Project Planning & Control

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Lecture – 53
Lesson - 05
PERT Example Problem – Summary

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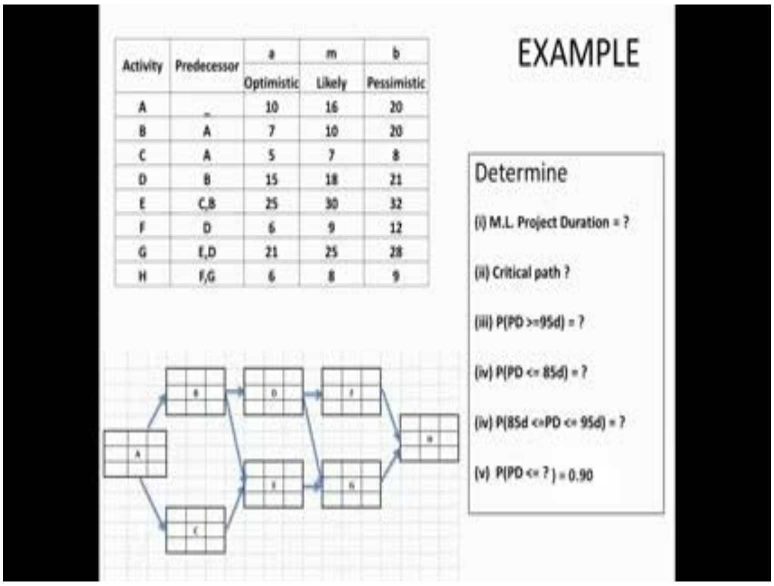

So, let us workout an example here, so here we have like we said the inputs are as activities, so here we have activity A to H, given you the predecessor relationship here, the optimistic duration in days, the most likely duration, the pessimistic duration. Now, so this is your basic input information and what is asked from this, so what are the questions we can ask when we given this. One is, we can look for the most likely project duration, so that is one of the results we will have to get, we can certainly get what the critical path is.

Now, here are some interesting questions which PERT will give you, we should be able to ask, what is the probability that the project duration is greater than or equal to 95 days. We could not ask such a question in CPM, or what is the probability that the project duration is less than or equal to 85 days or what is the probability that the project duration lies between 85 and 95 days or given that I want, given that my probability is 90%, what is my confidence of finishing my project at a 90% probability. What is the duration I should finish my project with 90% probability?

So, all of these types of questions can be asked from your basic PERT data. Now, what we have gone down here and done is we have actually put up the network, so this is the same network, the activity predecessor relationship is used to put the network.

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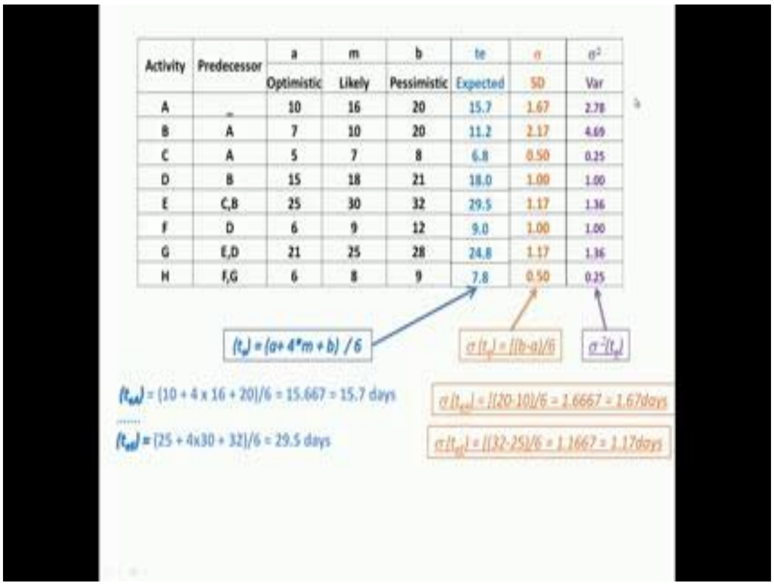

Activity	Predecessor	a	m	b	te	σ	σ²
		Optimistic	Likely	Pessimistic	Expected	SD	Var
A	-	10	16	20	15.7	1.67	2.78
B	A	7	10	20	11.2	2.17	4.69
C	A	5	7	8	6.8	0.50	0.25
D	B	15	18	21	18.0	1.00	1.00
E	C,B	25	30	32	29.5	1.17	1.36
F	D	6	9	12	9.0	1.00	1.00
G	E,D	21	25	28	24.8	1.17	1.36
H	F,G	6	8	9	7.8	0.50	0.25

And we go on to the next step in PERT, which is to calculate the expected value. So, here you can see I have used this formula to calculate the values in this column and you will be able to associate it. So, if I take for example 15.7 here, A is 10 plus 4 into 16 plus 20 divided by 6 gives me 15.7 which is the expected value. Similarly, I have given you.. worked it out for E here; that is 25 plus 4 into 30 plus 32 gives you 29.5.

So, you have calculated similarly for all the other activities, calculate expected duration and then, we move on to calculating the standard deviation for the activities. So, here we apply this formula and what we get here again is for, , the activity A, we have basically set B minus A which is 20 minus 10 divided by 6. So, this gives me, it is a rounded of the value of the standard deviation of the activities and from the standard deviation we square it to get the variance values for each of the activities. So, with this the table of, we have taken the basic input data, convert it to expected duration, the standard deviation and finally, the variance of the activities.

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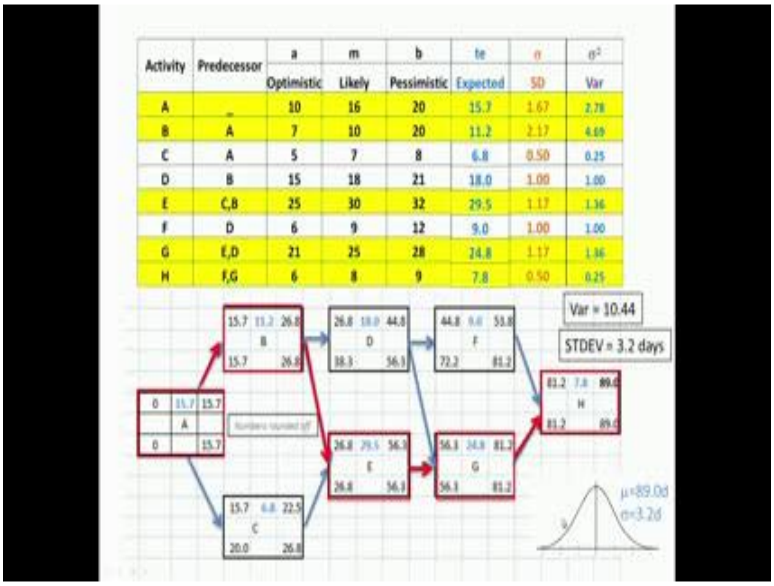

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Now, in the next step what we do is, like we discussed in the step-wise procedure, we take the expected values. So, for example, A is 15.7 here, for B 11.2 comes there, for C 6.8 comes there, for D it is 18, we have just used the expected values as the basic duration of the activity and we have done the forward pass and the backwards pass.

So, you will get the early start, the early finish for all of the activities, you can see it is the early start, early finish and some of these values are rounded, rounded off. So, you might not, you will see, based on what it is not exactly, what it is, it is a rounded off value which I have here and when you add the activities, so if I look at this you can take a guess at what the critical path is, you should be able to do that.

But, ultimately we land up with the project duration of H finishes on 89 and that is a project duration. We saying 89 days is the project duration, but like we discussed it is not just the project duration, it happens to be the mean of the project duration, it happens to be the mean of the normal distribution showing the project duration. Now, we do more computations, so for example, we can take we can find the critical paths, so you can see the critical path goes through A B E G H.

And now, like we discussed earlier, I will have to find the variance of this 89 being the mean, what is the variance of this distribution. We remember it is the sum of the variances. So, I have to take the sum of the variance of A B E G H and sum it up and I get a total variance of 10.44, so that is the variance along this path and I take the square root of that to get the standard deviation as 3.2.

So, my project duration now is defined as a normal distribution with a mean of 89 and the standard deviation of 3.2. So, this has characterised my project duration, the distribution by giving the mean and it is the standard deviation. So, now, we go further, so this has given me, so this is really the core of PERT, all of the assumptions we have used, everything has now been applied and we are now, to be able to use this data to answer questions we are basically using basic probability and statistics approaches, which is to read the values of a normal curve. But up to here is where PERT contribution has been done.

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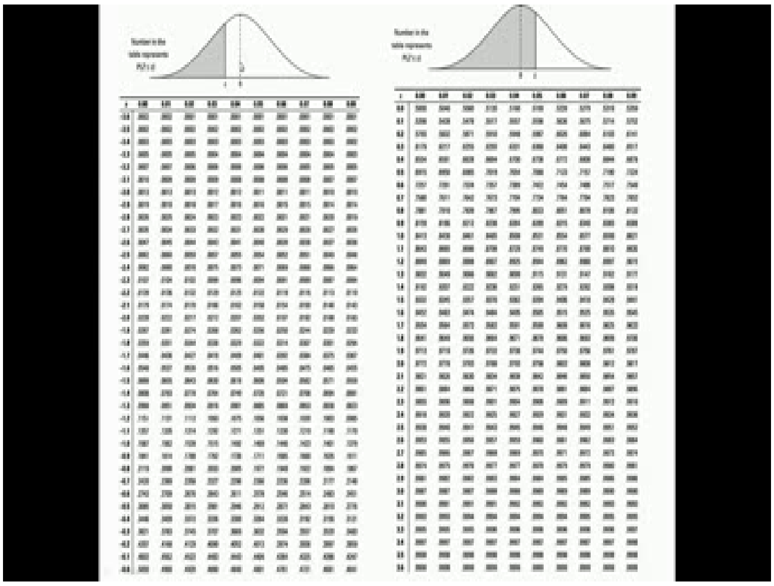

Now, you might.. you have to.. like we discussed in the initial part of this lecture, be familiar how to read a normal distribution value; the z values. So, here you have, I gave you the curve.. the table in two parts; one part you can see the distribution here and it gives you the area for this level of the area. So, here given a certain distance of z from the mean, what is the area under the curve is described here.

Here it gives you the, shows the shaded area and discusses what the shaded area is based on the distance again from the mean. Now, you should be familiar with how to interpret these two and we will use, we will be using this curves for further analysis.

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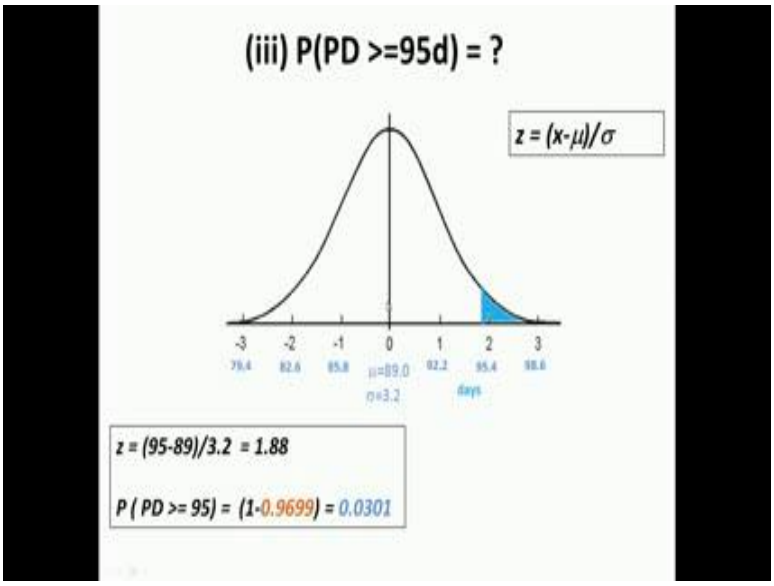

So, remember the first one of the other questions we asked in the, from a probabilistic perspective, what is the probability that the project duration is more than 95 days or it is greater than or equal to 95 days. So, if we take, so, remember, so this is back to a normal distribution of project duration, we had a mean of 89, we had a standard deviation of 3.2 and we have now looked at the standard deviation here.

So, we have one first standard deviation, second, third and we have now , the values here are the days to which the, for the corresponding standard deviation, so these are the project duration days. So, when we say that project duration of 95 days, it falls somewhere here. So, to find the probability of that, basically what we are saying is, what is the area of this, this shaded part of the curve.

So, my 95 days is here and for the project duration to be greater than 95 it has to fall in this area, so what is the area of this part of the curve. So, we go to the tables, as you have to go to the tables, but to go to the table we really have to first find what the z value here, yes. So, the z is actually the number of standard deviations from the mean, so here we have 95 as the mean.

So, we have basically, I am sorry, 95 as a value, we have 89 as the mean, I am basically using just formula which says, it is a value minus mean divided by sigma is the z value 1.88 and what we do is, so the 1.88 is right here; that is the distance from the mean. It is 1.88 standard deviations from the distance from the mean; that is where 95 days in this normal curve.

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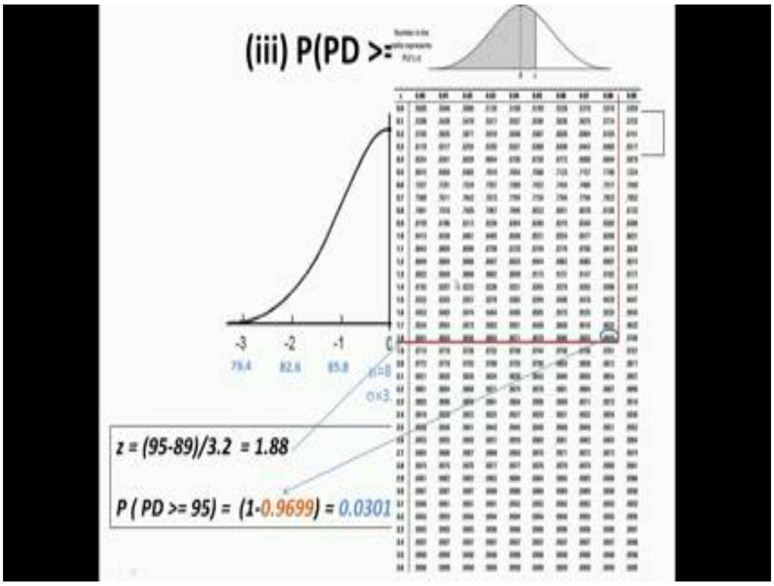

And I actually go to the normal table and look at 1.88, so here we have the table 1.88, 1.88 and I get the value 0.9699, which is now you can see 0.9699 represents this whole area. So, the area I want is the area beyond that, which will be 1 minus 1.9699, so the probability of project duration being greater than or equal to 95 is 0.0301. So, I hope you followed how that computation was done to be able to get this value, but basically, what we are doing is, transforming the 95 into the number of standard deviations away from the mean and using that to go to the normal distribution table to find the probability value.

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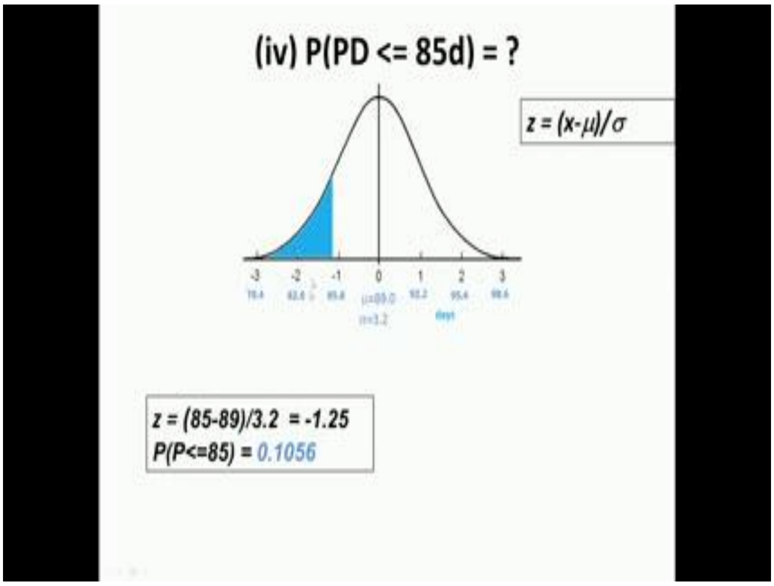

Now, we had another question which was what is the probability that the project duration is less than or equal to 85 days. So, it is the same process we follow but remember now the shaded area we want is this; that is the probability we are looking for is the probability that the project duration is less than 85 days less than or equal to 85 days.

So, when we go with this, we are again applying the same formula here; that is we are looking at 85 minus 89, because the value of x is 85, the value of the mean is mean 89 divided by the standard deviation, so this is that minus 1.25 standard deviations away from the mean, so that is the value of x from the mean.

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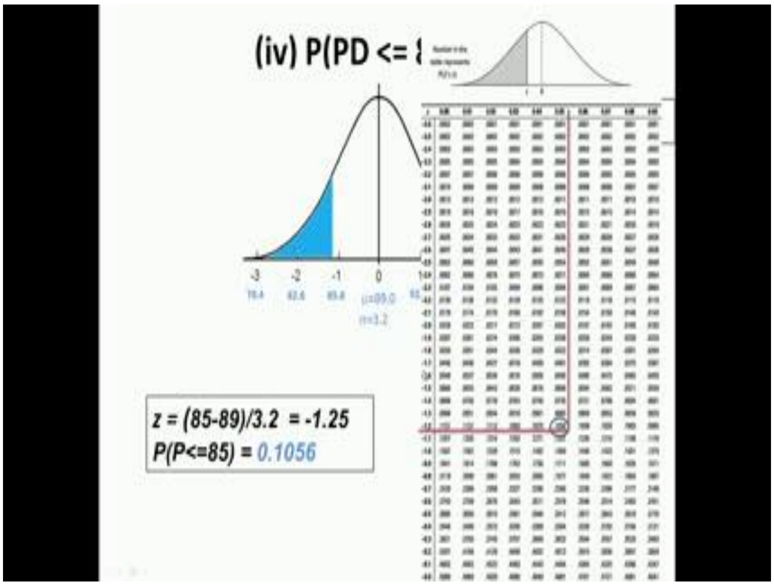

And now I go to the normal table with this portion shaded and you can see I am looking at -1.25 and I get 0.1056. So, 0.1056 is the shaded area, so I can read this directly off, so the probability of the project duration being less than or equal to 85 is 0.1056. So, this is again very similar to the earlier question, it is just that the area shaded is different and I will have to use the appropriate distribution curve to read it directly off.

I could use the other normal distribution table also, but it, I will have to do a little, few more operations to get to this number. So, we can use both tables or you can use one of the tables, but you should be familiar as to how to use it effectively.

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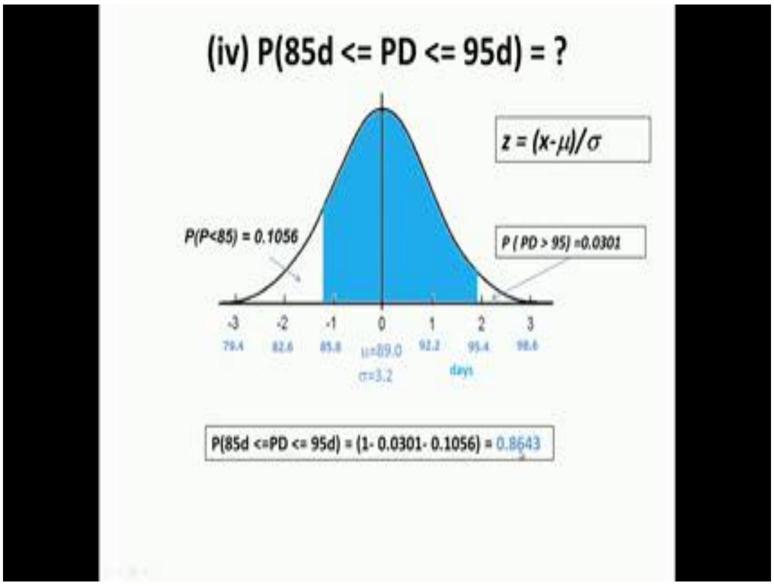

Now, we go to the third probability question which was, what is the probability the project duration lies between 85 and 95. So the shaded area here is again, what is represented by this relation. So, we want the probability it lie between it should be greater than an equal to 85 and less than or equal to 95 and this shaded area we can calculate from are earlier values. We had, remember this was we got this was a point this area here was 0.0301 the area here was 0.1056 and now we know they area under the whole curve is 1.

So, if we subtract one from, I mean to subtract from 1 these to area we will get the resulting area which is shaded here, which is 0.848643. So, we are saying the probability of the project duration lying between these to values is 0.8643. So, you can see that we have started answering probabilistic based questions based from project prospectus.

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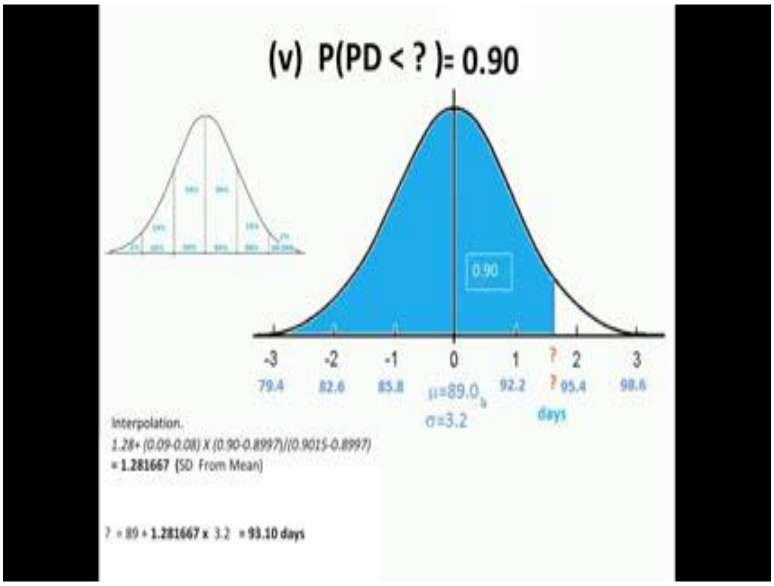

Now, we come to different questions, so here I am giving you a probability. I am saying that I want to know within what duration my project can be completed and I want the 90 percent the certain that it'll be completed in that duration. So how do we do this? So, basically, if we go back to the normal curve, we can see that what we want is, the shaded area is here it shows 90 percent of the area shaded and what we want to know is find the duration for which this 90 percent of this area would be covered.

So, we have to a work it the other way, so if we actually take, for example, the typical normal distribution with percentages of areas given. So, you can see the first.. that the stand the, so we have divided into 1, 2, 3 standard deviations and 3 standard deviations in this end first 2% of the area and this area is 14% and this area is 34%. So, as you cumulative you get 2, 16, 50 you can see 84 and mean am saying 90 90 it should less on or equal to 90 we are looking at, close away, I mean not it should be a when we say we want a 90 percent certainty, we are looking at somewhere between 84 and 98, which is what we show between the first between one and second standard deviation and that is, what we are kind of highlighted here.

So, we are now trying to find the value at.. of the that.. how many standard deviations first we have found is this point from the mean and then, the convert the standard deviation into the duration.

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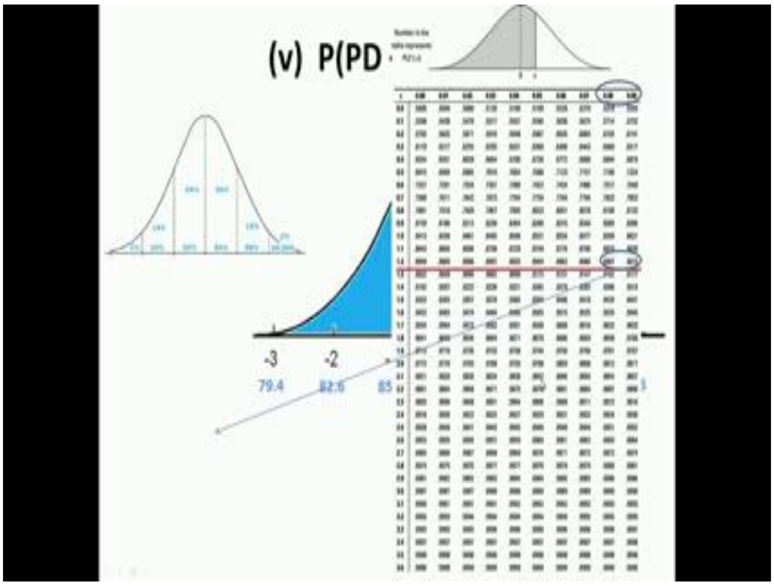

So, the first step here is to be able to see where we are as far as the number of standard deviations we are from it. So, we look at we go for the 90 percent of the 0.90, so I do not have a 0.90 exactly, but I have a 0.9015 and a 0.8997; so obviously, 0.90 will lie between these two values and what have to do is do is an interpolation. So, here we find of this is the equivalent of the one point between 1.2 and 1.29 lies my 90 value. So, we just do an interpolation, we can see the basic interpolation formulation here, which is we typical interpolation formulation, which you can do.

And we will find that when we were solve.. when we kind of solve this value is so many standard deviations from the mean. Now, what we need do is convert that in to a duration which we have be to able to say, so the duration basically is 89 plus so many standard deviations multiplied by the standard deviations as the duration of which have 90 percent chance of finish in the project with it. So, given that if I am 90 percent confident that I will finish the project will 92.87 days.

So, this has given in illustration is to how both the normal distribution of project duration can be used both, not only to estimate the duration given a particular probability, but also the probability given a particular duration.

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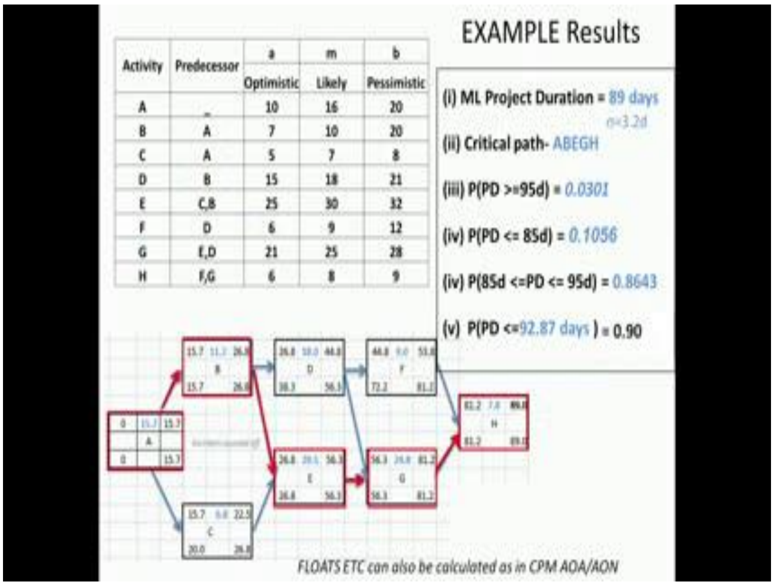

So, if you look at a summary of the results from the examples, so we can see that we took the basic information, we can we added the expected value, the way mean, the standard deviation, the variance, we use the expected value here to calculate the expected project duration that we came the mean for our normal distribution. We found the variance of the normal distribution, then converted that to the standard deviation, through the characterizing in a normal distribution we were able to get all of these values which step by step.

Now, in addition to this floats just as we did in CPM, floats and other things can also be calculated, but we are not getting into that in this session. So, now, I hope this has given a broad perspective on how enough we can do PERT calculations and enough involved, but an interesting question we can ask from doing PERT certainly add a lot of, can a lot of insights into how the project is going to go and we will able to map the uncertainties and all the other issues with it.

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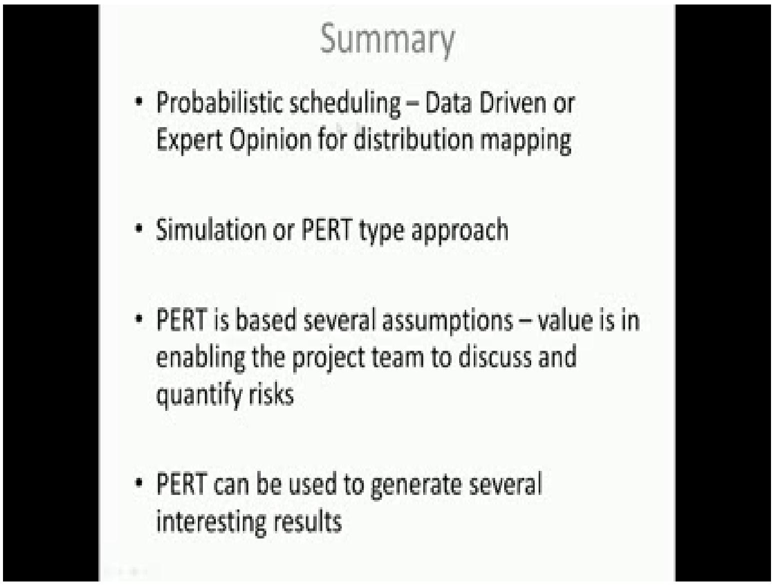

Now, we had looked at probabilistic scheduling, right now, we went to it detail from the exPERT opinion and PERT perspective, but do realise the data-driven prospective is also very important and it can really add a lot of value to a project whether the data exist. And between simulation and PERT, PERT was very popular, it is really a very good methodical technique, but with computing power increasing more and more, people are shifting to simulation-based techniques.

One of the key reason of that is PERT that several assumptions and we saw several these assumptions and these assumptions certainly limit the value of the results from PERT, but while the value in the results in limited there is a lot of value still in PERT to be able to get experts together, get the team together to quantify their risks, to articulate what the risks actually are, that is lot of benefit in that.

And as we have seen PERT can be use generates several interesting results and as we go forward I think the introduction to PERT is really useful, because it teaches you do thinks in terms of probability, think in terms of project duration or project events and the probability that is a associated with it. In CPM we never had a chance to do that, so here in becomes introduction to probabilistic thinking for projects which has very, very important in managing projects. So, if you have questions and discussions please post them online and we will take it up as you post it.

Thank you.