Stochastic Structural Dynamics

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Lecture No. # 32

Probabilistic Methods in Earthquake Engineering-1

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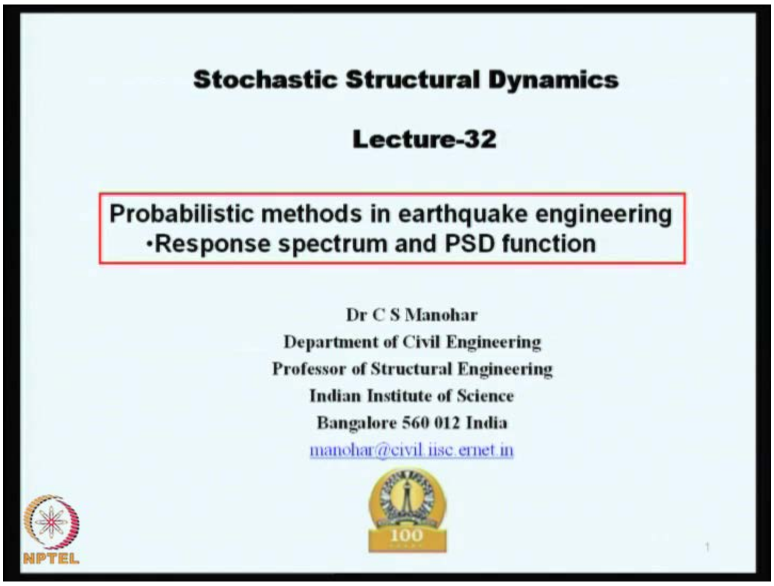

So far in this course, we have been focusing on analysis of the response moments, probability distribution function of the state variables and reliability measures, using both analytical and Monte Carlo simulation methods. So, in the remaining part of this course, we will discuss few applications. So, we will begin in this lecture, a discussion on applications to problems in earthquake engineering.

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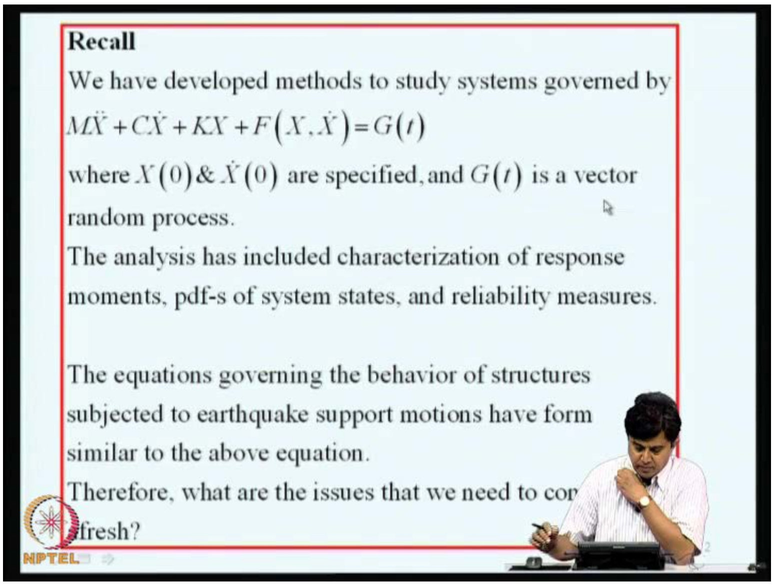

So, we can quickly recall, that we have now developed methods to study systems governed by a general multi degree freedom system, MX double dot plus CX dot plus KX plus some non-linear terms is equal to some random excitation with initial conditions specified, and G of t is a vector random process. We have considered several issues like a randomness being present in M C K as well, in addition to randomness in G of t, etcetera. The analysis has included characterization of response moments, probability density functions of system states, and reliability measures like, first passage times, extreme values, and so on and so forth.

Now, if we consider the behavior of structures acted upon by earth quake ground motions, the governing equations would again be in general of this form. So, we do have now all the tools necessary to analyze this problem. So, the question now arises, what are the issues that we need to consider apart from analyzing this equation.

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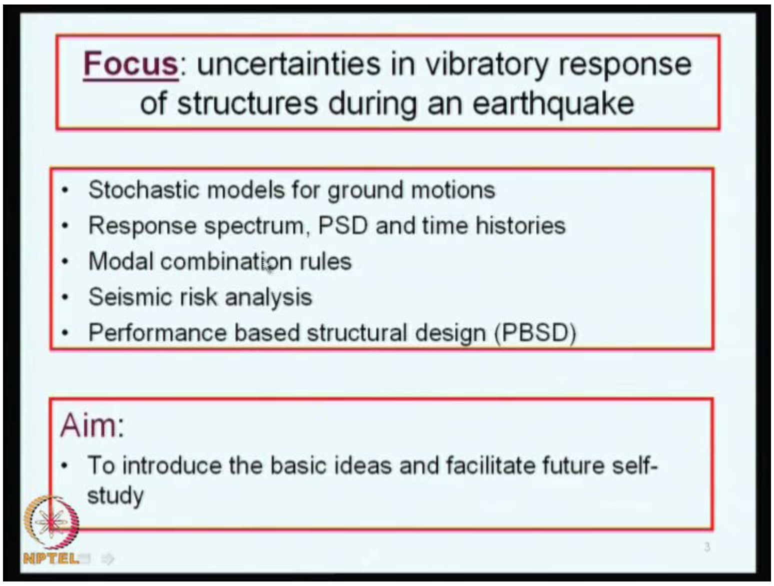

So, we will focus our discussion on uncertainties in vibratory response of structures during an earthquake. The dangers due to an earthquake include several features like, vibration of structures and response amplification due to the dynamical behavior, and problems associated with soil behavior, foundations, liquefaction, slope stability, so on and so forth. So, we will be not considering all aspects of this problem, but we will limit our attention only to the vibratory response of structures during earthquake.

So, this would require models for ground motions, since there exists considerable uncertainties in ground motions. We need to adopt probabilistic models for that; one of the common ways of specifying earthquake ground motion, is through a set of response spectra; this is one of the traditional methods of specifying earthquake ground motions. We will talk more about this during this lecture, and alternative representations would involve power spectrum functions and set of time histories.

So, we will investigate what is the relationship between these three alternative ways of specifying earthquake ground motions. Then, there are issues about using response spectrum base methods, for analyzing multi degree freedom systems, and that lead to questions on what are known as model combination rules. This again you will be one of the topic that we will discuss; then, we will spend some time on questions of Seismic risk analysis, where we will try to model various sources of uncertainties and address issues related to reliability of the structure. And there are other topics like, performance based structural design, etcetera, and I may make some remarks on these issues.

So, the basic aim of this lectures would be to introduce the basic ideas and facilitate future self-study; so, each of these topics are covered in great detail in existing literature. So, what I will be doing is, to give a glimpse of the basic issues, so that you could go back and study them in greater detail, if there is an interest.

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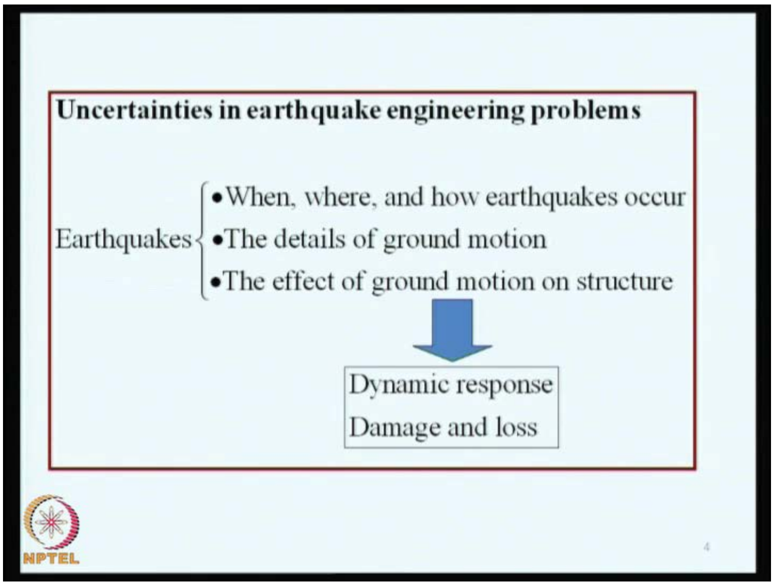

So, if you look at the basic source of uncertainties in earthquake engineering problems, there are basically three sources. First one is, when, where and how earthquakes occur; the next one is given that an earthquake has occurred, what are the details of the ground motion at a given side; and the next is, what is the effect of these ground motions on engineering structures, that includes characterization of dynamic response damage and loss.

The first of these questions has strong interface with earth science research. And we will have to borrow some of the findings from studies conducted by g of s is geologies, zoologies, and so on and so forth. So, as structure engineers, our interest should begin by with questions on details of ground motion, and how they will affect the motion of the structure. So, to start with, we will discuss these issues, and then, briefly return to the first issue and see how all of them tie up.

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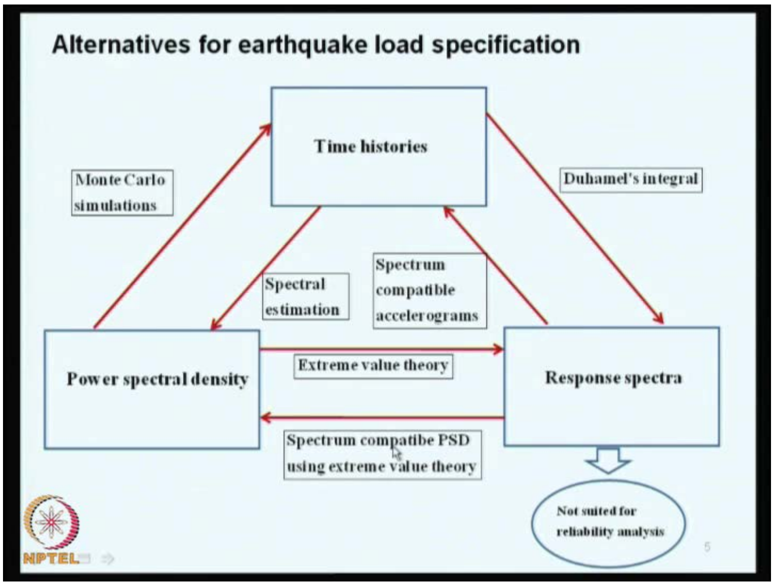

The earth quake ground accelerations can be characterized in terms of a set of time histories. And if you model these ground ensemble of time histories has a random process, we can talk about the power spectral density functions, assuming that they are stationary, etcetera. More sophisticated models for earth quake ground motions are possible, that includes for non-stationarity, and so on and so forth. There is yet another way of specifying earthquake ground motions, and that is through a set of what are known as response spectra; this is probably the most popular way of specifying earthquake ground motions. And most of the design course etcetera provide earthquake loads specification through a set of response spectra.

So, we can begin by asking the question n, how these three alternate ways of specifying earthquake ground motions are related. We can quickly consider the relationship between set of time histories and power spectral density functions. Suppose if you have a set of time histories using methods of statistics, we can perform this spectral estimation and get an estimate for power spectral density function.

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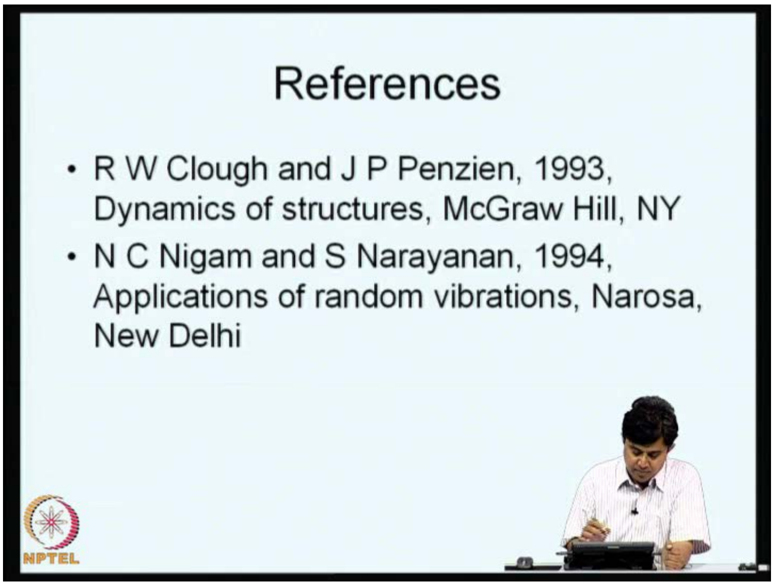

Similarly, if we have a model for the power spectral density function, we can use Monte Carlo simulation methods and get an answer below time histories. So, this route, the relationship between time histories and power spectral density functions were already explore. So, what needs to be done right now, is to find the relationship between response spectra and the power spectral density functions, and response spectra and a set of time histories. So, this is what we will explore in this lecture. So, there are couple of references, the books by Clough and Penzien on dynamics of structures, and the book by Nigam and Narayanan on applications of random vibrations provide some of the basic material, that I am using during these lectures.

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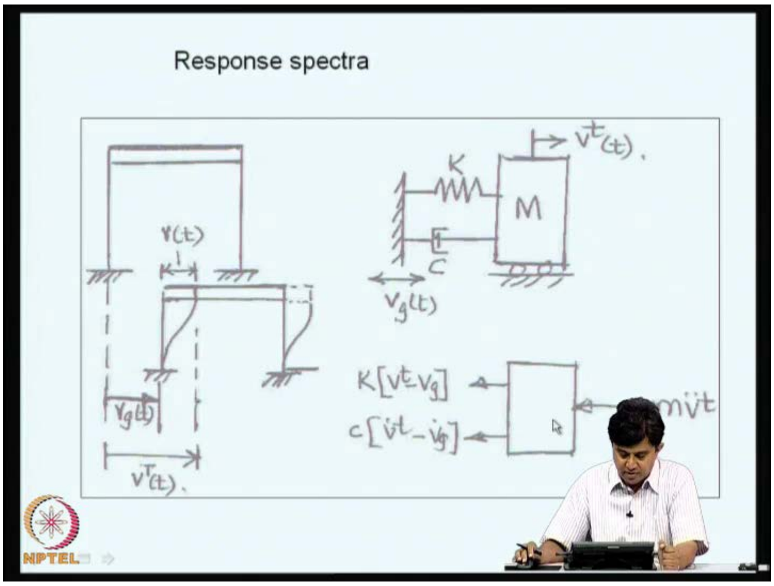

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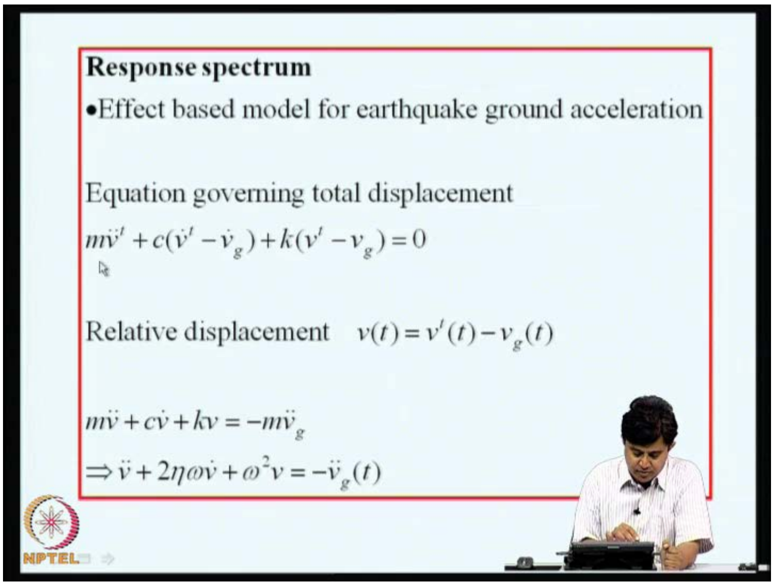

So, we will quickly review the notion of response spectra; so, here we consider earthquake ground displacement as V g of t and we will consider the action of this earthquake ground displacement on a single degree freedom system, whose mass is M, stiffness is K and damping is C. This single degree freedom system can notionally represent a single base, single portal frame; so, this V g of t is a ground displacement, V t of t is the total displacement of the slap, and V of t is the this is the relative displacement. So, if you model the behavior of this portal frame as a single degree freedom system, we can draw the free body diagram and set of the governing equation of motion; and that governing equation of motion for the total displacements is shown here, mv t double dot plus c into v t dot minus v g dot plus k into v t minus v g equal to 0.

Now, the equation for relative displacement, that is, v of t is v t of t minus v g of t can be derived and we get v double dot plus 2 eta omega v dot plus omega square v is equal to minus v g double dot of t. In the response spectrum based approach for modeling earthquake ground motions, what we intend to do is, we model the earthquake ground motion based on its effect on a serious of single degree freedom systems. So, this way of modeling earthquake ground motion is a response base characterization.

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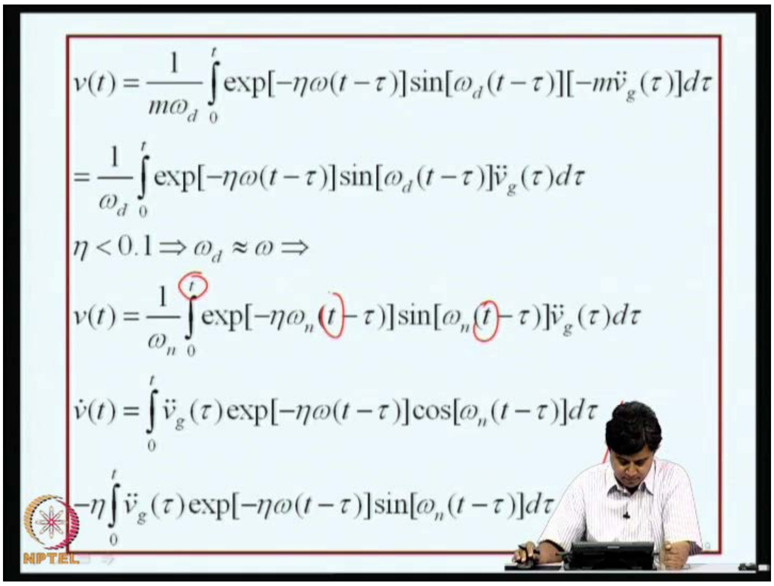

So, what I will get to the detail shortly; so, we will first analyze a quickly perform a simple analysis of this single degree freedom systems; alternate representation of the behavior of the single degree freedom system. Suppose, if you consider the expression for relative displacement, if you assume that system starts from rest, the Duhamel integral provides us with the expression for relative displacement. And if we simplify, this m will get cancelled with this m; and in most often in our discussions, this minus sign is not included, because ground acceleration being oscillatory in nature; this plus minus sign here could not make much difference. So, this minus sign would be omitted in further discussion.

So, if we do that, this will be the expression; certain simplifications are possible, if system damping is less than 10 percent, then the damp natural frequency can be approximated by the system natural frequency - un damped natural frequency; and with that, we get this expressions; so, omega n is the natural frequency.

Now, the associated with this expression for relative displacement, we can derive the expression for relative velocity by differentiating this expression with respect to t. The t appears here as a limit here; it appears here, it appears here and it appears here; so, we need to perform differentiation with respect to this t and we get two terms. Once we differentiate the integrand and next with respect to the limit; so, we get the expression for velocity relative velocity is as shown here.

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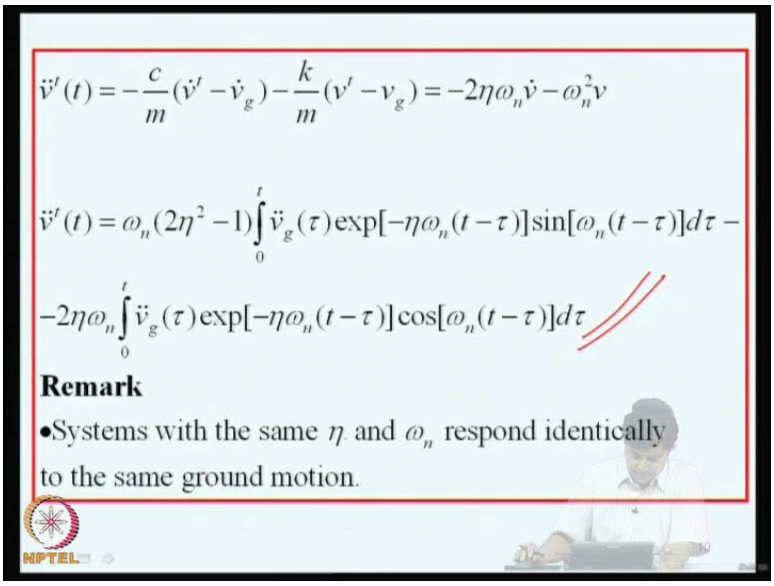

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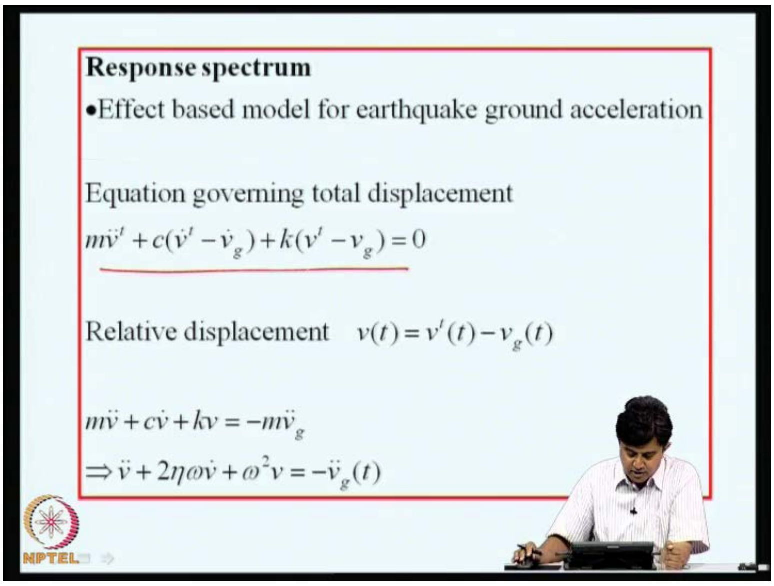

Now, the expression for total absolute acceleration can be derived, by considering this equation of motion. So, mv t double dot will be equal to these two terms taken on the right hand side; if we divided by m, it will be c v dot plus k v minus of that; so, that expression is written here, minus c by m v t dot minus v g dot minus k by m v t minus v g. So, this can be written as minus 2 eta omega n v dot minus omega n square v. So, using now the expression for v and v dot in terms of Duhamel integral, we can get now the expression for absolute acceleration. From this, we can make a simple observation, that systems is same damping ratio eta in natural frequency respond identically to the ground motions. For a given ground motion, all systems having the same natural frequency and same damping would respond identically.

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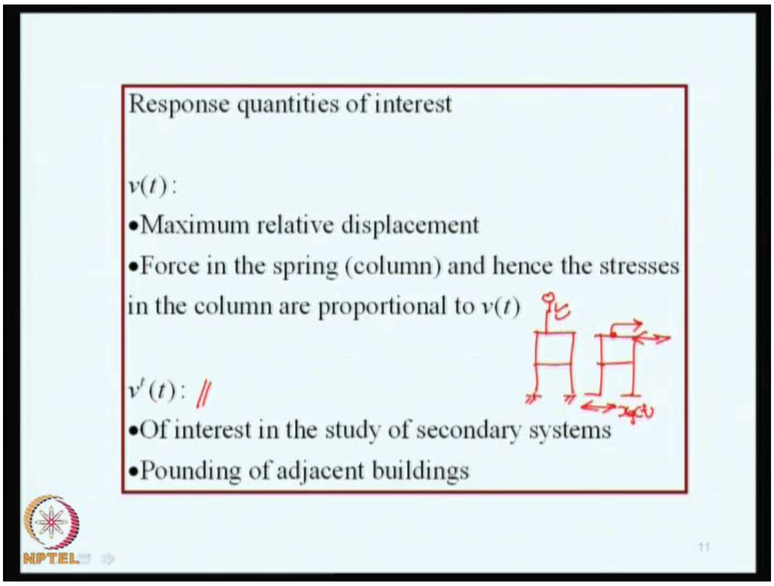

Now, what are the response quantities that are of interest to us. Relative displacement is one quantity. The maximum value of v of t would be of interest to us, because the force in the spring will be proportional to v of t k into v of t will be the force in the spring. And the force in the spring actually correspond to the column in the portal frame that I have mentioned. So, the stresses in the column etcetera would be dependent on relative displacement; therefore, we are interested in relative displacement. We are also interested in the absolute displacement; this can be of interest in several context.

For example, if there is a primary system, which can be a say a building structure on which there is a secondary system; this could be a machine component or some other sensitive equipment. And if you are interested in characterizing the Seismic behavior of this secondary system, then what we do is, we first analyze the primary system to the applied ground motions and find out the response of this floor, to this support displacement, and we treat this secondary system, as if acted upon by an earthquake, which is similar to the response of this floor.

So, if this is the objective of this is how we are, if the objective is to analyze the secondary systems, then we would be interested in the absolute displacement of the primary system. So, in that sense, we are interested in absolute displacement also.

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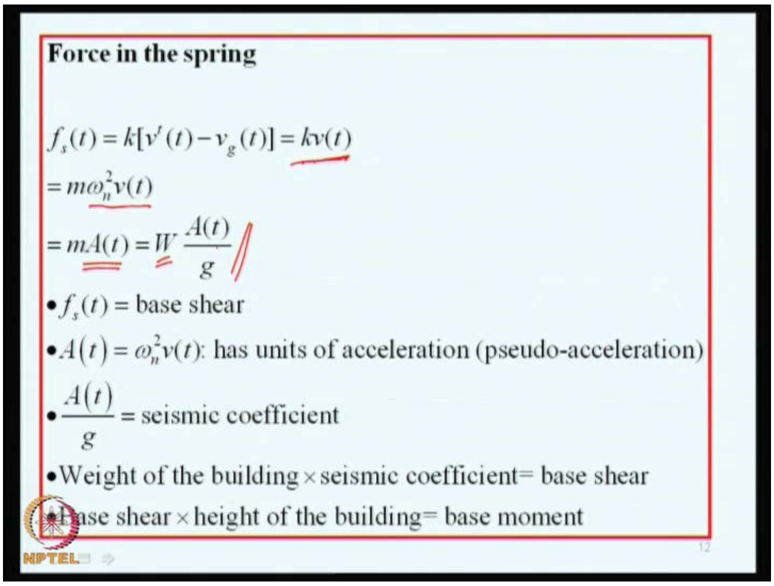

Now, let us take a look at the expression for force in the spring, that will be k into the relative displacement, that is kv t; and for k, I can write it as m omega n square v of t. Now, if we look at this quantity omega n square v of t, I can call it as A of t; and this A of t has units of acceleration, it is not the relative acceleration or the absolute acceleration of the mass, but instead it is the hypothetical quantity, whose units correspond to the units of an acceleration. Now, this m can be written as weight divided by acceleration due to gravity; therefore, the force in the spring can be expressed as a fraction of the weight of the structure and that fraction is A of t by g.

So, this f s of t is nothing but base shear, and A of t as I was mentioning has units of acceleration, we call it as Pseudo acceleration; and we also call A of t by g as use by Seismic coefficient, because this coefficient multiplied by the weight of the structure provides us the horizontal force, that is base shear.

Now, that is weight of the building into Seismic coefficient, it is the base shear. So, base shear into height of the building provides the base moment; so, in mu of all this, we are interested in quantity k into v of t. So, the definition of k into v of t now leads us to the notion of a Seismic coefficient.

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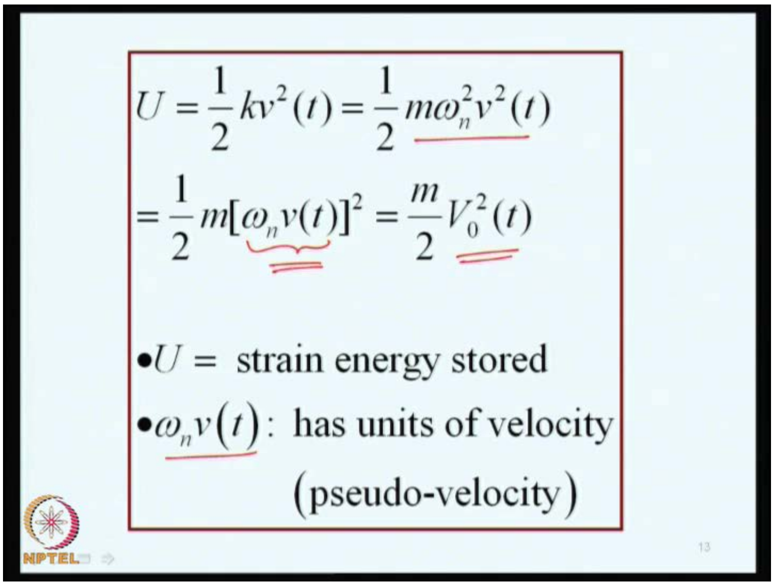

Now, if you look at the strain energy in the system, that is half k v square of t, again if you are adjust, I mean, if you express k as m omega n square, then I get an expression m omega n square v square of t. Now, this has a quantity omega n into v of t, which has units of velocity; this quantity has units of velocity and we call this quantity as Pseudo velocity. And the strain energy is proportional to this square of the Pseudo velocity; so, this quantity would also be of interest to us.

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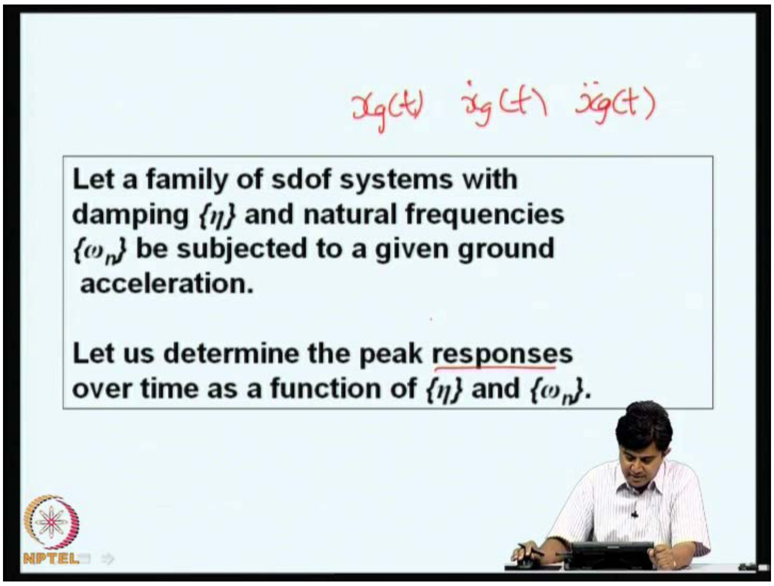

Now, what we do is, we are given an earthquake support displacement associated velocity and associated acceleration; what we will do is, we will consider that this ground displacement and ground motion will act on a family of single degree freedom systems with damping eta, eta 1, eta 2, eta 3, eta n and etcetera, and natural frequency omega n. So, we consider a family of single degree freedom system with different natural frequencies and damping.

So, what we will do is, we will subject each of this single degree freedom system to this ground motion and find out the maximum value of response for each one of the oscillator. The response here could be relative displacement, absolute acceleration, Pseudo velocity, Pseudo acceleration, so on and so forth. So, what we do is, we will find the peak responses over time as a function of damping and natural frequency; this response itself could be one of the quantities that we are interested.

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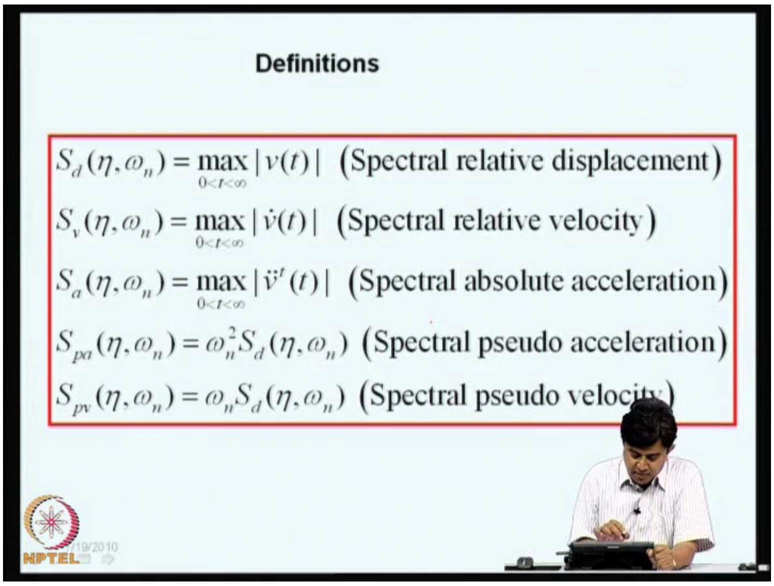

So, if we denote by S d of eta comma omega n, the maximum peak relative - maximum relative displacement - and we call it as spectral relative displacement. This maximum v dot of t, we call it as spectral relative velocity; and maximum of absolute acceleration, we call as spectral absolute acceleration. The omega n square into this relative displacement, we notice that it has units of acceleration and it is useful in characterizing base shear, we call it as spectral Pseudo acceleration. Similarly, omega n into the relative displacement, we call it as spectral Pseudo velocity. The word spectrum here means on the x axis, we have a frequency parameter.

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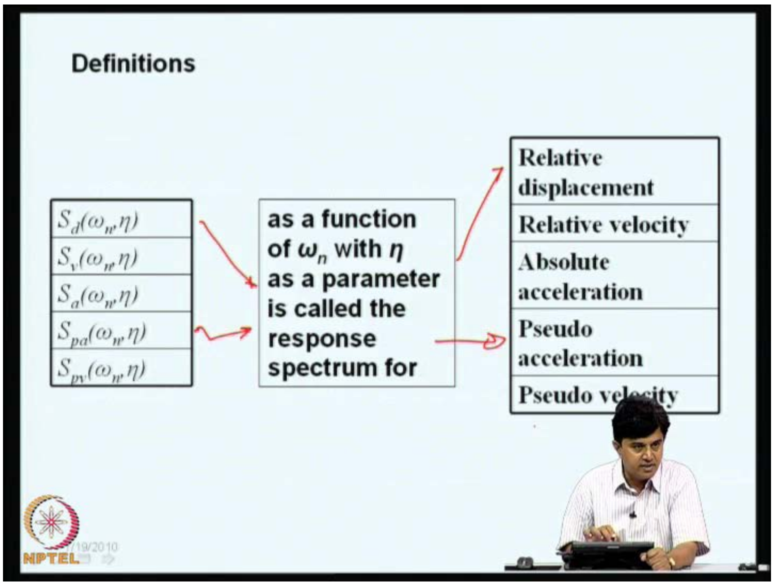

Now, the plot of S d of omega n comma eta as a function of omega n, with eta as a parameter is called the response spectrum for relative displacement. So, this we have to read in sequence. Similarly, the plot of, say for example, S pa omega n eta is the plot of S pa omega n comma eta, as a function of omega n with eta is a parameter is called the response spectrum for Pseudo acceleration. So, we define several response spectra; one for relative displacement, relative velocity, absolute acceleration, Pseudo acceleration, Pseudo velocity, etcetera.

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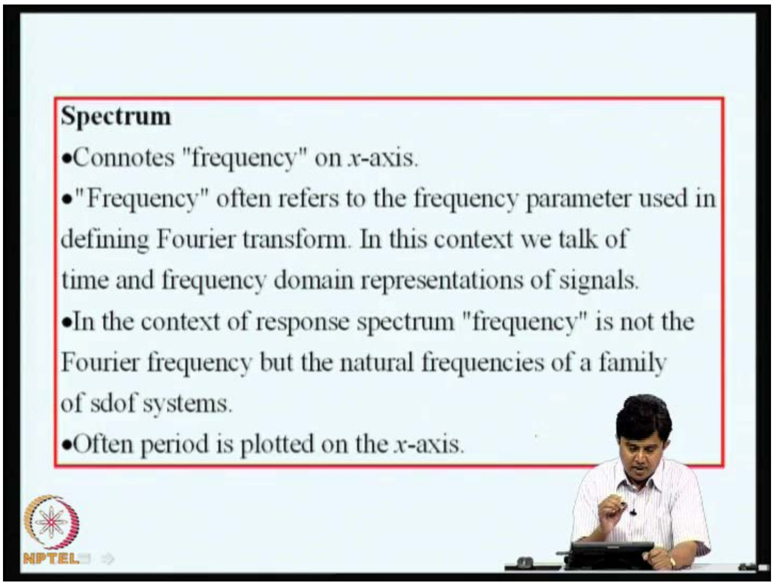

So, if you remarks, the word spectrum converts frequency on x axis; the word frequency itself has to be carefully understood here. Frequency often refers to the frequency parameter used in defining Fourier transform. In this context, we talk of time and frequency domain representation of signals, but however, in the context of response spectrum, frequency is not the Fourier frequency, but the natural frequencies of a family of SDOF systems. Often on the x axis, instead of showing frequency, either in radian per second or hertz, the associated period is plotted on the x axis and this period will have the units of time. So, the curve should not be mistaken for a time history; it is actually the time is the spectral time, it is not the real time.

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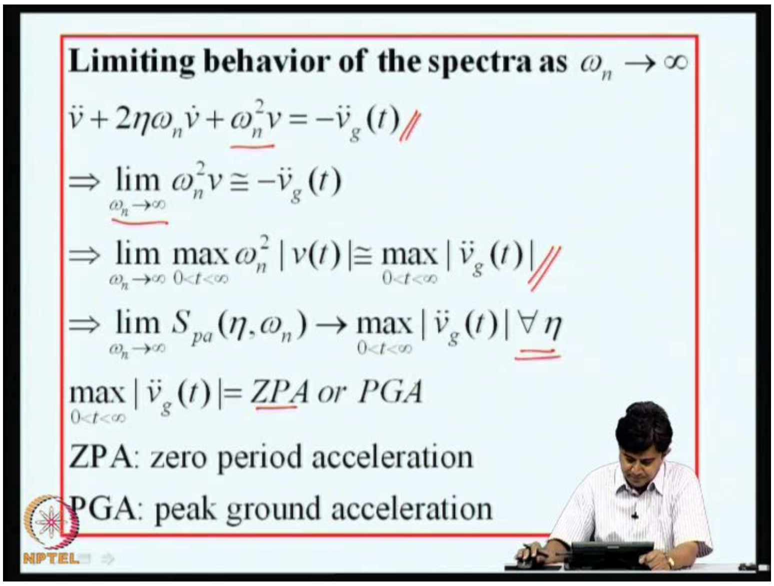

The response spectrum has several interesting properties; for example, if we consider the equation for relative displacement and consider the case where the natural frequency becomes very large, so the period of this structure becomes approaches 0; then, we see that, the term omega n square v would dominate the first two terms, and we can see that, omega n square v will be approximately equal to v g double dot of t; that is this term in relation to these terms will be very large; therefore, this will, we can approximate the l h s by omega n square v and this will be equal to v g double dot of t.

Now, if you now consider maximum over time of this quantity, you will see that, this will be nothing but the peak ground acceleration; this is the highest value of the ground acceleration. So, or in other words, the limiting value of the Pseudo acceleration response spectrum as omega n tends to infinity or period goes to 0 is nothing but the maximum value of ground acceleration. This is true for all damping values; it is independent of the damping of the single degree freedom system. This quantity is known as 0 period acceleration or the peak ground acceleration.

So, that would mean the Pseudo acceleration response spectrum for omega n tending's to infinity, should converts to the peak value of the ground acceleration, for all values of damping. If on x axis, instead of plotting frequency, if you plot period, then at 0 period, all the response spectra should be anchored at a single value, which corresponds to the peak ground acceleration of the signal, for which you are constructing the response spectrum.

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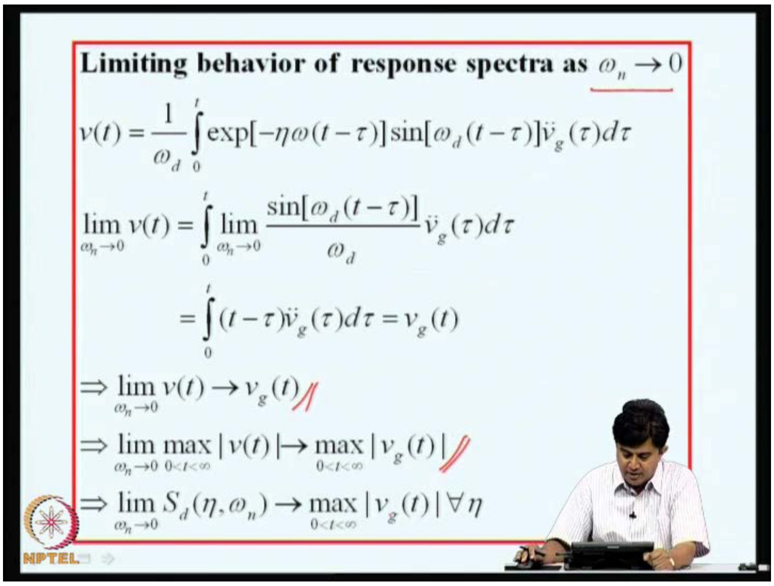

Similarly, if we now consider the other end as natural frequency goes to 0, that means, we are considering very flexible systems, we can show that the relative displacement v of t as omega n tends to 0 becomes equal to the ground displacement. So, thus, if you now consider limit of omega n tending to 0 and maximum value of the relative displacement, this will be equal to the maximum value of the ground displacement.

So, again, this would mean that, at omega n equal to 0, the relative displacement response spectra is anchored at the maximum value of the ground displacement. So, these things have to be checked, whenever we specify response spectra; and these limiting values are independent of damping. So, response spectra as I said, is a parameter family of curves, where parameter is a damping, and for all values of damping, this limiting values are applicable.

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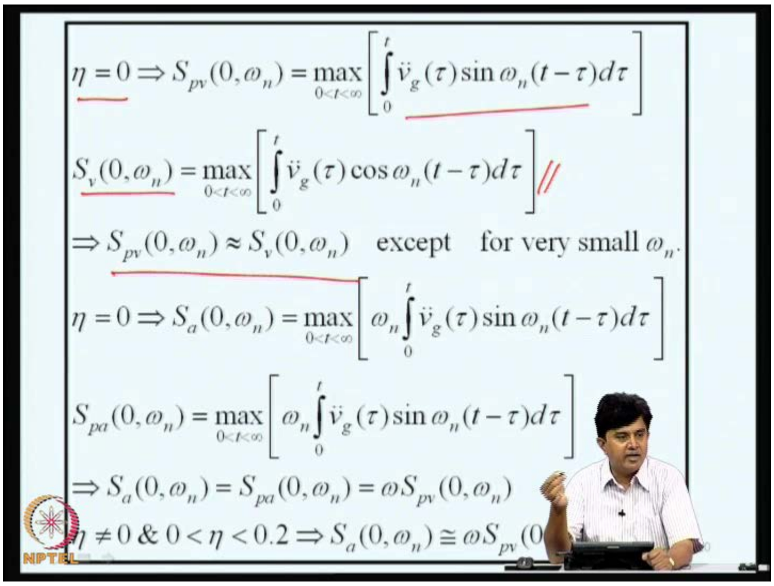

We can look at this Pseudo velocity and relative velocity; and if you look at the value of the Pseudo velocity at damping equal to 0, it will be equal to the maximum value of this integral; the terms involving damping will go to 0; this an un-damped system. And thus, it will be given by this; this is Pseudo velocity, this is the relative velocity. Now, you can see that, the difference between these two expression is associated with sin and cosine terms, and we can verify that, these two quantity is will be nearly equal except for very small omega n, that would mean the Pseudo velocity and a relative velocity response spectra are close to each other, for the systems which are un-damped, except when natural frequencies are very low.

Similarly, if you consider un-damped system and look at the absolute acceleration response spectrum, again if we carefully analyze the expressions, we can show that, the absolute acceleration spectrum will be equal to the Pseudo acceleration response spectrum; and for damping not equal to 0, we can approximately take the absolute acceleration response spectrum as omega times S pv. So, these are some of the properties of the response spectra, as omega n goes to 0, omega n goes to infinity, damping goes to 0, so on and so forth.

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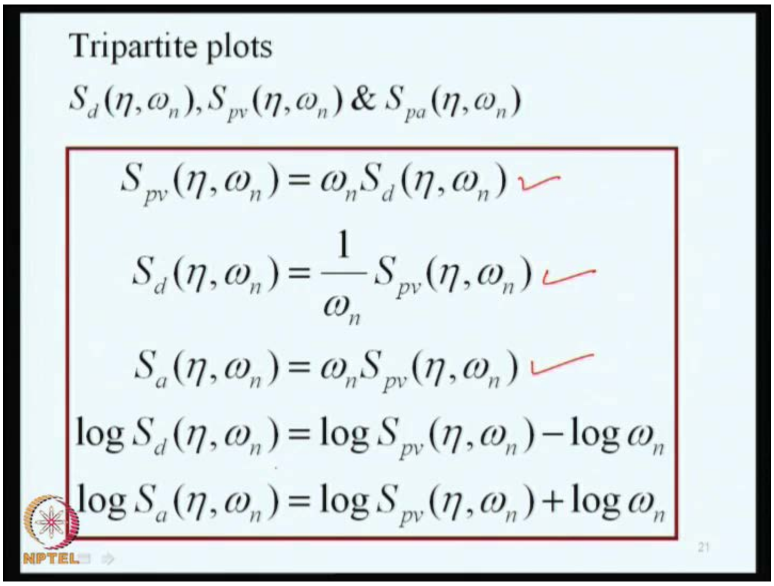

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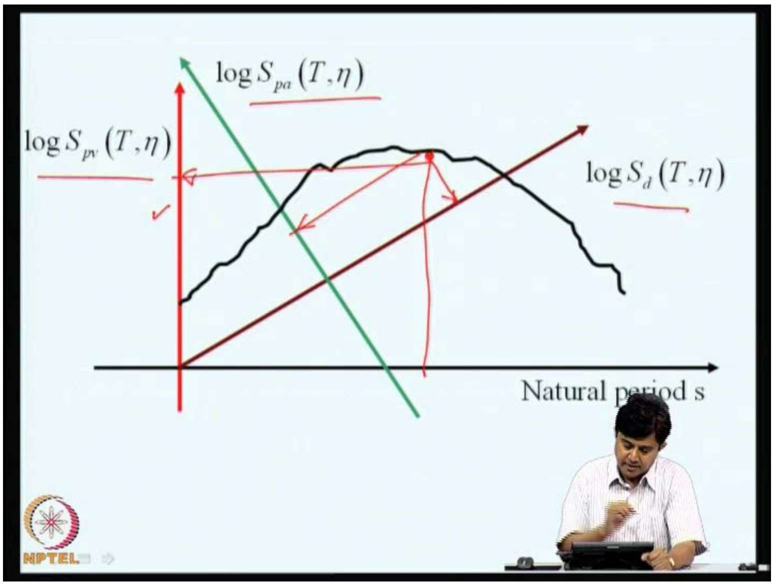

Often the response spectra are shown on a what are known as tripartite plots, and that is based on the observation, that S pv and S d are related through this expression, and S d and S pv are related to this expression, and S a and S pv are related to through this. So, if you take logarithms of this, these appear as straight lines; so, that would mean that we can show the response spectra in terms of the set of three curves. On one axis, we plot Pseudo velocity and other one the acceleration, and other one on the displacement; and on x axis, we plot the natural periods; that means, for a given value of the natural period, we can read three values on these axis as here; one is here, other one is here and other one is here; so, it is a compact way of displaying the response spectra.

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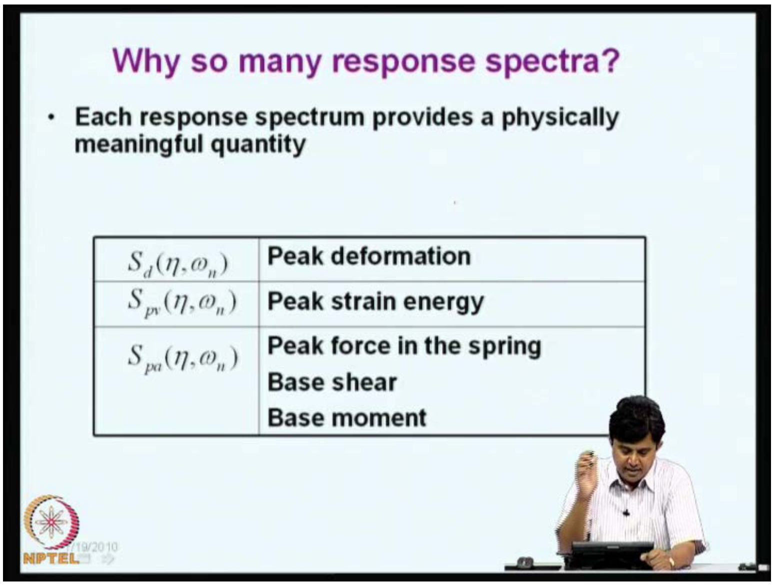

The question would arise, why we are defining so many response spectra; after all, we can define for displacement, be happy with that, or for acceleration, be happy with that. But as I have shown already, each response spectrum provides a physically meaningful quantity; for example, S d eta comma omega n is associated with peak deformation, S pv with peak strain energy, S pa with peak force in the spring, and it also leads to the definition of base shear and base moment. So, it is nice to be able to define all these quantities.

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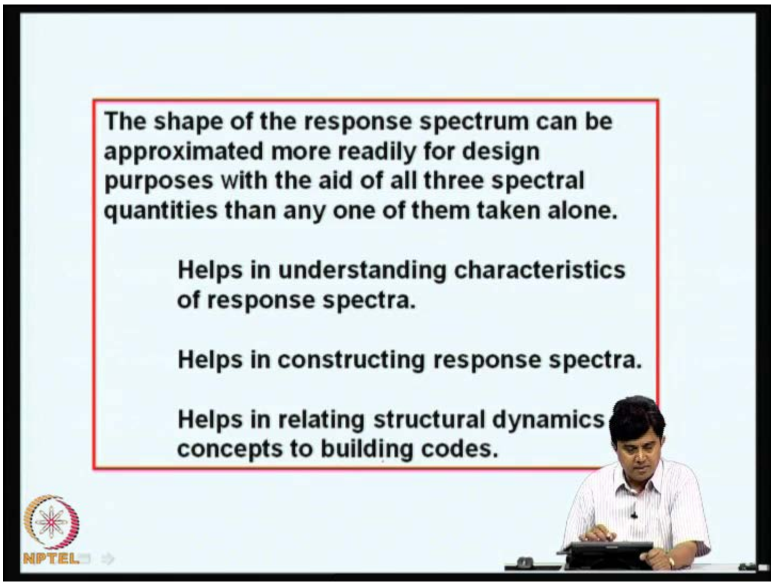

Also there are other reasons, I will just enlist them. The shape of the response spectrum can be approximated more readily for design purposes, with the aid of three spectral quantities than any one of them taken alone. This helps in understanding characteristics of response spectra and also it helps in constructing response spectra; more where it helps in relating structural dynamics concept to building provisions in the building codes.

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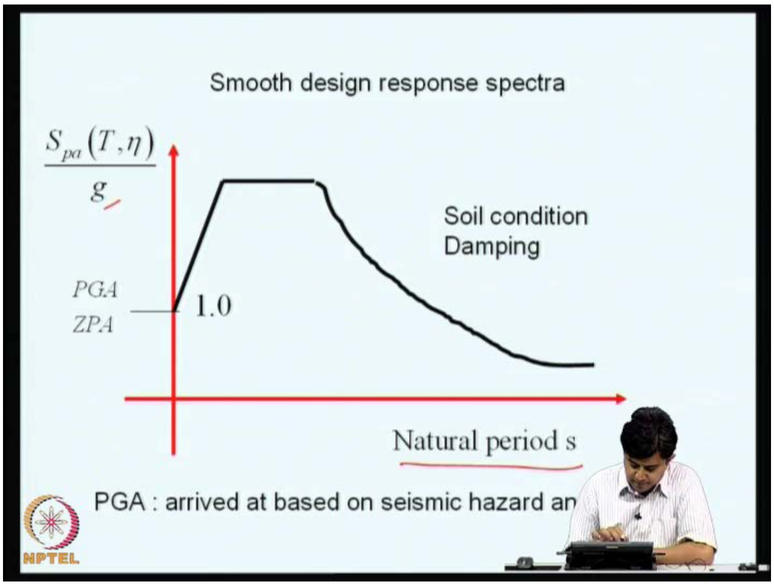

Now, in the building, I talk so far about response spectra define with respect to a given ground motions, but in codal practices, we consider a family of ground motions, and for each one, there will be a response spectra; and their specification of ground motion will be in terms of averages of this response spectra, suitably scale to display the level to which we would like to design the structure for earthquake loads.

So, typically in a design course, the response spectrum would be given as what are known as smooth design response spectra, a typical one is as shown here; on the x axis, we have natural period, and on the y axis, is the Pseudo acceleration normalize with respect to acceleration due to gravity, that would mean the response spectra for will be anchored at P ground acceleration. And it is anchored at a value of one in the code; in the codes of practice, the PGA will not be specified in this curve, but it will be separately specified, I will make some remark on that; and for different soil conditions and damping, there will be different graphs.

So, a user has to determine, this will provide a shape of the response spectrum; shape provides the frequency content essentially, but the amplitude, that is the highest value will have to be specified in terms of peak ground acceleration and this is arrived at based on Seismic hazard analysis, wherein we take into account uncertainties in various geological parameters, like, faulting mechanisms, wave propagation through soil, earth medium, so on and so forth; so, that would be specified separately.

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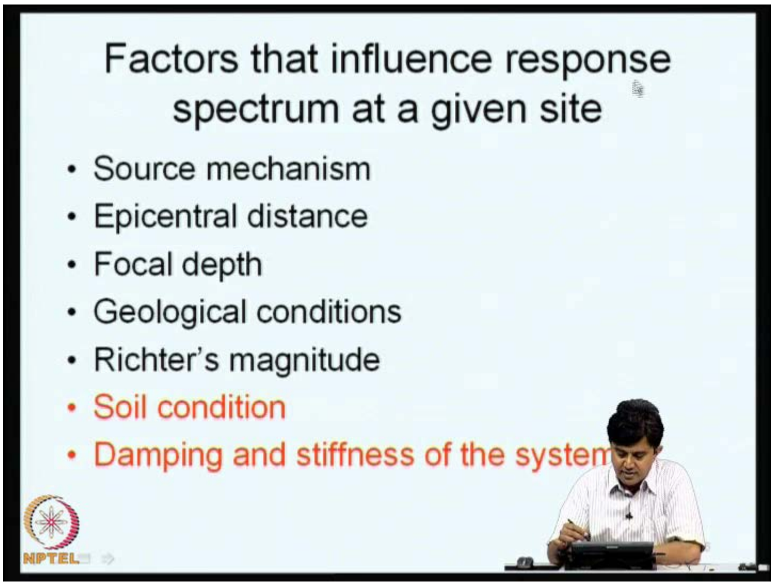

So, the factors that influence the response spectrum at a given side are, the source mechanism, the epicentral distance, focal depth, the geological conditions, the magnitude of the earthquake, so on and so forth. These factors that is the first five factors are not explicitly displayed on a smooth design response spectra; what this smooth design response spectrum essentially allows for is the influence of soil condition, and damping and stiffness of the system. These other factor which are important will have to be dealt with separately, in terms of specifying the peak ground acceleration. So, the frequency content and the shape would take into account, the influence of soil condition, damping and stiffness of the system. The geological influence of source mechanism, epicentral distance, magnitude, etcetera, will be accounted for in specifying the peak ground acceleration; so, this is a general approach that is often used.

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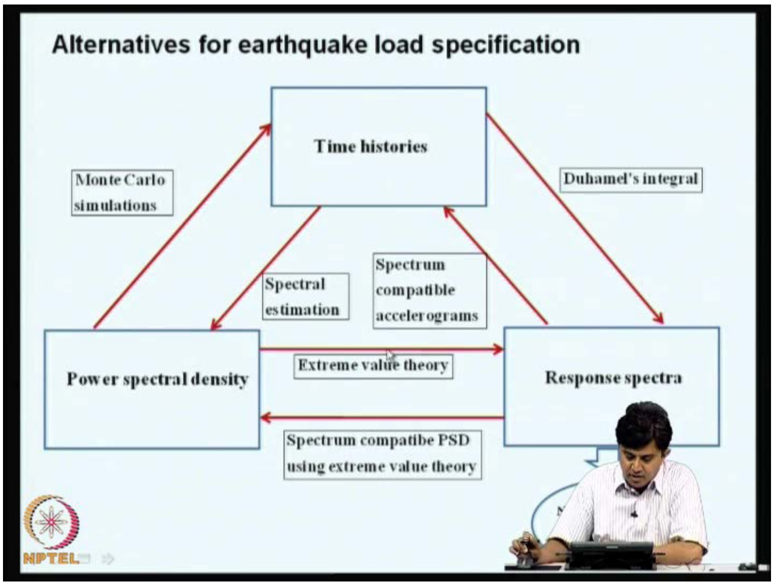

So, now we will return to this graph; I have now explained what are response spectra. Now, response spectra essentially represent the peak response of a single degree freedom - a family of single degree freedom system - to specify ground motion. So, consequently the relationship between response spectra and power spectral density function will be essentially through the theory of extreme values.

Suppose if we are given power spectral density function of the ground motion, how do we get response spectra? The response spectrum ordinates can be interpreted as the peak response, so over a duration. So, that thus the relationship between power spectral density and response spectra will be through extreme value theory, which involves spectral moment, shape factors, and so on and so forth. Similarly, if we are given response spectra and we want to generate a power spectral density, which is compatible with the given response spectra, we have to still use the extreme value theory, but (( )) in a straightly inverse form; we will see how it can be done.

The relationship between the ground motion and response spectra is through Duhamel integral, that I just now described. It is possible to generate a set of time histories, which are compatible with the given response spectra, using what are known as spectrum compatible accelerograms. There are the algorithms for generating that, this aspect I will not be covering, but I will be now discussing the relationship between power spectral density function and response spectra.

The response spectrum as such is not suited for analysis such as, reliability analysis; and if there is non-linear behavior in the system, we will have difficulties in using response spectra, unless we make certain assumptions. So, for purpose of non-linear analysis, it is best to generate time histories that are compatible with response spectra and do a time domain analysis. And similarly, for reliability analysis, it is best to use a compatible power spectral density function, and then, you can do Monte Carlo or subset simulations or whatever you want, based on these models.

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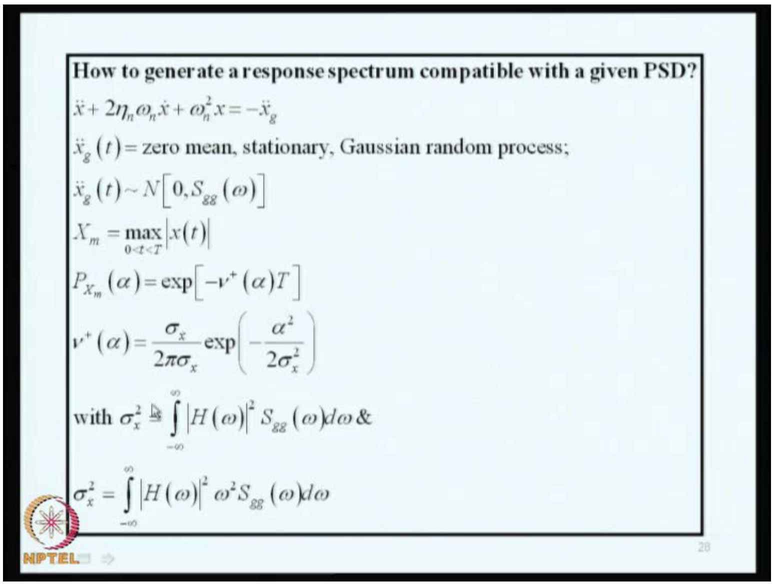

So, we will consider now a few questions; how to generate a response spectrum compatible with a given power spectral density function. That would mean, we will consider the dynamics of a single degree freedom system with natural frequency omega n and damping eta n, and subject it to the action of a ground acceleration, x g double dot of t. We model the ground acceleration as a 0 mean, stationary, Gaussian random process; it is described in terms of its power spectral density N, here is the normal probability distribution, this is 0 is the mean, this is the power spectral density function.

So, what we are interested? We are interested in maximum value of the absolute value of response in steady state over a duration capital T; this we already know, that through our analysis of extremes, we know that the probability distribution of this extreme is given in terms of rate of crossing of level alpha, multiplied by T; and this rate itself is given in terms of the so called spectral moments; so, this we are presently ready with.

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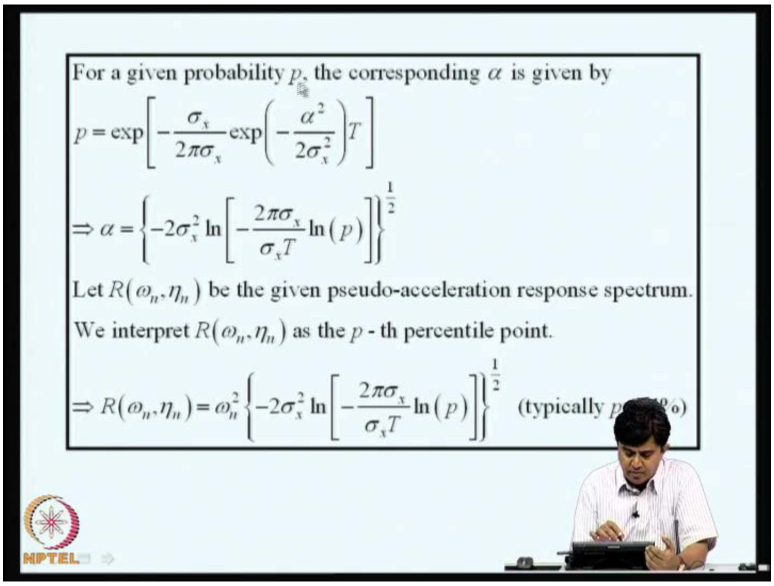

Now, for a given probability p, the corresponding value of alpha, I can be derived using this relation, p is a probability; so, we will need alpha now, at which level this probability is a valid. So, if you work out alpha, this is this inversion of this; and if we denote by R of omega n comma eta n is the given Pseudo acceleration response spectrum, we interpret R of omega n comma eta n as the pth percentile point. Consequently, we get R of omega n comma eta n itself nothing but this expression; this term is this, and I am multiplying by omega n square to get the Pseudo acceleration.

So, here, on the left hand side, we have the response spectrum that we are looking for, and on the right hand side, we have the natural frequency, and sigma x and sigma x dot, which in turn contain information on the input power spectral density, damping natural frequency, and duration over which we are taking the extreme value, and p is the probability, that is typically probability level is about 84 percent.

So, given power spectral density therefore I can get the compatible response spectrum. It could be also be interpreted as simply the mean value; this is now here it is given in terms of a percentile point, but we can simply say the expected peak value plotted as a function of omega n is our desired response spectrum. So, it is a matter of interpretation that we have to adopt.

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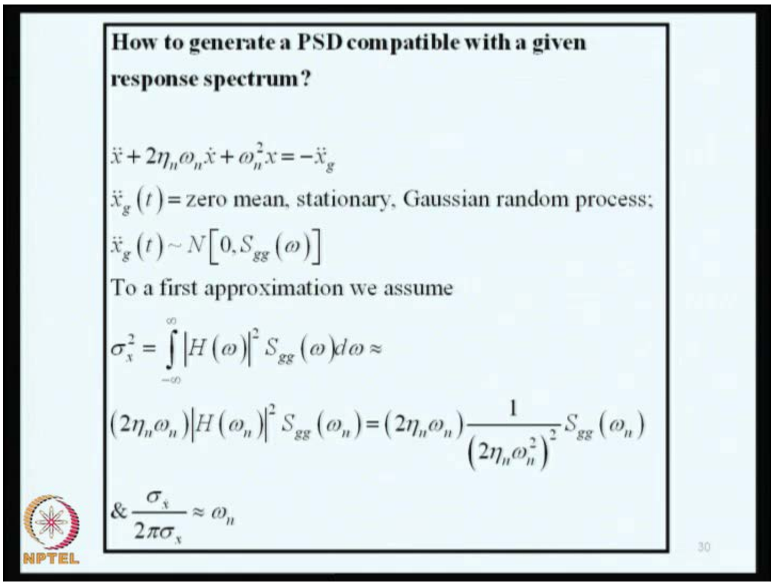

Now, we will consider the other problem, where suppose if you are given response spectrum, how do we generate a power spectral density function compatible with the given responsibility. So, again we consider the same problem, x g double dot, this is 0 mean, stationary, Gaussian random process with a specified power spectral density function. Now, to first approximation, this is the sigma x square is given by this; this is exact, but we approximate this by assuming that the power spectral density function is fairly flat in regions, where H of omega very sharply, that is near the natural frequencies, and therefore, we can replace this integral by this; this approximation I have discussed earlier, you know, while discussing linear random vibration analysis, and we get here sigma x square and sigma x dot.

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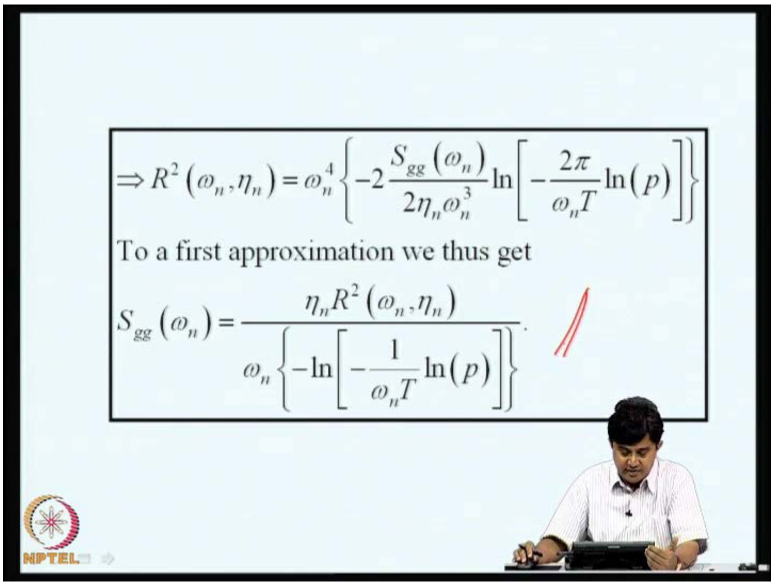

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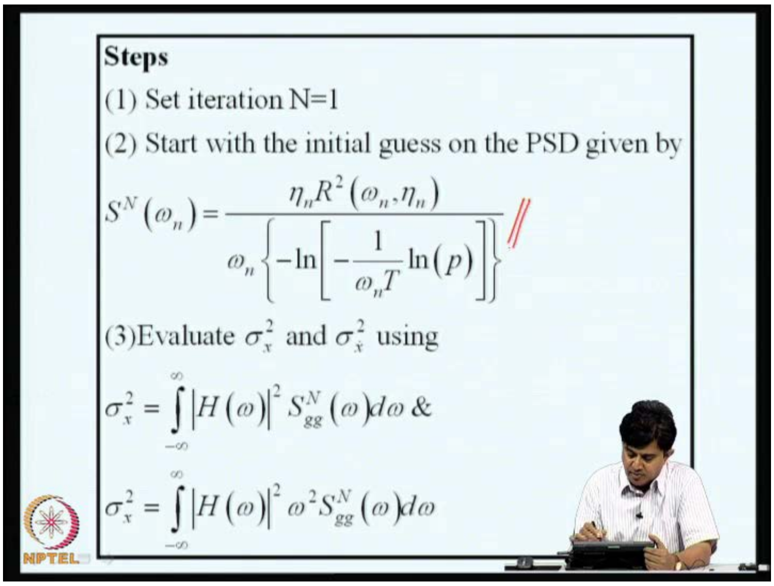

Now, we go to the definition of response spectrum in terms of pth percentile point. We have R square is equal to this. Now, to a first approximation, we take the power spectral density function from this formula and we approximate the power spectral density to be given by this. Equipped with first level, guess on power spectral density function, we know perform a set of iterations. So, we set the iteration number N equal to 1 and start with initial guess on power spectral density function given by this. And then, we evaluate sigma x square and sigma x dot square with rules of quadrature, no more we make the assumption that it is broad banded etcetera. So, we get better approximation to sigma x square and sigma x dot square.

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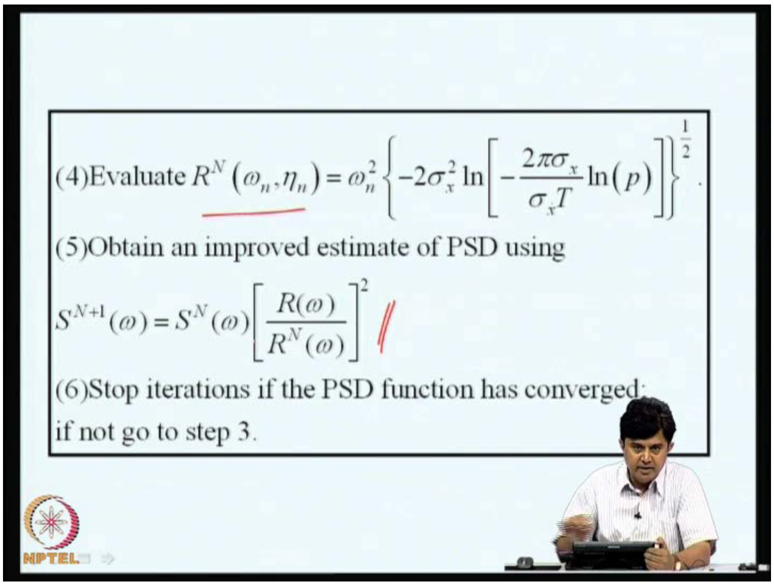

Based on that, we return to the definition of response spectra and get an improved approximation to the response spectra. And following this, we get the approximation to the power spectral density function and this is used as the approximation for that. And we check for convergence, and we stop if the PSD has converged, or if not we repeat the steps. So, we can write this has to be coded on a computer and this can easily be implemented.

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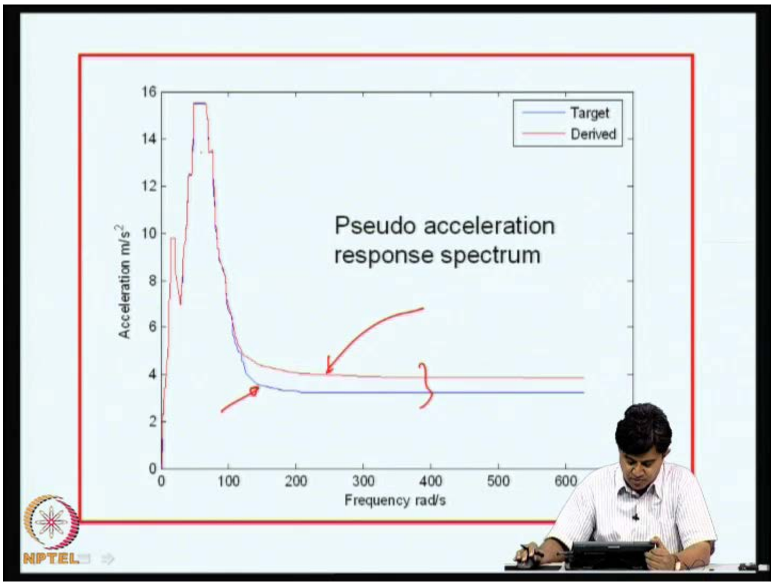

So, I will show an example, where the blue line here is the smooth design response spectrum, and the problem on hand is to generate the associated power spectral density function. So, what we have done here is, we start with this blue line and we find the corresponding compatible power spectral density function; and corresponding to that power spectral density function, we estimate the response spectrum and we compare what we have derived with the target. So, we can see here that, over the frequency range of interest, the two curves that is a blue and red are matching fairly well; there is a an error on the high frequency and this is not serious, because they will not be the dynamics is captured essentially in the regions, where there is significant peaks in the response spectrum.

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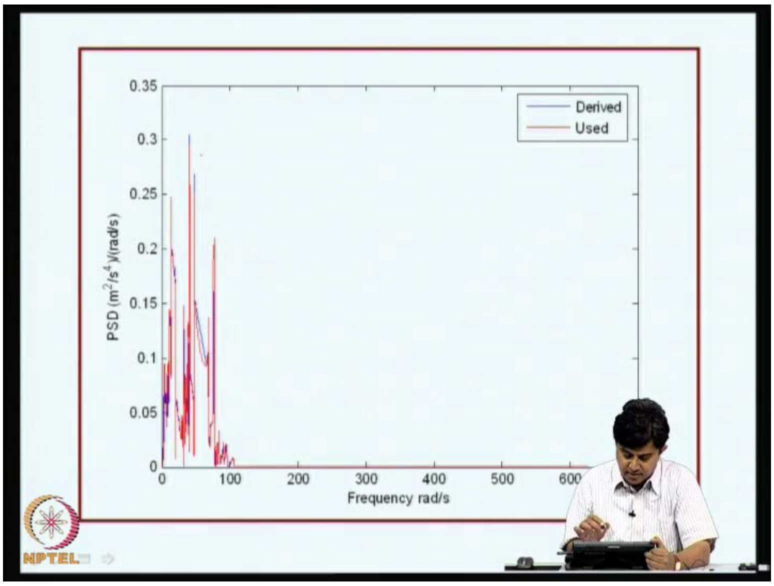

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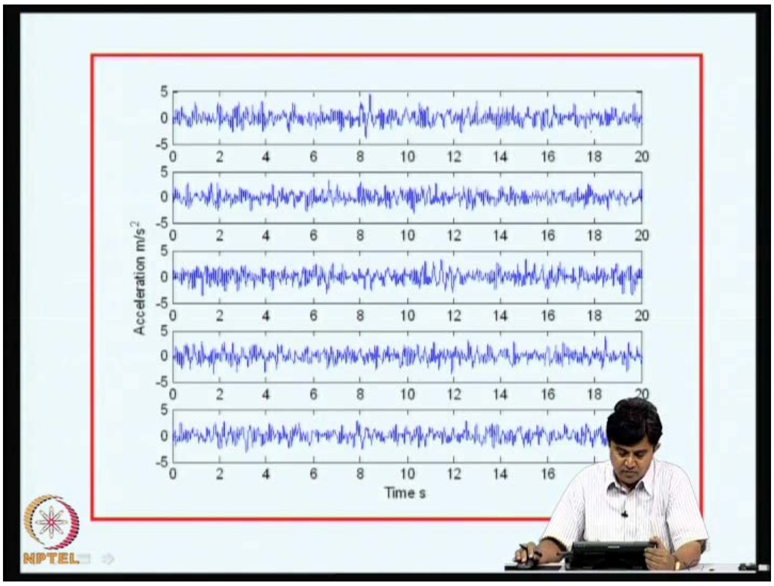

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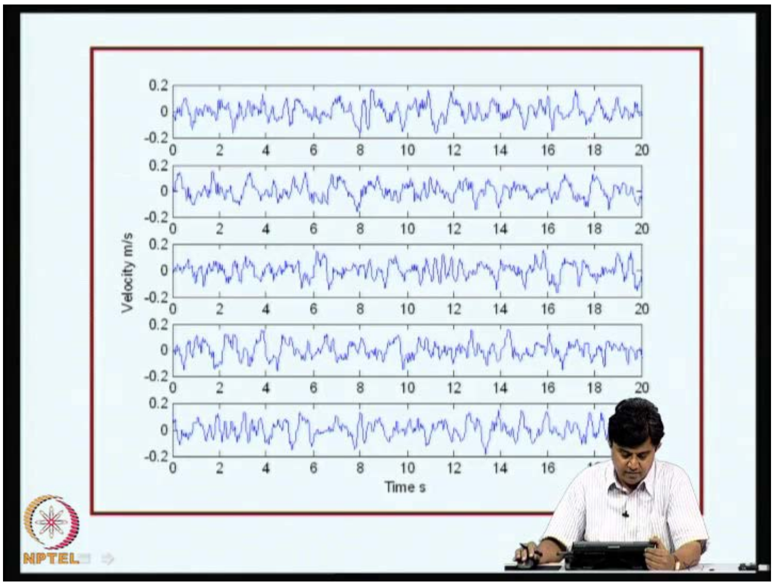

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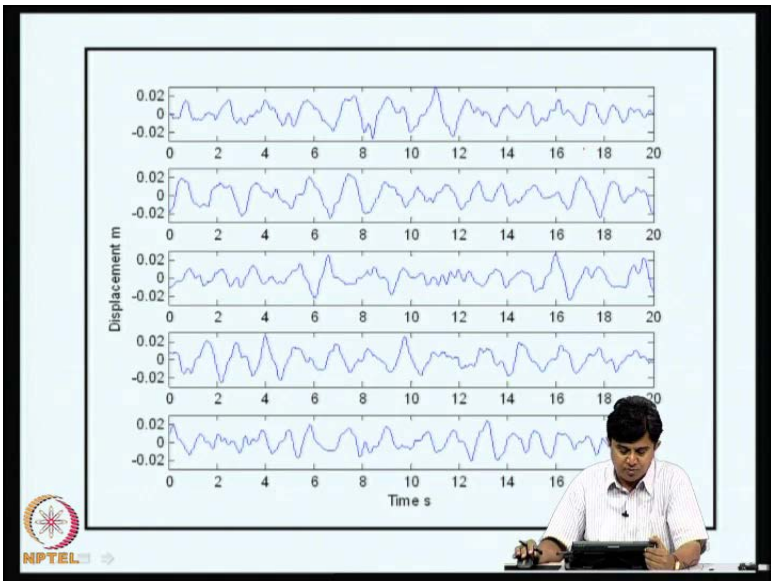

So, this is the comparison of the power spectral density function. Again, there is a derived one and the used one, they compare reasonably well. And this is an ensemble of compatible time histories; this is on acceleration, this is on velocity, this is on displacement. So, here we have use the Fourier representation for samples of stationary Gaussian random process, where assuming that the signal is mean square periodic and these are the samples.

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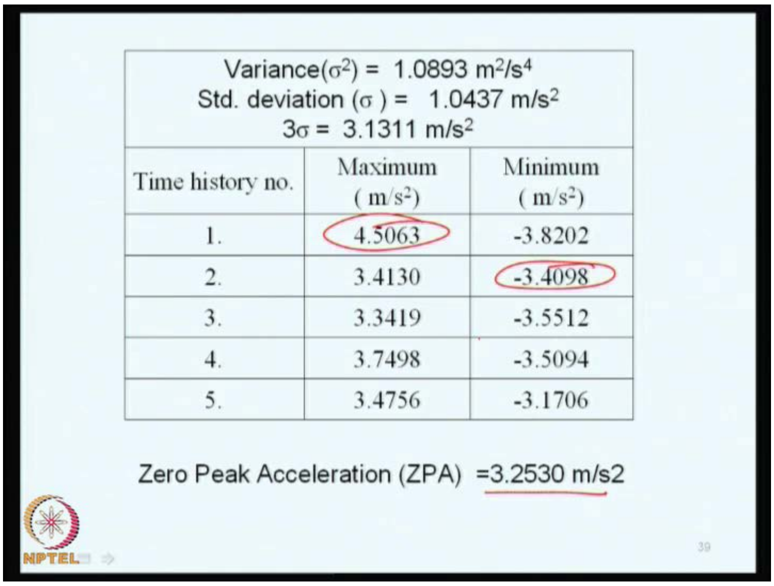

Variance(σ^2) = 1.0893 m^2/s^4
Std. deviation (σ) = 1.0437 m/s^2
3σ = 3.1311 m/s^2

Time history no.	Maximum (m/s^2)	Minimum (m/s^2)
1.	4.5063	-3.8202
2.	3.4130	-3.4098
3.	3.3419	-3.5512
4.	3.7498	-3.5094
5.	3.4756	-3.1706

Zero Peak Acceleration (ZPA) =3.2530 m/s2

So, we have compared here a few episodes of simulations and found out the actual peak ground acceleration, and compared with the 0 period acceleration implied in the response spectrum. And the 0 period acceleration is 3.253 meter per second square and different realizations of the maximum are shown here in this; this is the maximum here, this is the maximum here, so on and so forth. These are all different realizations of the peak ground acceleration for different samples.

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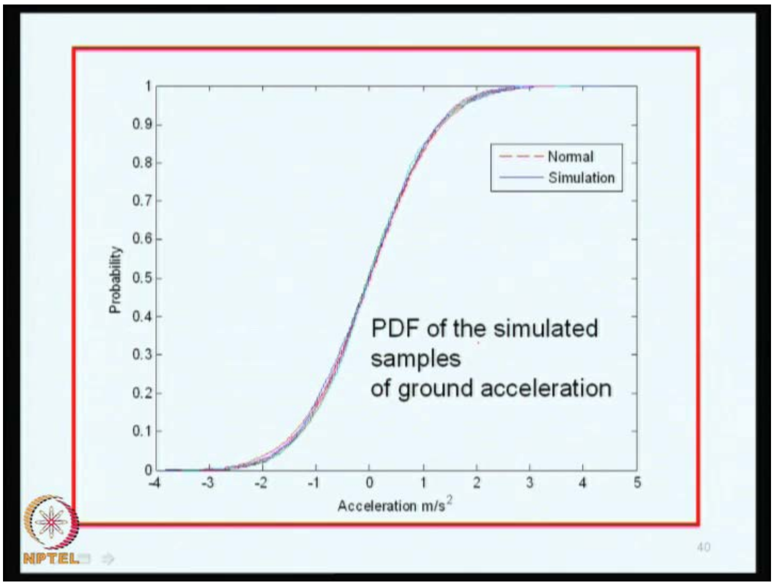

So, just to make sure that we are simulating correctly, in this slide, we are showing the probability distribution of the simulated samples of the ground acceleration. And there is a red line which corresponds to the normal distribution that the target, and blue is the simulation and they compare; I mean, these are different time histories are superposed on each other and they show reasonably good mutual agreement.

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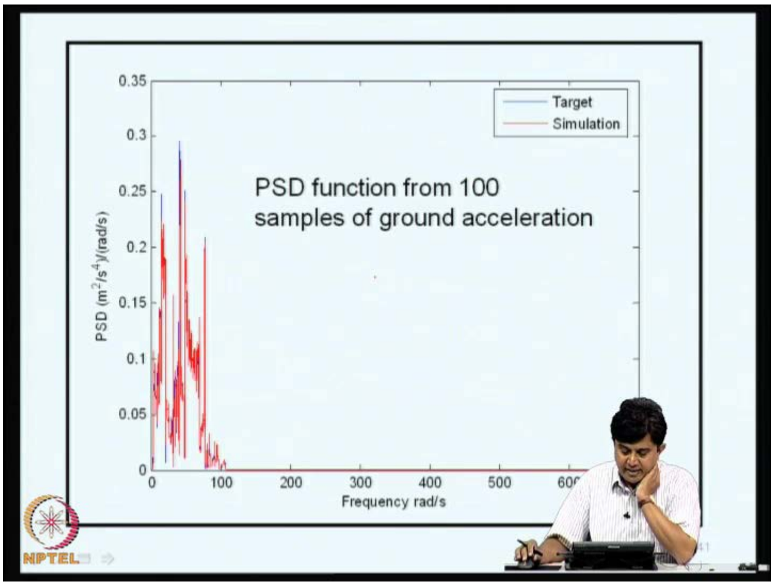

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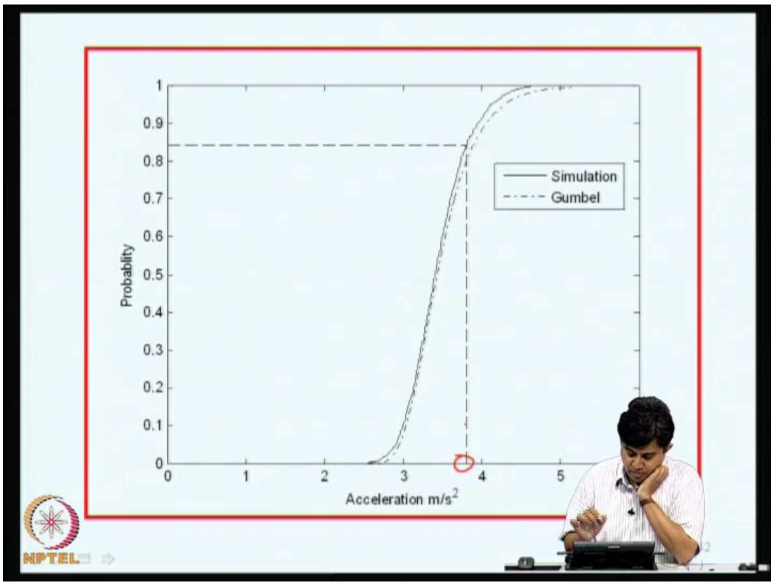

We also simulated the power spectral density functions from hundred samples of ground acceleration and compared it with the target. And here again we see reasonably good match. This is the plot of extremes of the ground acceleration; just to show that we are taking 84 percentile point to define our peak ground acceleration, and that matches, this graph shows that we are getting a reasonably good match for that number.

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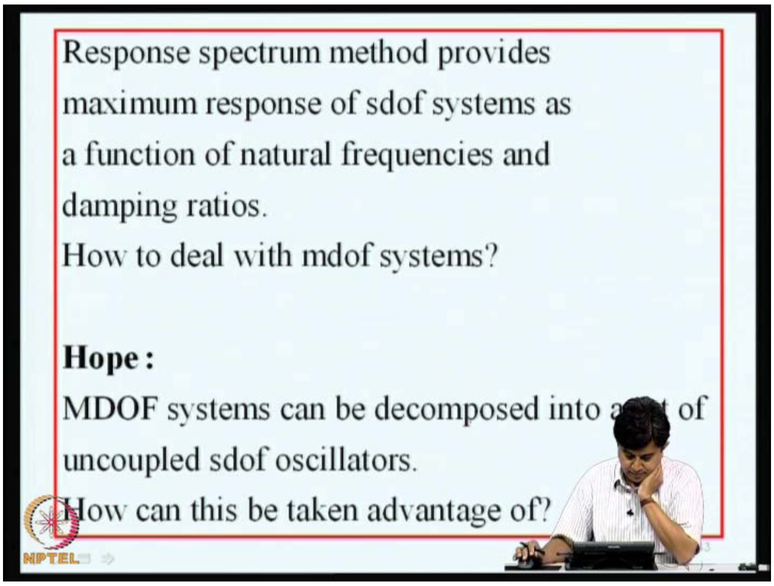

Now, in our discussion on response spectrum, what we have done so far is that, we have considered response of single degree freedom system to the given ground motions. So, the response spectrum method provides maximum response of SDOF systems as a function of natural frequencies and damping ratios. Now, we seldom need to considers single degree freedom systems, often we have to deal with multi degree freedom system. So, how do we deal with multi degree freedom systems, when earth quake loads are specified as set of response spectrum?

Now, the hope here is, that the multi degree freedom systems can be decomposed into a set of uncoupled oscillators; so, that we know. So, for each of the generalized coordinates using the given response spectrum, we can find out the peak values. Now, the fact that multi degree freedom systems can be decomposed into a set of single degree freedom system is an important advantage. Now, how best we can use this, when loads are specified in terms of response spectrum. That is the question we need to consider.

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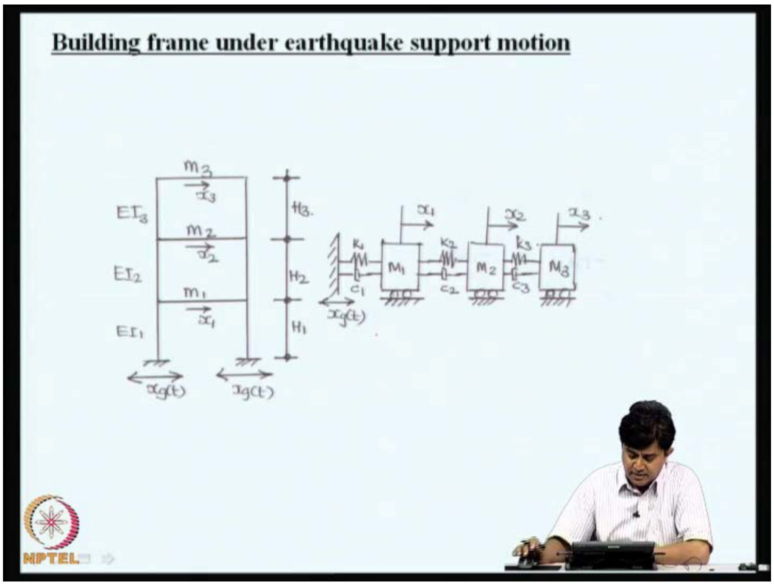

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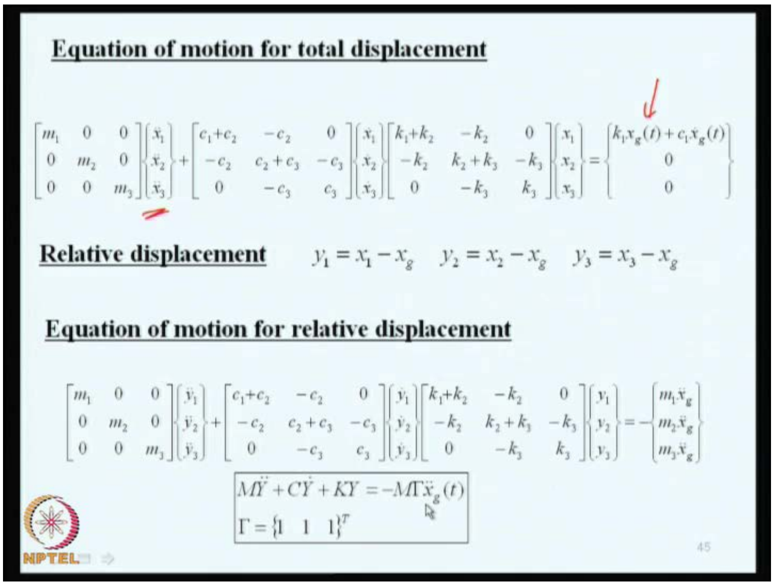

Now, to clarify some of the notations etcetera, we consider a three degree freedom system; this we have discussed earlier on a few occasions. And this is the shear building model for that, and we can write the equations of motion; for total displacement, here where the excitation now will appear in terms of support displacement and velocity, or we can set up the equation for relative displacement, in which case the support motions appear as accelerations as shown here. So, the governing motion, governing equation of motion for relative displacement will be MY double dot plus CY dot plus KY equal to minus M into an influence vector x t double dot of t; this we have seen on few occasions earlier.

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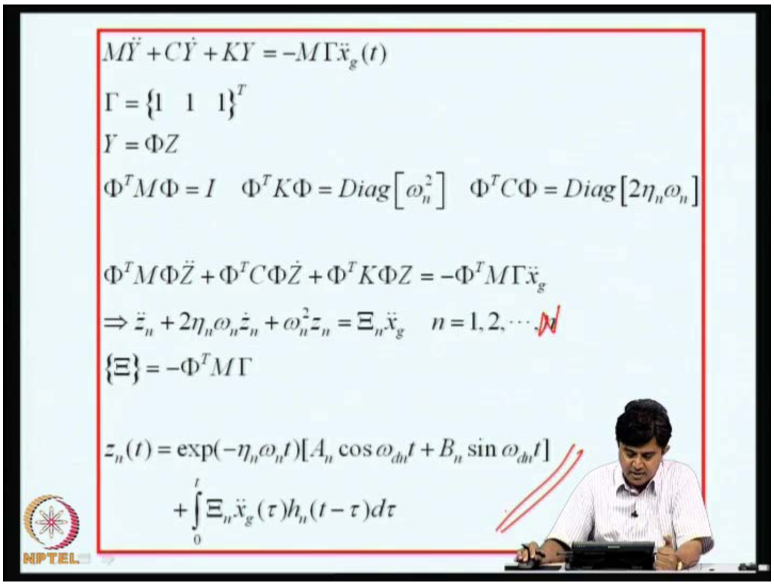

Now, we will just go through some of the steps involved in the solution. So, we make the substitution Y equal to phi Z, where phi is a model matrix and we uncouple the equation using the orthogonality properties. And we get the set of equations for the generalized coordinates which are uncouple, and this capital XIN is the so called model participation factor and this determines what fraction of ground acceleration acts on the nth oscillator.

So, this can be solved in principle using the theory of ordinary differential equation, that Duhamel integral and so on, so forth. So, this also we have seen in principle, we can deal with the response of multi degree freedom system to support motions.

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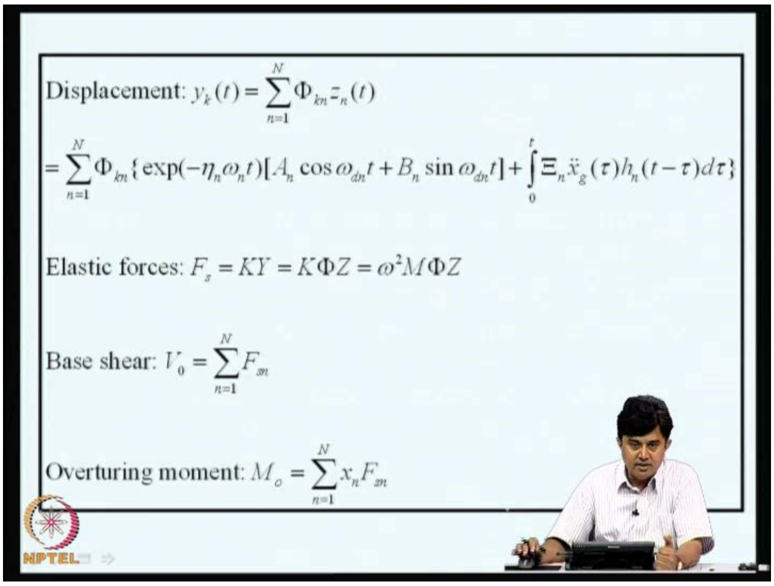

Now, structures displacement at kth degree of freedom is given by this, and this can be expressed terms of ground acceleration in this form. How would elastic forces? It is stiffness matrix into the relative displacement; and this again we can be obtained using this. How about base shear? It is some of the floor shears at various levels and this can be obtained, once we know the elastic forces. Similarly, over turning moment is the moment of these forces at different floor levels multiplied by the heights; x n is the height of the nth floor measure from the ground. So, these quantities are of interest, when we analyze the structure to support displacement.

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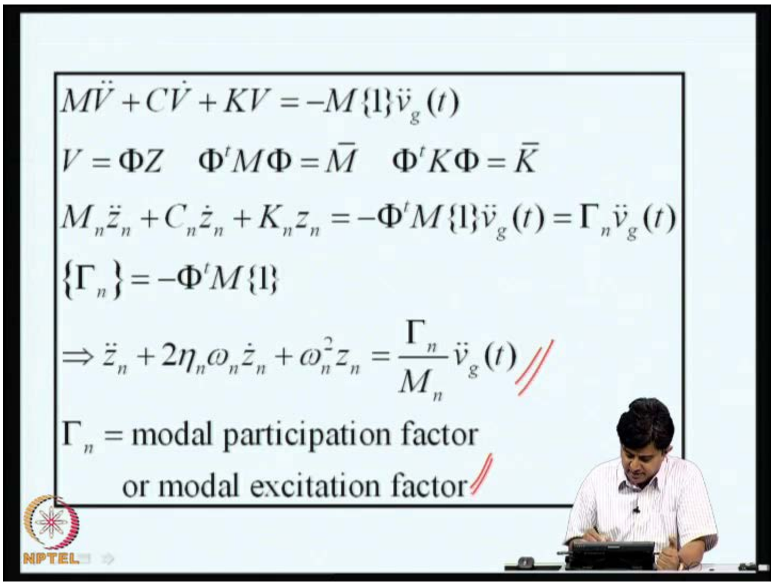

So, just again slight repetition here; this is the equation for the generalized coordinates and this gamma n by M n is known as a model participation factor or model excitation factor.

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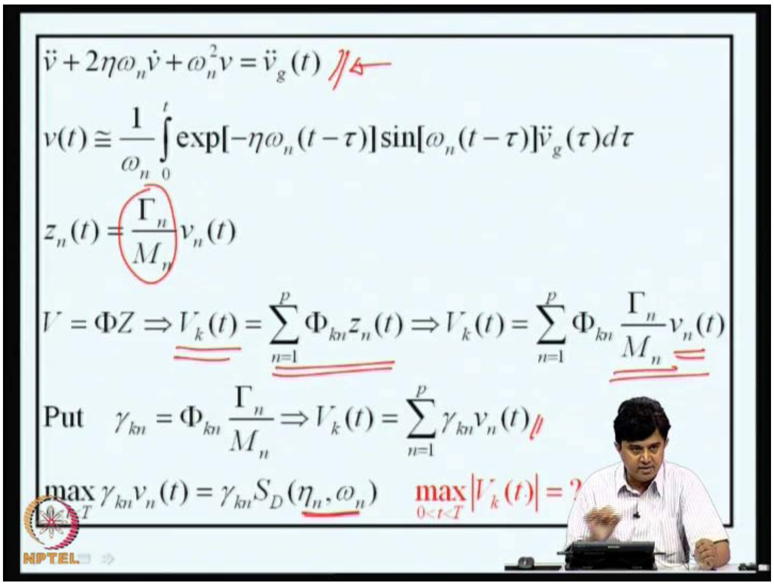

So, what we are interested in multi degree freedom system. So, we are interested in, suppose the kth displacement relative displacement at the kth floor level, we have to find this quantity. Now, in for z n of t, I will write in terms of the participation factor; and I define this v of t here, this is the response of the system to applied support displace support acceleration; and you remember that, this is the family of single degree freedom system, that we have used in defining response spectrum.

So, this is actually the quantity that is used in response spectrum, that is what I will multiply now, because the excitation is multiplied by this factor, I need to find the time history was a generalized coordinates, I need to simply multiply this v n of t by this participation factor.

Now, I put gamma k n, I will introduce all this and I will introduce a single number, I call it as gamma k n; and in terms of v n of t, the response is given as here. Now, if you are interested in now maximum value of v k of t, that is what we are interested in. now, we know the maximum value of these terms inside the summation. So, we know that the maximum value of gamma k n v n of t is nothing but gamma k n into S d eta n omega n; so, this is the given response spectrum. Therefore, what I know is the maximum value of the term inside the summation, but what I am interested is a maximum value of the sum. So, this question still remains; we have not yet answered this.

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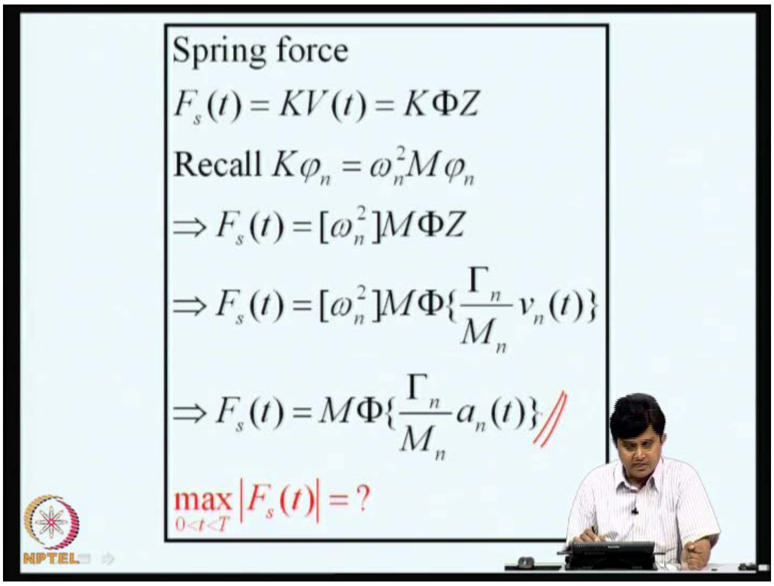

Similarly, the quantities spring force is K into V, and if we simplify this, again we get the spring force - the vector of spring forces - in terms of a n of t which are generalized coordinates. No, this is a n of t is the Pseudo acceleration; and the question is again this is a matrix product, therefore, a summation is implied. So, the question is again, what is maximum value of the vector of support spring forces?

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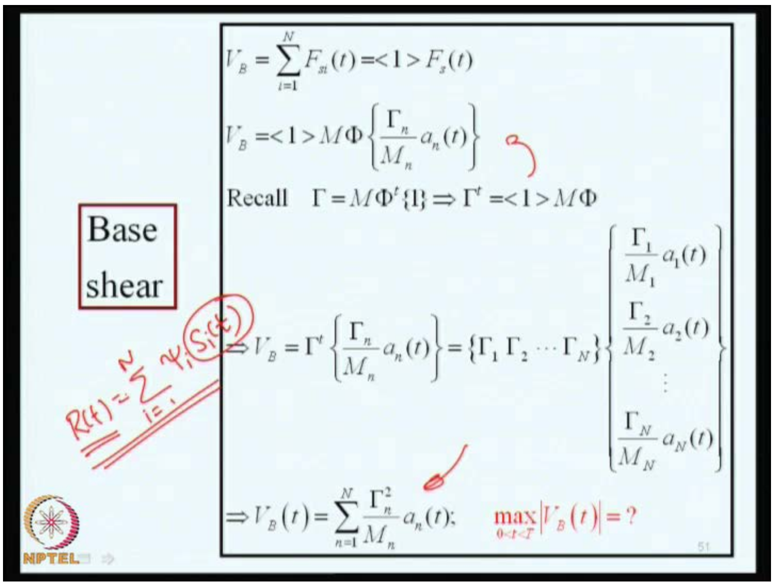

Now, let us look at base shear, which is sum of all these sum of F s i of t. So, this summation can be written as, row of 1 into F x of t. So, V b is given by, for F x of t, I will write this. And we know this gamma is M phi transpose into 1; so, gamma transpose is this. And if you substitute this into this, we get this expression, and therefore, the base shear is given in terms of a n of t, which is omega n square into v. So, we know the maximum value of again this term, in terms of the Pseudo acceleration response spectrum; we can derive this, but the question which still remain, what is the maximum value of V b of t?

So, for each of these quantities, displacement or spring force or the base shear, we are getting a general form of the expression is, say, response R of t is some kind of summation i equal to 1 to n some psi i into some S i of t. This is the generic form we are getting. The maximum value of S i of t can be estimated based on the given response spectrum, but it does not tell us what is the maximum value of R of t. So, that is the general question that we are coming across for each of the situations.

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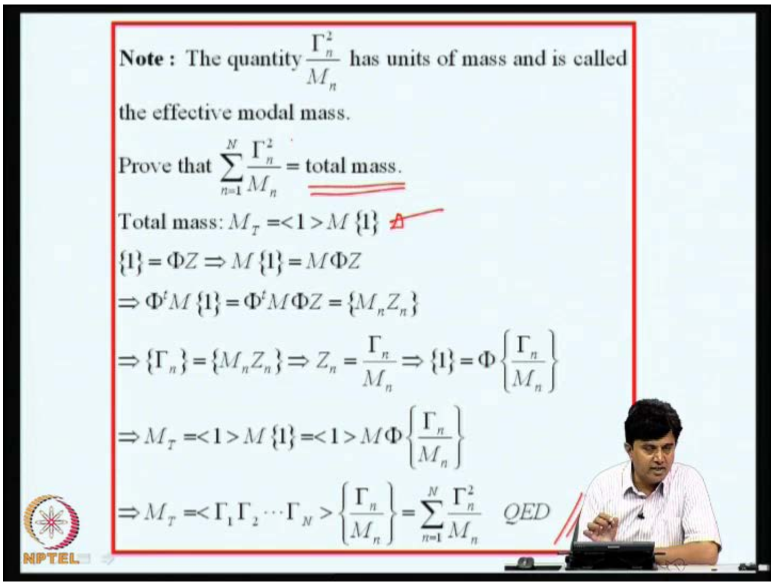

Now, I will come to this question slightly later, but we will see some of the properties of representing the response in terms of response spectrum. Now, you will look at the quantity gamma n square by M n, we can verify that it has units of mass and we call it as effective modal mass. We can prove that, the some of this effective modal mass will be equal to the total mass; this is a very useful quantity, when we are modeling; we can first discuss how this can be proved. The total mass is given by 1 into M into 1 row row 1 mass matrix into 1, that is simply it adds up the all the mass elements. So, we can use now, for 1, we will do a modal decomposition and write it in terms of phi Z, and pre-multiply by M, I get M into 1 is M phi Z; and again pre-multiply by phi transpose, I get this is orthogonal with diagonal terms of M n; so, this will be simply M n Z n.

So, now if you use all these this expressions and go back to this definition of total mass, we can show that the total mass is same as the sum of effective modal masses. Now, this is I said is useful in modeling, because one of the question that we need to consider is, how many modes we should include in analyzing a multi degree freedom system? So, a role that we can adopt is, we should include as many modes as it requires to capture 90 percent of the mass of the structure. So, you can go on evaluating the effective modal mass and you can retain as many modes, as is needed to capture above 90 percentage of the mass of the structure; so, that can be one of the criteria.

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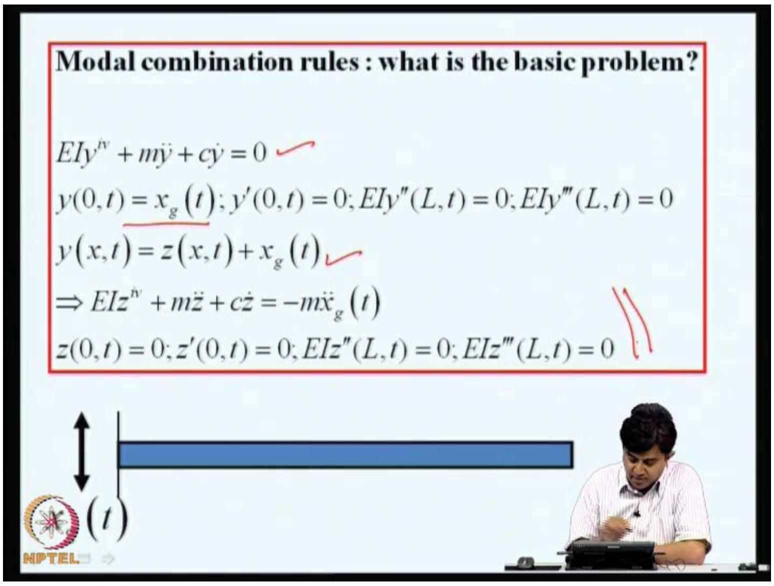

Now, I will try to answer the question that I have been posing, on how to find the maximum value of a quantity, which is expressed in terms of a summation, where the maximum value of that inside the summation are known. So, just to make that point clear, we will consider a cantilever beam under support displacement, and we know that the governing equation for relative displacement can be derived. First, we can write the equation for absolute displacement, which is written here, and this has time dependent boundary conditions; and through this transformation, we can convert them into homogenous boundary conditions, and in homogenous hand side and we get this equation.

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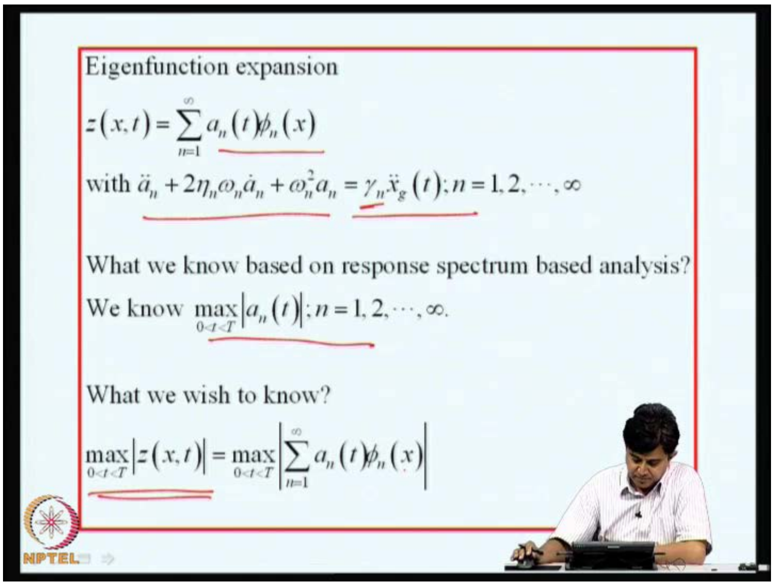

Now, we for Z of x comma t, we can perform a modal expansion in terms of generalized coordinates n of t and the [mosses] phi n of x, and this leads to a set of uncoupled oscillators; this we have seen on few occasions earlier.

Now, what do we know based on response spectrum based analysis. We know the maximum value of the response of a family of single degree freedom system so the given ground accelerations. So, what we know is maximum value of a n of t; from the response spectrum occurs, I can multiply the maximum value of the ordinate at omega n and eta n by this participation factor gamma n, and I can evaluate this. But what we wish to know is the maximum value of this summation.

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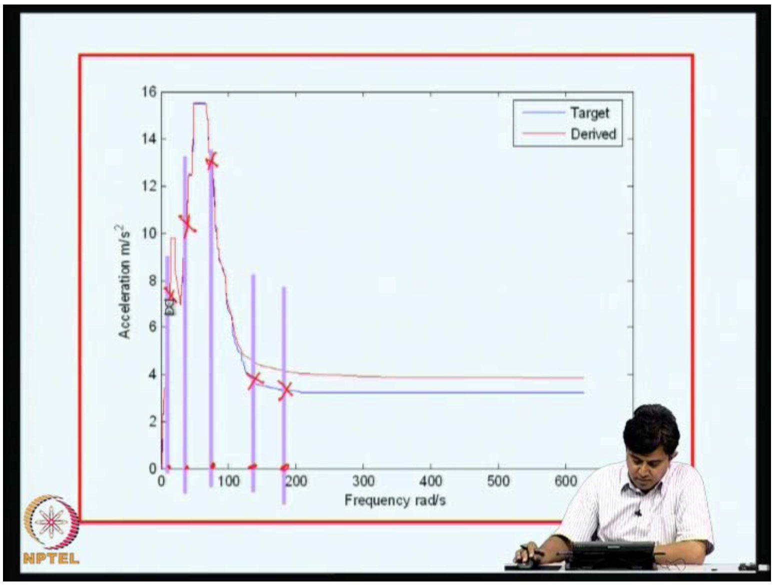

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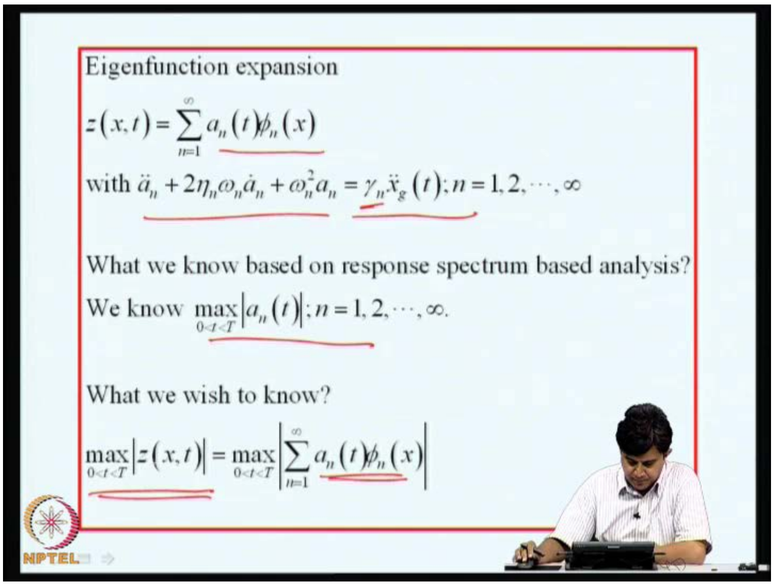

So, I need maximum value of absolute value of Z of x comma t, which is maximum value of this summation, that would mean, suppose you focus on the blue line which is the response spectrum, and these lines vertical lines here are the lines drawn at the system natural frequencies. Suppose these are the system natural frequencies, then I know that the response that we need to consider are here. So, this, this, this crosses provide us with the individual maximum of the generalized coordinates; using that, I can find out the maximum value of the terms inside the summation, but what I am interested is maximum value of the sum.

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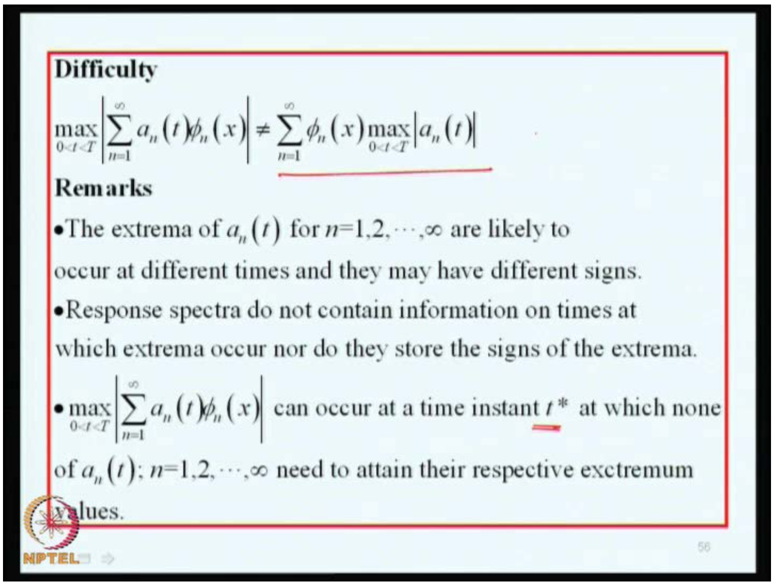

So, clearly the maximum value of this sum is not equal to the maximum value of, you know, you cannot take the maximum value inside this and write this in this form, simply because, the extremes of a n of t for n is equal to 1 to infinity are likely to occur at different times and they may have different signs; one of that could be positive and other could be negative. The response spectra do not contain information on times at which extreme occur, nor do they store the sign of the extreme; so, that information is lost in the definition of response spectra.

Even if you have stored this, there will be still difficulty, because so suppose the maximum value of this sum, suppose if it occurs at a time instant t star, this t star need not coincide with any of the time instant at which individual a n of t reach the respective maximum values. So, there can be altogether a new t star, at which the sum reaches its maximum; at that t star, there is no reason why any of these a n of t need to reach the respective maximum values. So, that would mean, there is a basic difficulty in using response spectrum based methods for multi degree freedom systems.

If all a n of t reach maximum value at same time and they all of them have same sign, then this right hand side will be equal to the left hand side, but that is very unlikely to happen. So, we have to leave a certain approximations, if you have to response spectrum base methods or Seismic response analysis.

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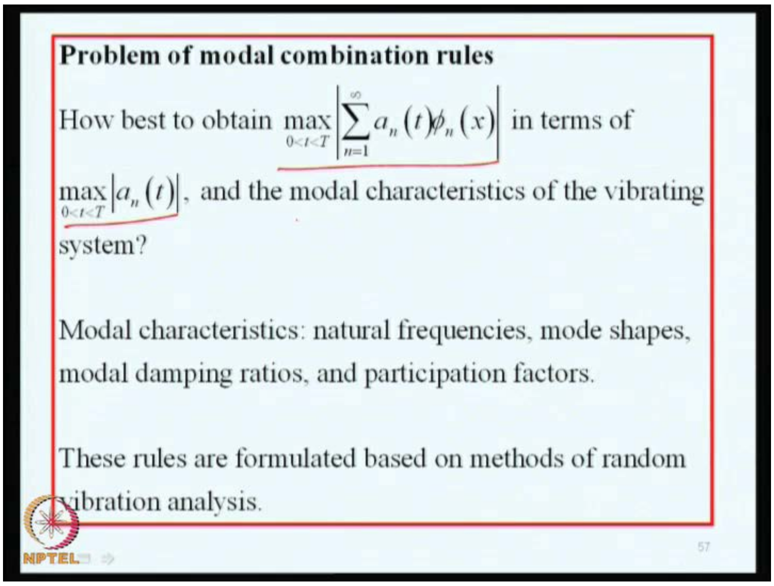

So, to deal with this, we consider what are what are known as modal combination rules; these rules basically address this question, how best to obtain the maximum value of this sum, in terms of maximum values of the terms inside the summation.. And of course, a modal characteristics of the vibrating system which are encapsulated in phi n of x. So, these modal characteristics are natural frequencies, mode shapes, modal damping ratios and participation factors.

Now, the rules for formulating these combinations are based on the application of principles of random vibration analysis. So, this is where the probabilistic analysis comes to the rescue of a deterministic analysis. So, what we will do is, we will close the lecture at this juncture. And in next lecture, we will consider the problem of modal combination rules, and discuss how principles of linear random vibration analysis can be applied to resolve this issue in an optmal manner. So, at this point, we will conclude this lecture.