in terms of power spectral density functions, along the principal axes. But instead of power spectral density function, if response spectra are specified, then we need to evolve the modal combination rules, which now should take into account, not only the combination over different modes but also combination over different excitation components.

So, when principal axes of excitation and structure axes coincide or when excitation intensities along three axes are the same, a combination rule with, say for example, SRSS for combination over excitation components, and CQC rule for combining over modal contributions can be obtained.

Now, more general forms which take into account, the value of theta can also be developed for during these combinations. Now, this I will not get into the details, because we have already discussed one, and how to arrive at combination rules, once we obtained the power spectral density of the response quantity of interest.

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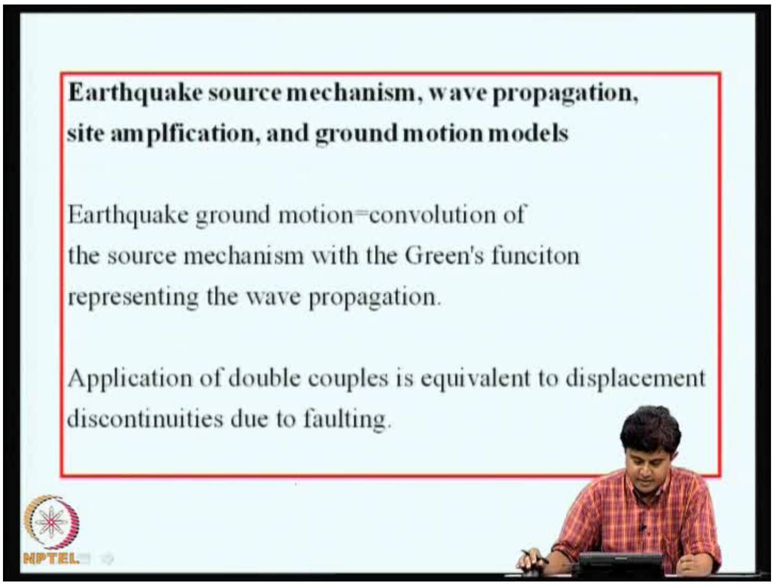

Now, in the existing literature, there are power spectral density function modals, which take into account issue such as, earthquake source mechanism, wave propagation, site amplification, and I mean ground motion models take into account these features. Now, the earthquake ground motion is pursued to be the convolution of source mechanism, with the greens function representing the wave propagation.

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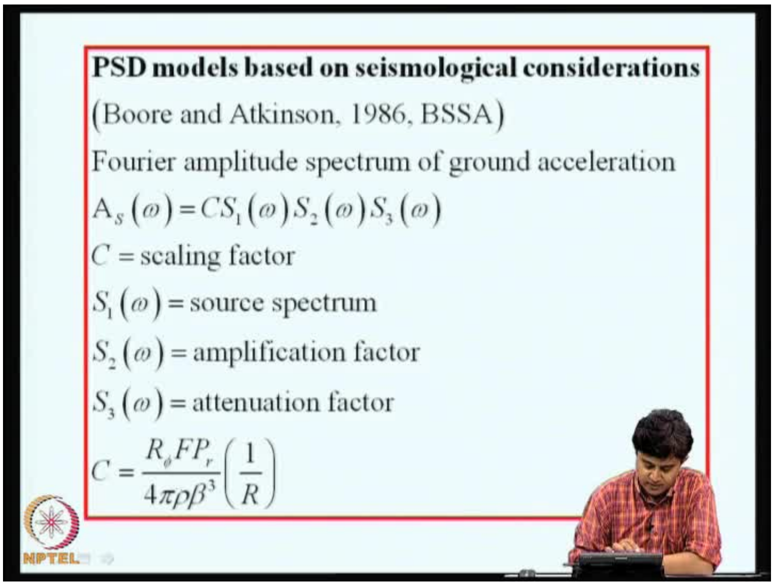

Now, application of double couples is equivalent to displacement discontinuities due to faulting; this one of the modals that is often used; the relative displacement along the fault is replaced by action of double couples and based on that greens functions are derived. So, I would like to just mention, without getting into too many details, one of the power spectral density function model based on seismological considerations, that is proposed by Boore and Atkinson.

These authors suggest that, the Fourier amplitude spectrum of ground acceleration denoted as a S of omega can be expressed as, a constant C into product S 1 of omega S 2 of omega S 3 omega; C is a scaling factor, S 1 of omega is known as source spectrum, S 2 of omega is a amplification factor, S 3 of omega is the attenuation factor. I mean these are the functions that allow for effect of these quantities.

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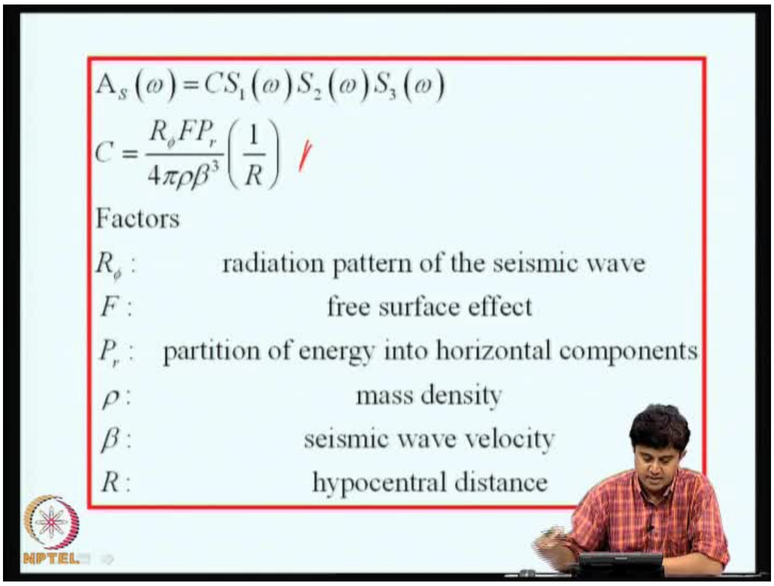

C itself is a function of several variables. C is shown here, there are various factors; for examples, r phi is a factor that takes into account in radiation pattern of the seismic waves; this capital F is the so called free surface effect. There are other thing which we need not we need not get into the physics of this problem, but the point that is being made is that, the power spectral density for strong motion - strong ground motion - can be related to several of this seismological factors, which are displayed here; for example, mass density of the medium, the seismic wave velocity, the hypo central distance, so on and so for.

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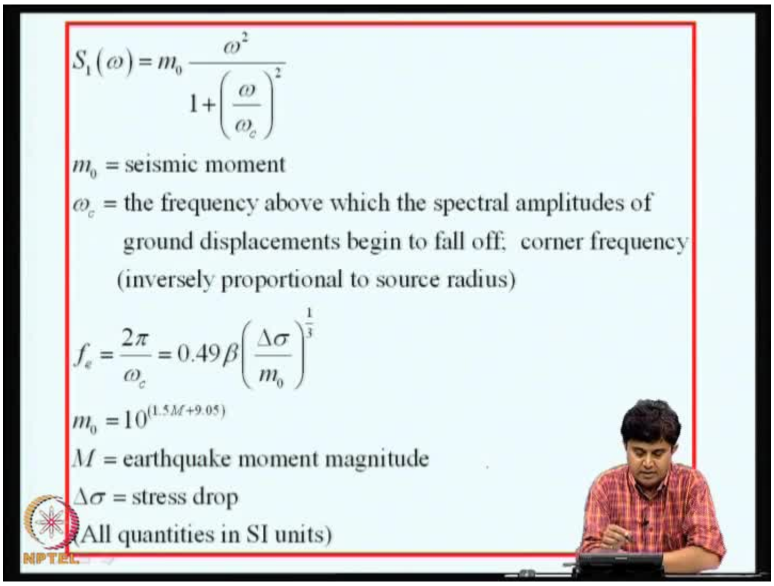

This S 1 of omega which is the source spectrum takes into account, say, the magnitude and quantities like that. So, this m naught is the seismic moment, omega c is the frequency above which the spectral amplitudes of ground displacements began to fall off, so called corner frequency, it is inversely proportional to the source radius.

This is related to the stress drop and magnitude of the earthquake. So, where this m not is a function of these quantities, where M is earthquake moment magnitude, delta sigma is the stress drop and all these quantities are in SI units. The point that again I would like to emphasize, the issues not on how these expressions are obtained, but to take (( )) of the fact that, there are models in the available literature, which relates the power spectral density function to several of the model parameter associated with describing earthquakes.

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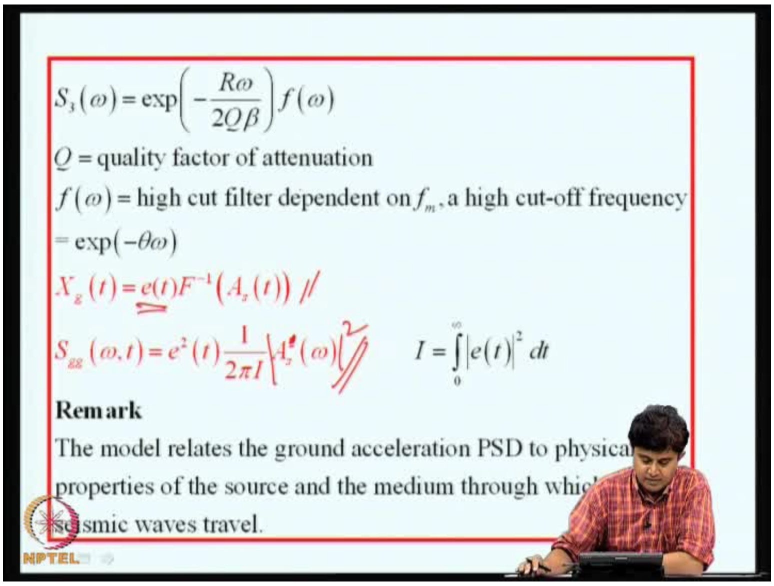

Now, for the site effect, that is S 2 amplification factor S 2 of omega, one could use several models; for example, simple Kanai Tajimi or clough and Penzien model could be used. And S 3 of omega which is related to attenuation; this Q is a quality factor of the attenuation, that is Q factor for the medium; f of omega is a high cut filter dependent on f m, a high cut off frequency which is written in this form. Now, at the end of this, we can get the Fourier transform of the ground motion, following several of this physical arguments; we could take the Fourier transform of that and get a time domain signal multiplied by an envelope e of t, and we can define evolutionary power spectral density function model e square of t into 1 by 2 phi i a square of omega.

The point that is made is, that the model relates the ground acceleration PSD to physical properties of the source and medium through which the seismic waves travel. So, to develop an understanding this type of models, we need to interact with seismology, and understand the physics of wave propagation, faulting, and so on and so forth. The thing is such effects have been made and there are models which take into account the outcome of such efforts.

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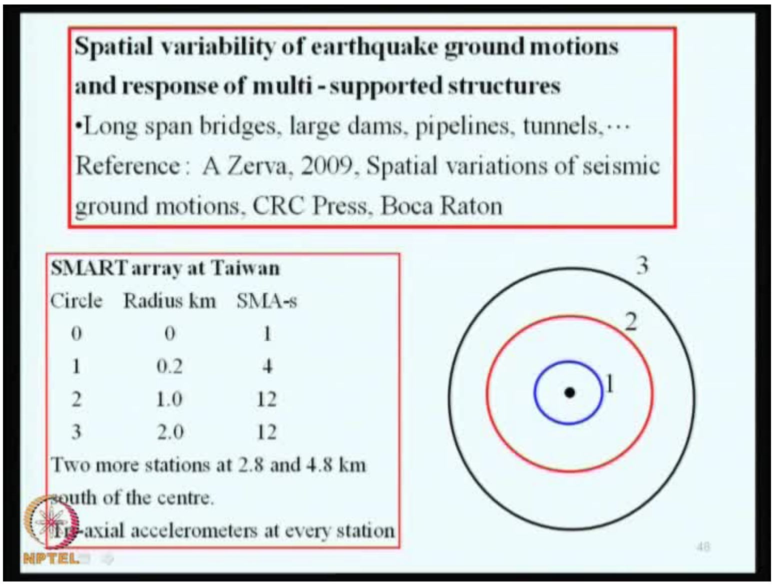

Now, we move on to another topic, that is related to the problem of spatial variability of ground motions and response to multi supported structures. All structures are multiply supported; when we talk about multiply supported structures in the context of earthquake engineering, what is meant is that, different support suffers different types of support motions.

So, such structure, for example, are long span bridges, large dams, pipelines, tunnels, etcetera. So, given the large extent of - the large spatial extent of - this type of structures, different supports, say for example of a bridge, which may be say one and half kilometers long, different support would be subjected to different support motions in the event of an earthquake. And the question is, are there any specific phenomenological issues, that need to be taken into account while considering response of this long span structures?

Activities related to modeling of spatial variability of earthquake ground motions, began with strong motion instrumentation, which were densely configured in certain areas on the earth crust. What is shown here is, one of the dense arrays in place called Taiwan, in the country Taiwan; it has three concentric rings and a central point, and these rings have radius of 200 meters, 1 kilo meter and 2 kilo meter.

There is a strong motion seismograph at this point, and there are four strong motions seismograph along this circle, 12 along this circle, 12 along the outer circle. So, that means, within the radius of 2 kilometers, there are about 30 or so seismic the triaxial a strong motion instruments. And in the event of an earthquake, all of them will record the event, and we will be able to get the spatial variability in ground motions, through a study of this records. There are few more station recording instruments that have been added to this array; the point is that, within reasonably small area, there are a large number of strong motions accelerographs which record the same event.

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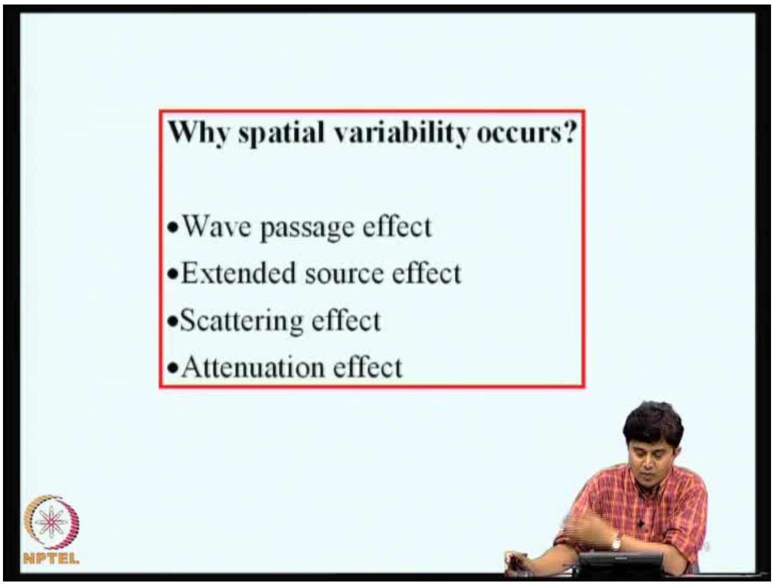

Now, why do spatial variability occur? There are different effects; we will give certain heuristic explanation for this: one, vertice known as wave passage effect; other one is known as extended source effect; third one is scattering effect and the last one is attenuation effect.

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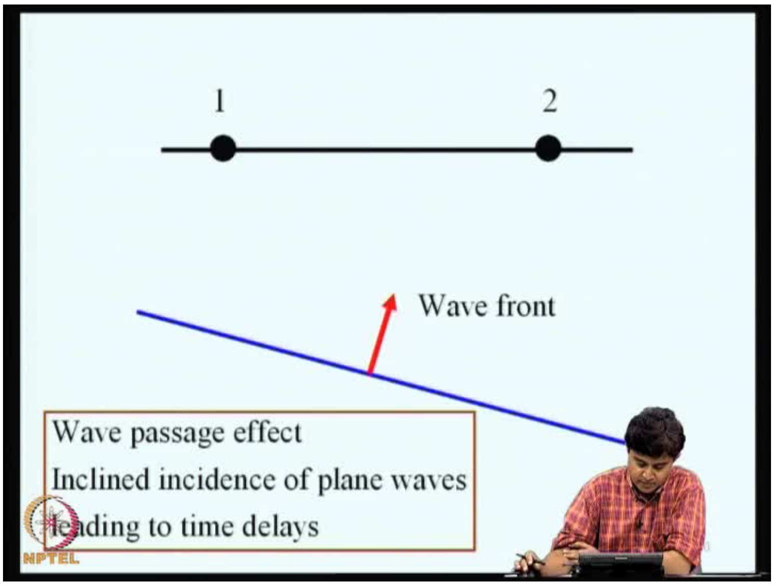

A wave passage effect refers to, a situation where a plane waves incident at an angle, as shown here, this is the wave front moving in this direction; 1 and 2 are the observation stations. So, the wave front would arrive at station 1 earlier than station 2; therefore, there is a time lack between arrival of waves at stations 1 and 2; so, that effect is known as wave passage effect. This is one of the things that can be modeled, a simple way would be to assume that, if motion at 1 is X of t, this could be X of t minus tau; that means, the same ground motion arrives at 2, the motion at 1, arrives at 2, but after that the time delay tau. So, in fact, early studies on spatial variability began with this type of simplified models.

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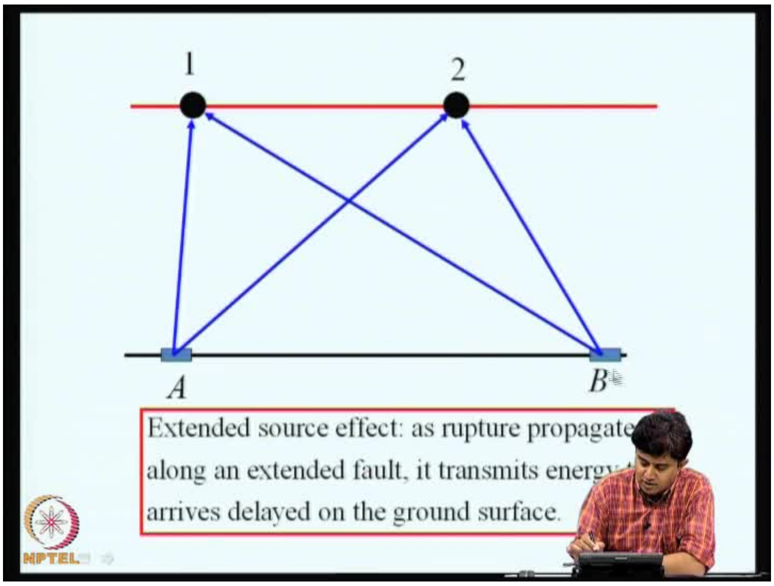

Now, the next effect is, what is known as extended source effect. Now, in the event of an earthquake, it is well known that, a rupture propagates along a fault line; and as energy is released at different instants in time along the same along this same fault line, for example, from point A on the fault line, waves starts propagating in the medium much before the waves start propagating from point B.

So, consequently what happens? From A, this wave would reach here, and from B, this wave would reach here after some time and it will pass through a different medium; therefore, it will have different characteristics. And similarly, from point A, the wave goes here and this will also receive waves from point B.

So, as the as the earthquake process develops, the waves arriving at 1 and 2 will be super position of several of this complex effects, which each packet of that is emanating from different points along this line A B.

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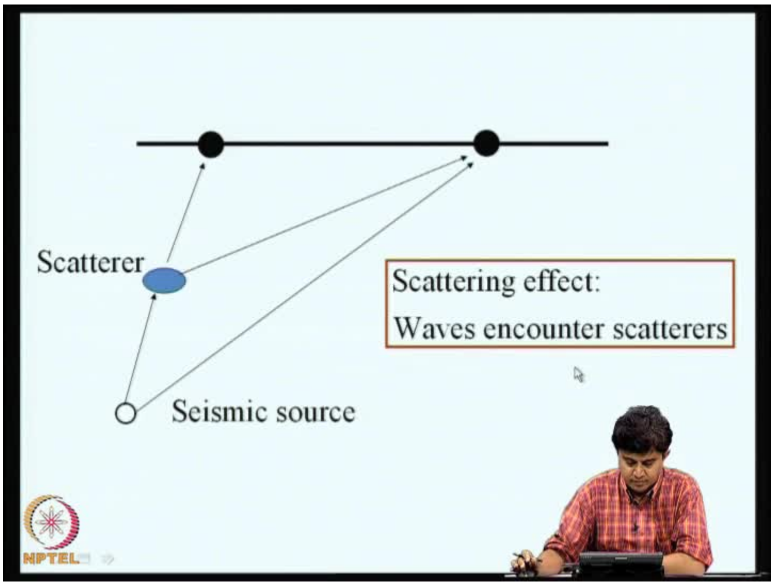

Now, we know that the ground medium is inhomogeneous; so, there could be what are known as scattering. So, as waves propagate through heterogeneous medium, there will be scattering; this is highlighted somewhat dramatically here, where there is discrete discontinuity – inhomogeneity. So, as if this is a seismic source, the waves hit here, and they propagate here, and they get deflected here and some of the waves directly reach here. So, consequently, the details of ground motion at this point and this point could also vary.

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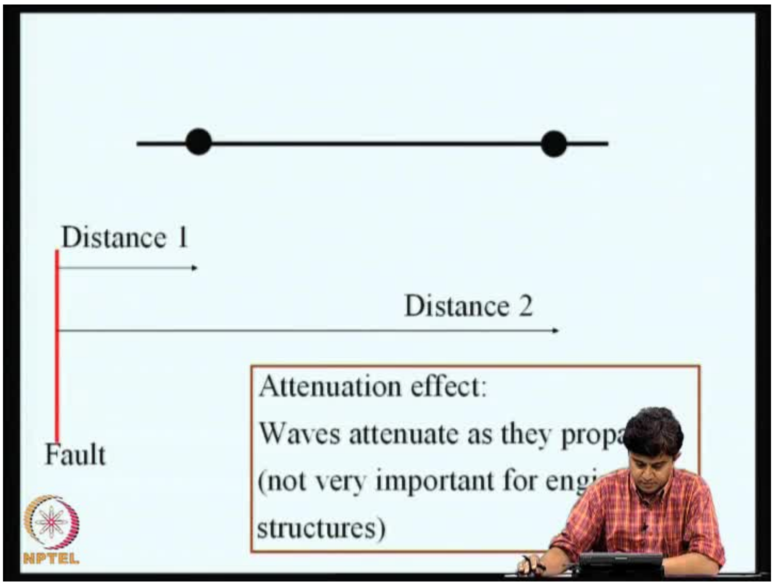

Now, the other effect the last effect is attenuation effect; for example, if waves are propagating for a fault line, they have to reach this station; that distance they the need to travel is a distance 1, and to reach this station, it is distance 2. So, as waves propagate, they attenuate; and the amount of attenuation is the function of the distance, through which the waves travel. So, consequently, the amount of attenuation that would have taken place, when waves arrive at this station, would be quite different from the attenuation that would take place as a travel through longer distance. So, there will be differences in details of ground motions at stations at 1 and 2, because of this effect.

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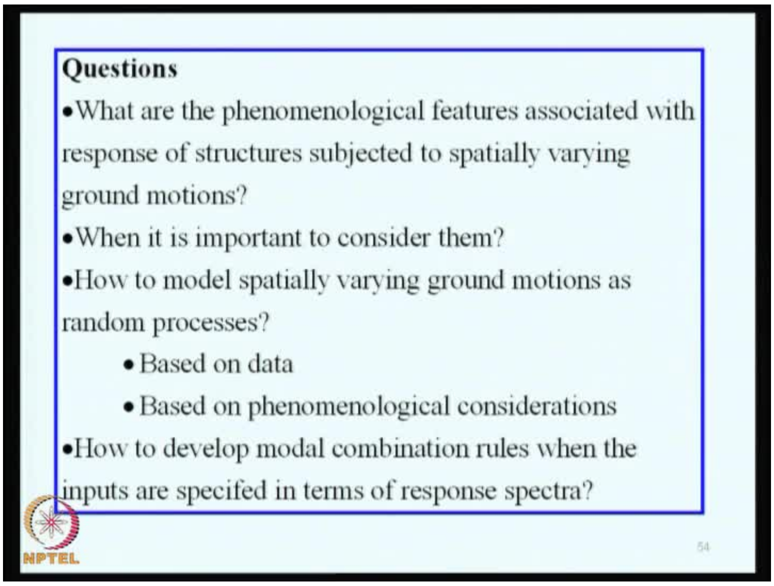

Now, the question that we need to ask, if we want to study the spatial variability, what are the phenomenological features associated with response of structures subjected to spatially varying ground motions? When it is important to consider them? Now, how to model spatially varying ground motions as random processes? So, you will have to develop a random process model, which takes into account this kind of the four phenomenological features that we just now mention. This modeling could be purely based on recorded instrumental data or based on phenomenological considerations.

Now, again the question that we would like to ask is, how to develop model combination rules when the inputs are specified in terms of response spectrum? Now, here, you recall, when we talked about response spectra, we talked about single component of ground motion. So, the only combination that was needed was over the modes. But when we talked about three component ground motions, there are summation over with three components of ground motion and the modes.

Now, here, there will be multiple components at each support. So, there will be a next layer of summations, which will be over different excitation components at different supports. So, that is the summation over supports, summation over components, summation over modes. So, how best to develop the model combination rules? These are important, because the response spectrum based methods for specifying earthquake ground motions, has come to be accepted as one of the most popular ways of specifying earthquake induce loads. And therefore, one has to rationalize rationally develop the combination rules, and for this, the principles of random vibration analysis forms the basis. Although the application of response spectrum base method themselves could have a deterministic approach, it may be a deterministic approach; the basis of that depends on random vibration principles.

So, in the next part of the lecture, we will consider these questions and we will conclude the present lecture at this juncture.